Narrow resonances in hadronic light-by-light scattering

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KEK and Nagoya University

Analytic HLbL estimate in the White Paper (WP)

→ T. Aoyama *et al.*, Phys. Rept. **887** (2020) 1-166

	$10^{11} \times a_\mu$	$10^{11} \times \Delta a_\mu$
π^0, η, η'-poles	93.8	4.0
pion/kaon box	-16.4	0.2
S-wave $\pi\pi$ rescattering	-8	1
scalars, tensors	-1	3
axials	6	6
light quarks, short distance	15	10
c-loop	3	1
HLbL total (LO)	92	19

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HLbL contribution of higher resonances

WP estimate based on different models:

- scalar meson contribution:
  - → V. Pauk, M. Vanderhaeghen (2014) $a_\mu^{\text{scalars}} = [-3.1(8), -0.9(2)] \times 10^{-11}$
  - → M. Knecht et al. (2018) $a_\mu^{\text{scalars}} = [-(2.2^{+3.2}_{-0.7}), -(1.0^{+2.0}_{-0.4})] \times 10^{-11}$
- tensor meson contribution:
  - → I. Danilkin, M. Vanderhaeghen (2017) $a_\mu^{\text{tensors}} = 0.9(1) \times 10^{-11}$
- axial-vector meson contribution:
  - → V. Pauk, M. Vanderhaeghen (2014) $a_\mu^{\text{axials}}[f_1, f_1'] = 6.4(2.0) \times 10^{-11}$
  - → F. Jegerlehner (2017) $a_\mu^{\text{axials}}[a_1, f_1, f_1'] = 7.6(2.7) \times 10^{-11}$
  - → P. Roig, P. Sánchez-Puertas (2020) $a_\mu^{\text{axials}}[a_1, f_1, f_1'] = (0.8^{+3.5}_{-0.8}) \times 10^{-11}$
- axial-vector contribution in interplay with short-distance constraints (SDCs):
  - → J. Leutgeb, A. Rebhan (2020) $a_\mu^{\text{axials}}[a_1, f_1, f_1'] = 17.4(4.0) \times 10^{-11}$
  - → L. Cappiello et al. (2020) ↪ "data-driven", adjusted normalization (52% saturation of LSDC)

How to improve WP estimate of resonances?

- how to reduce model uncertainties?
- consistent inclusion in dispersive framework?
- short-distance constraints?
- beyond narrow resonances?

Resonance contributions to HLbL

- **unitarity**: resonances unstable, not asymptotic states
  $\Rightarrow$ do not show up in unitarity relation
- **analyticity**: resonances are poles on unphysical Riemann sheets of partial-wave amplitudes
  $\Rightarrow$ describe in terms of multi-particle intermediate states that generate the branch cut
- realistic in the case of resonant $\pi\pi$ contributions in S-wave (f_0) and D-wave (f_2)
- axial-vector mesons would appear as resonance in 3π channel $\Rightarrow$ need to rely on **narrow-width (NW) approximation**

Narrow resonances

- in the NW limit, imaginary part from unitarity relation reduces to **δ-function**:

$$\mathrm{Im}_s \Pi^{\mu\nu\lambda\sigma} = \pi\delta(s-M^2)\mathcal{M}^{\mu\nu}(p\to q_1,q_2)^* \mathcal{M}^{\lambda\sigma}(p\to -q_3,q_4)\,,$$

$$\mathcal{M}^{\mu\nu}(p\to q_1,q_2) = i\int d^4x e^{iq_1\cdot x}\langle 0|T\{j^\mu_{\mathrm{em}}(x)j^\nu_{\mathrm{em}}(0)\}|p\rangle$$

- project onto tensor decomposition for HLbL and plug into dispersion relation for scalar functions:

$$\check{\Pi}_i(s) = \frac{1}{\pi}\int ds' \frac{\mathrm{Im}\check{\Pi}_i(s')}{s'-s}$$

- δ-function, Cauchy kernel, and polarization sum combine to propagator-like structure
- dispersive result may differ from propagator models by non-pole terms

Narrow resonances

- decompose $\mathcal{M}^{\mu\nu}$ into Lorentz structures $\times$ **transition form factors** (TFFs)
- in the NWA, dispersive definition only involves on-shell meson $\Rightarrow$ **only physical TFFs** enter

Sum rules and basis (in)dependence

- HLbL tensor basis involves structures of **different mass dimension**
- scalar coefficient functions of higher-dimension structures asymptotically fall off faster
- implies **sum rules** for those coefficient functions:

$$0 = \frac{1}{\pi} \int ds' \, \mathrm{Im} \check{\Pi}_i(s')$$

- guarantees **basis independence** of entire HLbL

Sum rules and basis (in)dependence

- sum-rule contribution of single-particle state (resonance):

$$\mathrm{Im}\check{\Pi}_i(s') \sim \pi\delta(s'-M^2)\mathcal{F}(q_1^2,q_2^2)\mathcal{F}(q_3^2,0)$$
$$\Rightarrow \frac{1}{\pi}\int ds'\,\mathrm{Im}\check{\Pi}_i(s') \sim \mathcal{F}(q_1^2,q_2^2)\mathcal{F}(q_3^2,0) \neq 0$$

- sum rules **not fulfilled** by resonances
  $\Rightarrow$ NW contribution to HLbL is **basis dependent**
- basis dependence only needs to cancel in sum over intermediate states
- only pseudoscalars do not contribute to sum rules
  $\Rightarrow$ unambiguous

Kinematic singularities

- HLbL coefficient functions $\check{\Pi}_i$ free from kinematic singularities in Mandelstam variables $\Rightarrow$ enables dispersive treatment $\rightarrow$ Colangelo, Hoferichter, Procura, Stoffer, JHEP **04** (2017) 161
- not free from kinematic singularities in q_i^2, but **residues vanish** due to sum rules
- kinematic singularities can be subtracted, but introduce **additional ambiguities** if sum rules are violated

Light-cone expansion for TFFs

→ Hoferichter, Stoffer, JHEP **05** (2020) 159

- tensor decomposition for scalar, axial-vector, and tensor meson TFFs derived with Bardeen–Tung recipe:

  → Bardeen, Tung, Phys. Rev. **173** (1968) 1423

$$\mathcal{M}^{\mu\nu}(p[,\lambda] \to q_1, q_2) \propto \sum_i T_i^{\mu\nu[\alpha]}[\epsilon_\alpha^\lambda(p)]\mathcal{F}_i(q_1^2, q_2^2)$$

- **no Tarrach ambiguities** appear for scalar, axial vector, or tensor meson TFFs

  → Tarrach, Nuovo Cim. **A28** (1975) 409

- absence of kinematic singularities guaranteed
- for many TFFs experimental information is scarce

  → talk by B. Kubis

Light-cone expansion for TFFs

→ Hoferichter, Stoffer, JHEP **05** (2020) 159

- **asymptotic behavior** of TFFs can be derived using light-cone expansion → Brodsky, Lepage (1979, 1980, 1981)
- general structure:
$$\mathcal{F}(q_1^2, q_2^2) \sim F^{\text{eff}} M^{2n-1} \int_0^1 du \frac{\phi(u)}{(uq_1^2 + (1-u)q_2^2)^n} + \mathcal{O}(Q^{-2(n+1)})$$
$$= \frac{F^{\text{eff}} M^{2n-1}}{Q^{2n}} f(w) + \mathcal{O}(Q^{-2(n+1)})$$
F^{eff}: effective decay constant; $Q^2 = \frac{q_1^2+q_2^2}{2}$; $f(w)$ with $w = \frac{q_1^2-q_2^2}{q_1^2+q_2^2}$ determined using asymptotic form of wave functions $\phi(u)$
- goes beyond the strict OPE limit, Q^2 scaling rigorous (w dependence less so)

Light-cone expansion for TFFs

$\rightarrow$ Hoferichter, Stoffer, JHEP **05** (2020) 159

- pseudoscalars: $\mathcal{F} \sim \frac{1}{Q^2}$
- scalars: $\mathcal{F}_1^S \sim \frac{1}{Q^2}$, $\mathcal{F}_2^S \sim \frac{1}{Q^4}$
- axial vectors: $\mathcal{F}_1^A = \mathcal{O}(Q^{-6})$, $\mathcal{F}_{2,3}^A \sim \frac{1}{Q^4}$
- tensors: $\mathcal{F}_1^T \sim \frac{1}{Q^4}$, $\mathcal{F}_{2,3,4,5}^T \sim \frac{1}{Q^6}$
- BL scaling reproduced in all cases by quark model

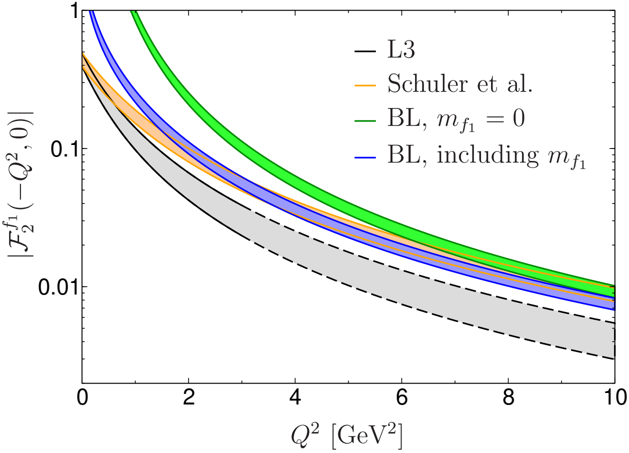
  $\rightarrow$ Schuler et al., Nucl. Phys. B **523** (1998) 423

- holographic QCD models for axial TFFs agree with BL scaling, both Q^2 and w dependence

  $\rightarrow$ Leutgeb, Rebhan, PRD **101** (2020) 114015

Light-cone expansion for TFFs

→ Hoferichter, Stoffer, JHEP **05** (2020) 159



Dispersive evaluation of $f_0(980)$ contribution

$\rightarrow$ Danilkin, Hoferichter, Stoffer, arXiv:2105.01666 [hep-ph]

- $\pi\pi$ rescattering previously limited to $f_0(500)$

  $\rightarrow$ Colangelo, Hoferichter, Procura, Stoffer, JHEP **04** (2017) 161,

  PRL **118** (2017) 232001

- extension up to ~ 1.3 GeV by using coupled-channel $\gamma^*\gamma^* \rightarrow \pi\pi/\bar{K}K$ S-waves for $I = 0$

  $\rightarrow$ Danilkin, Deineka, Vanderhaeghen, PRD **101** (2020) 054008

- covers $f_0(980)$, dispersive description of resonance in terms of $\pi\pi/\bar{K}K$ rescattering

Dispersive evaluation of $f_0(980)$ contribution

$\rightarrow$ Danilkin, Hoferichter, Stoffer, arXiv:2105.01666 [hep-ph]

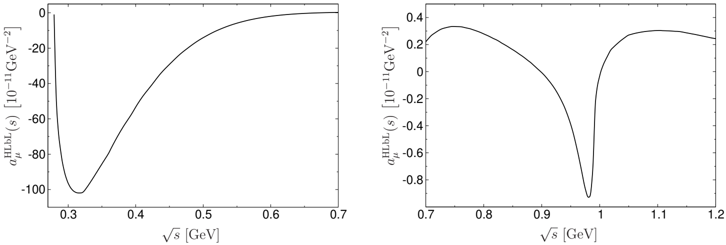
- sum-rule violations in S-wave rescattering are very small
- result largely **basis independent**
- together with $I = 2$ leads to

$$a_\mu^{\text{HLbL}}[S\text{-wave rescattering}] = -8.7(1.0) \times 10^{-11}$$

Dispersive evaluation of $f_0(980)$ contribution

→ Danilkin, Hoferichter, Stoffer, arXiv:2105.01666 [hep-ph]

- dispersive $f_0(980)$ contribution estimated from deficit in shape of integrand:



$$a_\mu^{\mathrm{HLbL}}[f_0(980)]_{\text{rescattering}} = -0.2(1) \times 10^{-11}$$

Dispersive evaluation of $f_0(980)$ contribution

→ Danilkin, Hoferichter, Stoffer, arXiv:2105.01666 [hep-ph]

- dispersive $f_0(980)$ contribution can be compared to **NWA in the same basis** for HLbL
- using TFFs from quark model → Schuler et al. (1998)

$$a_\mu^{\mathrm{HLbL}}[f_0(980)]_{\mathsf{NWA}} = -0.37(6) \times 10^{-11}$$

with $M_{f_0(980)} = 0.99$ GeV, $\Gamma_{\gamma\gamma}[f_0(980)] = 0.31(5)$ keV

- differences to NW estimates of → Knecht et al., PLB **787** (2018) 111 mainly due to propagator model, corresponding to a different HLbL basis
- comparison to → Pauk, Vanderhaeghen, EPJC **74** (2014) 3008 difficult due to kinematic singularities

Dispersive evaluation of $f_0(980)$ contribution

→ Danilkin, Hoferichter, Stoffer, arXiv:2105.01666 [hep-ph]

- NWA for $a_0(980)$:

$$a_\mu^{\mathrm{HLbL}}[a_0(980)]_{\mathsf{NWA}} = -\left([0.4, 0.6]^{+0.2}_{-0.1}\right) \times 10^{-11}\,,$$

  where TFF scale is given by $[M_\rho, M_S]$

- leads to

$$a_\mu^{\mathrm{HLbL}}[\text{scalars}] = -9(1) \times 10^{-11}$$

- even heavier scalars: small contribution around -1×10^{-11}, but **very uncertain** two-photon coupling (not seen prominently in $\gamma\gamma$ reactions)
  $\Rightarrow$ better treat in some form in asymptotic matching

Axial-vector contributions

- axial vectors play a prominent role in fulfilling SDCs
  $\rightarrow$ talks by G. Colangelo and A. Rebhan
- input for TFFs rather uncertain: data situation best for $f_1(1285)$ $\rightarrow$ talk by B. Kubis
- how to deal with broad $a_1(1260)$? using NWA and $SU(3)$?
- inclusion in dispersive framework previously hampered by kinematic singularities

Axial-vector contributions

- new basis **solves issue with kinematic singularities** for axial vectors
- **axial-vector poles** in **transverse part** of HLbL
- **longitudinal part**: axial-vector pole in Mandelstam variable s cancels with numerator in $g-2$ limit $s \to q_3^2$, but leaves **well-defined non-pole contribution**

$$\bar{\Pi}_1^{\text{axial}} = \frac{G_2(q_1^2, q_2^2) G_1(q_3^2)}{M_A^6} ,$$

$$G_1(q_3^2) = \mathcal{F}_1(q_3^2, 0) + \mathcal{F}_2(q_3^2, 0) ,$$

$$G_2(q_1^2, q_2^2) = (q_1^2 - q_2^2)\mathcal{F}_1(q_1^2, q_2^2) + q_1^2 \mathcal{F}_2(q_1^2, q_2^2) + q_2^2 \mathcal{F}_2(q_2^2, q_1^2)$$

$\to$ Colangelo, Hagelstein, Hoferichter, Laub, Stoffer, arXiv:2106.13222 [hep-ph]

- basis dependence due to sum-rule violations restricted by **absence of kinematic singularities**

Axial-vector contributions

- consistent inclusion in dispersive framework now possible
- main conceptual problem for narrow axial vectors solved; challenge is **input for TFFs**

Axial-vector contributions

- HLbL contribution very sensitive to **asymptotic behavior** of TFFs $\to$ talk by B. Kubis
- VMD model with asymptotic constraints
  $\to$ Zanke et al., arXiv:2103.09829 [hep-ph] points to $f_1(1285)$ contribution of symmetric TFF of a couple of units in 10^{-11}
- strong sensitivity to antisymmetric TFFs and **large uncertainties** $\Rightarrow$ need to be controlled in combination with SDCs on HLbL

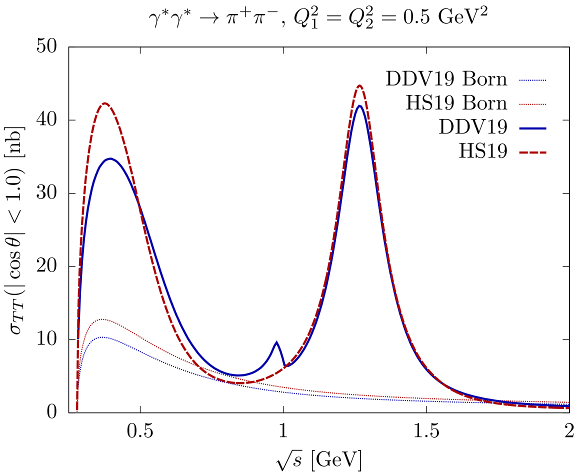
Tensor-meson contributions

- similarity to $f_0(980)$ and S-waves: $f_2(1270)$ contribution can be compared from NWA and $\pi\pi$ **rescattering**
- $\gamma^*\gamma^* \to \pi\pi$ helicity partial waves solved with Omnès methods including D-waves

$\to$ Hoferichter, Stoffer, JHEP **07** (2019) 073

$\to$ Danilkin, Deineka, Vanderhaeghen, PRD **101** (5) (2020) 054008

Tensor-meson contributions



→ T. Aoyama *et al.*, Phys. Rept. **887** (2020) 1-166

Tensor-meson contributions

- both NW tensor-meson contribution and $\pi\pi$ D-wave contribution to HLbL are affected by **kinematic singularities**
- two options:
  1. impose **sum rules** at a level sufficient to control ambiguities from residue subtraction
  2. change tensor basis or dispersive framework to avoid singularities in the first place
- both directions are pursued and work in progress

Conclusions

- contributions from hadronic states in the $1-2$ GeV range responsible for a substantial fraction of HLbL uncertainty
- WP: resonance and SDC uncertainties are **added linearly**
- recent improvement on scalar $f_0(980)$ contribution: **dispersive treatment** in terms of coupled-channel $\pi\pi/\bar{K}K$ rescattering
- higher scalars very uncertain $\gamma\gamma$ coupling $\Rightarrow$ better treated in asymptotic matching

Conclusions

- conceptual obstacles for inclusion of **axial vectors** in NWA in dispersive framework resolved
- given data situation and asymptotic constraints, prospects best for a phenomenologically driven determination of $f_1(1285)$ contribution
- **tensor mesons**: compare NWA with $\pi\pi$ **rescattering**: $\gamma^*\gamma^* \to \pi\pi$ D-waves solved with Omnès methods

Open questions and challenges

- kinematic singularities still affect tensor-meson contribution
  $\Rightarrow$ subtraction introduces ambiguity due to **sum-rule violations**
- TFF input for axial vectors requires more work to control uncertainties
- effects of NWA for broader resonances need to be addressed
- **sum-rule violations** and ambiguities due to **basis dependence** affect all narrow resonances (apart from pseudoscalars)
  $\Rightarrow$ need to be dealt with globally for entire HLbL

Backup

Dispersive evaluation of $f_0(980)$ contribution

→ Danilkin, Hoferichter, Stoffer, arXiv:2105.01666 [hep-ph]

Λ [GeV]		0.89	2.0
pion (+ kaon) Born terms (S-waves)		-11.4	-11.8
S-wave $I=0$ rescattering		-10.0	-9.8
sum rule pion (+ kaon) Born terms (S-waves)	$++,++$	8.0	8.4
	$00,++$	-9.2	-9.6
	total	-1.2	-1.2
sum rule S-wave $I=0$ rescattering	$++,++$	6.9	6.8
	$00,++$	-7.3	-7.2
	total	-0.4	-0.4

Heavier scalars

- two-photon coupling rather uncertain
- using quark-model TFFs:

$$a_\mu^{\text{HLbL}}[f_0(1370)] = -(1.5^{+0.7}_{-0.4}) \times 10^{-11} \quad \big[-(0.6^{+0.3}_{-0.2}) \times 10^{-11}\big],$$
$$a_\mu^{\text{HLbL}}[a_0(1450)] = -(0.5^{+0.2}_{-0.1}) \times 10^{-11} \quad \big[-(0.2^{+0.1}_{-0.05}) \times 10^{-11}\big]$$

- numbers in brackets: TFF scale set by M_ρ
- $SU(3)$ relation for $\Gamma_{\gamma\gamma}$ of the $f_0(1370)$