

On the isospin-violating phase of ρ - ω mixing

- 3π channel parameterized by ρ - ω mixing parameter $\epsilon_{\rho\omega}$

$$F_{\pi}^V(s) = \left(1 + \frac{s}{M_{\omega}^2 - s - iM_{\omega}\Gamma_{\omega}} \epsilon_{\rho\omega} \right) \Omega(s) G_{\text{in}}(s)$$

- $\epsilon_{\rho\omega}$ real, apart from **isospin-breaking corrections**

$$\omega \rightarrow X \rightarrow \rho \quad X = \pi^0\gamma, \pi\pi\gamma, \eta\gamma$$

- Implemented via

$$\epsilon_{\rho\omega} \rightarrow \text{Re } \epsilon_{\rho\omega} + i \text{Im } \epsilon_{\rho\omega} \frac{\left(1 - \frac{M_{\pi^0}^2}{s}\right)^3}{\left(1 - \frac{M_{\omega}^2}{s}\right)^3} \theta(s - M_{\pi^0}^2)$$

to ensure correct thresholds

- **Narrow-width estimate**

$$|\text{Im } \epsilon_{\rho\omega}^X| = \frac{\sqrt{\Gamma(\omega \rightarrow X)\Gamma(\rho \rightarrow X)}}{3M_V}$$

leads to 2.7° ($\pi^0\gamma$), 0.7° ($\pi\pi\gamma$), and 0.2° ($\eta\gamma$)

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	χ^2/dof	M_ω [MeV]	$10^3 \times \text{Re } \epsilon_{\rho\omega}$	δ_ϵ [°]	$10^{10} \times a_\mu^{\pi\pi} _{\leq 1 \text{ GeV}}$
SND06 + CMD2	1.28	781.75(22)(1)	1.97(4)(2)		498.5(3.4)(2.6)
	1.05	782.39(23)(2)	1.92(4)(3)	9.9(1.8)(0.4)	497.3(3.1)(3.9)
BaBar	1.14	781.86(14)(1)	2.04(3)(2)		501.9(3.3)(2.0)
	1.14	781.93(18)(4)	2.03(4)(1)	1.3(1.9)(0.7)	501.9(3.3)(1.8)
KLOE''	1.20	781.81(16)(3)	1.98(4)(1)		491.8(2.1)(1.8)
	1.13	782.42(23)(5)	1.95(4)(2)	6.1(1.7)(0.6)	490.8(2.1)(1.7)
BESIII	1.12	782.17(33)(7)	2.01(19)(9)		490.8(4.8)(3.9)
	1.02	783.05(48)(2)	1.98(19)(7)	17.7(7.0)(1.2)	490.3(4.6)(3.0)
SND20	2.93	781.79(30)(6)	2.04(6)(3)		494.2(6.7)(9.0)
	1.87	782.37(28)(6)	2.01(5)(2)	10.2(2.4)(1.4)	494.9(5.3)(3.1)
Combination w/o SND20	1.25	781.72(8)(3)	2.02(2)(3)		494.5(1.5)(2.3)
	1.20	782.12(12)(4)	1.96(2)(2)	4.6(9)(8)	494.2(1.4)(2.1)

- Most fits improve moderately by allowing for a phase δ_ϵ
 \hookrightarrow **huge effect for SND20**
- Preferred value of δ_ϵ varies a lot among the different data sets
- δ_ϵ correlated with M_ω BaBar 2012, $\Delta M_\omega \simeq \frac{\Gamma_\omega}{2} \delta_\epsilon$, but required value of $\delta_\epsilon \simeq 10^\circ$ for agreement with $e^+e^- \rightarrow 3\pi, \pi^0\gamma$ too large to be explained by $X = \pi^0\gamma, \pi\pi\gamma, \eta\gamma$