Flavor Structures and Majorana Nature of Massive Neutrinos

ν

Zhi-zhong Xing

(IHEP, Beijing)

Nagoya Sumo, Summer 1997

- ★ 1996 — 1998 — 2015
- ★ It is a μ-τ symmetry !
- ★ The ν mass ordering and octant of θ_{23}
- ★ On $0\nu2\beta$: a discovery or a null result?

@ Flavor Physics & CP Violation, Nagoya, 25—29/5/15

July 1996

I arrived in Nagoya in April 1996, and gave a seminar on neutrinos at TMU (Minakata san) in July 1996 —— my 1[st] talk in Japan.

H. Fritzsch, Z.Z. Xing, hep-ph/9509389, Phys. Lett. B 372 (1996) 265

$$M_l = \frac{c_l}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \Delta M_l$$

$$M_\nu = c_\nu\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + \Delta M_\nu$$

symmetry breaking

$$\Delta M_l = \frac{c_l}{3}\begin{pmatrix}-i\delta_l & 0 & 0\\ 0 & i\delta_l & 0\\ 0 & 0 & \varepsilon_l\end{pmatrix}$$

$$\Delta M_\nu = c_\nu\begin{pmatrix}-\delta_\nu & 0 & 0\\ 0 & \delta_\nu & 0\\ 0 & 0 & \varepsilon_\nu\end{pmatrix}$$

For the first time, lepton flavor mixing with 2 large + 1 small angles:

$$U \approx \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{pmatrix} + i\sqrt{\frac{m_e}{m_\mu}}\begin{pmatrix}\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}}\\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0\\ 0 & 0 & 0\end{pmatrix} + \frac{m_\mu}{m_\tau}\begin{pmatrix}0 & 0 & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}}\\ \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{12}} & \frac{1}{\sqrt{3}}\end{pmatrix}$$

Our predictions:

$$\theta_{12} \simeq 42^\circ\ , \quad \theta_{23} \simeq 52^\circ\ , \quad \theta_{13} \simeq 4^\circ\ , \quad \delta \simeq \pm 90^\circ$$

Part A

January 1997

My daughter was born in Nagoya. (We left in March 1998)

June 1998

Y. Suzuki
June 4

Solar ν's

Atmospheric ν's

T. Kajita
June 5

"Modest" Conclusions

(1) Flux: $\phi^{8B} = 2.44 \pm 0.05 (stat.) ^{+0.09}_{-0.07} (syst.) \times 10^6 /cm^2/s$

(0.368 for BP95, 0.474 for BP98)

(2) No seasonal variations.

(3) (D-N)/(D+N) = −0.023 ± 0.020 (stat) ± 0.014 (syst)

no difference:

excluded regions extended into "small angle sol"

No core enhancement found.

(4) Day-Night + E-shape analysis.

(a) "No oscillation" is disfavoured @ 1~5% C.L.

♣ (b) L.A. solution is disfavoured @ 1~5% C.L.

♣ (c) V.O. regions are favoured (than MSW regions) @ 95% C.L.

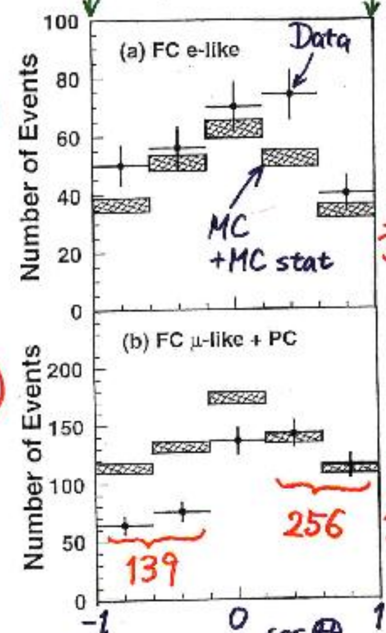
(MSW is ok for 99% C.L.)

Super-K

Neutrino98
TAKAYAMA

Yes,
2 large angles!

Zenith angle dependence
(Multi-GeV)



χ^2(shape) = 2.8/4 dof

$\frac{Up}{Down} = 0.93^{+0.13}_{-0.12}$

χ^2(shape) = 30/4 dof

$\frac{Up}{Down} = 0.54^{+0.06}_{-0.05}$

(6.2σ !!)

※ Up/Down syst. error for μ-like

Prediction (flux calculation ······ ≲ 1% ; 1km rock above SK ···· 1.5%) 1.8%

Data (Energy calib. for ↑↓ ···· 0.7% ; Non ν Background ····· < 2%) 2.1%

Part A

Back to June 1962

Progress of Theoretical Physics, Vol. 28, No. 5, November 1962

Remarks on the Unified Model of Elementary Particles

Ziro MAKI, Masami NAKAGAWA and Shoichi SAKATA

Institute for Theoretical Physics
Nagoya University, Nagoya

(Received June 25, 1962)

A particle mixture theory of neutrino is proposed assuming the existence of two kinds of neutrinos. Based on the neutrino-mixture theory, a possible unified model of elementary

$$\nu_1 = \nu_e \cos\delta + \nu_\mu \sin\delta,$$

$$\nu_2 = -\nu_e \sin\delta + \nu_\mu \cos\delta.$$

I did not know this MNS ν mixing until I left Nagoya

1998 —— 2015

We've learnt a lot from ν oscillations:

$$\Delta m^2_{21} \; , \; |\Delta m^2_{31}| \; , \; \theta_{12} \; , \; \theta_{13} \; , \; \theta_{23}$$

It's more exciting that the SM is incomplete, although the Higgs has been discovered.

But a number of burning questions:

- ♣ **the Majorana nature?**
- ♣ **the absolute ν mass scale?**
- ♣ **the ν mass hierarchy?**
- ♣ **the octant of θ_{23} ?**
- ♣ **the Dirac phase δ ?**
- ♣ **the Majorana phases? ……**

There are many other open questions about ν's in particle physics, cosmology, astrophysics …..

Global fit

F. Capozzi et al (2014) —— the standard parametrization:

Parameter	Best fit	1σ range	2σ range	3σ range
		Normal neutrino mass ordering $(m_1 < m_2 < m_3)$		
$\Delta m^2_{21}/10^{-5}$ eV^2	7.54	7.32 — 7.80	7.15 — 8.00	6.99 — 8.18
$\Delta m^2_{31}/10^{-3}$ eV^2	2.47	2.41 — 2.53	2.34 — 2.59	2.26 — 2.65
$\sin^2\theta_{12}/10^{-1}$	3.08	2.91 — 3.25	2.75 — 3.42	2.59 — 3.59
$\sin^2\theta_{13}/10^{-2}$	2.34	2.15 — 2.54	1.95 — 2.74	1.76 — 2.95
$\sin^2\theta_{23}/10^{-1}$	4.37	4.14 — 4.70	3.93 — 5.52	3.74 — 6.26
$\delta/180^\circ$	1.39	1.12 — 1.77	0.00 — 0.16 $\oplus$ 0.86 — 2.00	0.00 — 2.00
		Inverted neutrino mass ordering $(m_3 < m_1 < m_2)$		
$\Delta m^2_{21}/10^{-5}$ eV^2	7.54	7.32 — 7.80	7.15 — 8.00	6.99 — 8.18
$\Delta m^2_{13}/10^{-3}$ eV^2	2.42	2.36 — 2.48	2.29 — 2.54	2.22 — 2.60
$\sin^2\theta_{12}/10^{-1}$	3.08	2.91 — 3.25	2.75 — 3.42	2.59 — 3.59
$\sin^2\theta_{13}/10^{-2}$	2.40	2.18 — 2.59	1.98 — 2.79	1.78 — 2.98
$\sin^2\theta_{23}/10^{-1}$	4.55	4.24 — 5.94	4.00 — 6.20	3.80 — 6.41
$\delta/180^\circ$	1.31	0.98 — 1.60	0.00 — 0.02 $\oplus$ 0.70 — 2.00	0.00 — 2.00

the PMNS matrix looks very different from the CKM matrix

CKM vs PMNS

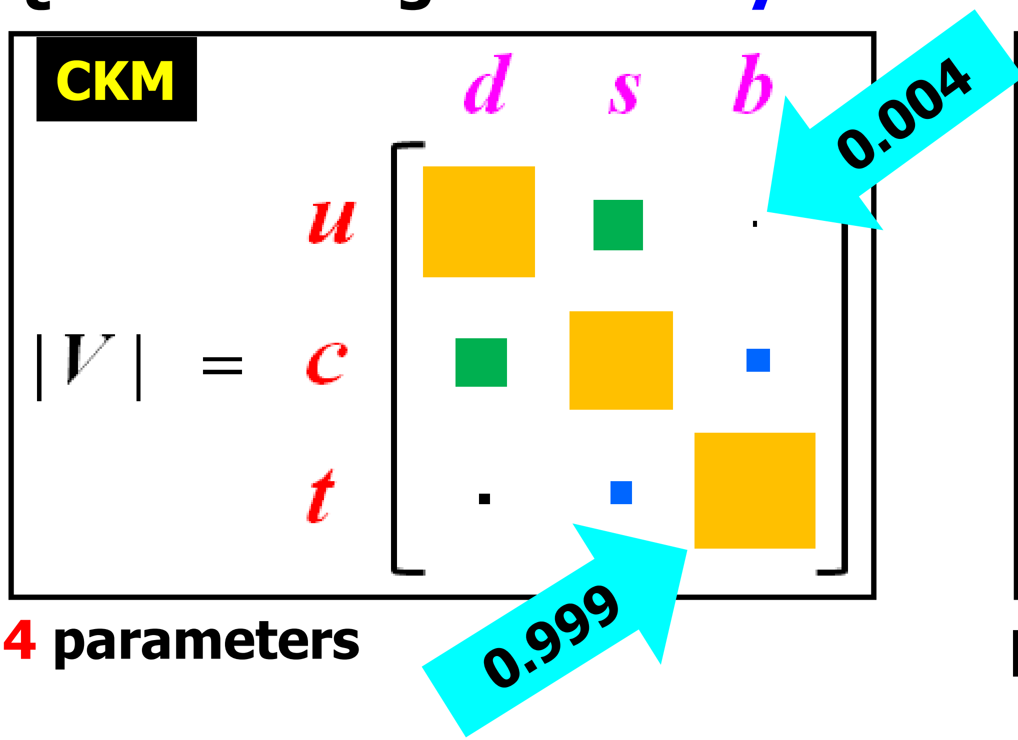
$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{(u\ \ c\ \ t)_L}\,\gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu + \overline{(e\ \ \mu\ \ \tau)_L}\,\gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu\right] + \text{h.c.}$$

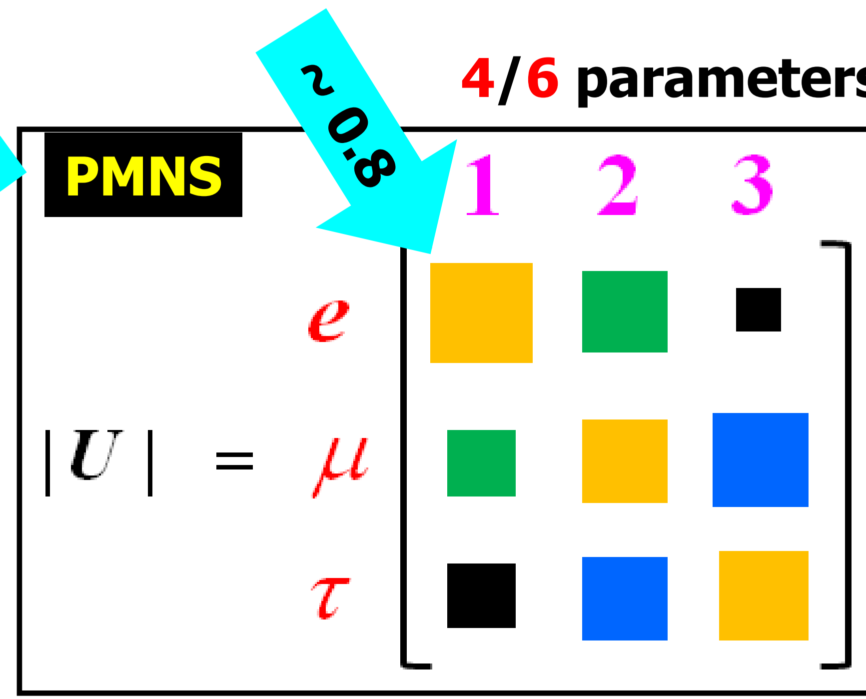
CKM PMNS

Be careful when discussing something like the Q—L complementarity!

Quark mixing: hierarchy!

4/6 parameters





4 parameters

Lepton mixing: anarchy?

Behind the PMNS

Behind the observed pattern of lepton flavor mixing is an approximate (or a partial) μ-τ flavor symmetry!

$$|U_{\mu 1}| \simeq |U_{\tau 1}|\ , \quad |U_{\mu 2}| \simeq |U_{\tau 2}|\ , \quad |U_{\mu 3}| \simeq |U_{\tau 3}|$$ ⟹

It is very likely that the PMNS matrix possesses an exact μ-τ symmetry at a given energy scale, and this symmetry must be softly broken —— shed light on flavor structures

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu$$

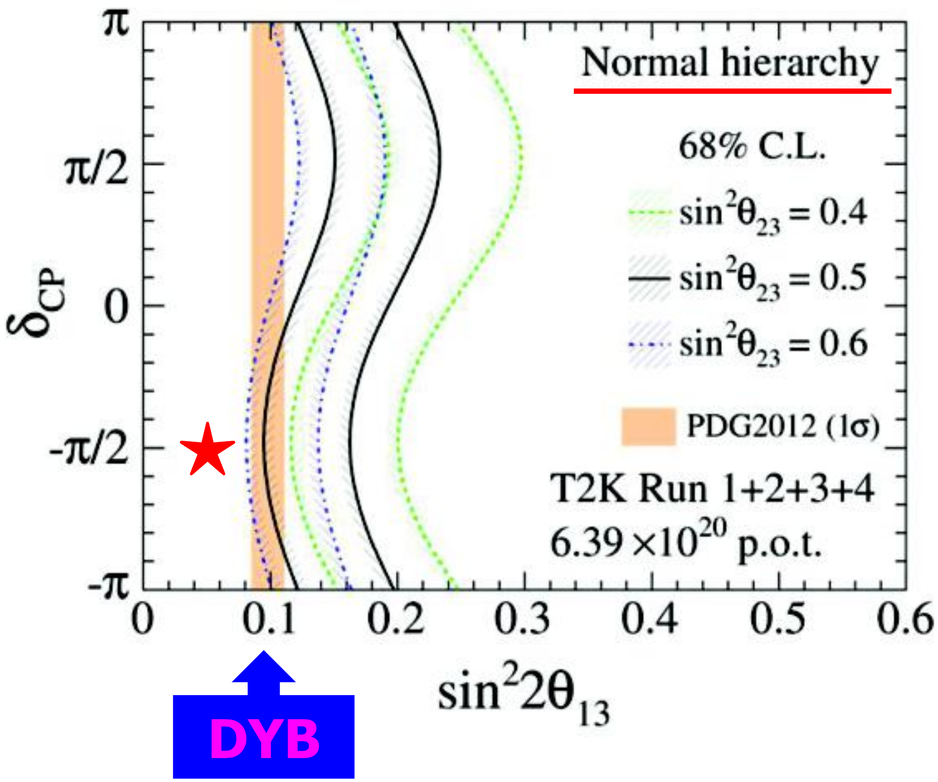
Conditions for the exact μ-τ symmetry in the PMNS matrix:

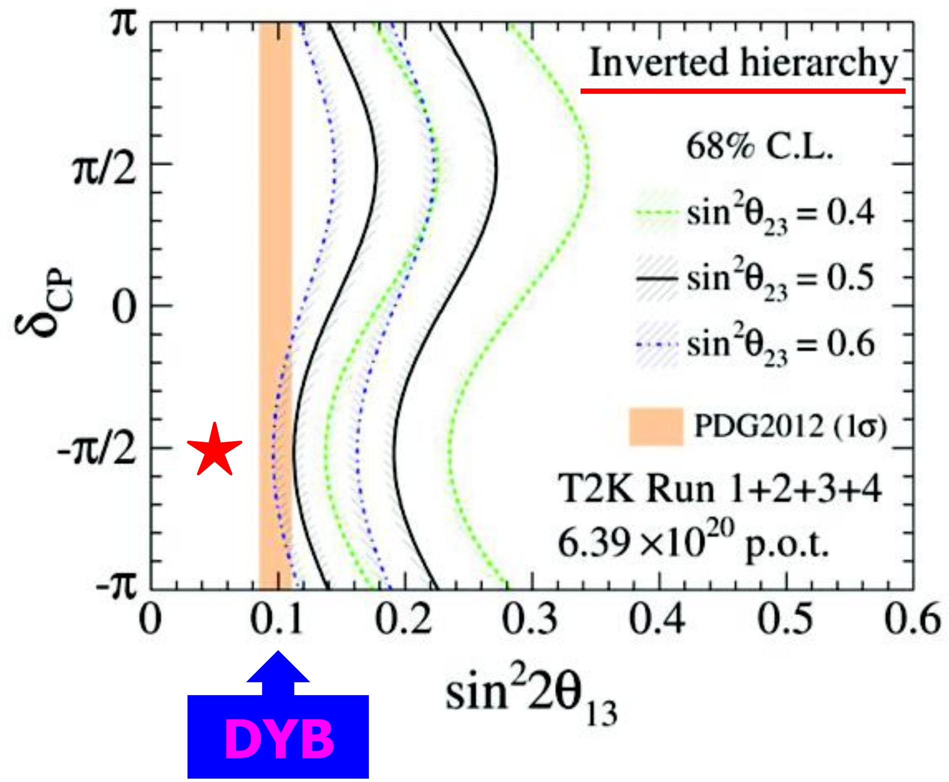
$$|U_{\mu i}| = |U_{\tau i}| \quad \Longrightarrow \quad \left\{ \begin{array}{l} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} \delta = +\pi/2 \\ \theta_{23} = \pi/4 \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} \delta = -\pi/2 \\ \theta_{23} = \pi/4 \end{array} \right.$$

Current data: **ruled out** **not sure** **favored**

T2K vs DYB

The T2K observation of a relatively strong $\nu_\mu \to \nu_e$ appearance plays a crucial role in the global fit to make θ_{13} consistent with the Daya Bay result and drive a slight but intriguing preference for $\delta \sim -\pi/2$.





DYB's good news: θ_{13} unsuppressed
T2K's good news: δ unsuppressed

Life is easier for probing CP violation, ν mass hierarchy ....

μ-τ flavor symmetry

In the flavor basis, the Majorana ν mass matrix can be reconstructed:

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T$$

→ **μ-τ symmetry**

μ-τ permutation symmetry

$$M_\nu = \begin{pmatrix} C & D & D \\ D & A & B \\ D & B & A \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau$

$$\begin{cases} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{cases}$$

T. Fukuyama, H. Nishiura
hep-ph/9702253

Bimaximal, Tribimaximal …

μ-τ reflection symmetry

$$M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$

$$\begin{cases} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{cases}$$

K. Babu, E. Ma, J. Valle
hep-ph/0206292

TM1, Tetramaximal …

$$\frac{1}{2}\,\overline{(\nu_e \;\; \nu_\mu \;\; \nu_\tau)_{\rm L}}\, M_\nu \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix}_{\rm R}$$

Current data

↕

Larger ← **μ-τ symmetry breaking** → **Softer**

Model building

The **flavor symmetry** is a powerful **guiding principle** of model building.

The **flavor symmetry** could be

- ♣ Abelian or non-Abelian
- ♣ Continuous or discrete
- ♣ Local or global
- ♣ Spontaneously or explicitly broken

S_3 , S_4 , A_4 , Z_2 , $U(1)_F$, $SU(2)_F$, ...

Advantages of choosing a **global + discrete** flavor symmetry group G_F .

- ♣ No Goldstone bosons
- ♣ No additional gauge bosons mediating harmful FCNC processes
- ♣ No family-dependent D-terms contributing to sfermion masses
- ♣ Discrete G_F could come from some string compactifications
- ♣ Discrete G_F could be embedded in a continuous symmetry group

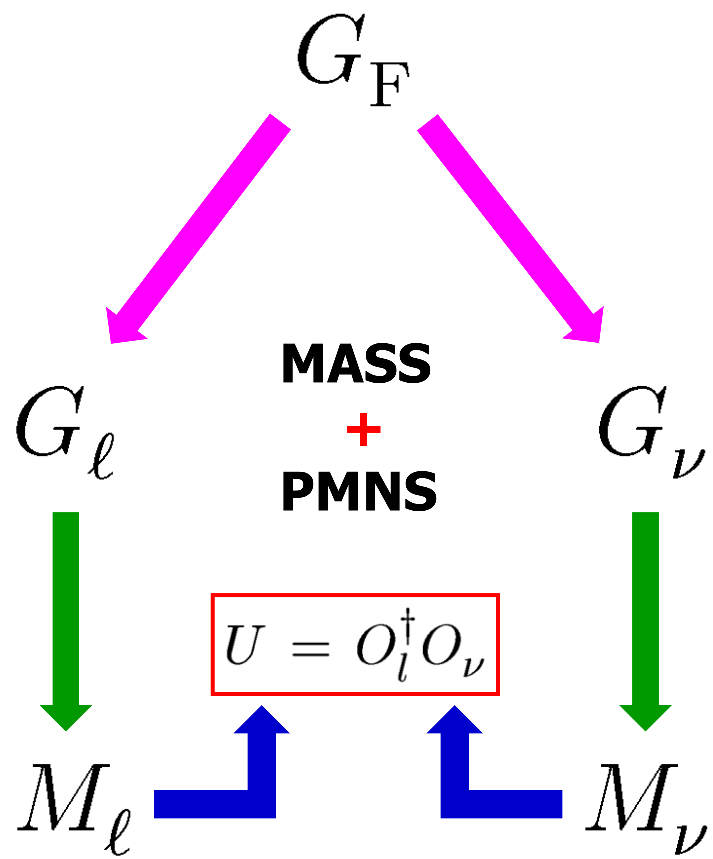
SUSY

Flavor symmetries

Some small discrete groups for model building (Altarelli, Feruglio 2010).

Group	d	Irreducible representation
$D_3 \sim S_3$	6	$1, 1', 2$
D_4	8	$1_1, \ldots, 1_4, 2$
D_7	14	$1, 1', 2, 2', 2''$
A_4	12	$1, 1', 1'', 3$
$A_5 \sim PSL_2(5)$	60	$1, 3, 3', 4, 5$
T'	24	$1, 1', 1'', 2, 2', 2'', 3$
S_4	24	$1, 1', 2, 3, 3'$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, 1_9, 3, \bar{3}$
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1}', 3, \bar{3}$

Too many possibilities, but the μ-τ symmetry inclusive

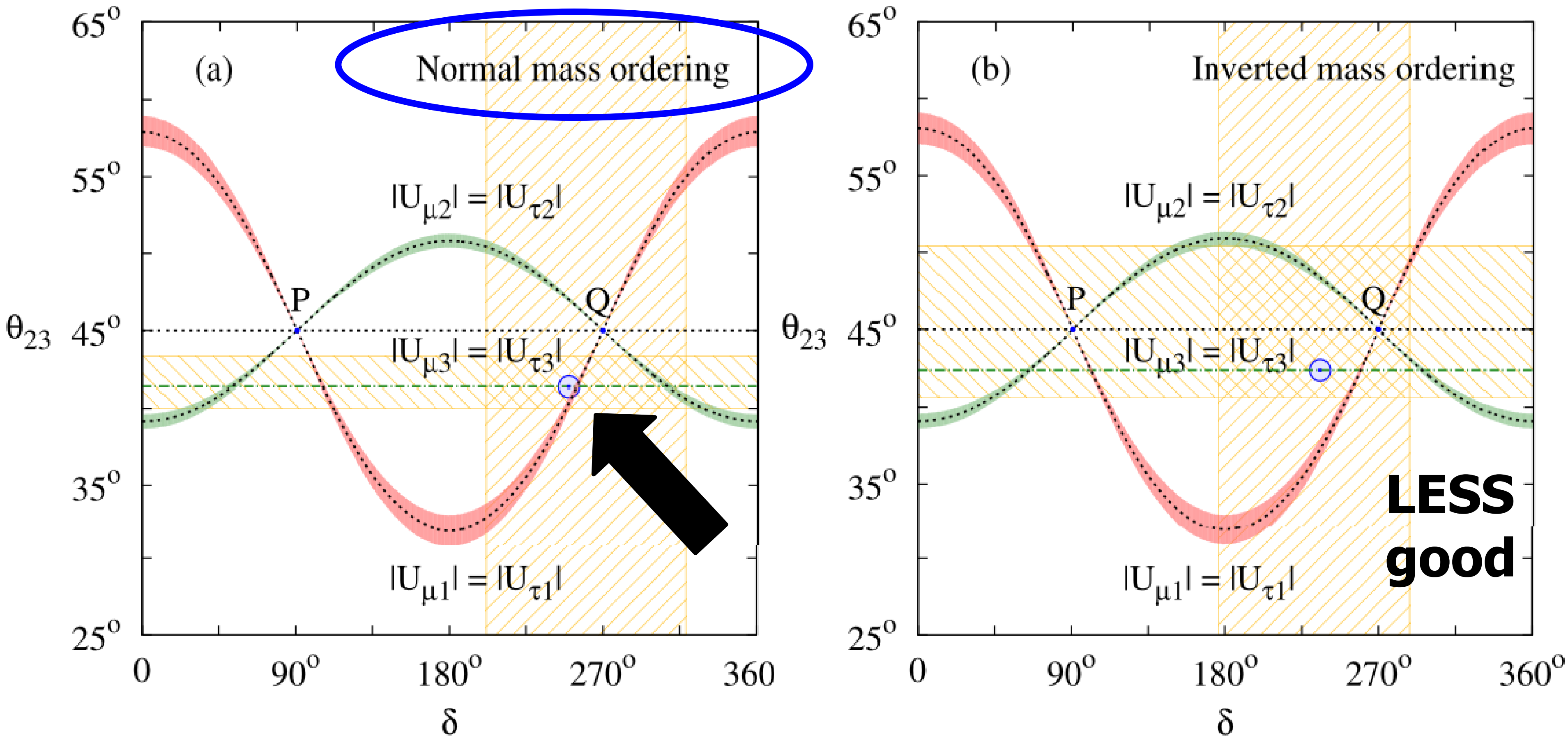


Generalized CP combined with flavor symmetry to predict the phase δ.

Part C

A partial μ-τ symmetry?

$|U_{\mu 1}| = |U_{\tau 1}|$ → $$\cos\delta = \frac{1}{2}\left(\sin^2\theta_{13} - \tan^2\theta_{12}\right)\cot\theta_{12}\csc\theta_{13}\cot 2\theta_{23}$$



Fogli et al's best fit: $(\theta_{23}, \delta) \simeq (41.4^\circ, 250^\circ)$ **in normal ordering**

Partial μ-τ prediction: $(\theta_{23}, \delta) \simeq (41.4^\circ, 255^\circ)$ **(Xing, Zhou 14).**

An example

The so-called TM1 neutrino mixing pattern is of this kind (Xing, Zhou 06; Lam 06):

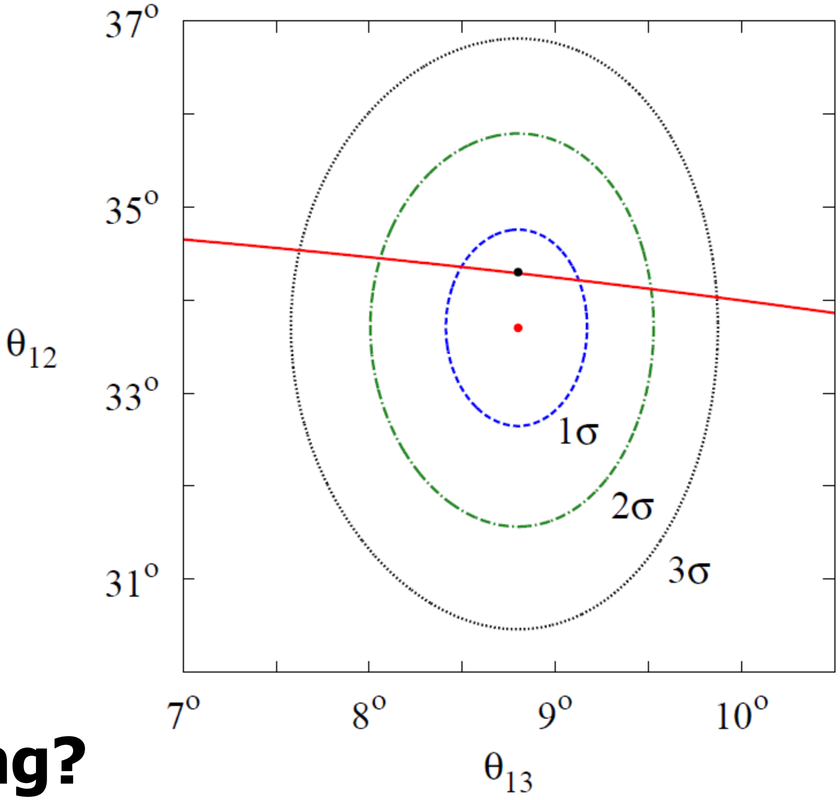
$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} & \frac{\sin\theta}{\sqrt{3}}e^{-i\phi} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{\sin\theta}{\sqrt{2}}e^{i\phi} & \frac{\cos\theta}{\sqrt{2}} + \frac{\sin\theta}{\sqrt{3}}e^{-i\phi} \\ \frac{1}{\sqrt{6}} & -\frac{\cos\theta}{\sqrt{3}} - \frac{\sin\theta}{\sqrt{2}}e^{i\phi} & \frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{3}}e^{-i\phi} \end{pmatrix} P_\nu$$

Model building based on A4 / S4 flavor symmetry

Lam 06; Antusch et al 12

Its most striking consequence

$$\sin^2\theta_{12} = \frac{1}{3}\left(1 - 2\tan^2\theta_{13}\right)$$



★ **Data will keep changing!**

★ **Global fit will get credit?**

★ **The octant of θ_{23} is really correlated with the mass ordering?**

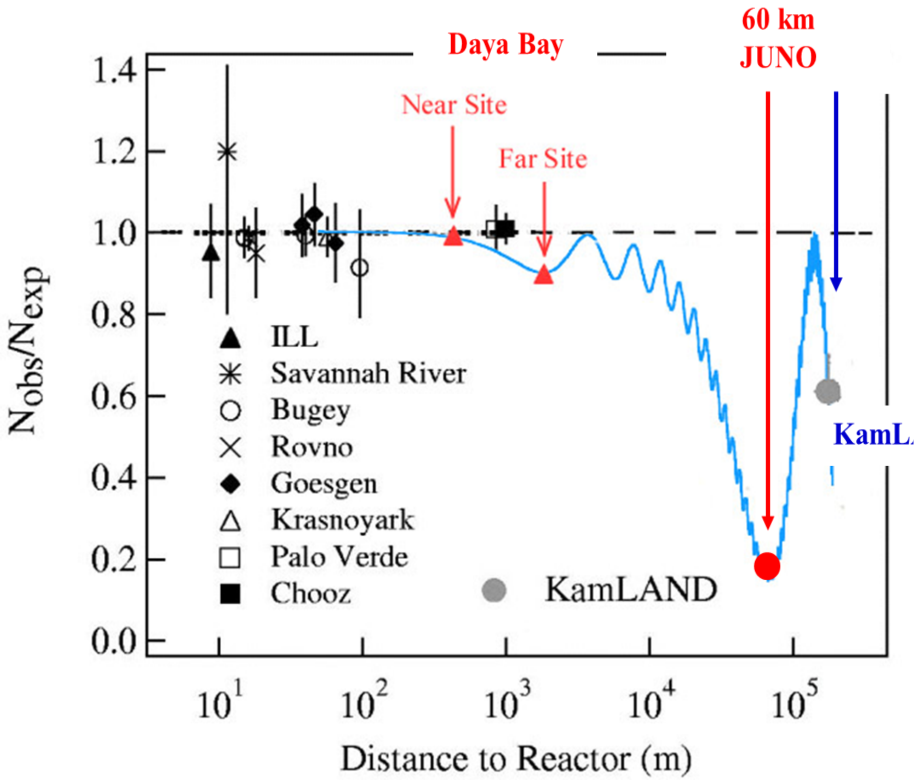
ν mass ordering

Accelerator/atmospheric: terrestrial matter effects play crucial roles.

$$\Delta m_{31}^2 \mp 2\sqrt{2} G_F N_e E$$

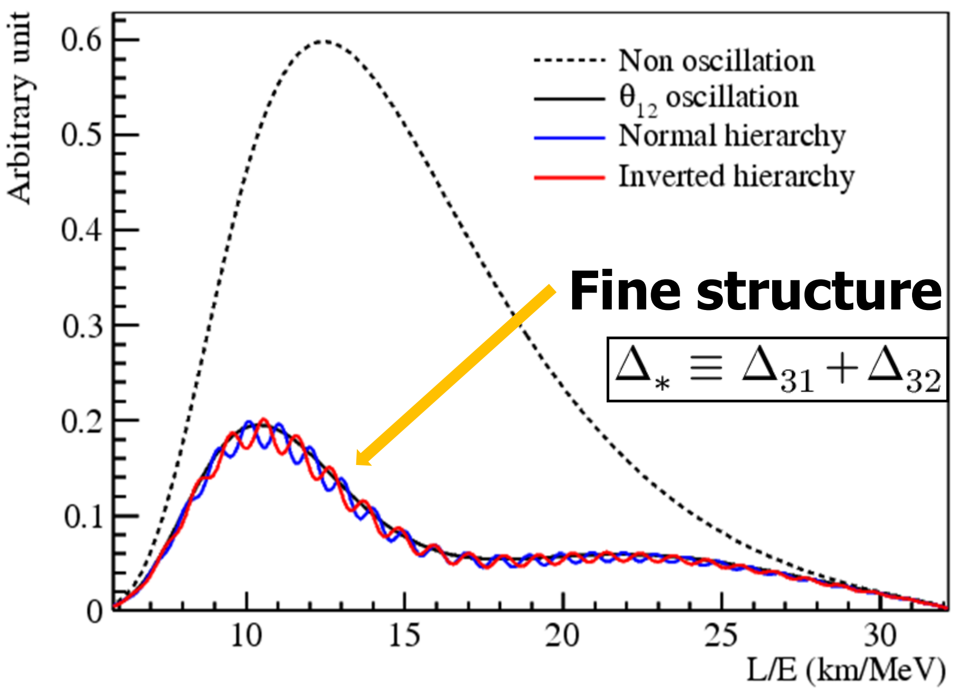
θ_{23} ← **its octant matters a lot**

Reactor (JUNO): Optimum baseline at the valley of Δm_{21}^2 oscillations, corrected by the fine structure of Δm_{31}^2 oscillations.



JUNO's idea

$$\Delta_{ji} \equiv \Delta m_{ji}^2 L/(4E)$$



$$P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin^2 \Delta_{21} - \frac{1}{2} \sin^2 2\theta_{13} \left[1 - \cos \Delta_* \cos \Delta_{21} + \cos 2\theta_{12} \sin \Delta_* \sin \Delta_{21}\right]$$

Part C

JUNO in progress

	Daya Bay	Yangjiang	Taishan
Status	running	construction	construction
Power/GW	17.4	17.4	18.4

- Idea 2008
- 20 kton LS detector
  3% E-resolution
- Approved 2/2013
- Groundbreaking 1/2015

Part C

Scale of μ-τ symmetry?

Assume the μ-τ symmetry to be exact at the seesaw scale, and then its slight breaking is due to *RGE* running effects.

Define three μ-τ asymmetries:

$$\Delta_1 \equiv |U_{\tau 1}|^2 - |U_{\mu 1}|^2 = \left(\cos^2\theta_{12}\sin^2\theta_{13} - \sin^2\theta_{12}\right)\cos 2\theta_{23} - \sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}\cos\delta$$
$$\Delta_2 \equiv |U_{\tau 2}|^2 - |U_{\mu 2}|^2 = \left(\sin^2\theta_{12}\sin^2\theta_{13} - \cos^2\theta_{12}\right)\cos 2\theta_{23} + \sin 2\theta_{12}\sin\theta_{13}\sin 2\theta_{23}\cos\delta$$
$$\Delta_3 \equiv |U_{\tau 3}|^2 - |U_{\mu 3}|^2 = \cos^2\theta_{13}\cos 2\theta_{23} \ ,$$

The one-loop RGEs for the Dirac neutrinos in the MSSM:

$$16\pi^2 \frac{\mathrm{d}\Delta_1}{\mathrm{d}t} = -y_\tau^2 \left[\xi_{21}\left(|U_{\tau 1}|^2\Delta_2 + |U_{\tau 2}|^2\Delta_1 + |U_{e3}|^2\right) + \xi_{31}\left(|U_{\tau 1}|^2\Delta_3 + |U_{\tau 3}|^2\Delta_1 + |U_{e2}|^2\right)\right]$$
$$16\pi^2 \frac{\mathrm{d}\Delta_2}{\mathrm{d}t} = +y_\tau^2 \left[\xi_{21}\left(|U_{\tau 1}|^2\Delta_2 + |U_{\tau 2}|^2\Delta_1 + |U_{e3}|^2\right) - \xi_{32}\left(|U_{\tau 2}|^2\Delta_3 + |U_{\tau 3}|^2\Delta_2 + |U_{e1}|^2\right)\right]$$
$$16\pi^2 \frac{\mathrm{d}\Delta_3}{\mathrm{d}t} = +y_\tau^2 \left[\xi_{31}\left(|U_{\tau 1}|^2\Delta_3 + |U_{\tau 3}|^2\Delta_1 + |U_{e2}|^2\right) + \xi_{32}\left(|U_{\tau 2}|^2\Delta_3 + |U_{\tau 3}|^2\Delta_2 + |U_{e1}|^2\right)\right]$$

$$y_\tau^2 = \left(1 + \tan^2\beta\right) m_\tau^2 / v^2 \qquad \xi_{ij} \equiv (m_i^2 + m_j^2)/\Delta m_{ij}^2 \text{ with } \Delta m_{ij}^2 \equiv m_i^2 - m_j^2$$

For the Majorana neutrinos, two extra phases enter RGEs.

Numerical example (1)

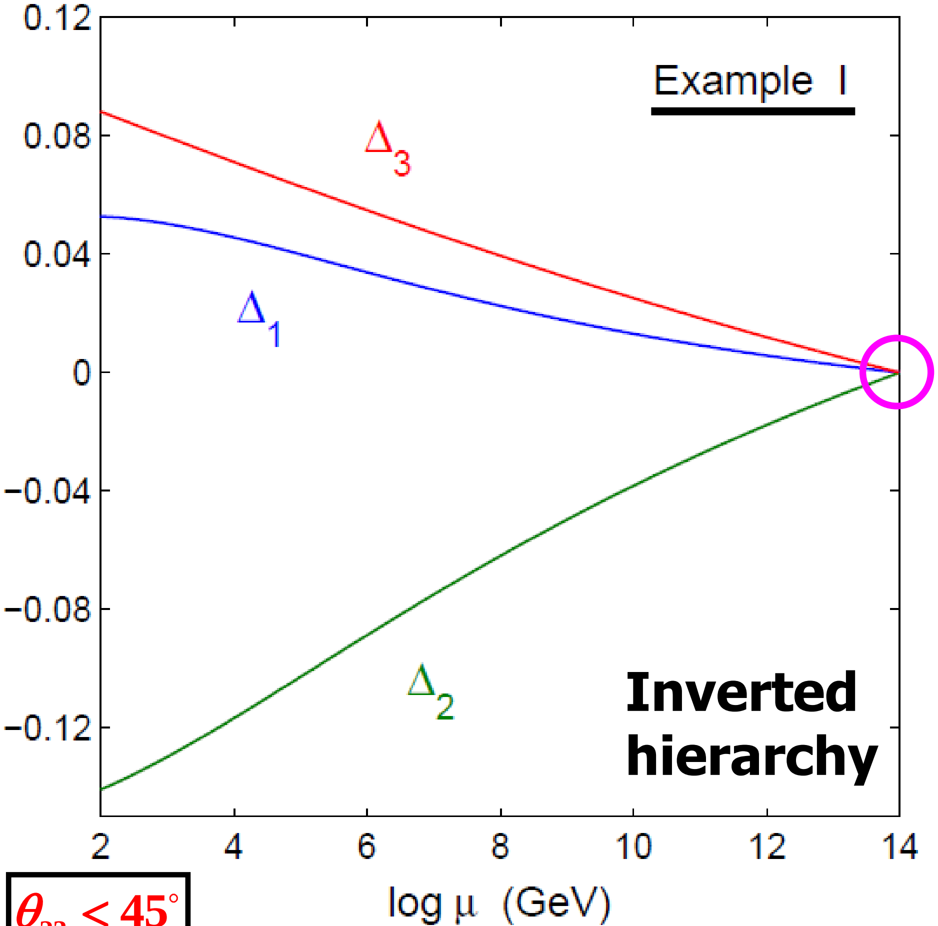
For illustration (Luo, Xing, 2014): μ-τ reflection symmetry.

Dirac neutrinos:

$\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{\rm EW} \sim 10^2$ GeV in the MSSM with $\tan\beta = 31$

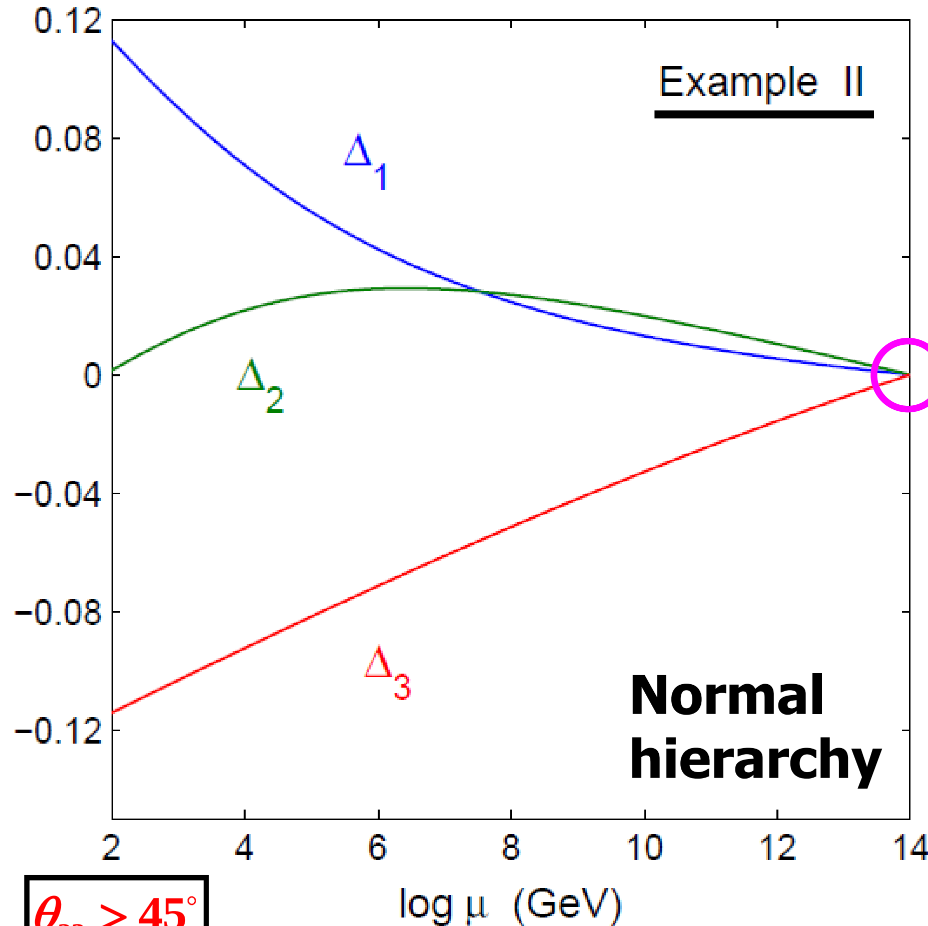
Parameter	Example I (Capozzi *et al* [3])		Example II (Forero *et al* [4])	
	Input ($\Lambda_{\mu\tau}$)	Output ($\Lambda_{\rm EW}$)	Input ($\Lambda_{\mu\tau}$)	Output ($\Lambda_{\rm EW}$)
m_1 (eV)	0.100	0.093	0.100	0.093
Δm^2_{21} (eV^2)	1.82×10^{-4}	7.54×10^{-5}	1.96×10^{-4}	7.60×10^{-5}
Δm^2_{31} (eV^2)	-2.60×10^{-3}	-2.34×10^{-3} IH	3.00×10^{-3}	2.48×10^{-3} NH
θ_{12}	10.8°	33.6°	10.3°	34.6°
θ_{13}	9.4°	8.9°	8.4°	8.8°
θ_{23}	45.0°	42.4°	45.0°	48.4°
δ	270°	236°	270°	237°
$\mathcal{J}$	−0.015	−0.029	−0.013	−0.029
Δ_1	0	0.053	0	0.114
Δ_2	0	−0.141	0	0.001
Δ_3	0	0.088	0	−0.115

Dirac neutrinos



$\theta_{23} < 45^\circ$

$$\Delta_1 \equiv |U_{\tau 1}|^2 - |U_{\mu 1}|^2$$
$$\Delta_2 \equiv |U_{\tau 2}|^2 - |U_{\mu 2}|^2$$
$$\Delta_3 \equiv |U_{\tau 3}|^2 - |U_{\mu 3}|^2$$



$\theta_{23} > 45^\circ$

Preliminary observations: Soft μ-τ reflection symmetry breaking can link the octant of θ_{23} to the ν mass hierarchy.

Numerical example (2)

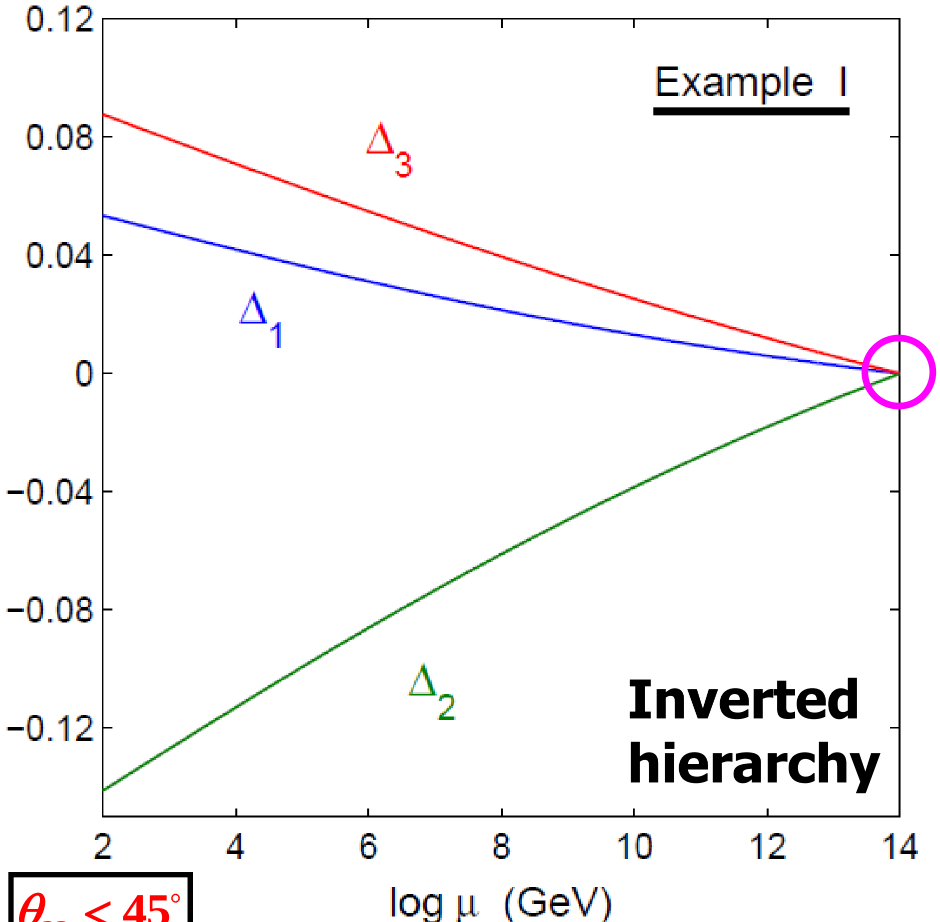
Majorana neutrinos: again about μ-τ reflection symmetry.

$\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{\rm EW} \sim 10^{2}$ GeV in the MSSM with $\tan\beta = 30$

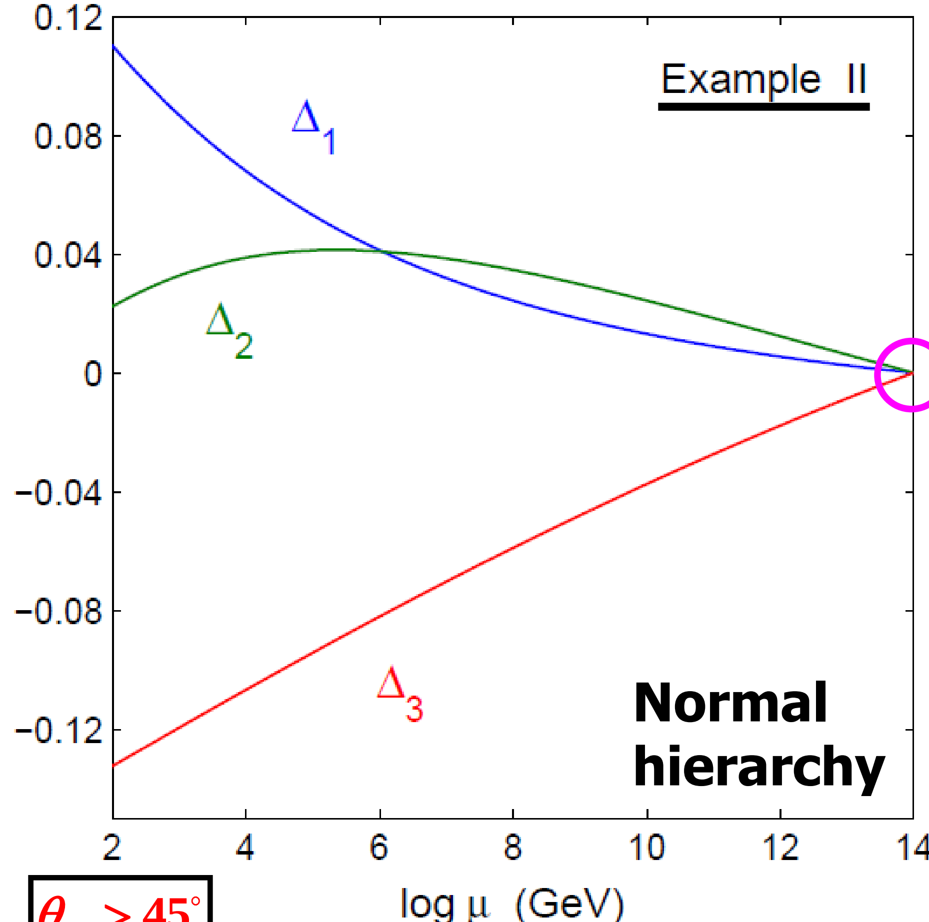
Parameter	Example I (Capozzi *et al* [3]) Input ($\Lambda_{\mu\tau}$)	Example I (Capozzi *et al* [3]) Output ($\Lambda_{\rm EW}$)	Example II (Forero *et al* [4]) Input ($\Lambda_{\mu\tau}$)	Example II (Forero *et al* [4]) Output ($\Lambda_{\rm EW}$)
m_1 (eV)	0.100	0.087	0.100	0.087
Δm^2_{21} (eV^2)	1.70×10^{-4}	7.54×10^{-5}	2.12×10^{-4}	7.60×10^{-5}
Δm^2_{31} (eV^2)	-2.98×10^{-3}	-2.34×10^{-3} **IH**	3.50×10^{-3}	2.48×10^{-3} **NH**
θ_{12}	35.2°	33.7°	32.1°	34.6°
θ_{13}	10.1°	8.9°	6.9°	8.8°
θ_{23}	45.0°	42.4°	45.0°	48.9°
δ	270°	236°	270°	241°
ρ	−82°	−66°	−76°	−45°
σ	19°	27°	17°	29°
$\mathcal{J}$	−0.040	−0.029	−0.027	−0.030
Δ_1	0	0.054	0	0.111
Δ_2	0	−0.142	0	0.022
Δ_3	0	0.088	0	−0.133

Majorana neutrinos

$$\Delta_1 \equiv |U_{\tau 1}|^2 - |U_{\mu 1}|^2$$
$$\Delta_2 \equiv |U_{\tau 2}|^2 - |U_{\mu 2}|^2$$
$$\Delta_3 \equiv |U_{\tau 3}|^2 - |U_{\mu 3}|^2$$



$\theta_{23} < 45^\circ$



$\theta_{23} > 45^\circ$

Preliminary observations: Soft μ-τ reflection symmetry breaking can link the octant of θ_{23} to the ν mass hierarchy.

1937: Majorana

★ Theory of the Symmetry of Electrons and Positrons

Ettore Majorana

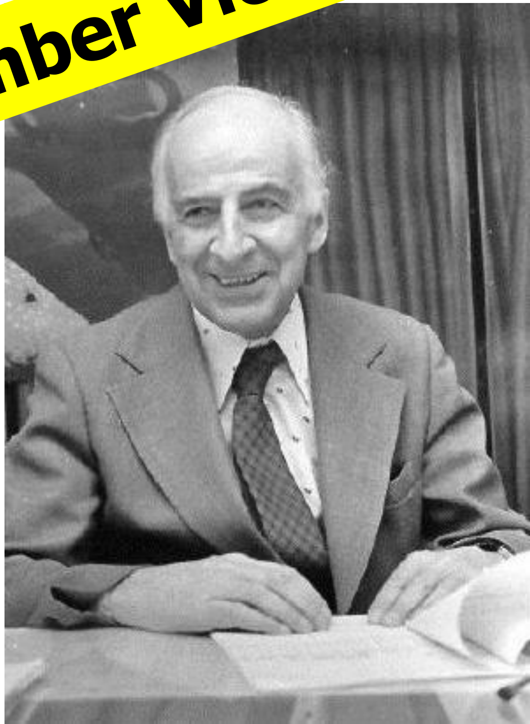
Nuovo Cim. 14 (1937) 171

Are massive neutrinos and antineutrinos identical or different —— a fundamental puzzling question in particle physics.

Lepton Number Violation

★ Mesonium and Anti-mesonium

Bruno Pontecorvo

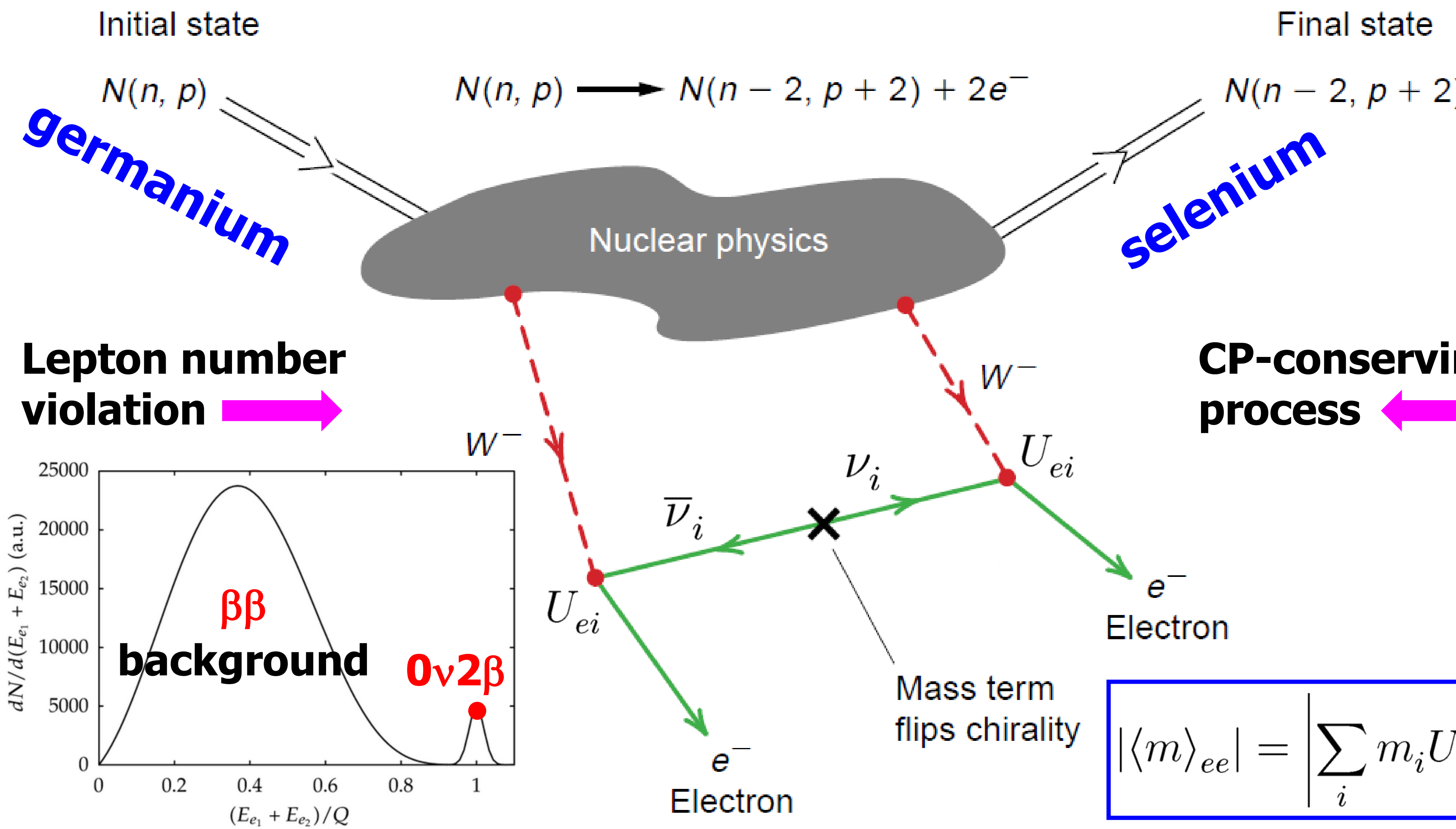


Zh. Eksp. Teor. Fiz. 33 (1957) 549
Sov. Phys. JETP 6 (1957) 429

If the two-component neutrino theory turned out to be incorrect and if the conservation law of neutrino charge didn't apply, then neutrino -antineutrino transitions would in principle be possible to take place in vacuum.

1939: 0ν2β decays

The neutrinoless double beta decay can happen if massive neutrinos are the Majorana particles (W.H. Furry 1939):



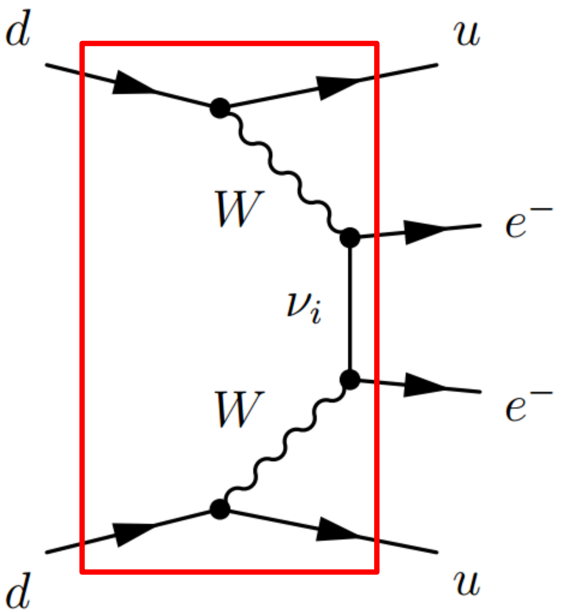
Lepton number violation ⇒

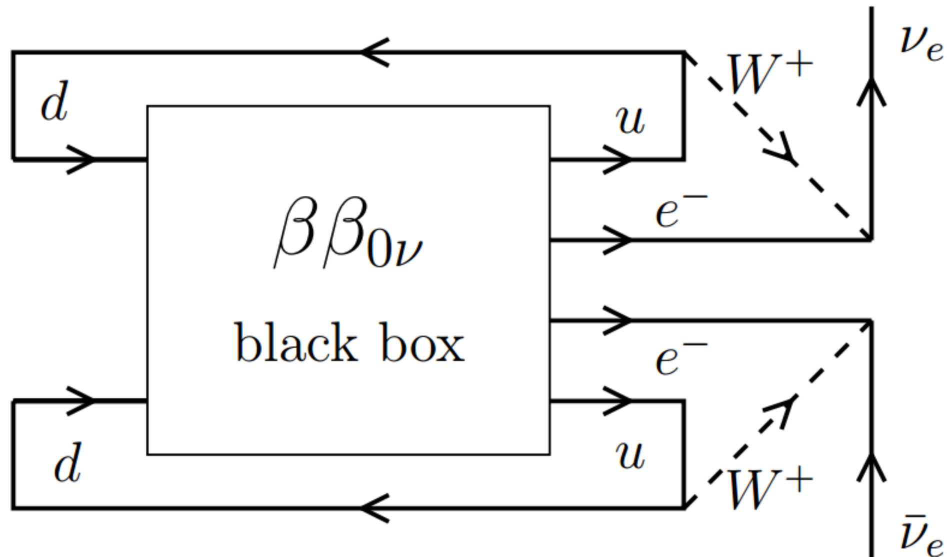
CP-conserving process ⇐

$$|\langle m \rangle_{ee}| = \left| \sum_i m_i U_{ei}^2 \right|$$

Schechter-Valle theorem

THEOREM (1982): if a 0ν2β decay happens, there must be an effective Majorana mass term.





Bruno Pontecorvo's Prediction

That is why we want to see 0ν2β

Four-loop ν mass:

$$\delta m_\nu = \mathcal{O}(10^{-24}\,\mathrm{eV})$$

(Duerr, Lindner, Merle, 2011)

Note: The black box can in principle have many different processes (new physics). Only in the simplest case, which is most interesting, it's likely to constrain neutrino masses

Ge + Xe

GERDA has killed the Heidelberg-Moscow's claim on 0ν2β.

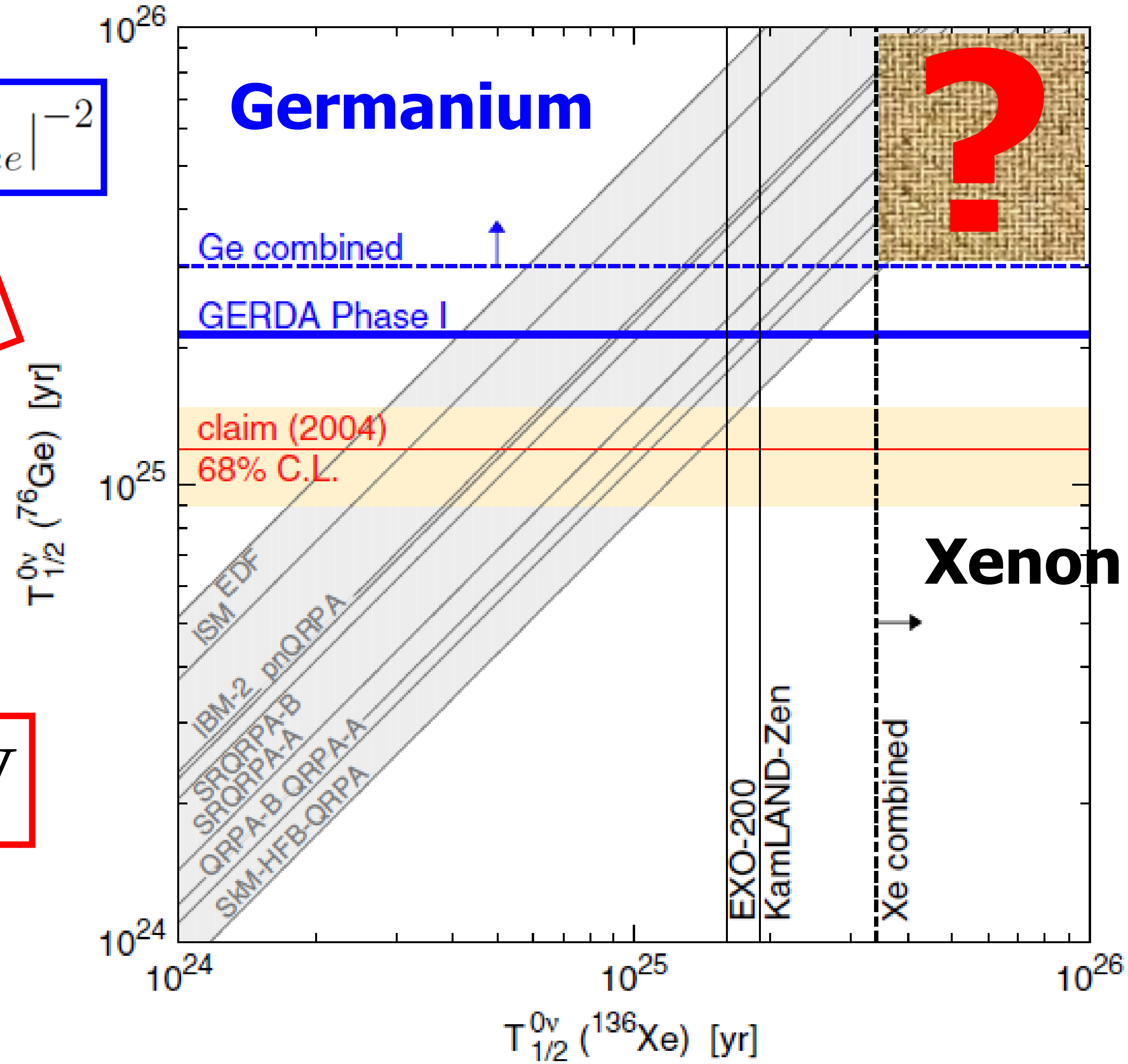
PRL **111**, 122503 (2013)

$$T^{0\nu}_{1/2} = \left(G^{0\nu}\right)^{-1} \left|M^{0\nu}\right|^{-2} \left|\langle m\rangle_{ee}\right|^{-2}$$

$$T^{0\nu}_{1/2} > 3.0 \times 10^{25}\ \text{yr (90\% C.L.).}$$

$$\left|\langle m\rangle_{ee}\right| < 0.2 \rightarrow 0.4\ \text{eV}$$

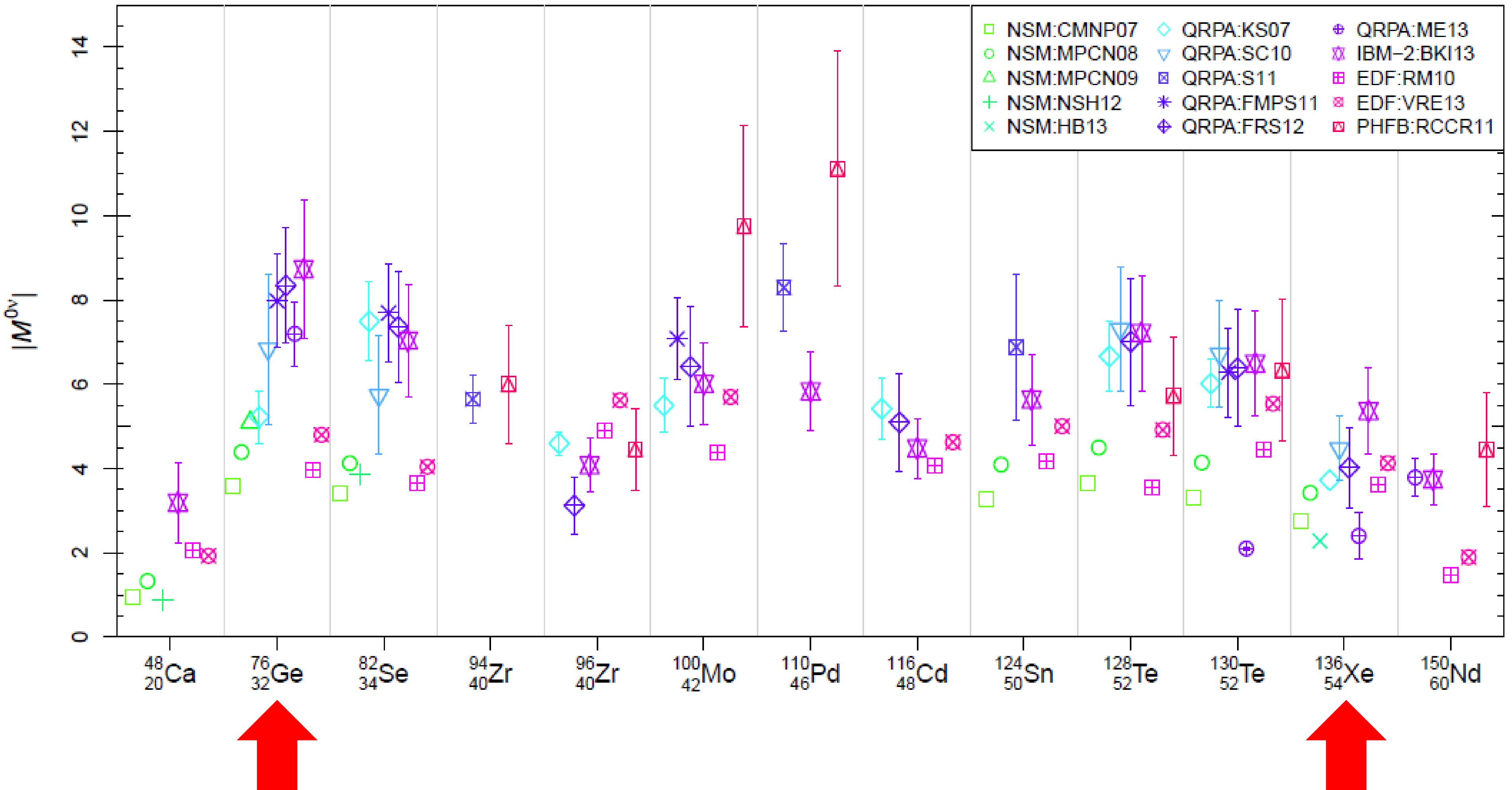
$$\left|\langle m\rangle_{ee}\right| = \left|\sum_i m_i U^2_{ei}\right|$$



Part D

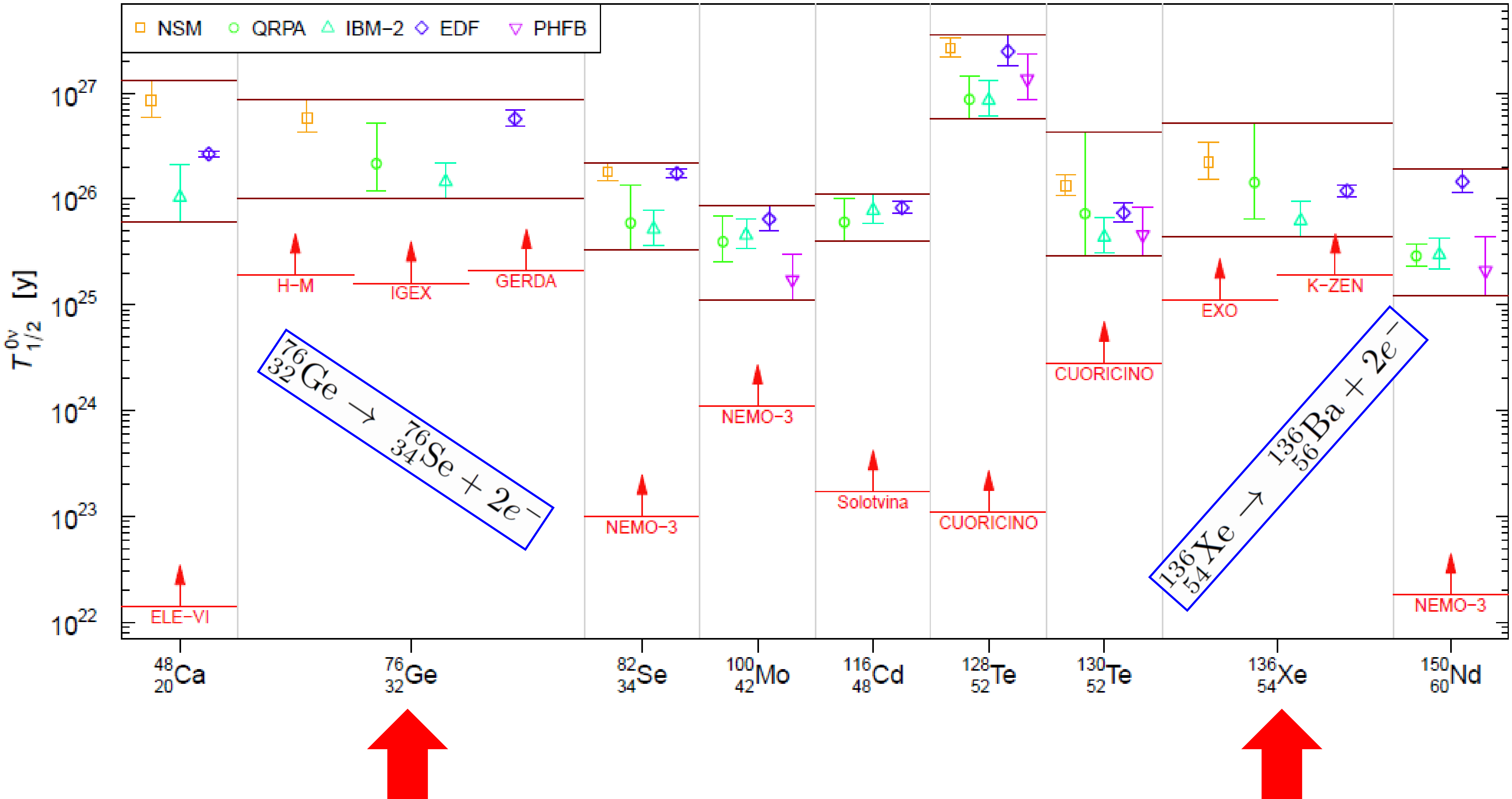
Nuclear matrix elements

Unfortunately, nuclear matrix elements can be calculated only based on some models which describe many-body interactions of nucleons in nuclei. Since different models focus on different aspects of nuclear physics, large uncertainties (a factor of 2 or 3) are unavoidable.



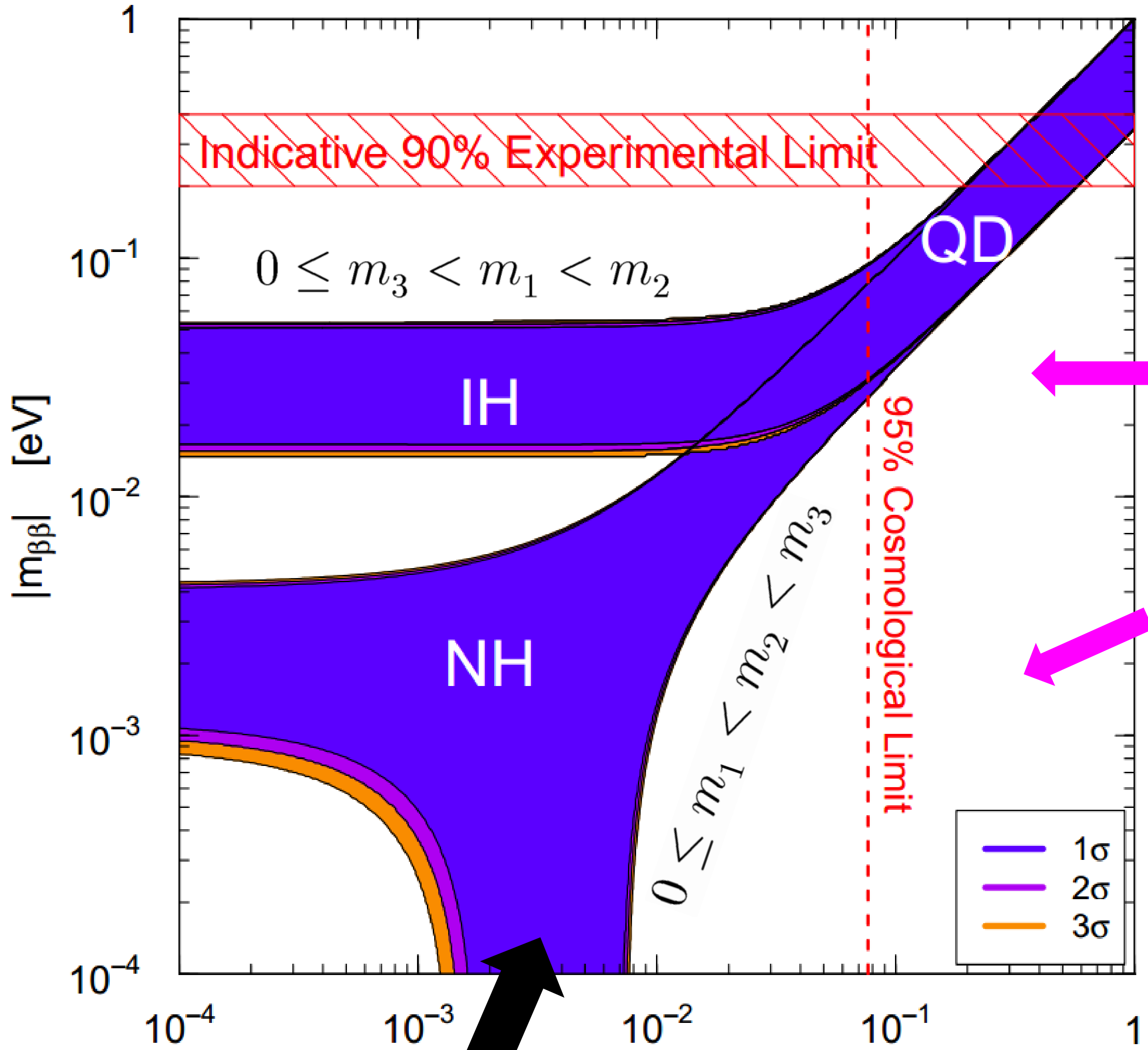
Half-life

Comparing the 90% C.L. experimental lower limits on the half-life of a 0ν2β-decaying nuclide with the corresponding range of theoretical prediction, given a value of 0.1 eV for the effective Majorana neutrino mass term (Bilenky and Giunti, 1411.4791).



Part D

Vissani graph



The dark well

The effective mass

$$|\langle m \rangle_{ee}| = \left| \sum_i m_i U_{ei}^2 \right|$$

Maury Goodman asks

An intelligent design?

Vanishing $0\nu2\beta$ mass?

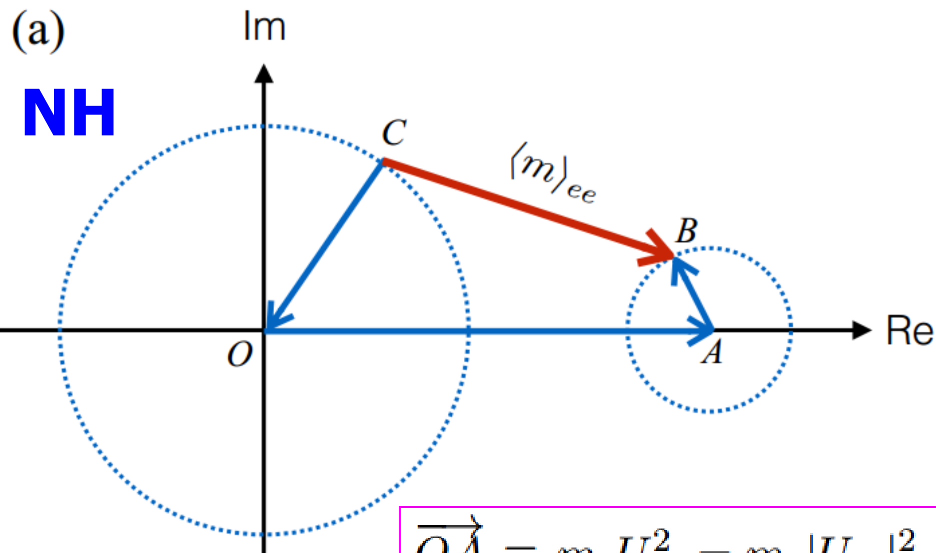
Xing, hep-ph/0305195

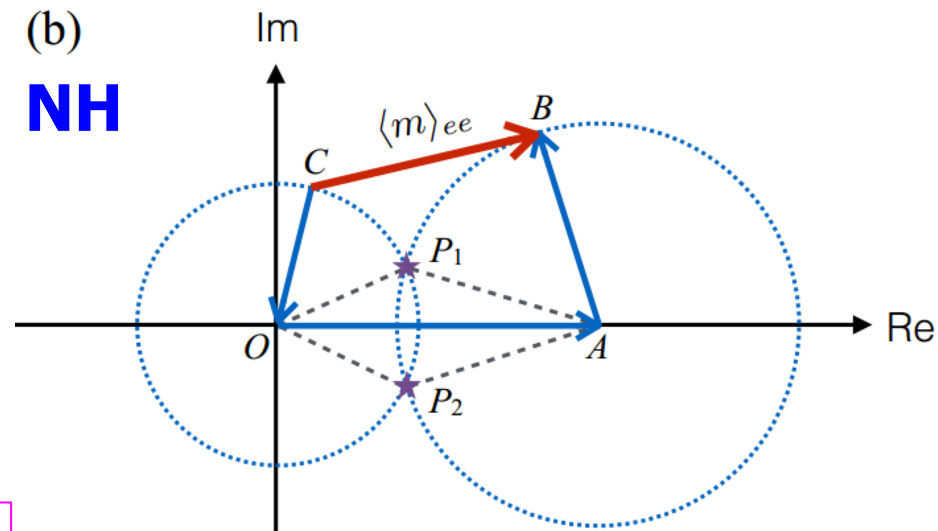
CP phases also matter

Burning Question:

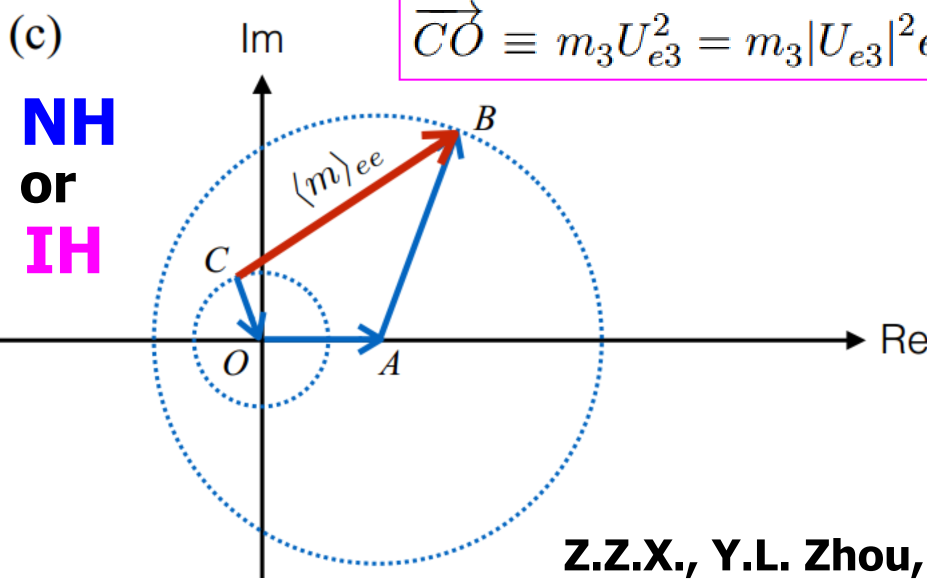
how to interpret a discovery or a null result of $0\nu2\beta$?

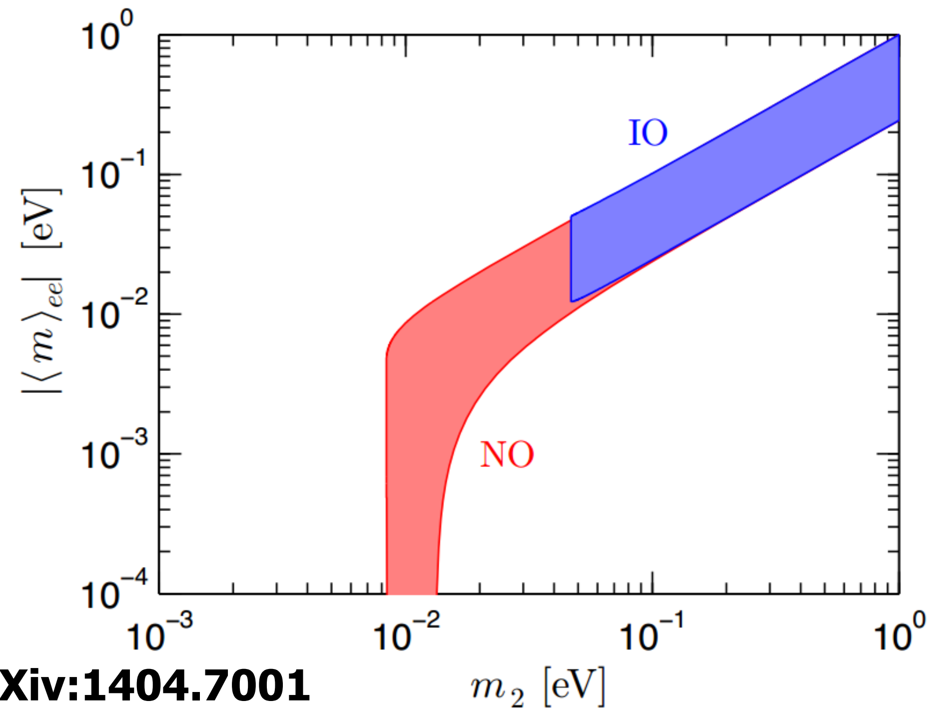
Coupling-rod diagram





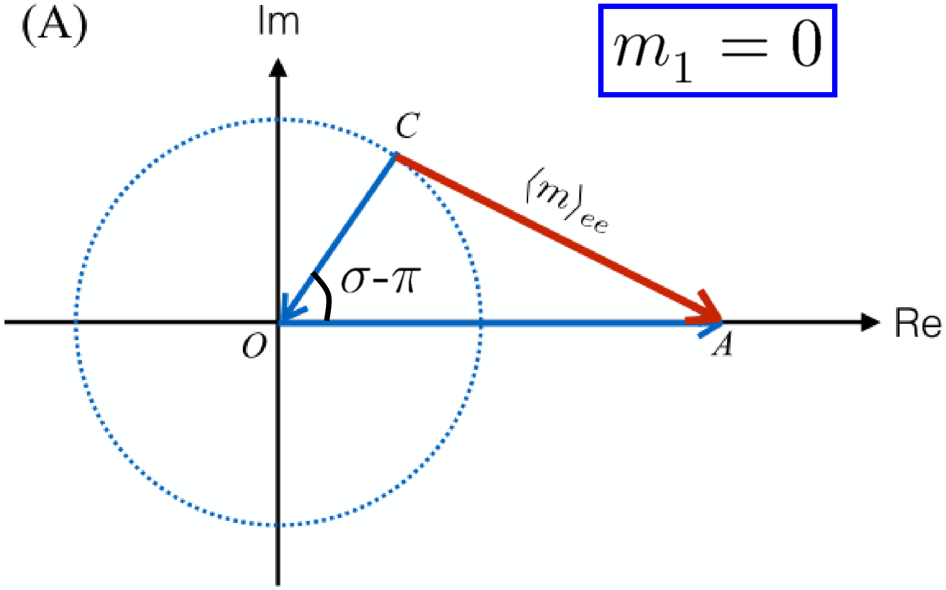
$$\overrightarrow{OA} \equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2 \; ,$$
$$\overrightarrow{AB} \equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{i\rho}$$
$$\overrightarrow{CO} \equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{i\sigma}$$

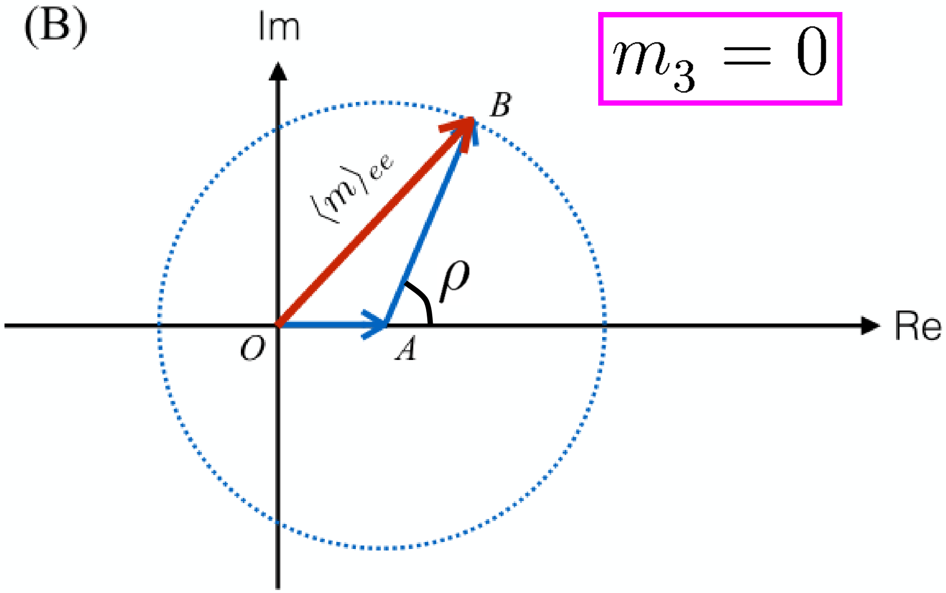




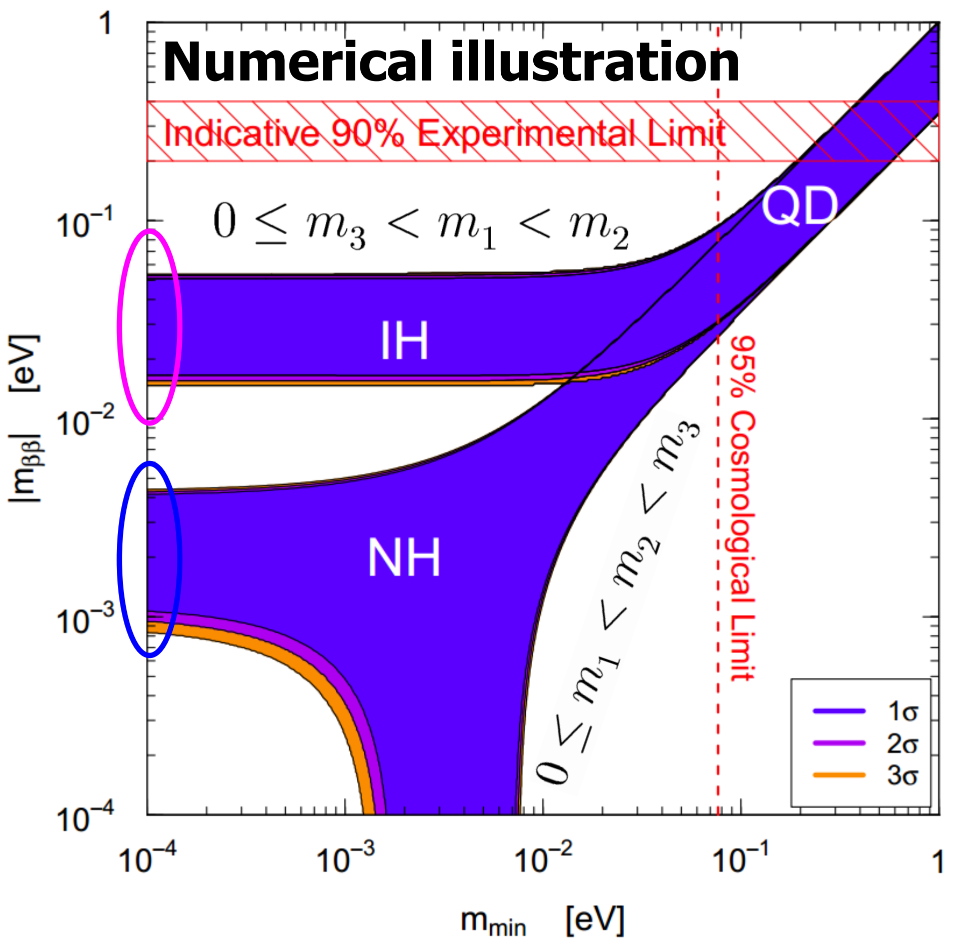
Z.Z.X., Y.L. Zhou, arXiv:1404.7001

Occam's razor

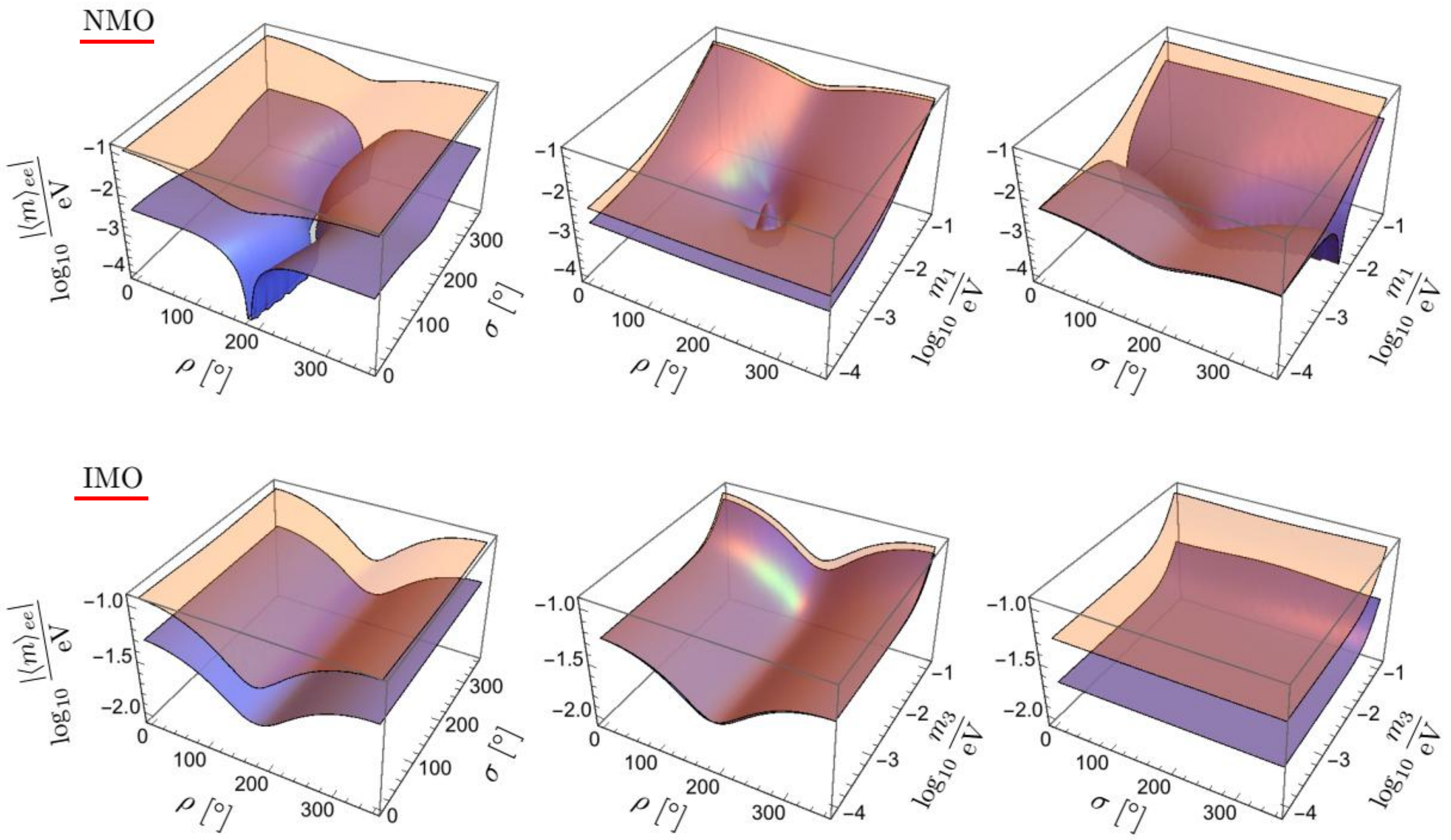




$$\overrightarrow{OA} \equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2 \, ,$$
$$\overrightarrow{AB} \equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{i\rho}$$
$$\overrightarrow{CO} \equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{i\sigma}$$



3-d description



Lower bound: blue; upper bound: light orange. Clearer sensitivities to mass and phase parameters (Xing, Zhao, Zhou, arXiv:1504.05820)

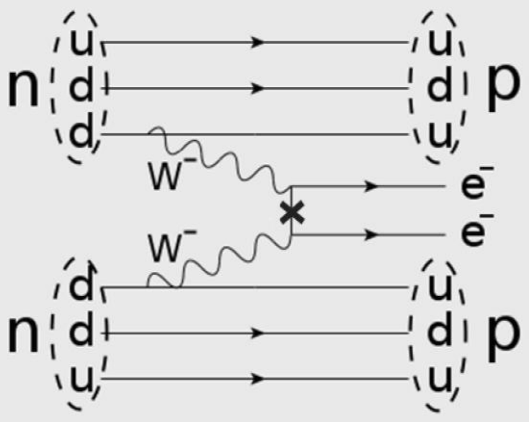
New physics?

Type (A): NP directly related to extra species of neutrinos.

Example 1: heavy Majorana neutrinos from type-I seesaw

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$\Gamma_{0\nu\beta\beta} \propto \left| \sum_{i=1}^{3} m_i U_{ei}^2 \; - \; \sum_{k=1}^{n} \frac{R_{ek}^2}{M_k} M_A^2 \mathcal{F}(A, M_k) \right|^2$$



In most cases the heavy contribution is negligible

Example 2: light sterile neutrinos from LSND etc

$$\langle m \rangle'_{ee} \equiv \sum_{i=1}^{6} m_i U_{ei}^2 = \underline{\langle m \rangle_{ee}} \left(c_{14} c_{15} c_{16}\right)^2 + \underline{m_4 \left(\hat{s}^*_{14} c_{15} c_{16}\right)^2 + m_5 \left(\hat{s}^*_{15} c_{16}\right)^2 + m_6 \left(\hat{s}^*_{16}\right)^2}$$

In this case the new contribution might be constructive or destructive

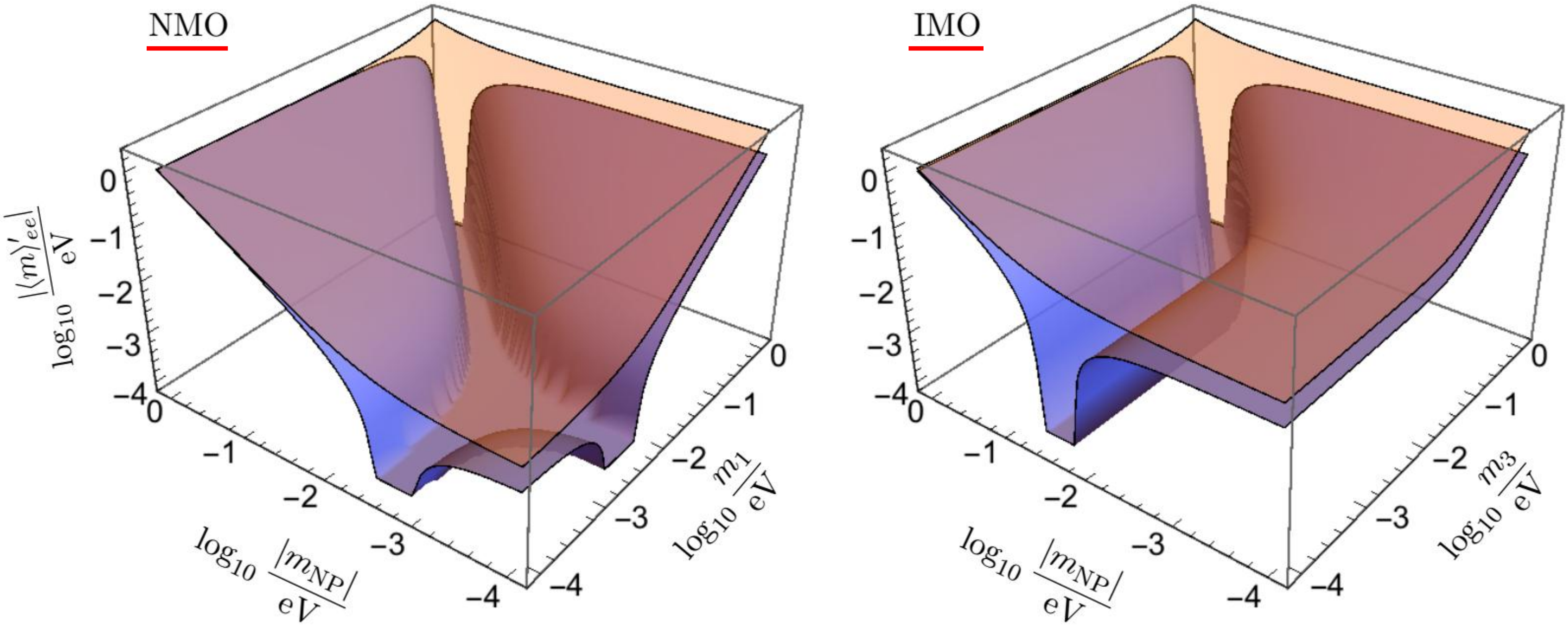
Type (B): NP has little to do with the neutrino mass issue.

SUSY, Left-right, and some others that I don't understand

Possible effects

New physics effects:

$$\langle m\rangle'_{ee} = m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 + m_{\rm NP}$$



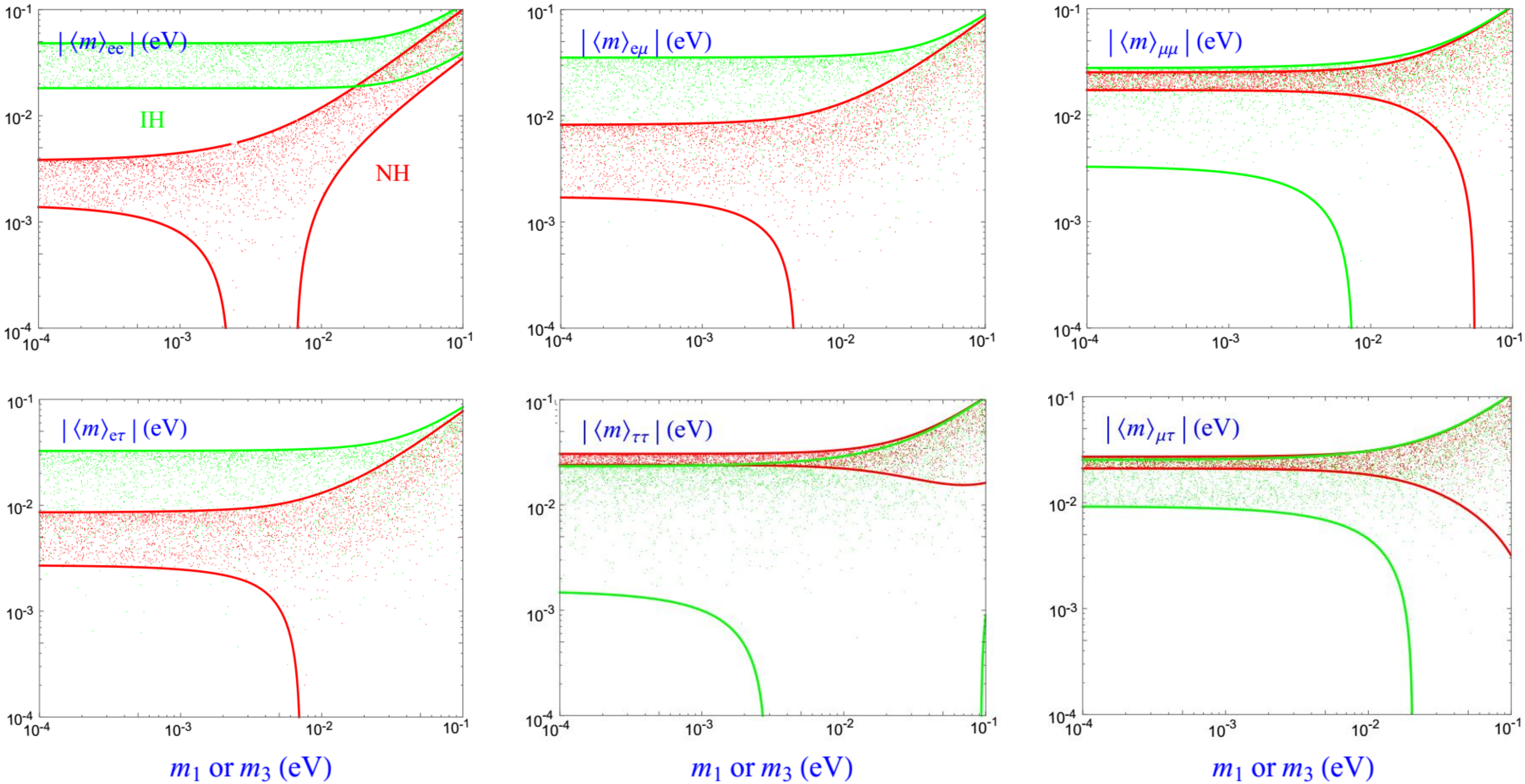
Lower bound: blue; upper bound: light orange. Clearer sensitivities to mass and phase parameters (Xing, Zhao, Zhou, arXiv:1504.05820)

$$\left|\langle m\rangle'_{ee}\right|_{\rm upper} = m_1 \left|U_{e1}\right|^2 + m_2 \left|U_{e2}\right|^2 + m_3 \left|U_{e3}\right|^2 + \left|m_{\rm NP}\right| ,$$

$$\left|\langle m\rangle'_{ee}\right|_{\rm lower} = \max\left\{0,\ 2m_i \left|U_{ei}\right|^2 - \left|\langle m\rangle'_{ee}\right|_{\rm upper} ,\ 2\left|m_{\rm NP}\right| - \left|\langle m\rangle'_{ee}\right|_{\rm upper}\right\}$$

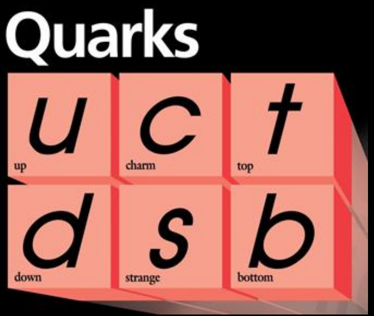
It is hard to tell much

More LNV processes



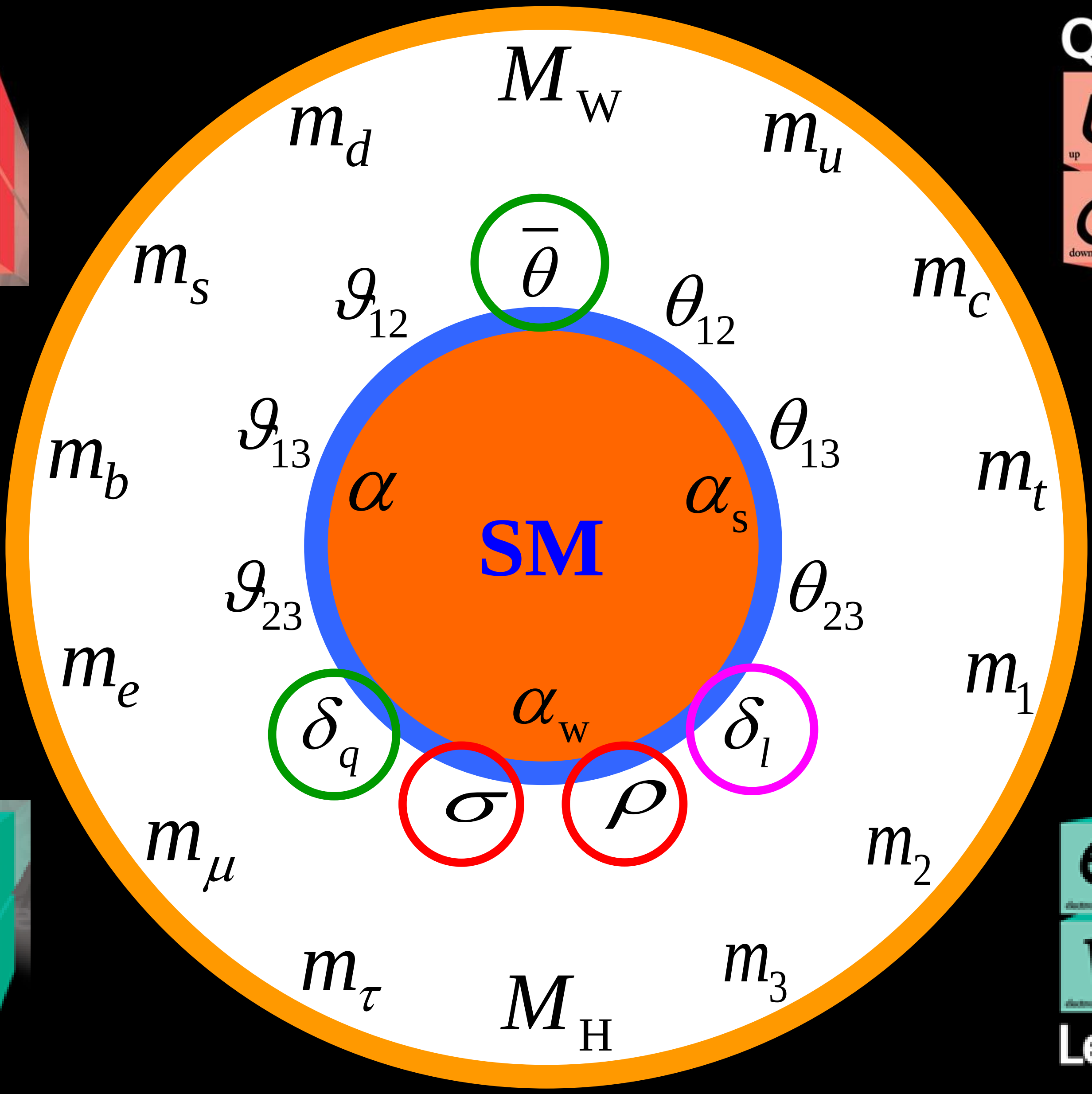
To identify the Majorana nature, CP-violating phases and new physics it is imperative to observe the 0ν2β decays and other lepton-number-violating processes (e.g., neutrino-antineutrino oscillations, the relic neutrino background, doubly-charged Higgs decays). None is realistic

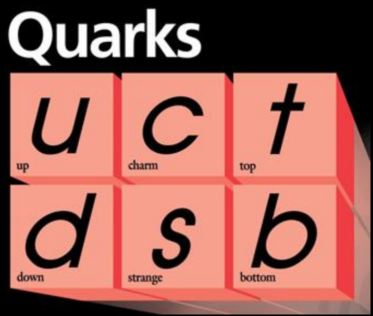
OUTLOOK: Success in FPCP is still a long way off



1/5

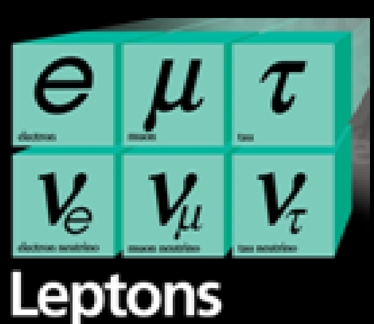
OK!

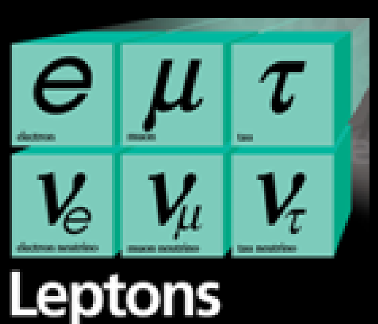




4/5

NO!





Martinus Veltman: Anyway, we go on until we go wrong