Implementation of HLbL short-distance constraints

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Outline of talk

1. 2006.00007 by Lüdtke & Procura:
   Model-independent Interpolation Approach (slides courtesy of Massimiliano Procura)
   to estimate numerical effects of LSDC
   — *lower but compatible with estimates from Regge model for excited pseudoscalars by Colangelo et al., in contrast to:*
2. 2005.11761 by Masjuan, Roig & Sanchez-Puertas:
   The interplay of axial mesons and short-distance constraints in $(g-2)_\mu$
   (slides courtesy of Pablo Sanchez-Puertas)
   with simple Regge model for tower of axials
3. Axial vector meson contributions from (chiral) holographic QCD:
   1912.01596 by Leutgeb & Rebhan
   1912.02779 by Cappiello, Catà, D'Ambrosio, Greynat & Iyer
4. Preliminary results from hQCD with finite quark masses (LR, 2107.nnnnn)
   with infinite towers of both pseudoscalars and axials

Interpolation approach

Jan Lüdtke & Massimiliano Procura, EPJ C 80 (2020) 1108 [arXiv: 2006.00007]

- Idea: satisfy all long- and short-distance constraints (SDCs) **without reference to hadronic models**
- Longitudinal SDCs: two-step **interpolation** procedure for $\bar{\Pi}_1(Q_1^2, Q_2^2, Q_3^2)$
  1. interpolate between MV constraint and pQCD quark loop (also away from $Q_1^2 = Q_2^2 = Q_3^2$)
  2. interpolate between these SDCs and the dispersive low-energy representation
- Evaluated the a_μ^{HLbL} integral using $\bar{\Pi}_1$ obtained in this way
- Obtained **conservative** error estimate (interpolation parameters, errors on TFFs, α_s corrections)
- Results:
  - longitudinal SDCs increase a_μ^{HLbL} by $(9.1 \pm 5.0) \times 10^{-11}$ compared to the contribution from the ground-state pseudoscalar poles
  - uncertainty **almost exclusively** due to 1–2 GeV region
  - dispersive input on resonances in this region can be straightforwardly included once available

Importance of axial-vector mesons

P. Masjuan, P. Roig, P. Sanchez-Puertas, arXiv:2005.11761

Interplay of transverse/longitudinal dof in $\langle VVA \rangle$

- MV's OPE[1] relates HLbL and $\langle VVA \rangle$ in certain kinematic limit
- Usually $\langle VVA \rangle$ basis as 1L+3T structures (anomaly $\rightarrow w_L \sim \frac{N_c}{4\pi^2 q_{12}^2}$)

$$\langle V_\mu(q_1) V_\nu(q_2) A_\rho(q_{12}) \rangle \sim \epsilon_{\mu\nu q_1 q_2} q_{12}^\rho w_L - t_{\mu\nu\rho}^{(+)} w_T^{(+)} - t_{\mu\nu\rho}^{(-)} w_T^{(-)} - \tilde{t}_{\mu\nu\rho}^{(-)} \tilde{w}_T^{(+)}$$

- But such construction (see also M. Knecht JHEP08 2020) implies *relations* (implicit as kinematic singularities) among tensor structures: Particularly, absence of massless poles but for pGBs (chiral limit) implies

$$(q_1^2 + q_2^2) w_T^{(+)}(q_1^2, q_2^2, 0) - (q_1^2 - q_2^2) w_T^{(-)}(q_1^2, q_2^2, 0) = 2N_c[1 - \tilde{F}_{P\gamma\gamma}(q_1^2, q_2^2)]$$

Anomaly requires an interplay of longitudinal and transverse dof!

- Intriguing? Not really, but crying out for axial-vector mesons!
  $\Rightarrow$ Interesting constraints and implications for model-building

[1] K. Melnikov and A. Vainshtein, Phys.Rev.D 70 (2004) 113006.

P. Masjuan, P. Roig, P. Sanchez-Puertas, arXiv:2005.11761

Example with axial-vector mesons and pseudo-Goldstones

- Axial TFFs (usually $C_S \to 0$ since $q_{12} \cdot \varepsilon_A = 0$ “unphysical” ... but care!):

$$\mathcal{M}_A^{\mu\nu\rho} = i\epsilon^{\mu\alpha\rho q_1}(q_{2\alpha}q_2^\nu - g_\alpha^\nu q_2^2)B_2 + i\epsilon^{\nu\alpha\rho q_2}(q_{1\alpha}q_1^\mu - g_\alpha^\mu q_1^2)\bar{B}_2 + i\epsilon^{\mu\nu q_1 q_2}[\bar{q}_{12}^\rho C_A + q_{12}^\rho \cancel{C_S}]$$

- They also contribute to w_L in $\langle VVA\rangle$!!

$$\{w_T^{(+)}, w_T^{(-)}, \tilde{w}_T^{(-)}\} \sim \frac{\{B_{2S}, B_{2A} - C_A, -B_{2A}\}}{q_{12}^2 - m_A^2} m_A F_A^a, \quad w_L \sim -\left[C_S + \frac{q_1^2 + q_2^2}{q_{12}^2} B_{2S} - \frac{q_1^2 - q_2^2}{q_{12}^2}(B_{2A} - C_A)\right]\frac{F_A^a}{m_A}$$

- Requires infinite number of axial-vector mesons (OPE $\Rightarrow B_{2S}(q^2, q^2) \sim q^{-4}$)
- In our basis $C_S = 0$, but others require “unphysical” longitudinal FF$\neq 0$ **[important result!]**
- Axials contribution to w_L pole-less; suggestive to split as

$$D^{\mu\nu}(q^2) = \frac{-g^{\mu\nu} + q^\mu q^\nu m_A^{-2}}{q^2 - m_A^2} = -\frac{q^2 g^{\mu\nu} - q^\mu q^\nu}{m_A^2(q^2 - m_A^2)} + \frac{g^{\mu\nu}}{m_A^2} \equiv \bar{D}^{\mu\nu}(q^2) + \frac{g^{\mu\nu}}{m_A^2}.$$

- $\bar{D}^{\mu\nu}$ transverse axial pole (RχT) $\Rightarrow w_T^{(\pm)} \sim$ “Subtracted”; $g^{\mu\nu} \sim$ “Contact”
- Either allow nonpole axial contributions or include a contact term!

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- They also contribute to w_L in $\langle VVA\rangle$!! Including pGBs ($m_{\rm pGB} \to 0$), the anomaly condition:

$$\frac{N_c\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2}\frac{\tilde{F}_{P\gamma\gamma}(q_1^2,q_2^2)}{q_{12}^2 - m_{\rm pGB}^2} - \sum_n \frac{F_{A_n}^a}{m_{A_n}}\left[C_S + \frac{q_1^2+q_2^2}{q_{12}^2}B_{2S} - \frac{q_1^2-q_2^2}{q_{12}^2}(B_{2A} - C_A)\right] = \frac{N_c\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2 q_{12}^2}$$

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Simple Regge model illustration

- Can illustrate with Regge model (TFF has appropriate high-energy scaling)

$$B_{2S}^{A_n^a}(q_1^2,q_2^2) = \frac{4F_A\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)m_{A_n^a}}{[q_1^2+q_2^2-(M_a^2+n\Lambda^2)]^2}, \quad \text{anomaly} \to \Lambda = \frac{4\pi F_A}{\sqrt{N_c}}, \quad B_{2S}^{n=0}(0,0) \to M_a$$

$a_\mu^{AV} =$ ($\times 10^{-11}$)

n	0	1	5	10	40	99	$(1-99)$
a_1	5.89	7.35	8.73	9.12	9.48	9.56	3.67
f_1	10.52	13.55	16.84	17.83	18.77	18.98	8.46
f_1'	1.97	2.35	2.69	2.77	2.85	2.87	0.90
Total	18.38	23.25	28.26	29.71	31.10	31.41	13.03

- Like in hQCD ($(28–41)\times 10^{-11}$), higher than current estimates due to high-energy scaling
- **Explains why RχT ($\sim$Subt) small & negative** (P. Roig,PSP, Phys.Rev.D 101 (2020))
- **Contact part dominates**
- Result close to full HW2(IR) below, while not for individual contributions
- **Suggestive to take a reasonable model for contact part:**
  - Fulfills the anomaly
  - Reliable $F_{\pi\gamma\gamma}(q_1^2,q_2^2)$ in fulfilling anomaly
  - Axial poles (appearing in subtracted part) ultimately fitted to experiment

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$a_\mu^{AV} =$

n	0	1	5	10	40	99	$(1-99)$
Std	18.4	23.3	28.3	29.7	31.1	31.4	13.0
Subt	-2.9	-3.7	-4.5	-4.8	-5.0	-5.1	-2.2
Cont	21.3	27.0	32.9	34.6	36.2	36.6	15.3

$\times 10^{-11}$

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Anomalous TFFs from holographic QCD

Pion TFF: [Grigoryan, Radyushkin, PRD76,77,78 (2007-8)]
[Cappiello, Catà, D'Ambrosio, PRD83 (2011)]
[J. Leutgeb, J. Mager, AR, PRD100 (2019) - 1906.11795]
Axial-vector TFF: [J. Leutgeb, AR, PRD101 (2020) - 1912.01596]
[Cappiello, Catà, D'Ambrosio, Greynat, Iyer, PRD102 (2020) - 1912.02779]

In bottom-up hQCD models, (as in the top-down string-theoretical Sakai-Sugimoto (SS) model,)
pions & (axial) vector mesons described by 5d-YM fields $\mathcal{F}^{L,R}_{MN} = \mathcal{F}^V_{MN} \mp \mathcal{F}^A_{MN}$

$$S_{\rm YM}^{U(N_f)\times U(N_f)} \propto \text{tr} \int d^4x \int_0^{z_0} dz\, e^{-\Phi(z)} \sqrt{-g}\, g^{PR} g^{QS} \left(\mathcal{F}^{(L)}_{PQ} \mathcal{F}^{(L)}_{RS} + \mathcal{F}^{(R)}_{PQ} \mathcal{F}^{(R)}_{RS} \right),$$

where $P, Q, R, S = 0, \ldots, 3, z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

conformal boundary at $z = 0$, either sharp cut-off of AdS_5 at z_0 (HW) or with nontrivial dilaton $z_0 = \infty$ (SW)

(SS: finite z_0, corresponding to point where D8 branes join; not asymptotically AdS_5 $\Rightarrow$ no matching to pQCD possible)

Chiral symmetry breaking either from extra bifundamental scalar field (HW1)
or through different boundary conditions for vector/axial-vector fields at z_0 (Hirn-Sanz (HW2), as in SS)

Anomalies uniquely from
Chern-Simons term: (by hand in bottom-up models, from D8 branes in SS model)

$$S^L_{\rm CS} - S^R_{\rm CS}, \quad S_{\rm CS} = \frac{N_c}{24\pi^2} \int \text{tr} \left(\mathcal{B}\mathcal{F}^2 - \frac{i}{2}\mathcal{B}^3\mathcal{F} - \frac{1}{10}\mathcal{B}^5 \right).$$

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Anomalous TFFs from holographic QCD

VMD:

- Electromagnetic background fields through non-normalizable modes for $\mathcal{B}^V$ with $\mathcal{B}^V_\mu|_{z=0} = e\mathcal{Q}A^{e.m.}_\mu$
- Bulk-to-boundary propagator $\mathcal{J}$ contains sum over infinite tower of vector mesons,

$$\mathcal{J}^{\mathrm{HW}}(Q,z) = Qz\left[K_1(Qz) + \frac{K_0(Qz_0)}{I_0(Qz_0)} I_1(Qz)\right], \qquad (M_1^V = m_\rho = 775\,\text{MeV} \Rightarrow z_0 = 3.103\,\text{GeV}^{-1}),$$

$$F_{\pi\gamma^*\gamma^*}(Q_1^2,Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\int_0^{z_0} \mathcal{J}(Q_1,z)\mathcal{J}(Q_2,z)\partial_z\alpha(z)dz$$

$$\mathcal{M}_{\mathcal{A}\gamma^*\gamma^*} \propto \epsilon^\mu_{(1)}\epsilon^\nu_{(2)}\epsilon^{*\rho}_{\mathcal{A}}\epsilon_{\mu\nu\rho\sigma}\left[q^\sigma_{(2)}Q_1^2 A(Q_1^2,Q_2^2) - q^\sigma_{(1)}Q_2^2 A(Q_2^2,Q_1^2)\right]$$

with asymmetric $A \leftrightarrow \bar{B}_2^{\text{Masjuan et al.}}$:

$$A(Q_1^2,Q_2^2) = \frac{2g_5}{Q_1^2}\int_0^{z_0}\left[\frac{d}{dz}\mathcal{J}(Q_1,z)\right]\mathcal{J}(Q_2,z)\,\psi^{\mathcal{A}}(z)\,dz$$

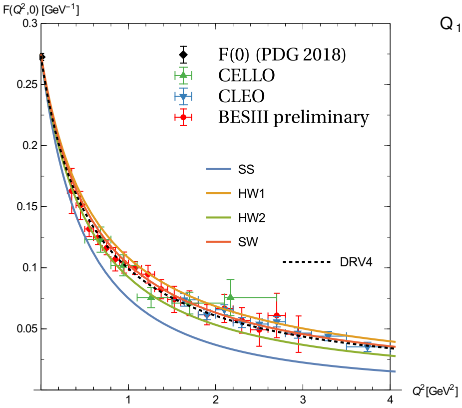
- **Landau-Yang theorem** realized by $\mathcal{J}'(Q,z)=0$ for $Q^2=0$
- Amazingly, bottom-up models with asymptotic AdS$_5$ geometry reproduce **asymptotic momentum dependence of pQCD (Brodsky-Lepage) for pions and axials!** (see Hoferichter & Stoffer, 2004.06127 for axials; $\rightarrow$ **talk by Bastian Kubis**)
- HW1: **correct asymptotic prefactor**
  HW2: with correct IR-fit of m_ρ and $f_\pi = 92.4$ MeV only 62% of LO pPQCD value!
  Caveat: real QCD approaches LO asymptotics certainly more slowly than hQCD!

Holographic TFFs and experimental data

Single-virtual pion TFF:

[J. Leutgeb, J. Mager & AR, 1906.11795]

(data from Danilkin et al., Prog.Part.Nucl.Phys. 107 (2019) 20)



Danilkin et al. (DRV) fit below 4 GeV^2

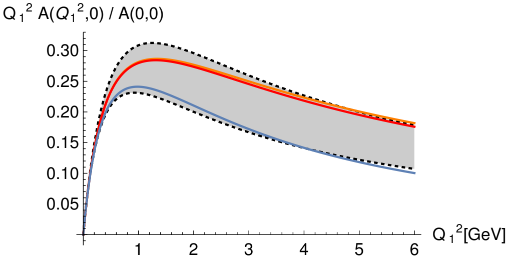
bracketed by HW1 and HW2!

Single-virtual axial TFF:

[J. Leutgeb & AR, 1912.01596]

dipole fit of L3 data for $f_1(1285)$ (gray band)

vs. SS, HW1, and HW2 models:



hQCD results:	HW1	HW2
$\lvert A(0,0) \rvert$ [GeV^{-2}]	21.04	16.63

$$A(0,0)^{\text{L3 exp.}}_{f_1(1285)} = 16.6(1.5)\,\text{GeV}^{-2}$$

Roig & Sanchez-Puertas, 1910.02881:

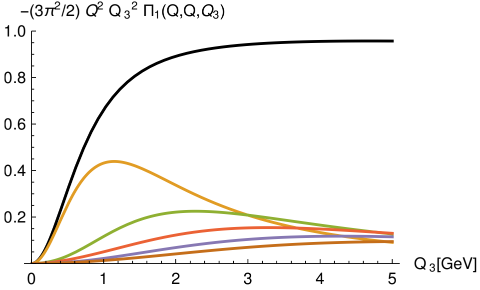
$$A(0,0)_{a_1(1230)} = 19.3(5.0)\,\text{GeV}^{-2}$$

Axial vector contributions to SDC

Infinite tower of axial-vector mesons responsible for satisfying the longitudinal SDC

- MV-SDC $\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$: 100% for HW1 and HW2(UV-fit)

HW2 model with $g_5^2 = 4\pi^2$ (UV-fit) at large $Q = 50$GeV and increasing $Q_3 \ll Q$:



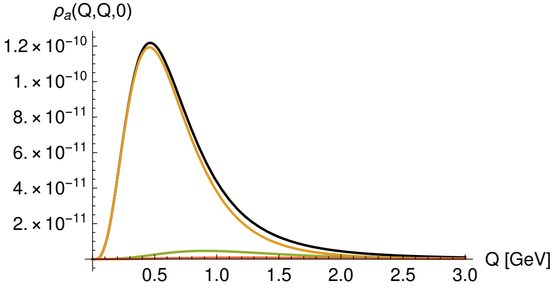
black line: infinite sum
colored lines: first 5 axial vector modes

- SDC for symmetric limit $Q_1^2 = Q_2^2 = Q_3^2 \to \infty$ satisfied qualitatively, but quantitatively only at max. 80% level (for HW1 and HW2(UV-fit))

Total axial-vector contributions to muon $g-2$

$$a_\mu^{\rm AV} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\, \rho_a(Q_1,Q_2,\tau)$$

E.g. at $\tau = 0$:



Strongly dominated by lowest axials, but nonnegligible contribution from higher modes:

	$j=1$	$j\leq 2$	$j\leq 3$	$j\leq 4$	$j\leq 5$	$a_\mu^{\rm AV}$
HW1	31.4	36.2	37.9	39.1	39.6	40.6 $\times 10^{-11}$
HW2	23.0	26.2	27.4	27.9	28.2	28.7 $\times 10^{-11}$

HW axial-vector results ≈ 60% longitudinal + 40% transversal (long. prop.: $q_{(3)}^\mu q_{(3)}^\nu/(M_n^A Q_3)^2$)

Masjuan et al.: HW2$^{j\leq 5}$ result $2.8 = 3.4(contact) - 0.6(transversal, subtracted)$

(contact part of prop. $\propto g^{\mu\nu}/(M_n^A)^2$)

Pseudoscalar plus axial vector contributions to a_μ

Our results [J. Leutgeb & AR, 1912.01596]: (combined with $a_\mu^{\rm PS}$ [1906.11795])

(z_0 s.t. $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV; degenerate a_1, f_1, f_1')

	HW1 (100% LSDC)	HW2 (62% LSDC)
$a_\mu^{\rm PS}[\pi^0 + \eta + \eta'] \times 10^{11}$	92 [61.3+16.7+14.2]	84 [59.2+15.9+13.4]
$a_\mu^{\rm AV}[L + T] \times 10^{11}$	41 [23+18]	29 [17+12]
$a_\mu^{\rm PS+AV} \times 10^{11}$	133	112

(compare with MV model: longitudinal contribution estimated $\sim$ 38 $\times 10^{-11}$)

independently at the same time:
[L. Cappiello, O. Cata, G. D'Ambrosio, D. Greynat, A. Iyer, 1912.02779]:

• **agreement**, but different parameters:

HW2$^{(1)}$: z_0 s.t. $m_\rho = 776$ MeV, $f_\pi = 93$ MeV, $f_{\eta'} = 74$ MeV
HW2$^{(2)}$: z_0 s.t. 100% UV limit (but $m_\rho = 987$ MeV !)

	HW2$^{(2)}$ (100% LSDC)	HW2$^{(1)}$ (62% LSDC)
$a_\mu^{\rm PS}[\pi^0 + \eta + \eta'] \times 10^{11}$	112 [75+21+16]	81 [57+14+10]
$a_\mu^{\rm AV}[L + T] \times 10^{11}$	32 [18$^{*)}$+14]	28 [14+14]
$a_\mu^{\rm PS+AV} \times 10^{11}$	144	110

$^{*)}$ 15 acc.to Colangelo et al. 2106.13222

HW models with finite quark masses (LR 2107.nnnnn)

HW1m: HW1 with finite quark mass; **HW3**: HW1 with HW2 b.c.;
Δ^+: scaling dimension of operator dual to Φ

Analytic result: **LSDC completely saturated by axials, no contribution from heavy PS!**
(as long as Δ^+ in holographically allowed range $2 \leq \Delta^+ < 4$)

preliminary results: (all: $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV – only isovector contributions)

model	PS	$n=1$	$n=2$	$n=3$	AV	$n=1$	$n=2$	$n=3$	$\Delta a^L_{\pi+a_1}$
HW1 chiral	m_π[MeV]	0|135	1888	2879	m_{a_1}[MeV]	1375	2154	2995	
$\Delta^+ = 3$	$a_\mu^\pi \cdot 10^{11}$	61.3	-	-	$4a_\mu^{a_1} \cdot 10^{11}$	31.4	4.7	1.8	5.4
HW1m	m_π[MeV]	**135***	1892	2882	m_{a_1}[MeV]	1367	2141	2987	
$\Delta^+ = 3$	$a_\mu^\pi \cdot 10^{11}$	62.0	0.6	0.1	$4a_\mu^{a_1} \cdot 10^{11}$	31.4	4.9	1.8	6.2
HW1m'	m_π[MeV]	**135***	1591	2564	m_{a_1}[MeV]	**1230***	1977	2901	
$\Delta^+ = 2.404$	$a_\mu^\pi \cdot 10^{11}$	60.4	1.4	0.2	$4a_\mu^{a_1} \cdot 10^{11}$	29.8	8.7	2.0	7.4
HW3m	m_π[MeV]	**135***	1715	2513	m_{a_1}[MeV]	1431	2421	3398	
$\Delta^+ = 3$	$a_\mu^\pi \cdot 10^{11}$	62.6	0.7	0.03	$4a_\mu^{a_1} \cdot 10^{11}$	32.7	3.4	1.8	6.1
HW3m'	m_π[MeV]	**135***	**1300***	2113	m_{a_1}[MeV]	1380	2355	3345	
$\Delta^+ = 2.399$	$a_\mu^\pi \cdot 10^{11}$	62.0	1.4	0.01	$4a_\mu^{a_1} \cdot 10^{11}$	33.2	4.1	1.8	6.9

* fitted

$\Delta a^L_{\pi+a_1} = a_\mu[\pi_2 + \pi_3 + a_1^{(L)1\ldots3}] \cdot 10^{11}$

for comparison, LP interpolant: $\Delta a^L_{\pi+a_1} = 2.6(1.5)$

- Numerically: some *increase* compared to chiral model with m_π inserted in propagator by hand ($(a_\mu - a_\mu^{\rm chiral})_{\pi+a_1} = +(1.3\text{-}2.4) \times 10^{-11}$)
- Contribution of heavy PS *smaller* than in PS Regge model of Colangelo et al., where $\Delta a^{\rm PS}_{\pi(n>1)} = 2.7$ prior to OPE substitutions

Conclusions/Outlook

- hQCD is not QCD, but sophisticated toy model that can give clues on
  – how short-distance behavior can be implemented at the hadronic level
    - important fundamental role of axial-vector mesons $\leftrightarrow$ anomaly

  – semi-quantitative estimates of the ballparks to be expected
  (HW1–HW2 brackets experimental results for pion TFF!)
    - axial-vector contributions more important numerically than estimated previously (in all hQCD models; close to Regge model of Masjuan, Roig & Sanchez-Puertas)

$$a_\mu^{\mathrm{AV}}[L+T] = \mathbf{35(6)}\,[20(3)+15(3)]\times 10^{-11} \qquad \text{for HW1–HW2}$$

vs. WP: $a_\mu^{\mathrm{SDC+axials}} = \mathbf{21(16)}\,[15(10)+6(6)]\times 10^{-11}$

- hQCD models to be made more realisitic:
  - little change with u, d quark masses, but need SU(3) breaking mass terms
  - Witten-Veneziano mechanism for U(1)$_A$ anomaly
  - LO high-energy asymptotics approached perhaps too quickly, Regge trajectories unrealistic in HW models $\rightarrow$ improved hQCD models with numerically determined deviation from conformality?

To appear soon:

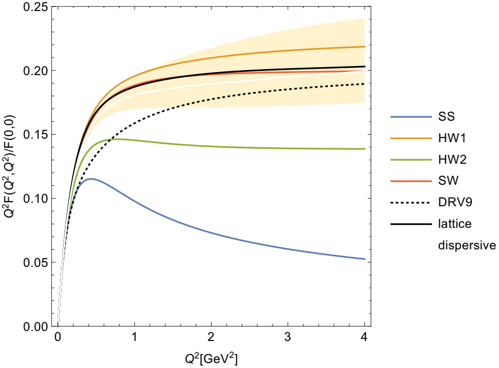
- Leutgeb & AR: hQCD(massive) results on excited pseudoscalars
- Cappiello, Catà, D'Ambrosio, Iyer: hQCD results on scalars

BACKUP SLIDES

Holographic pion TFF, doubly virtual

No experimental data yet for double-virtual pion TFF, but:

- results from dispersive approach M. Hoferichter et al., 1808.04823 and
- lattice extrapolations from A. Gérardin, H. B. Meyer, and A. Nyffeler, 1903.09471:



- HW1: (too) quickly approaches LO pQCD result (negative NLO corrections!)
- HW2: 62% LSDC falls short
- SW: (fortuitously?) close to lattice (89% of LO pQCD asymptotically)
- SS: wrong asymptotics, but below 0.3 GeV^2 closer to lattice than DRV interpolator