Lattice QCD calculation of direct CP violation and long distance effects in kaon mixing and rare decays

FPCP 2015

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Norman H. Christ

Columbia University

RBC and UKQCD Collaborations

Outline

- Lattice QCD in 2015
- First order electroweak:

  $K \to \pi\,\pi$ decay
- Second order electroweak
  - $K_L - K_S$ mass difference
  - Long distance parts of ε_K.
  - Rare kaon decays

RBC Collaboration

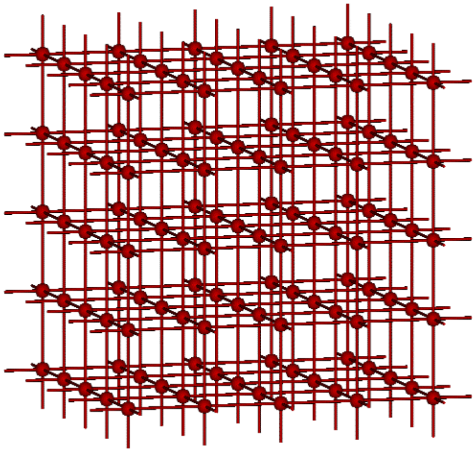
- BNL
  - Chulwoo Jung
  - Taku Izubuchi
  - Christoph Lehner
  - Amarjit Soni
- RBRC
  - Chris Kelly
  - Tomomi Ishikawa
  - Shigemi Ohta (KEK)
  - Sergey Syrityn
- Connecticut
  - Tom Blum
- Columbia
  - Ziyuan Bai
  - Xu Feng
  - Norman Christ
  - Luchang Jin
  - Robert Mawhinney
  - Greg McGlynn
  - David Murphy
  - Daiqian Zhang

UKQCD Collaboration

- Edinburgh
  - Peter Boyle
  - Julien Frison
  - Nicolas Garron (Plymouth)
  - Jamie Hudspith
  - Karthee Sivalingam
  - Oliver Witzel
- Southampton
  - Jonathan Flynn
  - Tadeusz Janowski
  - Andreas Juttner
  - Andrew Lawson
  - Edwin Lizarazo
  - Andrew Lytle (Mumbai)
  - Marina Marinkovic (CERN)
  - Antonin Portelli
  - Chris Sachrajda
  - Matthew Spraggs
  - Tobi Tsang

Lattice QCD 2015

Lattice QCD



- First-principles treatment of low-energy, non-perturbative QCD.
- All approximations understood and controlled:
  - Non-zero lattice spacing: $a \rightarrow 0$.
  - Finite volume: $L \rightarrow \infty$
  - Typically neglect E&M and $m_u \neq m_d$, $\alpha_{\mathrm{EM}} << 1$
- Supports not only rough phenomenology but also accurate theoretical physics (where it can be applied).
- Use chiral fermions (domain wall fermions) ensures chiral symmetry at finite lattice spacing

Current state-of-the-art

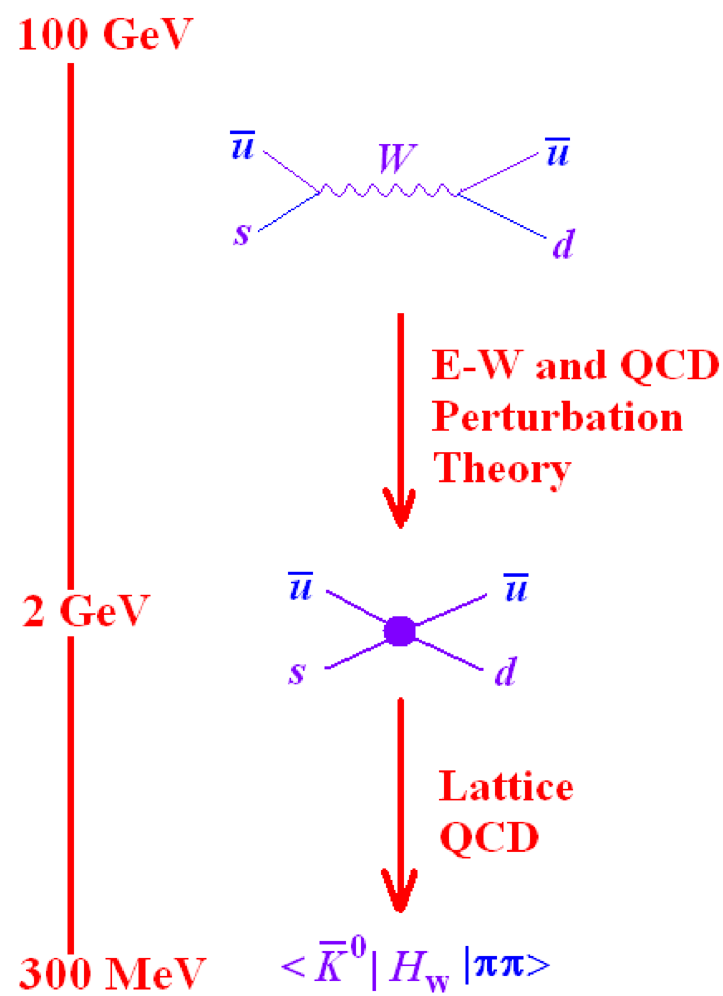
- Physical m_π=135 MeV and $L = 4 - 6$ fm.
- Large volume 48^3 x 96 and 64^3 x 128 ensembles.
- Complete set of measurements takes 5.3 hours on a 32-rack BG/Q machine (sustains 1 Pflops)
- Large collaboration essential:
  - Highly optimized code (64 threads, SPI comms., wide, vector SIMD)
  - Sophisticated algorithms (deflation, FG $(\Delta t)^3$ integrator)
  - Complex measurement strategies (NPR, G-parity BC, 4-pt functions, all-mode-averaging, all-to-all propagators)

ΔS=1 Weak Interactions

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{ud}} \frac{V_{ts}^*}{V_{us}^*} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



$K \rightarrow \pi\,\pi$ decay

$K \to \pi\pi$ phenomenology

- Final $\pi\pi$ states can have $I = 0$ or 2.

$$\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$
$$\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i e^{\delta_2-\delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} - \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}\right)$$

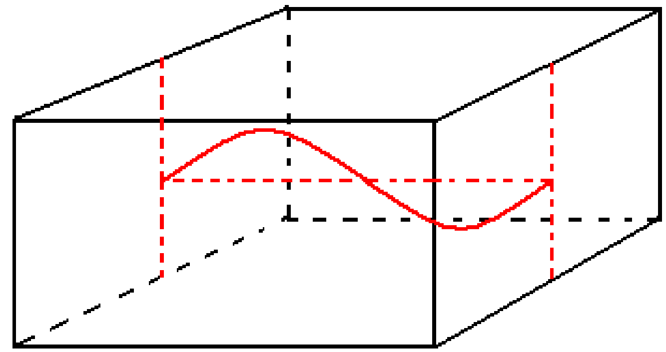
- $K^0 - \overline{K}^0$ mixing gives indirect CP violation:

$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im} M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\Gamma_{0\bar{0}}}{\mathrm{Re} M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\Gamma_{0\bar{0}}}\right\} + i\frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$$

Lattice Aspects

Physical $\pi\pi$ states – Lellouch-Luscher

- Euclidean $e^{-H_{QCD}t}$ projects onto $|\pi\pi(\vec{p}=0)\rangle$
- Exploit finite-volume quantization.
- Adjust volume so 1st or 2nd excited state has correct p.



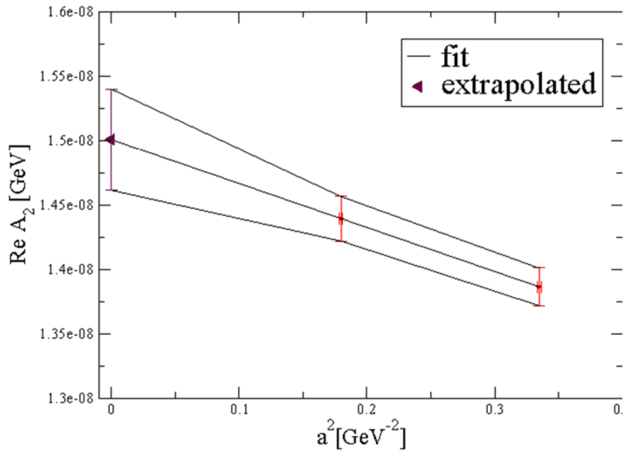
- Impose boundary conditions so ground state has physical p
  - $\Delta I = 3/2$: impose anti-periodic BC on d quark
  - $\Delta I = 1/2$: impose G-parity BC
- Correctly include π - π interactions, including normalization.

$$\Delta \mathbf{I} = \mathbf{3/2}$$

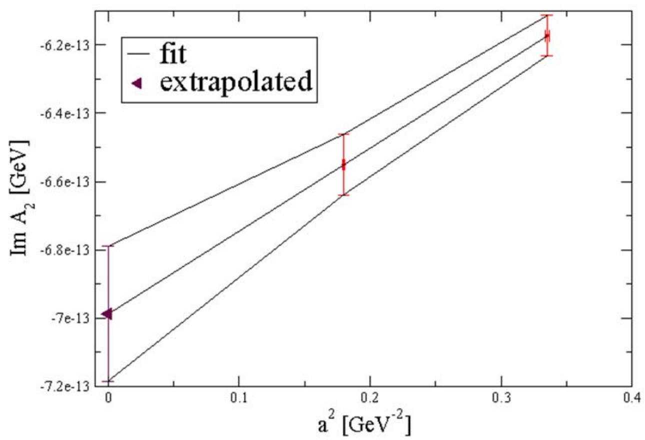
Δ I = 3/2: Continuum results

(Tadeusz Janowski)

- Use two new large ensembles to remove a^2 error (m_π=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV

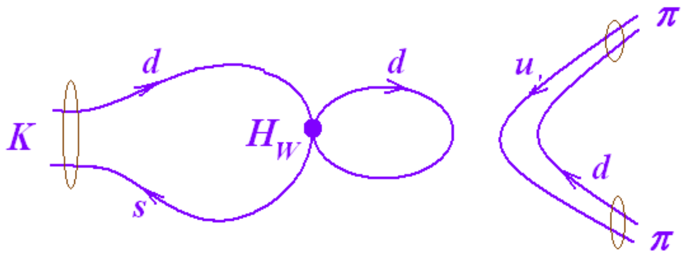


- Now continuum limit results: [Phys.Rev. D91 (2015) 7, 074502]
  - $\mathrm{Re}(A_2) = 1.50(4)_{stat}(14)_{sys} \times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2) = -6.99(20)_{stat}(84)_{sys} \times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2) = 1.479(4)\ 10^{-8}$ GeV



$\Delta I = 1/2$

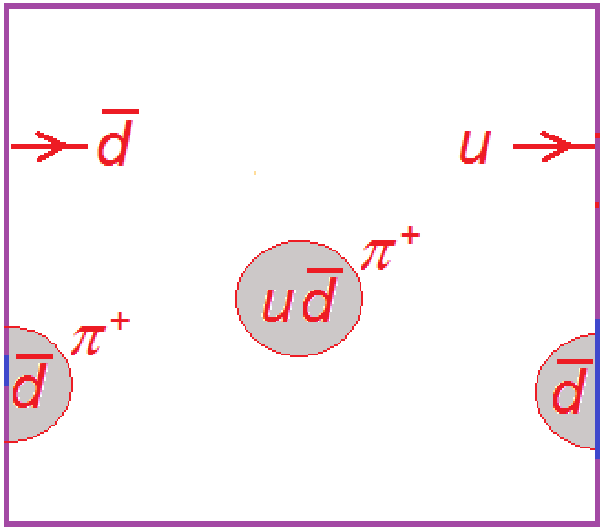
$\Delta I = 1/2 \;\; K \rightarrow \pi\pi$



- Made much more difficult by disconnected diagrams:
- Many more diagrams (48) than $\Delta I = 3/2$.
- Initial threshold decay calculation successful (Qi Liu)
  - Re (A_0): 25% stat errors
  - Im (A_0): 50% stat errors
- Recent threshold calculation of Ishizuka, et al. with Wilson fermions arXiv:1505.05289

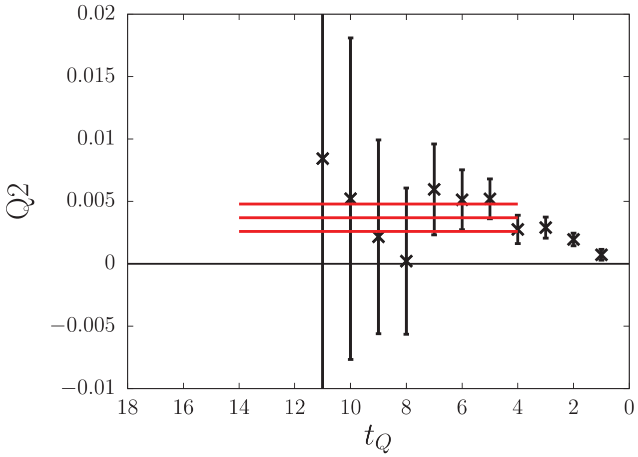
$\Delta I = 1/2$ $K \rightarrow \pi\pi$: Physical kinematics

- Goal is a 20% calculation of ε'/ε with all errors controlled
- Use $32^3 \times 64$ volume with $1/a = 1.379$ GeV
- Achieve $p = 205$ MeV from **G-parity** boundary conditions in 3 directions
- Requires new **G-parity** ensembles



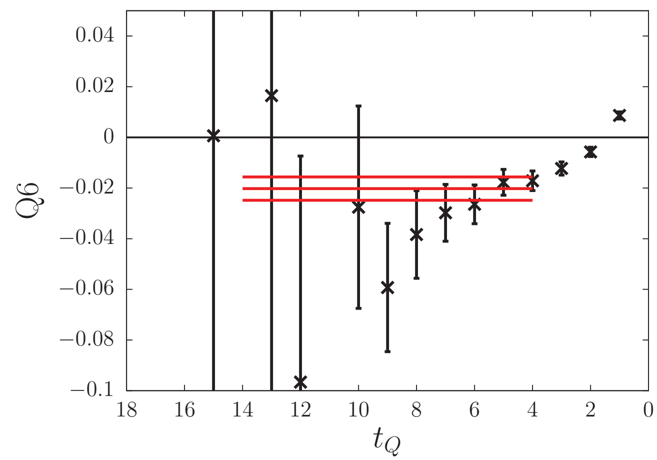
$\Delta I = 1/2 \;\; K \rightarrow \pi\pi$: Current status
(Chris Kelly & Daiqian Zhang)

Q2 - largest part of Re(A_0)



$\langle \pi\pi_{I=0}|Q_2|K\rangle = (4.23 \pm 1.14)\times 10^{-3}$

Q6 - largest part of Im(A_0)



$\langle \pi\pi_{I=0}|Q_6|K\rangle = (-1.89 \pm 0.46)\times 10^{-3}$

- 216 configurations
- First calculation nearly complete

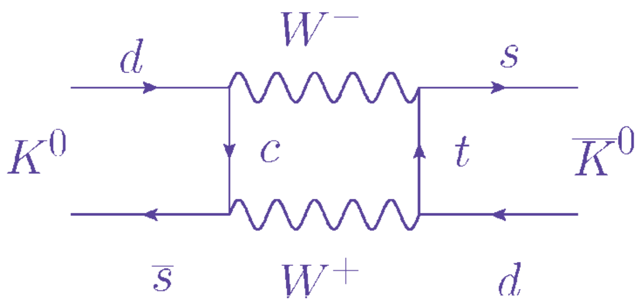
- $M_K = 490.6(2.4)$
- $E_{\pi\pi} = 498(11)$

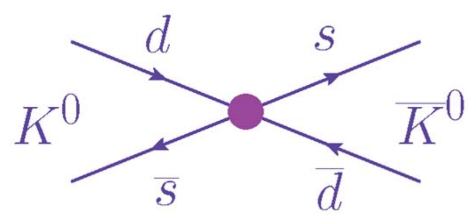
$K^0 - \overline{K}^0$ mixing

$K^0 - \overline{K^0}$ Mixing

- CP violating: $p \sim m_t$

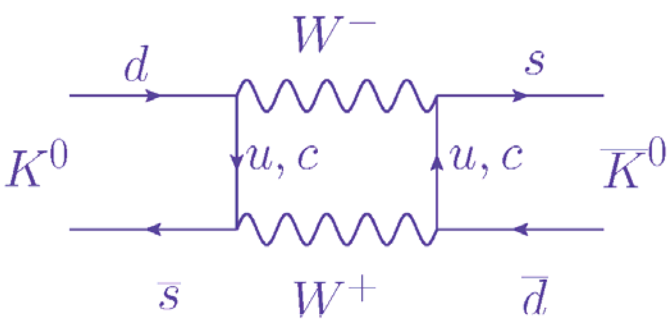
$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$

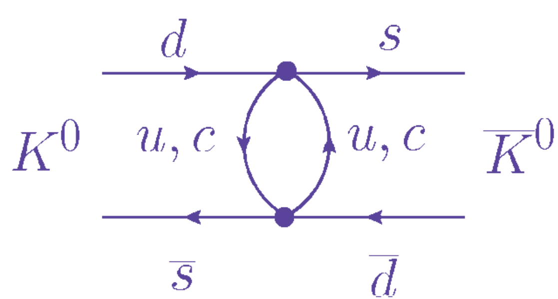




- CP conserving: $p \leq m_c$

$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$

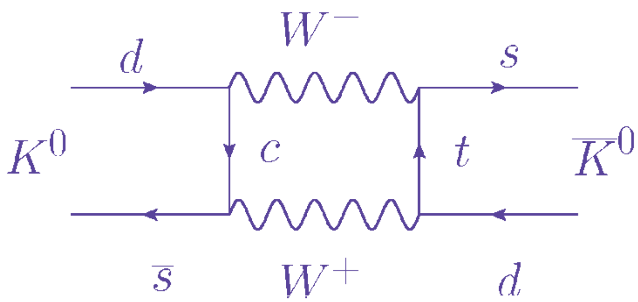




$K^0 - \overline{K^0}$ Mixing

- CP violating: $p \sim m_t$

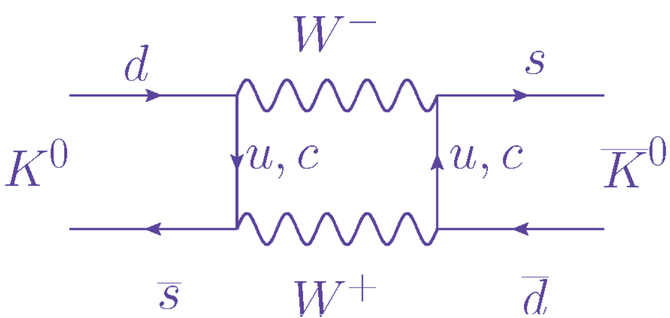
$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$



Long distance part is a small but important contribution

- CP conserving: $p \leq m_c$

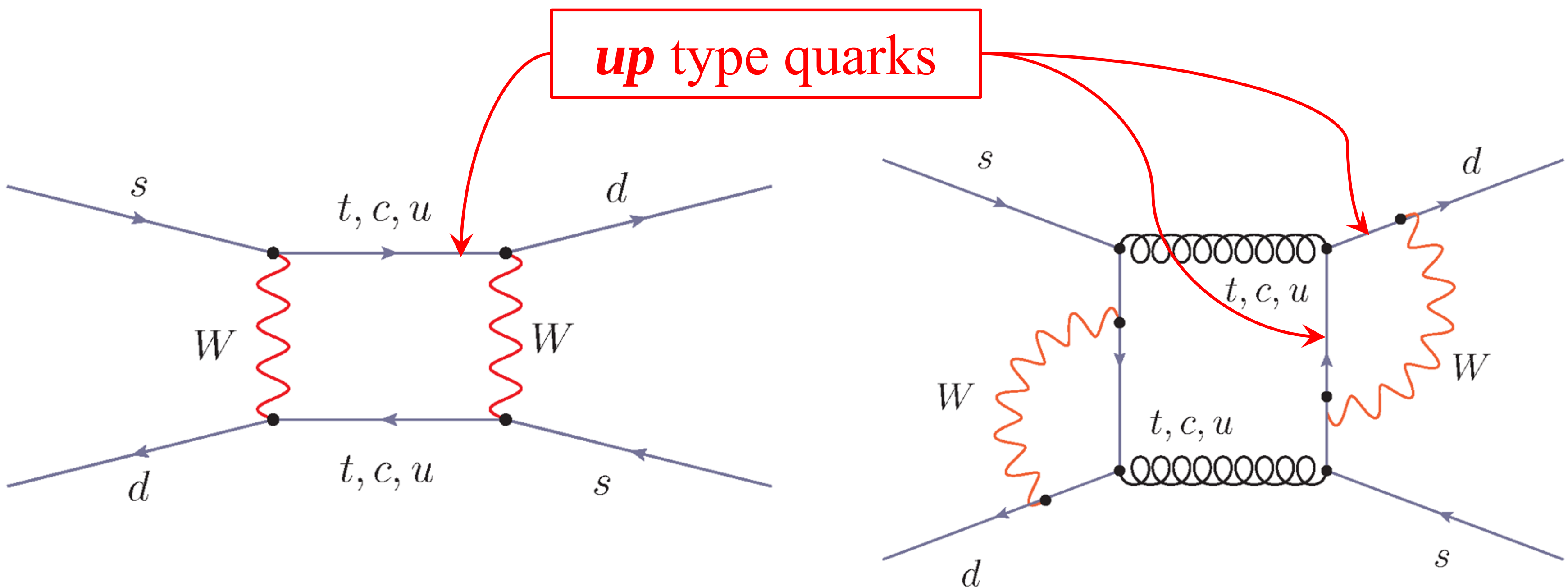
$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$



Long distance part is large. QCD perturbation theory fails at the 30% level.

Recall Standard Model Structure

- Two types of diagram (most gluons not shown):



Connected

(two quark lines are connected by W's)

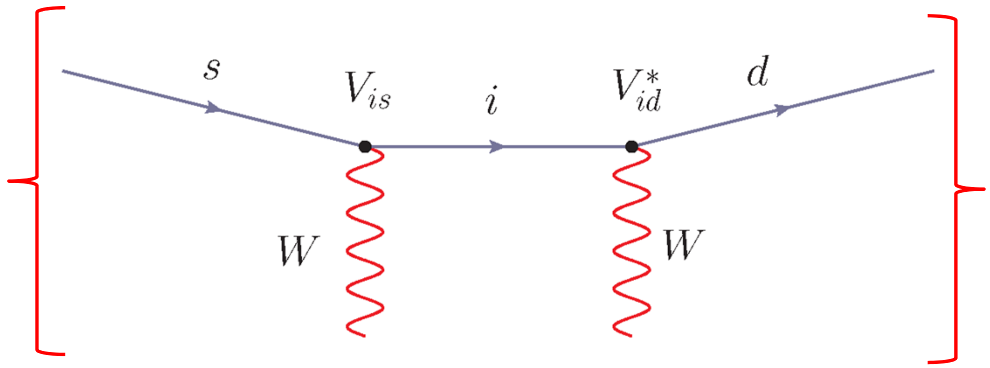
Disconnected

(each quark line is connected to itself by W's)

Standard Model Review

- Three up-type propagators:

$$\sum_{i=u,c,t}$$



- GIM subtraction:

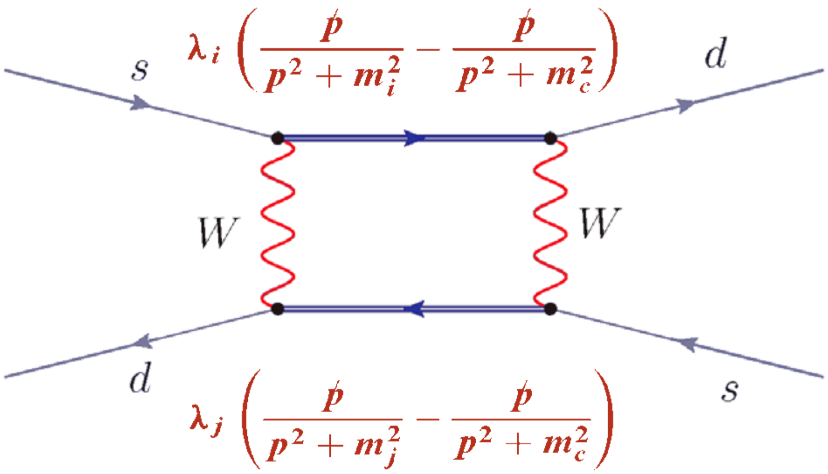
$$\sum_{i=u,c,t} \left\{ V^*_{i,d} \frac{\not p}{p^2+m_i^2} V_{i,s} - V^*_{i,d} \frac{\not p}{p^2+m_c^2} V_{i,s} \right\}$$

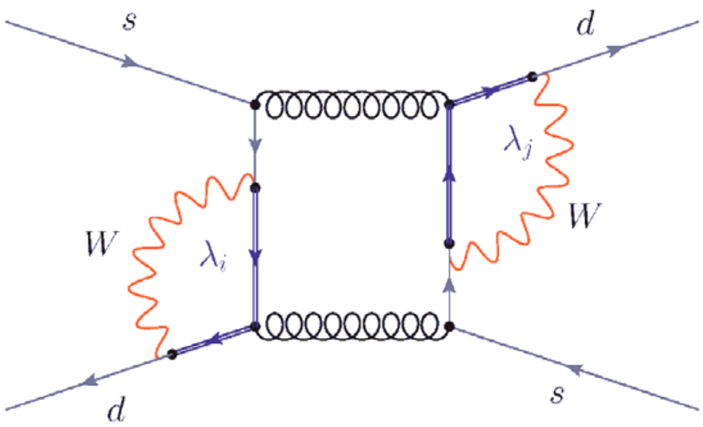
$$= \lambda_t \left\{ \frac{\not p}{p^2+m_t^2} - \frac{\not p}{p^2+m_c^2} \right\} + \lambda_u \left\{ \frac{\not p}{p^2+m_u^2} - \frac{\not p}{p^2+m_c^2} \right\}$$

$$\lambda_i = V^*_{i,d} V_{i,s}$$

Six contributions to ΔM_K and ε_K

- Six types of diagram:





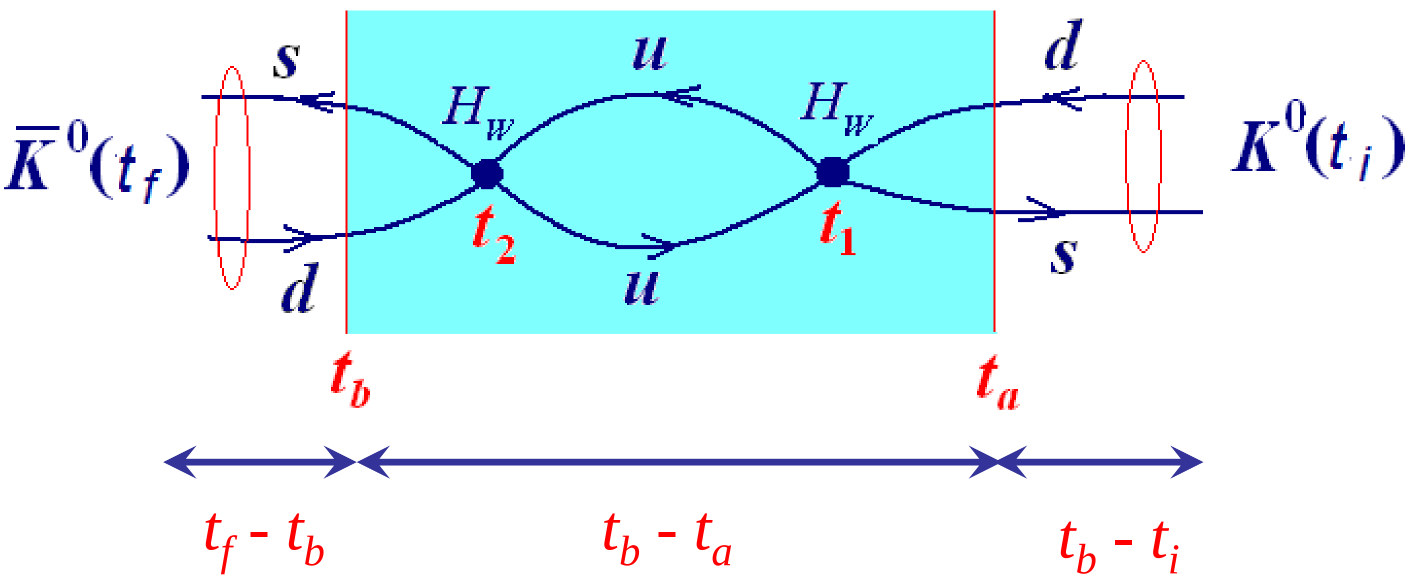
$\lambda_i \lambda_j = \lambda_t \lambda_t$, $\lambda_u \lambda_u$ and $\lambda_t \lambda_u$

- ΔM_K: $\lambda_u\,\lambda_u$ **term**
- ε_K: $\lambda_t\,\lambda_t$ **and** $\lambda_u\,\lambda_t$ **term**

Lattice Version

- Evaluate standard, Euclidean, 2nd order $K^0 - \overline{K^0}$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0'}(t_i)\right)|0\rangle$$



Interpret Lattice Result

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(\underbrace{-(t_b - t_a)}_{1.} \underbrace{- \frac{1}{M_K - E_n}}_{2.} \underbrace{+ \frac{e^{(M_K-E_n)(t_b-t_a)}}{M_K - E_n}}_{3.} \right)$$

1. $\Delta m_K{}^{\mathrm{FV}}$
2. Uninteresting constant
3. Growing or decreasing exponential: states with $E_n < m_K$ must be removed!

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} - 2 \frac{d(\phi + \delta_0)}{dk}\bigg|_{m_K} |\langle n_0 | H_W | K^0 \rangle|^2 \cot(\phi + \delta_0)\big|_{M_K}$$

(N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda, arXiv:1504.01170)

$K_L - K_S$ mass difference

Compute $\lambda_u \lambda_u$ term

- Use four-Fermi operators in the four-flavor theory:

$$Q_1^{qq'} = (\overline{q}_i d_i)_{V-A} (\overline{q}'_j s_j)_{V-A} \qquad Q_2^{qq'} = (\overline{q}_i d_j)_{V-A} (\overline{q}'_j s_i)_{V-A}$$

$$\mathcal{H}_W = \frac{G_F}{2} \sum_{q,q'=u,c} V_{qd} V^*_{q's} \left(C_1 Q_1^{qq'} + C_2 Q_2^{qq'} \right)$$

- Use Rome-Southampton NPR and 4-flavor RI/SMOM / $\overline{\text{MS}}$-NDR matching from Lehner and Sturm
- Assume Cabibbo unitarity:

$0 = \lambda_u + \lambda_c + \lambda_t \approx \lambda_u + \lambda_c$ where $\lambda_q = V_{qd} V_{qs}^*$

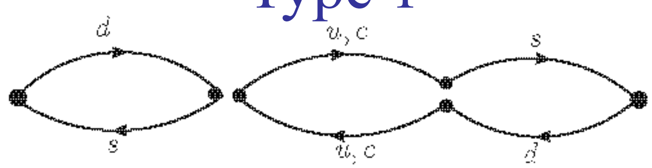
Lattice setup

- Must include charm quark (GIM *u*–*c* cancellation)
- Three calculations performed:

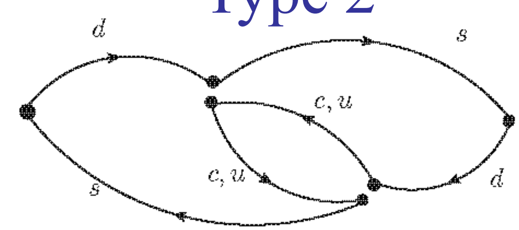
Jianglei Yu
- – $16^3 \times 32$, $m_p = 420$ MeV, types 1 & 2 (arXiv:1212.5931)
- – $24^3 \times 64$, $m_p = 330$ MeV, all graphs (arXiv:1406.0916)

Ziyuan Bai
- – $32^3 \times 64$, $m_p = 170$ MeV, all graphs

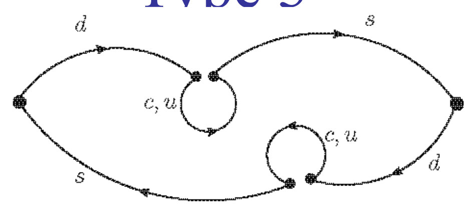
Type 1



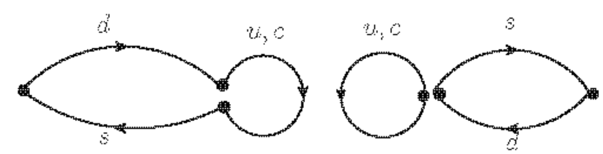
Type 2



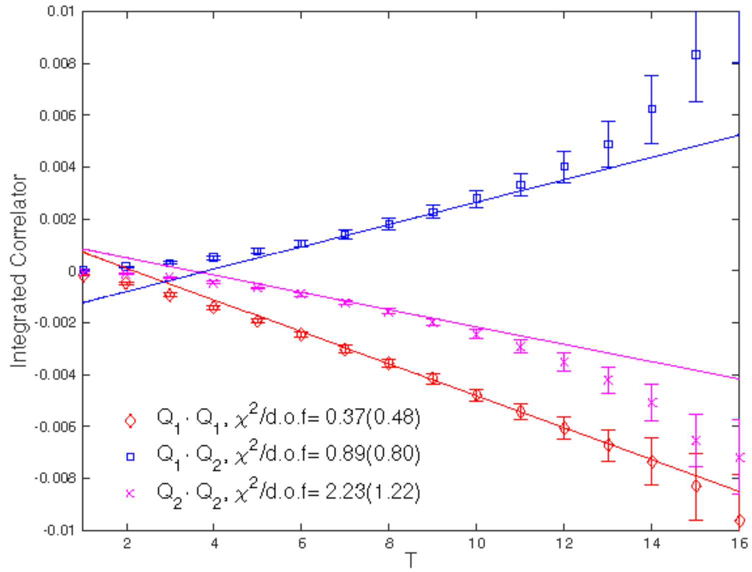
Type 3



Type 4



$m_\pi = 170$ MeV – 32^3 x 64 results
(Ziyuan Bai)



	ΔM_K x 10^{+12}
Types 1-4	5.76(73)
Types 1-2	4.19(15)
η	0
π	0.27(14)
$\pi\pi$, I=0	-0.097(49)
$\pi\pi$, I=2	-6.56(6) x 10^{-4}
Δ_{FV}	0.029(19)

- Use m_c = 750 MeV, fit for t ≥ 8
- Disconnected contribution small
- $\pi\pi$ contribution ~2% and FV correction ~0.5%

Long distance part of ε_K

ΔS = 1, Four flavor operators
(Ziyuan Bai)

- Choose appropriate N_f=4 effective Hamiltonian:

$$H_W^{\Delta S=1;\Delta C=\pm 1,0} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q,q'=u,c} V_{q's}^* V_{qd} \sum_{i=1}^{2} C_i Q_i^{q'q} + V_{ts}^* V_{td} \sum_{i=3}^{6} C_i Q_i \right\}$$

current x current:

$$\begin{aligned} Q_1^{q'q} &= (\bar{s}_i q'_j)_{V-A} (\bar{q}_j d_i)_{V-A} \\ Q_2^{q'q} &= (\bar{s}_i q'_i)_{V-A} (\bar{q}_j d_j)_{V-A} \end{aligned}$$

QCD penguin:

$$\begin{aligned} Q_3 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V-A} \\ Q_4 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V-A} \\ Q_5 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V+A} \\ Q_6 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V+A} \end{aligned}$$

Focus on $\lambda_t\lambda_u$ contribution to ε_K
(Ziyuan Bai)

- Construct $\boldsymbol{H_W}$(x) x $\boldsymbol{H_W}$(y) and extract the $\lambda_t\lambda_u$ term

$$H_W(x)H_W(y) = \frac{G_F^2}{2}\lambda_t\lambda_u \sum_{i=1}^{2}\sum_{j=1}^{6} C_i C_j Q_{ij}$$

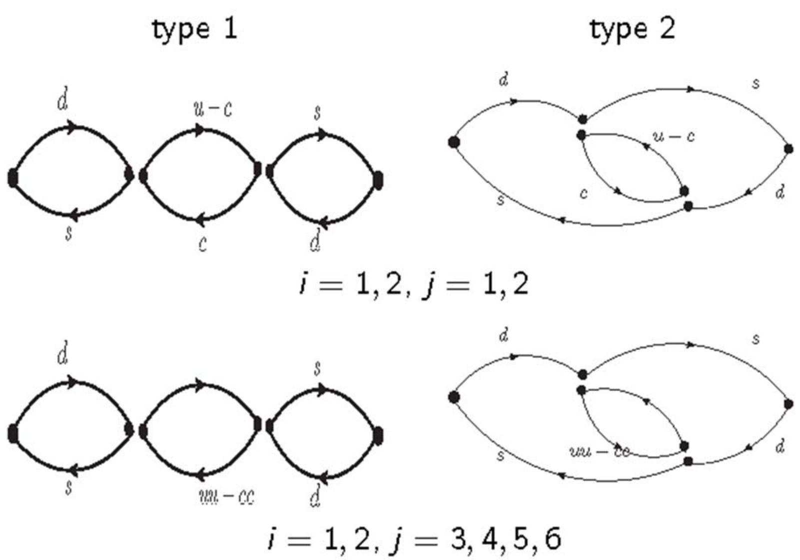
$$Q_{ij} = \begin{cases} 2Q_i^{cc}(x)Q_j^{cc}(y) - Q_i^{uu}(x)Q_j^{cc}(y) - Q_i^{cc}(x)Q_j^{uu}(y) & \text{if } j = 1,2 \\ \quad -Q_i^{uc}(x)Q_j^{cu}(y) - Q_i^{cu}(x)Q_j^{uc}(y) & \\ \left(Q_i^{cc}(x) - Q_i^{uu}(x)\right)Q_j(y) & \text{if } j = 3,\ldots,6 \\ \quad +Q_j(y)\left(Q_i^{cc}(y) - Q_i^{uu}(y)\right) & \end{cases}$$

- Identify five types of diagrams

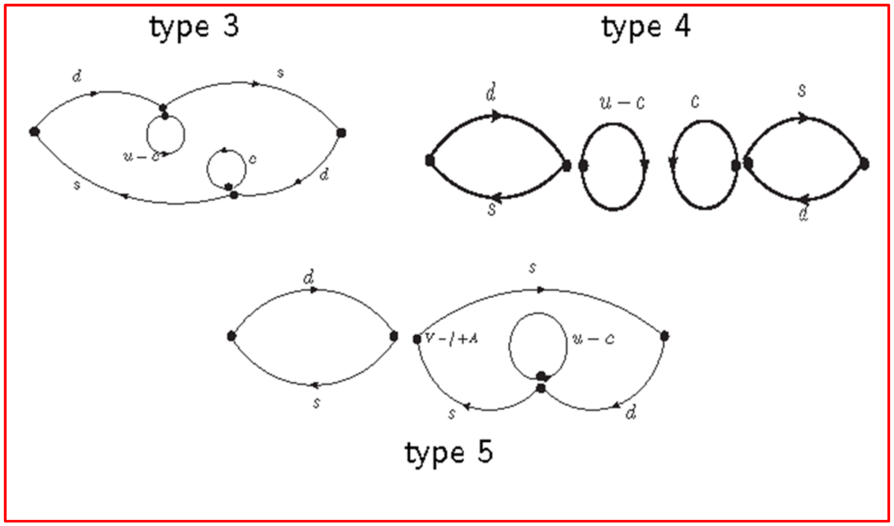
Diagrams for $\lambda_t \lambda_u$ contribution to ε_K

(Ziyuan Bai)

- Identify five types of diagrams

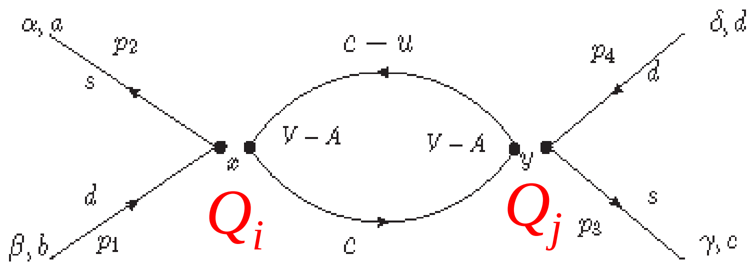


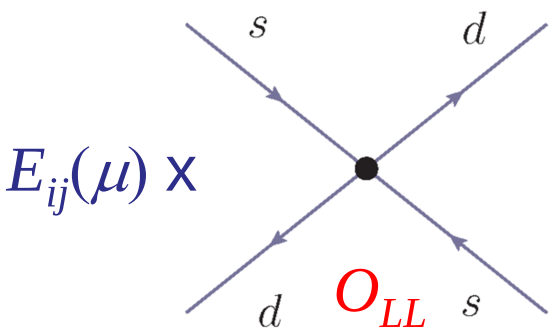
Omit from 1st study



Removing lattice short distance part
(Ziyuan Bai)

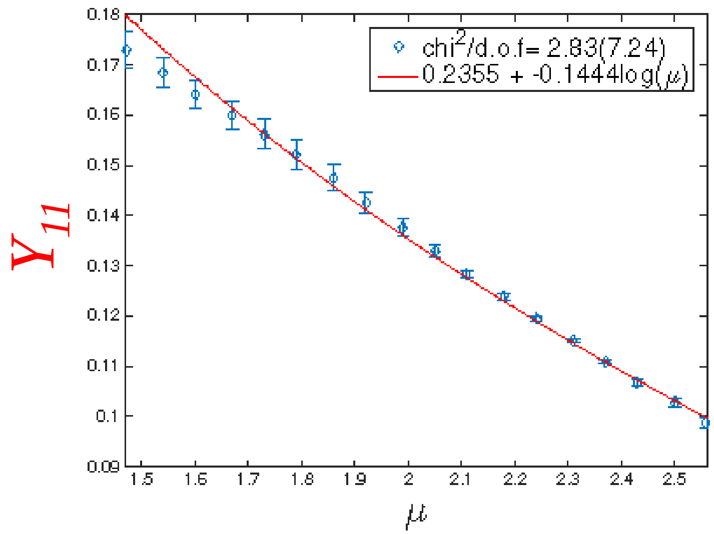
- Evaluate off-shell Green's function at $p_i^2 = \mu^2$
- Forces internal momentum also to the scale μ or greater
- This is a definition of the short-distance part of diagram.
- Add $E_{ij}(\mu)\,(\bar{s}\gamma^\nu(1-\gamma^5)d)\,(\bar{s}\gamma^\nu(1-\gamma^5)d)$ with $E_{ij}(\mu)$ chosen to make SD part agree with perturbation theory.
- $p_i^2 = 2p_i \cdot p_j = \mu^2$

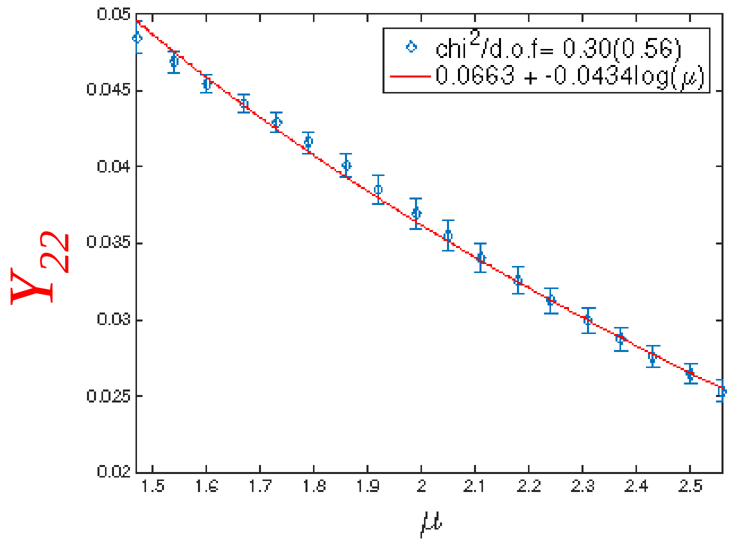




Short-distance lattice correction
(Ziyuan Bai)

- Results for short-distance coefficient E_{11} and E_{22} of O_{LL} for the products Q_1Q_1 and Q_2Q_2:





- Effect of a cutoff radius $|x - y| < R$ at $\mu = 1.93$ GeV

Cutoff	3	4	5	6	none
E_{11}^{lat}	0.1462	0.1501	0.1493	0.1489	0.1489
E_{22}^{lat}	0.0418	0.0427	0.0425	0.0425	0.0425

Progress toward long-distance part of ε_K

(Ziyuan Bai)

Preliminary

- Examine only type 1 and 2 diagrams
- Use C. Lehner's ***PhySyHCAl*** to add back the correct perturbative short distance part at LO.

μ (GeV)	Im $M_{0\bar{0}}^{ut,ld}$ (10^{-15} MeV)	Im $M_{0\bar{0}}^{ut,cont}$ (10^{-15} MeV)	Im $M_{0\bar{0}}^{ut}$ (10^{-15} MeV)
1.54	-0.871(30)	-4.772(56)	-5.642(64)
1.92	-1.065(30)	-4.546(54)	-5.601(62)
2.11	-1.151(31)	-4.435(52)	-5.586(61)
2.31	-1.226(31)	-4.350(51)	-5.576(60)
2.56	-1.302(30)	-4.208(50)	-5.511(58)

- Result:

tt ***ut***$_{\mathbf{sd}}$ ***ut***$_{\mathbf{ld}}$ **Im(A_0)**

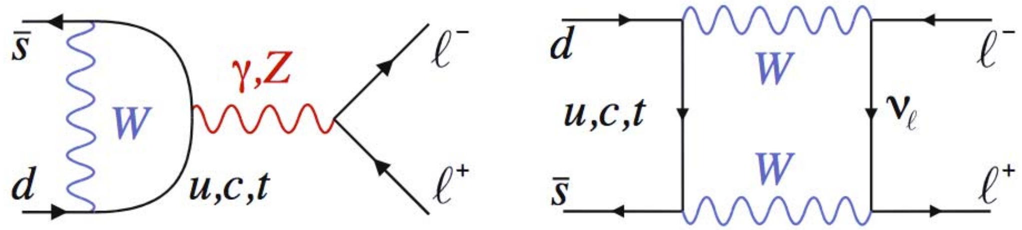
$$|\varepsilon_K| = (1.806 + 0.892 + 0.209 + 0.111) \times 10^{-3}$$

$$= 3.019 \times 10^{-3} \quad (2.228(11) \times 10^{-3} \text{ expt.})$$

Rare Kaon Decays

Rare Kaon Decays

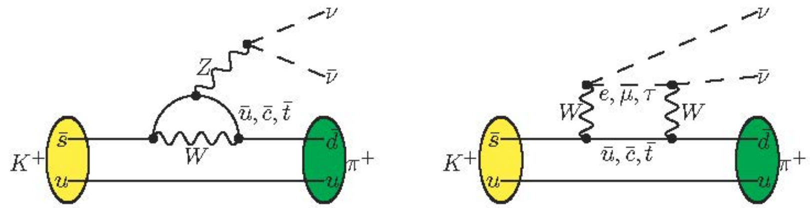
(Xu Feng, Antonin Portelli, Andrew Lawson)



- Can lattice methods be of use for rare *K* decays?
- $K_L \rightarrow \pi^0 + l + \bar{l}$: determine the sign of the indirect CP violating amplitude.
- $K^+ \rightarrow \pi^+ + \nu + \bar{\nu}$: calculate the long distance ($l \geq 1/m_c$) part of charm contribution. Small (≈4%) but leading theoretical uncertainty.

$K^+ \to \pi^+ + \nu\ \bar{\nu}$

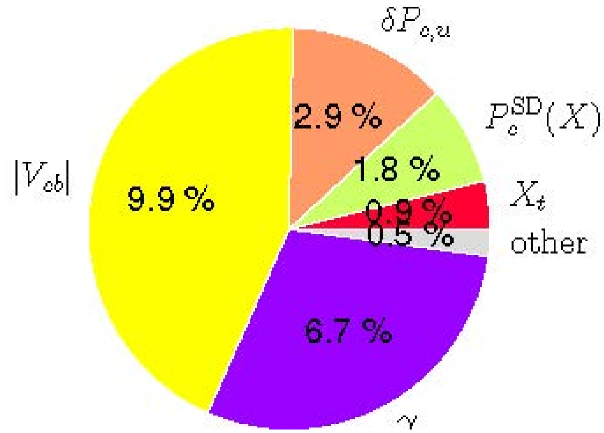
(Xu Feng)



- Estimate 3 contributions: top : charm-*sd* : charm-*ld*
  [Cirigliano et.al. Rev. Mod. Phys.]

$$\lambda_t \frac{m_t^2}{M_W^2} : \lambda_c \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c} : \lambda_u \frac{\Lambda_{\rm QCD}^2}{M_W^2} = 68\% : 29\% : 3\%$$

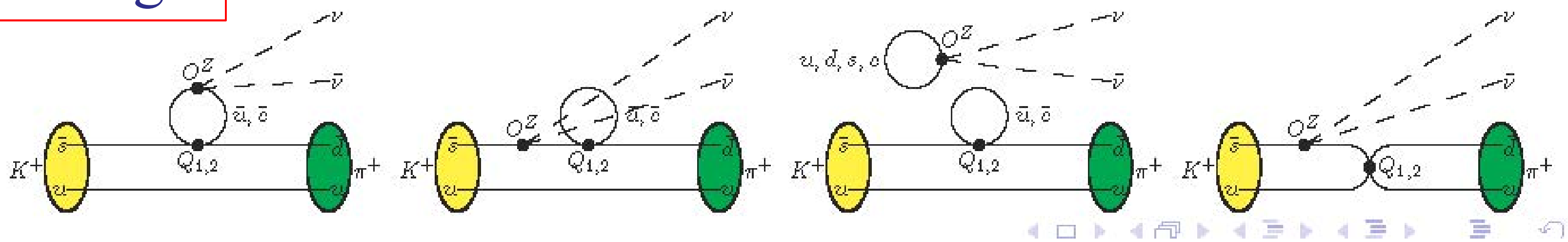
- Error budget
  [Buras, et.al. arXiv:1503.02693]



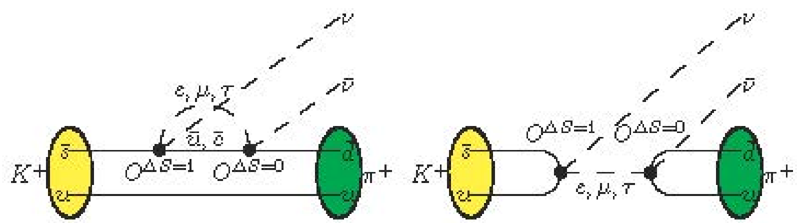
$K^+ \rightarrow \pi^+ + \nu\ \bar{\nu}$ - lattice details

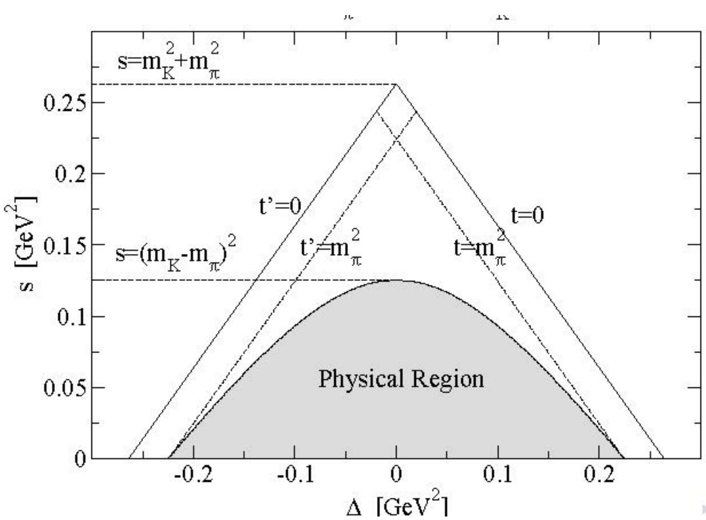
(Xu Feng)

Z- exchange



W- exchange



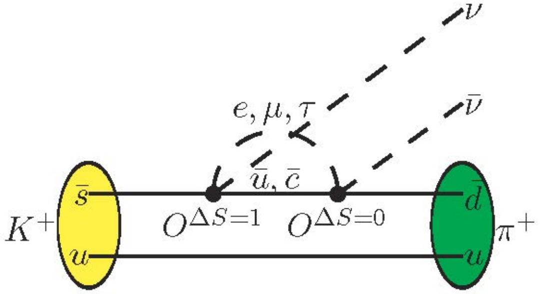


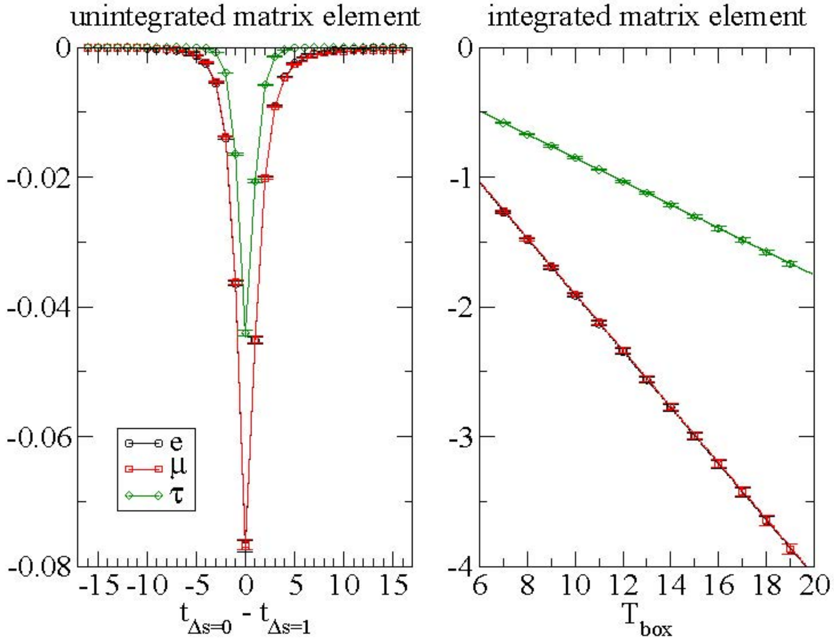
- Compute Dalitz plot distribution

$K^+ \rightarrow \pi^+ + \nu\ \bar{\nu}$ - results

(Xu Feng)

- Type 2 W-exchange





- type 2 diagram before SD subtraction

F^ℓ	lattice
e	$-2.164(31) \times 10^{-1}$
μ	$-2.164(31) \times 10^{-1}$
τ	$-9.03(14) \times 10^{-2}$

- type 2 diagram after SD subtraction, using $C^{lat}(\mu)$

F^ℓ	$\mu = 2$ GeV	$\mu = 3$ GeV
e	$-1.400(31) \times 10^{-1}$	$-1.849(31) \times 10^{-1}$
μ	$-1.402(31) \times 10^{-1}$	$-1.850(31) \times 10^{-1}$
τ	$-4.13(14) \times 10^{-2}$	$-6.68(14) \times 10^{-2}$

- Next use QCD pert theory to restore correct short distance part.

Outlook

- Physical pion masses, large volumes and accurate methods allow percent-level lattice calculations.
- Theoretical advances allow rescattering effects to be correctly computed in Euclidean space (so far only for low energy π–π states).
- $K \rightarrow \pi\pi$ decay and long-distance parts of 2nd order kaon decays and mixing is a practical target.
- Many critical quantities can now be computed:
  - $K \rightarrow \pi\pi$, $\Delta I = 3/2$ and 1/2, ε'/ε
  - $m_{K_L} - m_{K_S}$, long dist. contribution to ε
  - Long distance parts of $K \rightarrow \pi l \bar{l}$, $K \rightarrow \pi \nu\bar{\nu}$
  - QCD effects in g_μ - 2 from HVP and HLbL at $O(\alpha^3)$