Discussion: benchmarks

In memoriam Simon Eidelman

Tom Blum, Gilberto Colangelo, Aida El Khadra, Christoph Lehner, Laurent Lellouch, Harvey Meyer, Silvano Simula, Ruth Van de Water, Hartmut Wittig

Recommendations

Beyond the computation of full $a_\mu^{\text{LO-HVP}}$, we suggest that results be also given for intermediate quantities that may allow:

- crosschecks between collaborations
- self-consistency checks w/in a given calculation
- blinded comparisons w/ the R-ratio approach

Since the time-momentum-representation approach is most commonly used

$$a_{\ell,f}^{\text{LO-HVP}}(Q^2 \le Q^2_{\max}) = \lim_{a\to 0,\, L\to\infty,\, T\to\infty} \alpha^2 \left(\frac{a}{m_\ell^2}\right) \sum_{t=0}^{T/2}{}' K(tm_\ell, Q^2_{\max}/m_\ell^2)\ \mathrm{Re}C^f_{TL}(t)$$

suggest providing:

- $a_\mu^{\text{LO-HVP}}$ in standard euclidean time windows, see below (RBC/UKQCD '18)
- flavor-by-flavor in isospin limit
- $I = 1$ and $I = 0$ contributions, in particular because the latter is much less sensitive to FV and taste-breaking effects

Of course, sum of time windows must be consistent w/ total $a_\mu^{\text{LO-HVP}}$

Windows as functions of t and s

- Window functions (RBC/UKQCD '18):

$$\Theta(t; t_0, \Delta) \equiv \frac{1}{2}\left[1 + \tanh\left(\frac{t - t_0}{\Delta}\right)\right]$$

$$W(t; t_0, t_1, \Delta) \equiv \Theta(t; t_0, \Delta) - \Theta(t; t_1, \Delta)$$

- Standard (win): $W(t; 0.4\,\mathrm{fm}, 1\,\mathrm{fm}, 0.15\,\mathrm{fm})$

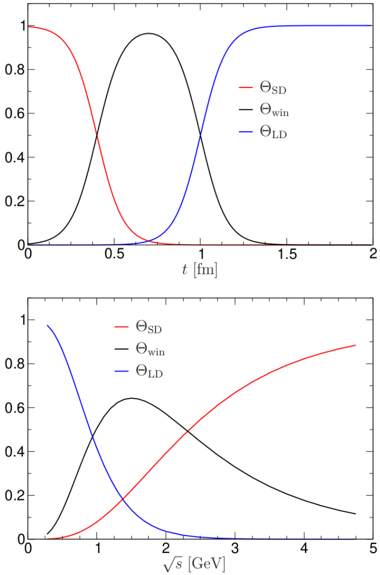
  $\rightarrow$ particularly good for lattice: small discretization and FV effects, very good signal, we should all agree

- Short-distance (SD): $W(t; 0\,\mathrm{fm}, 0.4\,\mathrm{fm}, 0.15\,\mathrm{fm})$

  $\rightarrow$ good signal but large discretization effects

- Long-distance (LD): $W(t; 1\,\mathrm{fm}, \infty, 0.15\,\mathrm{fm})$

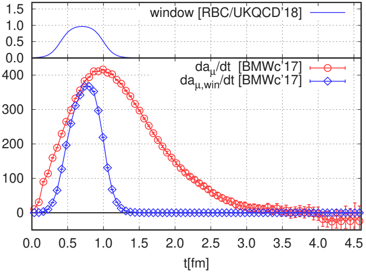
  $\rightarrow$ exponentially bad signal-to-noise, large FV (and taste-breaking for staggered) effects, can be alleviated by spectral decomposition of HVP correlator (Mainz '19, RBC '19)

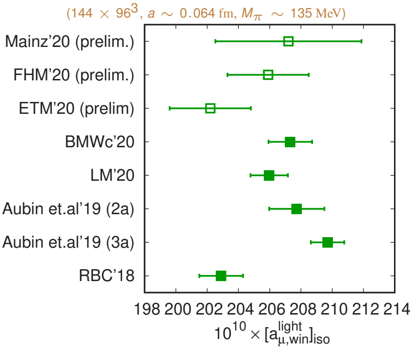


(M. Hoferichter, Nov. 2020 TI HVP workshop)

Standard window results

- Focus on individual flavors (including disconnected) in isospin limit
- Should allow very sharp comparisons (signficantly $< 1\%$) between lattice groups
  → test of various setups
- Comparison of light-quark contribution: agreement must be improved of differences understood (see also below)
- Once agreement for all flavor, QED and SIB contributions is found
  → particularly stringent comparison w/ R-ratio is possible





How to define isospin-symmetric QCD?

- Since most lattice comparisons will be made in context of isospin-symmetric QCD, we have to agree on what that is!
- To the precision required, only full QCD + QED computations w/ SIB corrections are unambiguous ...
- ... "pure QCD" ones are not, and QCD + qQED ones are in between
- Problem: in presence of QED, QCD parameters run differently
  - → QCD + QED and "pure QCD" parameters must be matched in a given renormalization scheme at a given scale
  - → on lattice, more convenient to match hadronic quantities (BMWc '13)
- QCD + QED to "pure QCD" and isospin-symmetric QCD matching proposal:
  - Fix scale by assuming w_0 in "pure QCD" is equal to QCD + QED value
  - Fix m_q by requiring mass of connnected, $q\bar{q}$, PS meson, M_{qq}, in pure QCD is equal to QCD + QED value
  - Define isospin-symmetric QCD by fixing light quark, l, mass in $N_f = 2+1+1$ simulations to obtain $2M_{ll}^{\text{iso},2} = M_{uu}^{\text{QC+ED},2} + M_{dd}^{\text{QC+ED},2}$
  - Requires prior calculation of $w_0^{\text{QC+ED}}$ and $M_{qq}^{\text{QC+ED}}$ (see e.g. BMWc '20) ...
  - ... or agreement on reference values for those quantities, e.g.

$$M_{ll}^{\text{iso}} = M_{\pi^0}\,, \quad M_K^{\text{iso},2} - \frac{1}{2}M_{ll}^{\text{iso},2} = \frac{1}{2}\left(M_{K^+}^2 + M_{K^0}^2 - M_{\pi^+}^2\right)\,, \quad M_{D_s}^{\text{iso}} = M_{D_s}$$

How to define isospin-symmetric QCD? (cont'd)

- To use QED and SIB corrections from an independent lattice calculation, must conform to the latter's isospin-symmetric QCD definition or one equal to it up to higher-order terms
- Providing $\partial a_\mu^{\text{LO-HVP}}/\partial M$, where M is the physical value of the scale setting or of a mass setting quantity, allows changing prescription *a posteriori*
- These issues are even more important if one wants to determine the lattice WA for $a_\mu^{\text{LO-HVP}}$ by adding the averages of invidual, flavor, QED & SIB contributions