Naohiro Osamura (M2, Nagoya U)

Loop-diagrammatic evaluation of QCD θ parameter and its application to the left-right symmetric model

arXiv[2211.XXXXX] in collaboration with
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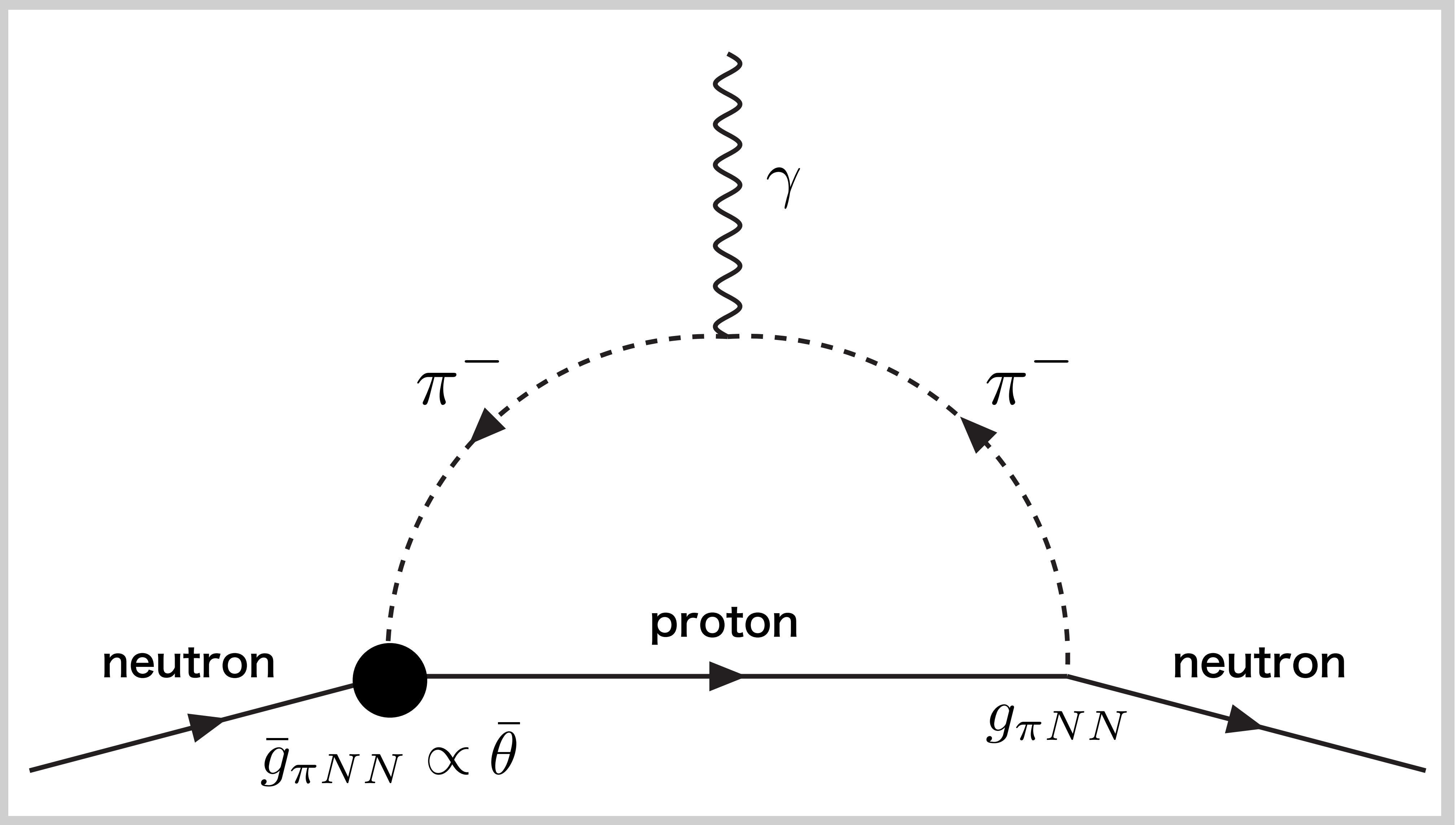
Contents

Strong CP problem

The chiral rotation induces $G\tilde{G}$ as a chiral anomaly.

$$\mathcal{L}_{\not{P},\not{T}} = -\sum_{q=\mathrm{all}} \mathrm{Im}(m_q)\bar{q}i\gamma_5 q + \theta_G \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \xrightarrow{\text{chiral rotation}} \bar{\theta}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

A physical parameter: $\bar{\theta} \equiv \theta_G - \sum_q \theta_q$



$d^{\mathrm{exp.}}_{\mathrm{neutron}} < 1.8 \times 10^{-26}\ e\ \mathrm{cm}$ ➡ $\bar{\theta} < 10^{-10}$

too small!

Strong CP problem

- **the CP phase in CKM matrix** $\underset{\sim O(1)}{}$ $>>$ $\bar{\theta}$ **?**
- **Find out the reason of such an ultra suppression in $\bar{\theta}$.**

Today's theme

- **Application of the conventional evaluation $\bar{\theta} = \theta_G + \arg\,\mathrm{Det}[\mathscr{M}_u \mathscr{M}_d]$ to loop-level should be modified!**

- **We estimated the $\bar{\theta}$ parameter induced in the LR model at loop-level by the new method.**

Parity solution of the strong CP problem

The parity symmetry forbids $G\tilde{G}$!!

$\bar{\theta}\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$: renormalizable P- and CP-odd gluonic operator

No axion!

SM

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

P violation

$$\bar{\theta}\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

Left-Right symmetric (LR) model

$$SU(3)_C \times SU(2)_L \times \boxed{SU(2)_R} \times U(1)_{B-L}$$

generalized parity $P_{\rm gen}$

$$\cancel{\bar{\theta}^{(0)}\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}} + \boxed{\delta\bar{\theta}\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}}$$

tree quantum correction

Left-Right symmetric model

◆ **matter content:**

	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$
$Q_L \equiv (u_L, d_L)^T$	$\square$	$\square$	$\mathbf{1}$	$1/6$
$Q_R \equiv (u_R, d_R)^T$	$\square$	$\mathbf{1}$	$\square$	$1/6$
H	$\mathbf{1}$	$\square$	$\mathbf{1}$	$1/2$
H'	$\mathbf{1}$	$\mathbf{1}$	$\square$	$1/2$
U_L	$\square$	$\mathbf{1}$	$\mathbf{1}$	$2/3$
U_R	$\square$	$\mathbf{1}$	$\mathbf{1}$	$2/3$
D_L	$\square$	$\mathbf{1}$	$\mathbf{1}$	$-1/3$
D_R	$\square$	$\mathbf{1}$	$\mathbf{1}$	$-1/3$

new particles **vector-like**

◆ **under P_{gen}:**

$$\vec{x} \leftrightarrow -\vec{x}$$
$$SU(2)_L \leftrightarrow SU(2)_R$$
$$Q_L,\ U_L,\ D_L,\ H \leftrightarrow Q_R,\ U_R,\ D_R,\ H'$$

◆ **mass matrix** $(i, j \ldots :$ light flavor$, a, b \ldots :$ heavy flavor$)$

$$\begin{aligned}-\mathcal{L}_Y = &\bar{Q}^i_L x^{ia}_u U^a_R \tilde{H} + \bar{Q}^i_R x^{ia}_u U^a_L \tilde{H}' + M^a_u \bar{U}^a_L U^a_R \\ &+ \bar{Q}^i_L x^{ia}_d D^a_R H + \bar{Q}^i_R x^{ia}_d D^a_L H' + M^a_d \bar{D}^a_L D^a_R \\ &+ \text{h.c.}\end{aligned}$$

$$\Rightarrow \quad \left(\bar{u}^i_L, \bar{U}^a_L\right) \begin{pmatrix} 0 & x^{ib}_u v \\ x^{\dagger aj}_u v' & M^a_u \delta^{ab} \end{pmatrix} \begin{pmatrix} u^j_R \\ U^b_R \end{pmatrix}$$

$$\left(\bar{d}^i_L, \bar{D}^a_L\right) \begin{pmatrix} 0 & x^{ib}_d v \\ x^{\dagger aj}_d v' & M^a_d \delta^{ab} \end{pmatrix} \begin{pmatrix} d^j_R \\ D^b_R \end{pmatrix}$$

θ parameter at tree-level

Intrinsic θ term

$$\mathcal{L}_\theta = -\frac{O(1)}{M_{\rm UV}^2}\left|H'\right|^2 G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \to -\frac{v'^2}{M_{\rm UV}^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$$\Rightarrow \theta \sim 10^{-10}\left(\frac{v'}{1.2\times 10^{13}\,\mathrm{GeV}}\right)^2\left(\frac{1.2\times 10^{19}\,\mathrm{GeV}}{M_{\rm UV}}\right)^2 \Rightarrow v' < 1.2\times 10^{13}\,\mathrm{GeV}$$

Mass matrix $(i,j\cdots:\ \text{light flavor},\ a,b\cdots:\ \text{heavy flavor},\ p,q:\ \text{all flavor})$

$$\left(\overline{u}^i_L, \overline{U}^a_L\right)\begin{pmatrix} 0 & x_u^{ib} v \\ x_u^{\dagger aj} v' & M_u^a \delta^{ab}\end{pmatrix}\begin{pmatrix} u^j_R \\ U^b_R\end{pmatrix} \equiv \overline{\mathcal{U}}^p_L \mathcal{M}_u^{(0)pq}\mathcal{U}^q_R$$

$$\Rightarrow \arg\det\left[\mathcal{M}_u^{(0)}\right] = -vv'\arg\left[x_u x_u^\dagger\right] = 0$$

SM

$$\mathrm{Im}\left[Y_u^{ij}\right]\bar{u}^i i\gamma_5 u^j v$$
$$+\mathrm{Im}\left[Y_d^{ij}\right]\bar{d}^i i\gamma_5 d^j v$$
$$\longrightarrow \arg\det\left[Y_u Y_d\right]$$

$$\boxed{\bar{\theta} = \cancel{\theta_G + \arg\det\left[\mathcal{M}_u^{(0)}\mathcal{M}_d^{(0)}\right]} + \arg\det\left[\delta\mathcal{M}_u^{(1)}\delta\mathcal{M}_d^{(1)}\right] + \arg\det\left[\delta\mathcal{M}_u^{(2)}\delta\mathcal{M}_d^{(2)}\right] + \cdots}$$

tree

Vanishing up to the 1-loop corrections to $\bar{\theta}$

K. S. Babu and R. N. Mohapatra , Phys. Rev. D 41 (1990), 1286

- **They showed vanishing the 1-loop corrections of CP-odd mass term contributing to $\bar{\theta}$.**

$$\bar{\theta} = \cancel{\theta_G + \arg\det\left[\mathcal{M}_u^{(0)}\mathcal{M}_d^{(0)}\right]} + \cancel{\arg\det\left[\delta\mathcal{M}_u^{(1)}\delta\mathcal{M}_d^{(1)}\right]} + \boxed{\arg\det\left[\delta\mathcal{M}_u^{(2)}\delta\mathcal{M}_d^{(2)}\right]} + \cdots$$

1-loop

The LR model has been expected as one of good solutions to the strong CP problem!

Vanishing up to the 1-loop corrections to $\bar{\theta}$

Question:

Is this evaluation of $\bar{\theta}$ correct?

$$\bar{\theta} = \cancel{\theta_G + \arg\det\left[\mathcal{M}_u^{(0)}\mathcal{M}_d^{(0)}\right]} + \underbrace{\cancel{\arg\det\left[\delta\mathcal{M}_u^{(1)}\delta\mathcal{M}_d^{(1)}\right]}}_{\text{1-loop}} + \boxed{\arg\det\left[\delta\mathcal{M}_u^{(2)}\delta\mathcal{M}_d^{(2)}\right]} + \cdots$$

Answer:

No! We propose a new method, diagrammatic evaluation.

Diagrammatic evaluation of $G\tilde{G}$

Fujikawa method (conventional method)

$$\mathcal{L}_\theta \ni \theta \frac{g_s^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} + m_R\bar{\psi}\psi + m_I\bar{\psi}i\gamma_5\psi$$

$$= \left(\theta - \frac{1}{2}\frac{m_I}{m_R}\right)\frac{g_s^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} + M\bar{\psi}_M\psi_M$$

where

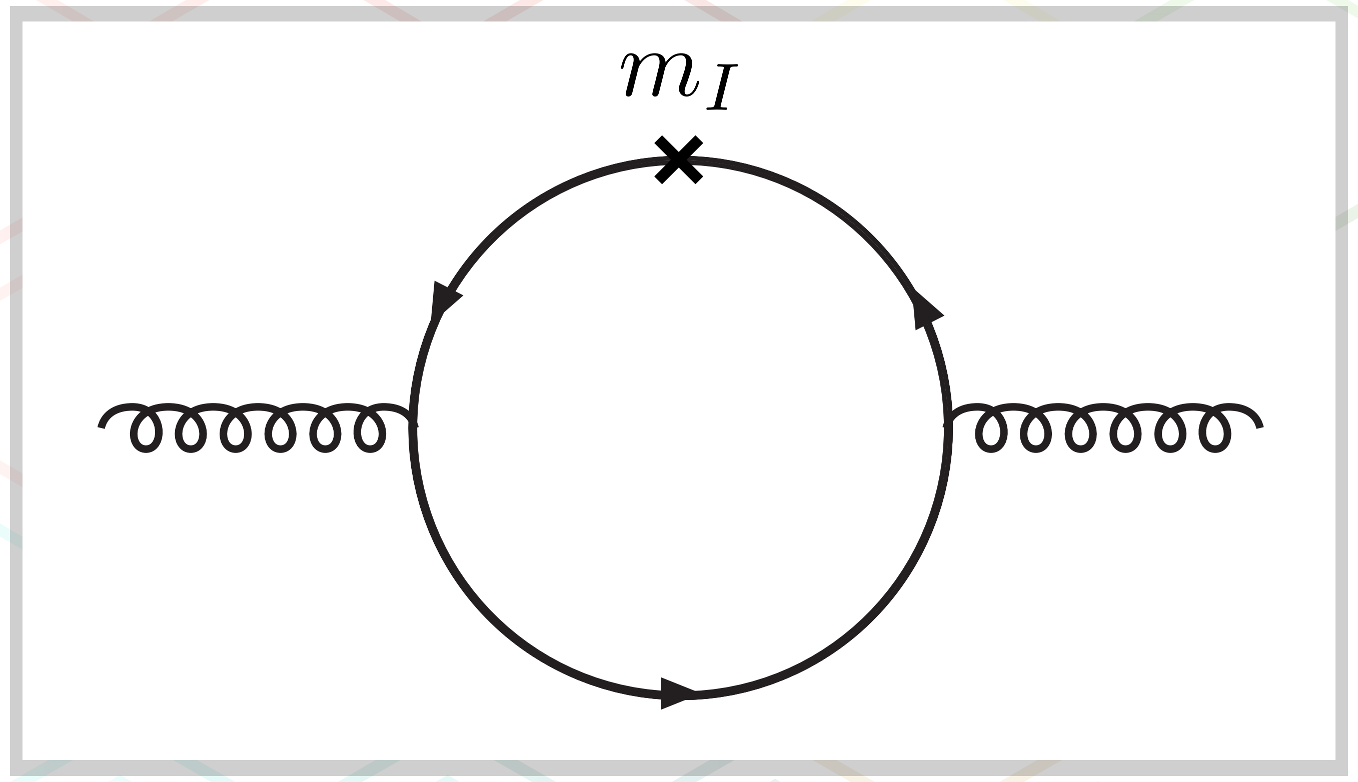
$$\psi = (1 + i\theta'\gamma_5)\psi_M, \quad \theta' = -\frac{1}{2}\frac{m_I}{m_R}$$

loop correction to CP-odd mass: $\bar{\theta} = \theta - \frac{1}{2}\frac{m_I}{m_R} \to \theta - \frac{1}{2}\frac{m_I}{m_R} - \frac{1}{2}\frac{\delta m_I}{m_R} \quad (m_I \to m_I + \delta m_I)$

diagrammatic evaluation (New!)

the method to calculate loop corrections to $G\tilde{G}$ directly

$$\theta \frac{g_s^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} \to (\theta + \delta\theta)\frac{g_s^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$$



Problem: $G\tilde{G}$ → **total derivative** → **non-perturbative**

Strategy: **Fock-Schwinger gauge** or **Operator Schwinger method**

The wrong point in the conventional method

Why is $\bar{\theta} = \theta_G + \arg\det\left[\mathcal{M}_u^{(0)}\mathcal{M}_d^{(0)}\right] + \arg\det\left[\delta\mathcal{M}_u\delta\mathcal{M}_d\right]$ wrong?

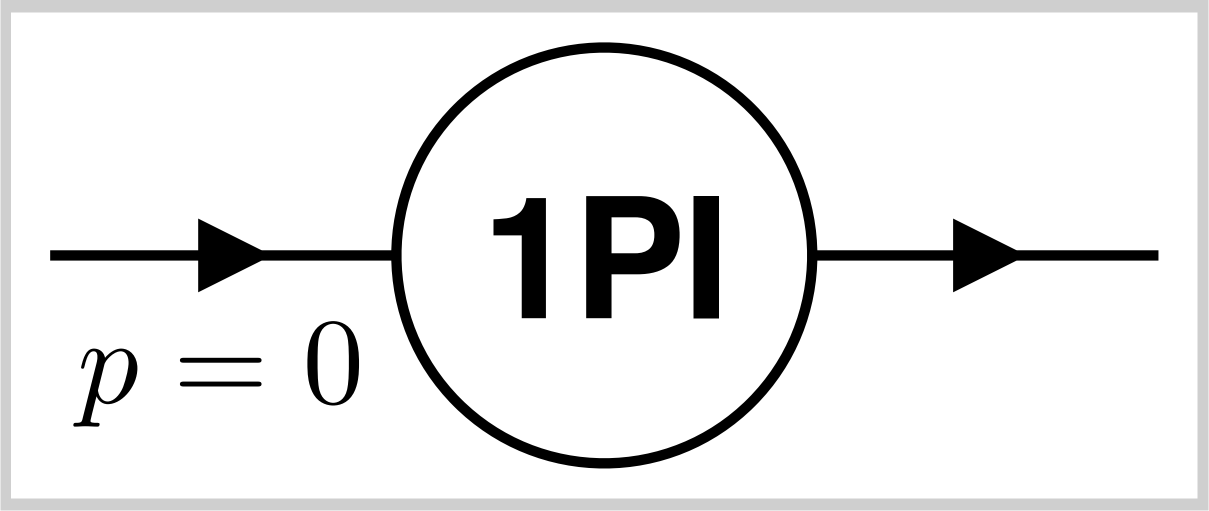
1. the momentum dependence in loop corrections: $\mathcal{M}^{(0)} + \delta\mathcal{M}(p)$

2. $G\tilde{G}$ is induced from the fermion-loop diagrams. $\theta_G \frac{g_s^2}{16\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$

3. The other contributions except the CP-odd mass are not considered.

1. the momentum dependence in loop corrections: $\mathcal{M}^{(0)} + \delta\mathcal{M}(p)$

The flow momentum is set for $p = 0$ in preceding studies.

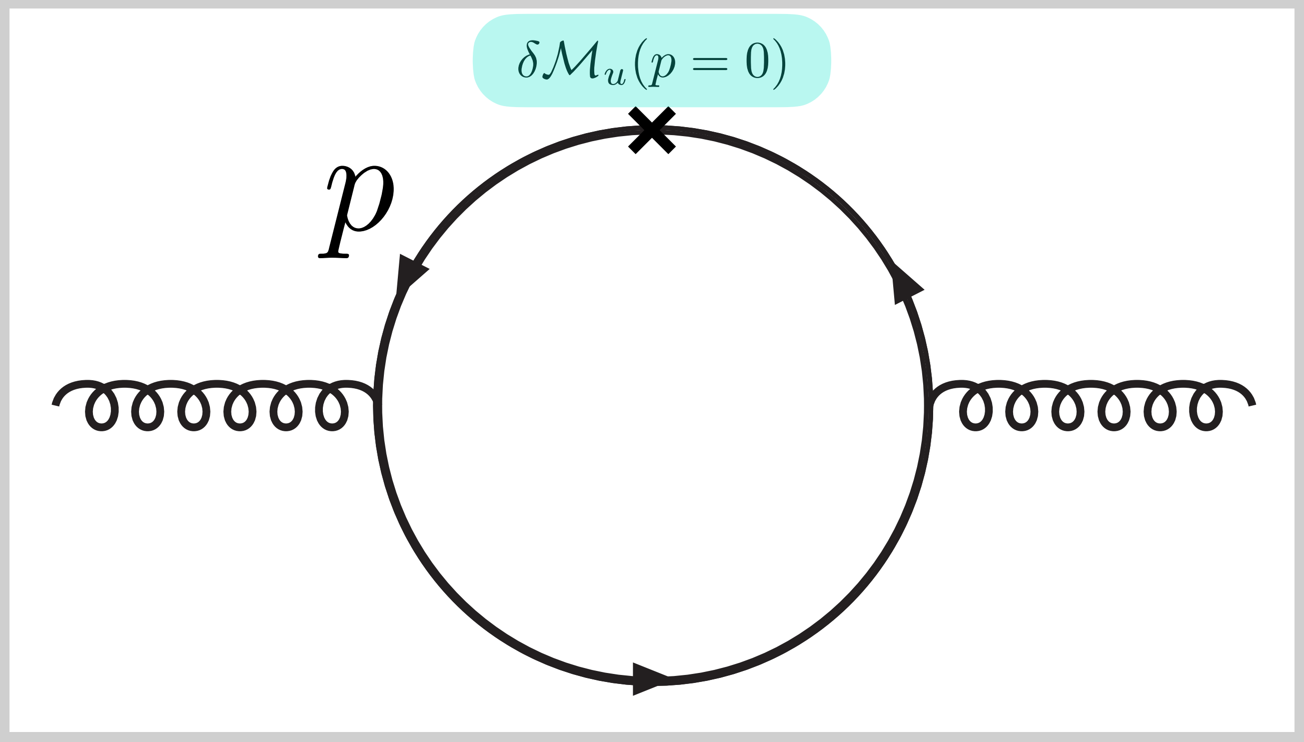


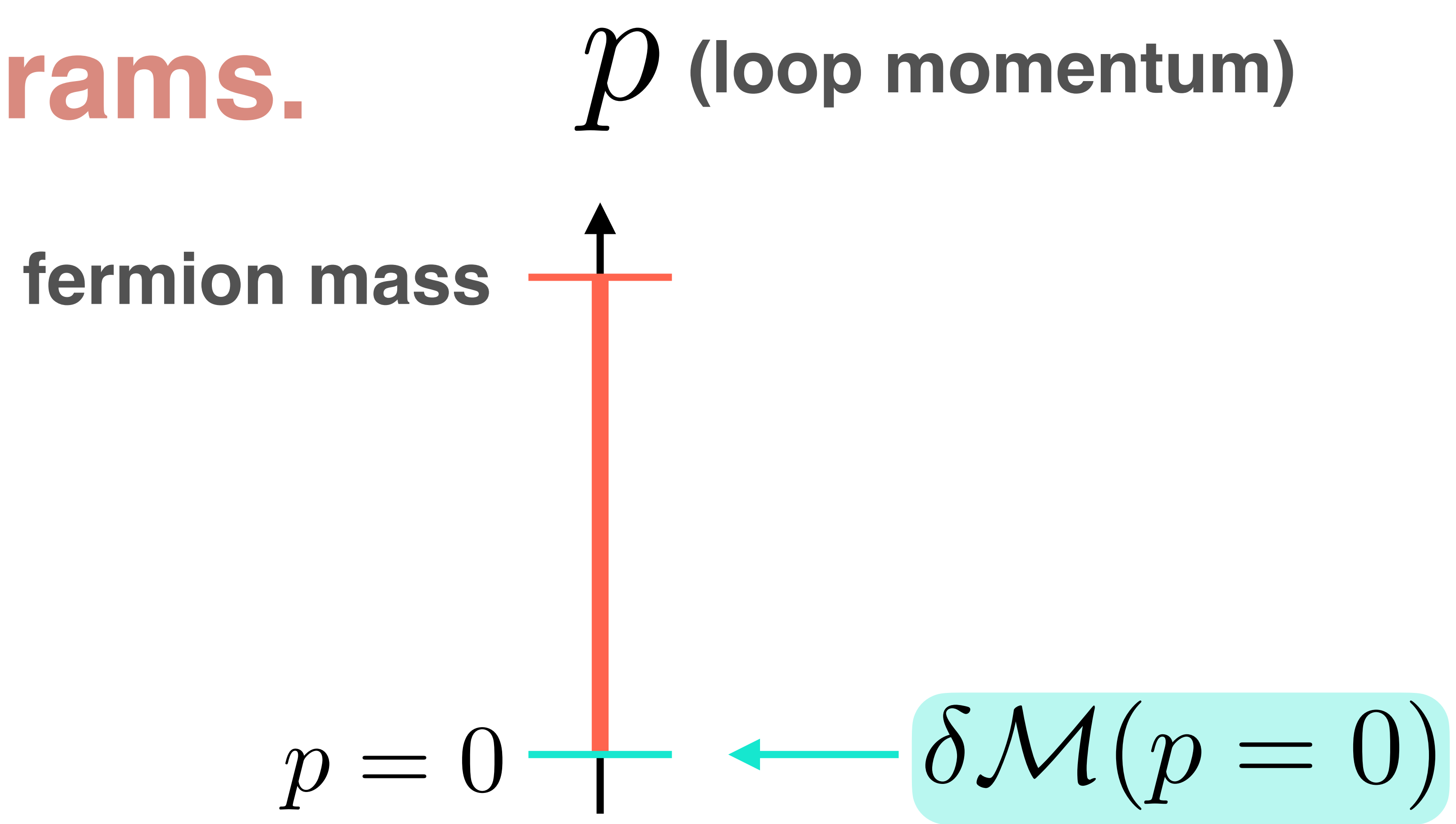
$\Rightarrow \delta\mathcal{M}(p = 0)$

The wrong point in the conventional method

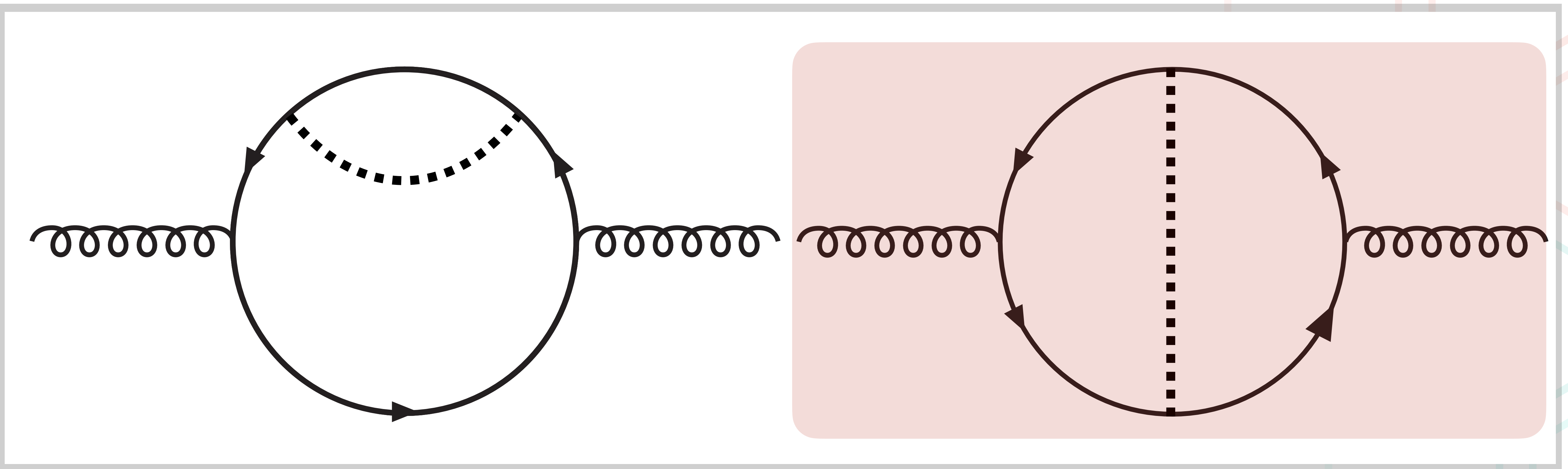
2. $G\tilde{G}$ is induced from the fermion-loop diagrams.

$\arg\det\left[\delta\mathcal{M}_u\delta\mathcal{M}_d\right]$





3. The other contributions except the CP-odd mass are not considered.



This type is not included in $\arg\det\left[\delta\mathcal{M}_u\delta\mathcal{M}_d\right]$.

CP violation in the LR model

$$\left(\overline{u}_L^i, \overline{U}_L^a\right)\begin{pmatrix} 0 & x_u^{ib} v \\ x_u^{\dagger aj} v' & M_u^a \delta^{ab}\end{pmatrix}\begin{pmatrix} u_R^j \\ U_R^b \end{pmatrix}$$

$$\left(\overline{d}_L^i, \overline{D}_L^a\right)\begin{pmatrix} 0 & x_d^{ib} v \\ x_d^{\dagger aj} v' & M_d^a \delta^{ab}\end{pmatrix}\begin{pmatrix} d_R^j \\ D_R^b \end{pmatrix}$$

seesaw $\Longrightarrow$

quark mass

$$V_q^\dagger \frac{m_q}{v} V_q = x_q \frac{v'}{M_q} x_q^\dagger$$

$\frac{v'}{M_q} \ll 1$

flavor basis ⟷ mass basis

CP phases in Yukawa couplings

$$x_u = \quad V_{\mathrm{CKM}}^\dagger \frac{\sqrt{m_u}}{\sqrt{v}} \overline{\Phi}(\theta_{u3}, \theta_{u8}) V_U \frac{\sqrt{M_u}}{\sqrt{v'}}, \qquad 1(V_{\mathrm{CKM}}) + 2(\theta_{u3}, \theta_{u8}) + 1(V_U) = 4$$

$$x_d = \quad \frac{\sqrt{m_d}}{\sqrt{v}} \overline{\Phi}(\theta_{d3}, \theta_{d8}) V_D \frac{\sqrt{M_d}}{\sqrt{v'}} \qquad 2(\theta_{d3}, \theta_{d8}) + 1(V_D) = 3$$

$$(V_{U/D} : \mathrm{CKM-like\ matrix})$$

combinations of Yukawa couplings ~CP-even~ (We have checked them analytically and numerically.)

$\mathcal{O}(x^2)$

$$\mathrm{Im}\left[x_u^{ia} x_u^{\dagger ai}\right] f(M_u^a) = 0$$

$$\mathrm{Im}\left[x_d^{ia} x_d^{\dagger ai}\right] f(M_d^a) = 0$$

$\mathcal{O}(x^4)$

$$\mathrm{Im}\left[x_u^{ia} x_u^{\dagger aj} x_u^{jb} x_u^{\dagger bi}\right] f(M_u^a, M_u^b) = 0$$

$$\mathrm{Im}\left[x_d^{ia} x_d^{\dagger aj} x_d^{jb} x_d^{\dagger bi}\right] f(M_d^a, M_d^b) = 0$$

$$\mathrm{Im}\left[x_u^{ia} x_u^{\dagger aj} x_d^{jb} x_d^{\dagger bi}\right] f(M_u^a, M_d^b) = 0$$

Non-vanishing combination

Non-vanishing combinations

- **upper bound**

$$\mathrm{Im}\,\mathrm{Tr}\left[\left(x_u^a x_u^{\dagger a}\right)\left[\left(x_u^b x_u^{\dagger b}\right),\left(x_u^c x_u^{\dagger c}\right)\right]\right] f(M_u^a, M_u^b, M_u^c) \quad \text{or} \quad \mathrm{Im}\,\mathrm{Tr}\left[\left(x_d^a x_d^{\dagger a}\right)\left[\left(x_u^b x_u^{\dagger b}\right),\left(x_u^c x_u^{\dagger c}\right)\right]\right] f(M_d^a, M_u^b, M_u^c)$$

for $M_u^1 \neq M_u^2$

- **lower bound**

$$\mathrm{Im}\ \mathrm{Tr}\left[(x_u x_u^\dagger)^2 (x_d x_d^\dagger)^2 (x_u x_u^\dagger)(x_d x_d^\dagger)\right] f$$

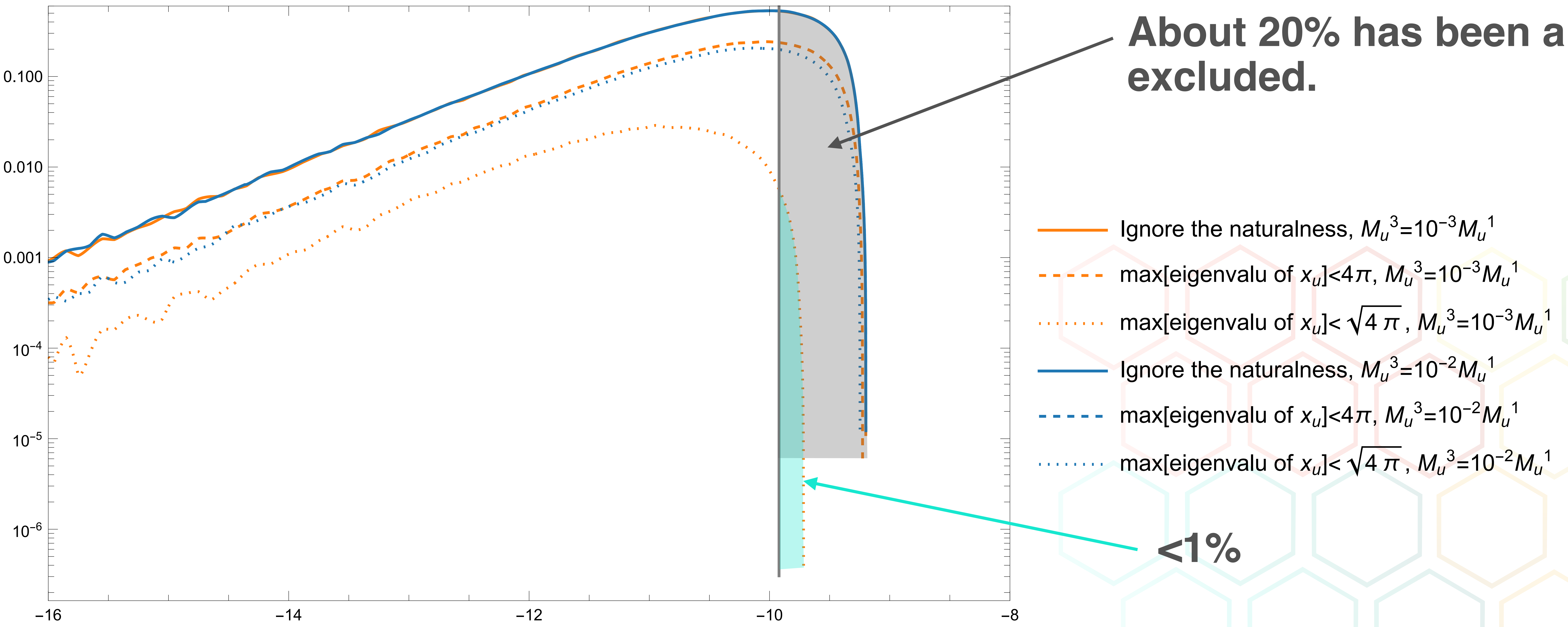
anti-commuting loop function is required.

$$\begin{aligned}
&\mathrm{Im}\,\mathrm{Tr}\left[\left(x_u^a x_u^{\dagger a}\right)\left(x_u^b x_u^{\dagger b}\right)\left(x_u^c x_u^{\dagger c}\right)\right] f(M_u^a, M_u^b, M_u^c) + \mathrm{Im}\,\mathrm{Tr}\left[\left(x_u^a x_u^{\dagger a}\right)\left(x_u^c x_u^{\dagger c}\right)\left(x_u^b x_u^{\dagger b}\right)\right] f(M_u^a, M_u^c, M_u^b) \\
&= \mathrm{Im}\,\mathrm{Tr}\left[\left(x_u^a x_u^{\dagger a}\right)\left(x_u^b x_u^{\dagger b}\right)\left(x_u^c x_u^{\dagger c}\right)\right] f(M_u^a, M_u^b, M_u^c) - \mathrm{Im}\,\mathrm{Tr}\left[\left(x_u^b x_u^{\dagger b}\right)\left(x_u^c x_u^{\dagger c}\right)\left(x_u^a x_u^{\dagger a}\right)\right] f(M_u^a, M_u^c, M_u^b) \\
&= \mathrm{Im}\,\mathrm{Tr}\left[\left(x_u^a x_u^{\dagger a}\right)\left(x_u^b x_u^{\dagger b}\right)\left(x_u^c x_u^{\dagger c}\right)\right]\left\{f(M_u^a, M_u^b, M_u^c) - f(M_u^a, M_u^c, M_u^b)\right\}
\end{aligned}$$

Upper bound -uuu-

CP phase

$$x_u = V_{\mathrm{CKM}}^{\dagger} \frac{\sqrt{m_u}}{\sqrt{v}} \bar{\Phi}\left(\theta_{u3}, \theta_{u8}\right) V_U \frac{\sqrt{M_u}}{\sqrt{v'}} \longrightarrow \mathrm{Im}\,\mathrm{Tr}\left[\left(x_u^a x_u^{\dagger a}\right)\left[\left(x_u^b x_u^{\dagger b}\right), \left(x_u^c x_u^{\dagger c}\right)\right]\right] f(M_u^a, M_u^b, M_u^c)$$



$\mathrm{Log}_{10}\delta\theta^{13}$ with fixing $M_u{}^2=10^{-1}M_u{}^1$ (PDF)

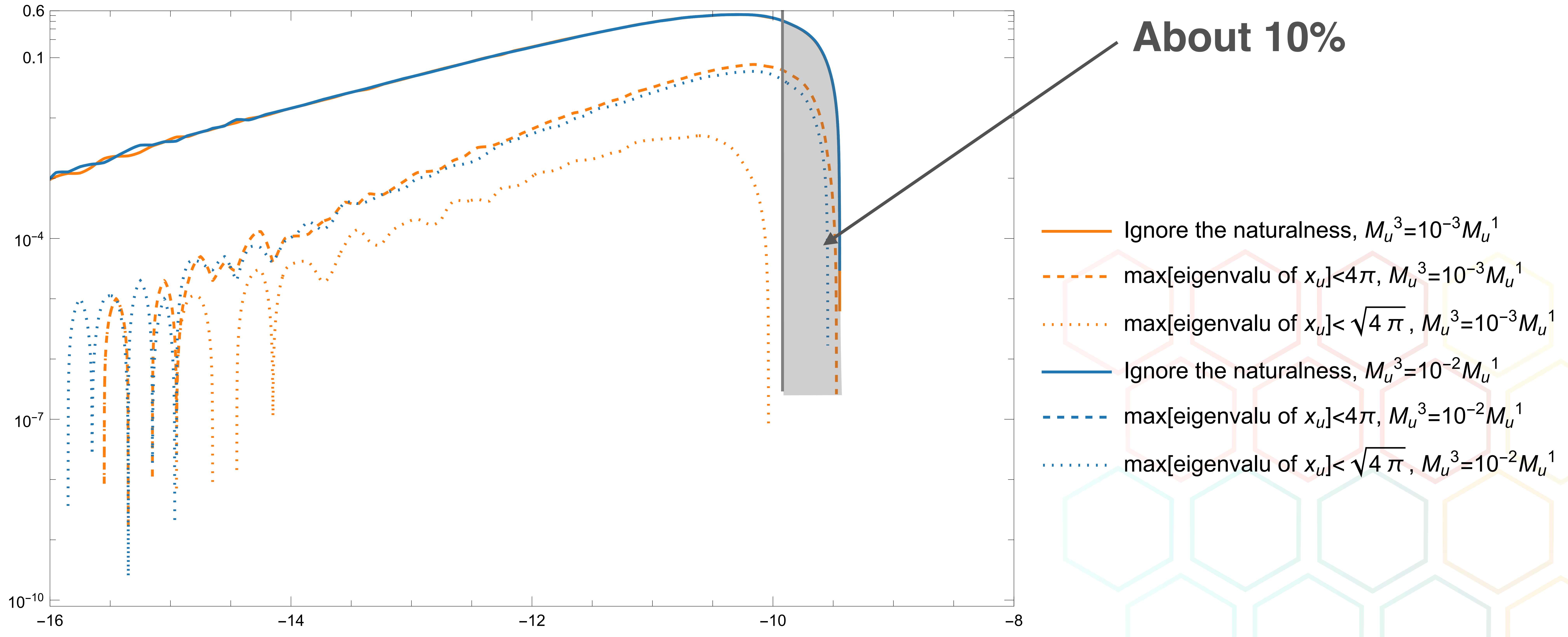
About 20% has been already excluded.

<1%

Upper bound -duu-

CP phase

$$x_u = V^{\dagger}_{\mathrm{CKM}} \frac{\sqrt{m_u}}{\sqrt{v}} \bar{\Phi}\left(\theta_{u3}, \theta_{u8}\right) V_U \frac{\sqrt{M_u}}{\sqrt{v'}} \longrightarrow \mathrm{Im\,Tr}\left[\left(x_d^a x_d^{\dagger a}\right)\left[\left(x_u^b x_u^{\dagger b}\right), \left(x_u^c x_u^{\dagger c}\right)\right]\right] f(M_d^a, M_u^b, M_u^c)$$



$\mathrm{Log}_{10}\delta\theta^{13}$ with fixing $M_u^{\ 2}=M_u^{\ 1}$ (PDF)

Lower bound ~complete degeneration~

When VL quark masses are degenerate completely, only one CP phase in the LR model is in $V_{\rm CKM}$.

$$v' = M_u^3 = M_u^1 = M_u^2 = M_d$$

$$x_u = \boxed{V_{\rm CKM}^\dagger} \frac{\sqrt{m_u}}{\sqrt{v}} \bar{\Phi}\left(\theta_{u3}, \theta_{u8}\right) V_U \frac{\sqrt{M_u}}{\sqrt{v'}} \longrightarrow \mathrm{Im\ Tr}\left[(x_u x_u^\dagger)^2 (x_d x_d^\dagger)^2 (x_u x_u^\dagger)(x_d x_d^\dagger)\right] f$$

Numerical result:

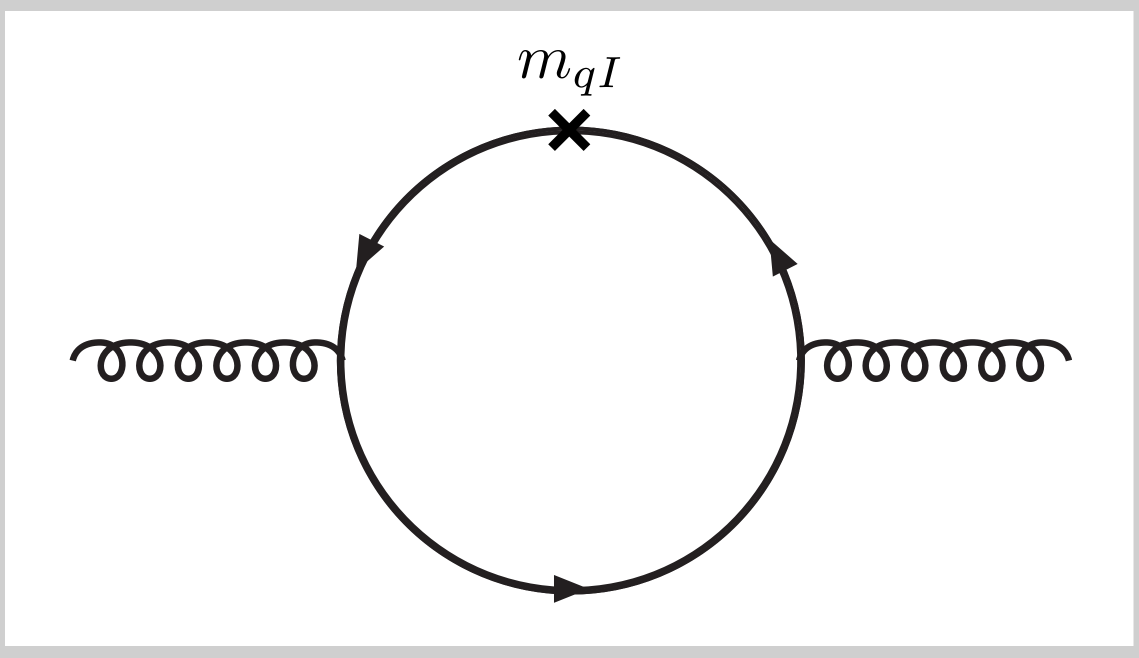
$$\delta\theta \sim \frac{1}{(16\pi^2)^2} \frac{M_u^3 M_d^3}{v'^4 \tilde{M}^2} \frac{m_t^2 m_c m_b^2 m_s}{v^6} J \sim 1 \times 10^{-20}$$

Summary

- We propose a new method, **diagrammatic evaluation**.

  ✗ $\bar{\theta} = \theta + \arg\det\left[\mathcal{M}_u^{(0)}\mathcal{M}_d^{(0)}\right] \cancel{+ \arg\det\left[\delta\mathcal{M}_u \delta\mathcal{M}_d\right]}$

  ✓ Fock-Schwinger gauge or Operator Schwinger method



- The contribution to $\bar{\theta}$ at two-loop level vanish.

- The upper bound of $\bar{\theta}$ induced at three-loop level is comparable to the experimental constraint $\bar{\theta} < 1.2 \times 10^{-10}$.

Backup

Strong CP problem

$\mathscr{L}_{\mathrm{QCD}}$ **is the CP violating theory with two CP violating sources.**

- $\theta \frac{\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$ **: renormalizable P- and CP-odd gluonic operator**

  θ **is unphysical parameter.**

notation:

$$G^a_{\mu\nu} \tilde{G}^{a\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$$

- $Y_u^{ij} \bar{u}_L^i u_R^j v + Y_d^{ij} \bar{d}_L^i d_R^j v + \mathrm{h.c.}$ **: Yukawa interactions.** $Y_{u/d}$ **are complex matrices.**

  Not hermitian!

$$Y_u^{ij} \bar{u}_L^i u_R^j v + Y_u^{\dagger ij} \bar{u}_R^i u_L^j v = \mathrm{Re}\left[Y_u^{ij}\right] \bar{u}^i u^j v + \underbrace{\mathrm{Im}\left[Y_u^{ij}\right] \bar{u}^i i\gamma_5 u^j v}_{\text{CP-odd quark mass}}$$ **(flavor basis)**

$$\xrightarrow{\text{chiral rotation}} M_u^i \bar{u}_M^i u_M^i$$ **(mass basis)**

Mass diagonalization

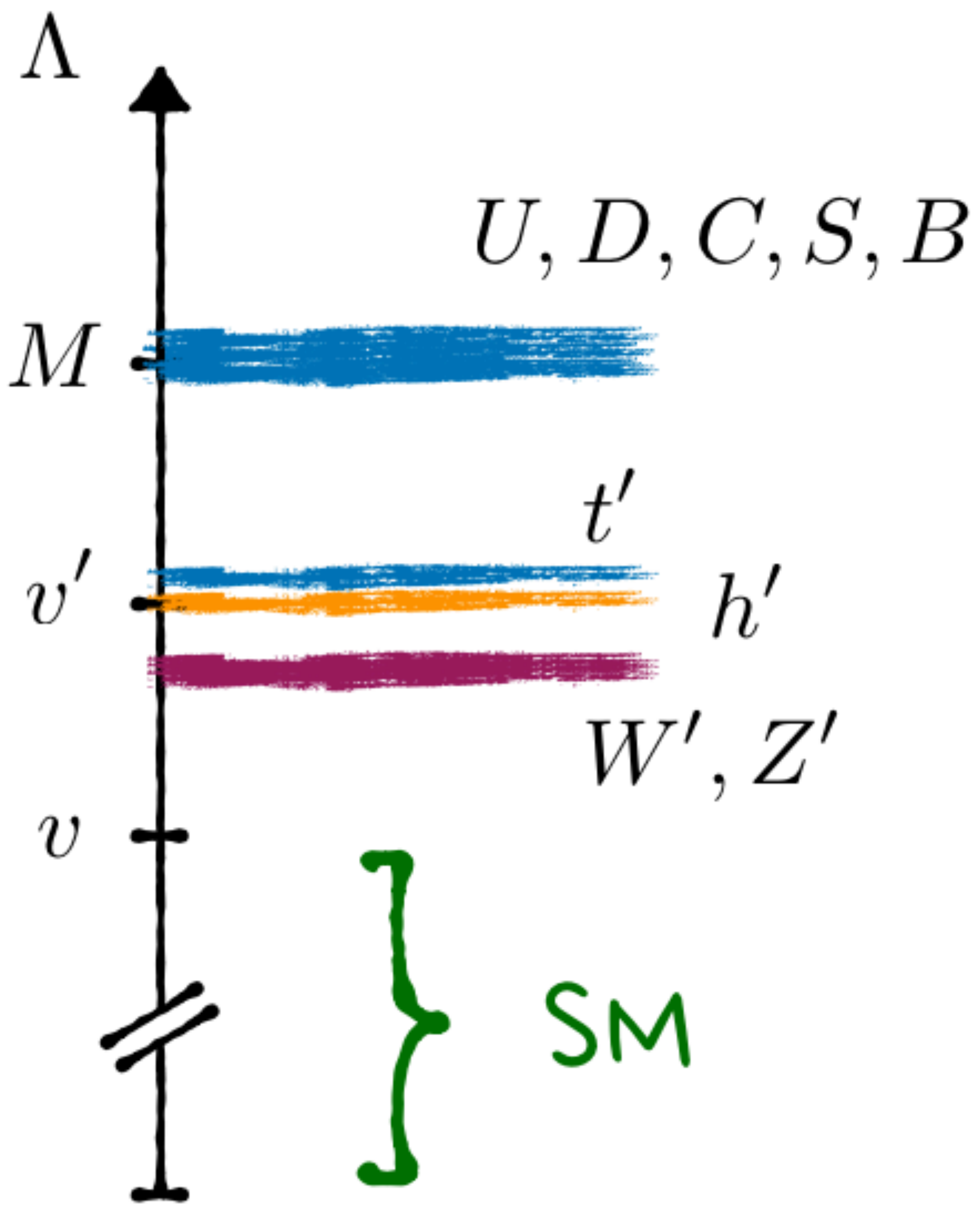
$$V_{qL}\mathcal{M}_q^{(0)}V_{qR}^\dagger = V_{qL}\begin{pmatrix} 0 & x_q v \\ x_q^\dagger v' & M_q \end{pmatrix} V_{qR}^\dagger = \mathrm{diag}(m_q, M_q)$$

$$V_{uL} = \begin{pmatrix} -V & V x_u \frac{v}{M_u} \\ \frac{v}{M_u} x_u^\dagger & 1 \end{pmatrix}, \qquad V_{uR} = \begin{pmatrix} V & -V x_u \frac{v'}{M_u} \\ \frac{v'}{M_u} x_u^\dagger & 1 \end{pmatrix},$$

$$V_{dL} = \begin{pmatrix} -1 & x_d \frac{v}{M_d} \\ \frac{v}{M_d} x_d^\dagger & 1 \end{pmatrix}, \qquad V_{dR} = \begin{pmatrix} 1 & -x_d \frac{v'}{M_d} \\ \frac{v'}{M_d} x_d^\dagger & 1 \end{pmatrix}$$

$$\Rightarrow \quad V_{qL} = \begin{pmatrix} 1 & O(\frac{v}{M_q}) \\ O(\frac{v}{M_q}) & 1 \end{pmatrix}, \quad V_{qR} = \begin{pmatrix} 1 & O(\frac{v'}{M_q}) \\ O(\frac{v'}{M_q}) & 1 \end{pmatrix}$$

Mass spectrum



$$\frac{M}{v'} < 100$$ **to realize m_b by the seesaw**

$$m_{W'} = \frac{g}{2} v' \qquad m_{Z'} = \frac{g v' \cos\theta_w}{2\sqrt{\cos 2\theta_w}} + \mathcal{O}\left(\frac{v^2}{v'^2}\right)$$

N. Craig, I. Garcia Garcia, G. Koszegi, and A. McCune, JHEP 09 (2021) 130

Preceding studies about LR model

K. S. Babu and R. N. Mohapatra , Phys. Rev. D 41 (1990), 1286

- They showed vanishing the 1-loop corrections of CP-odd mass term contributing to $\bar{\theta}$.

L. J. Hall and K. Harigaya, JHEP 10 (2018) 130

- They revealed the relationship between LR model and $SO(10)$ GUT.
- They estimated non-vanishing loop corrections to $\bar{\theta}$.

wrong point

$$\bar{\theta} = \theta + \arg\det\left[\mathcal{M}_u^{(0)}\mathcal{M}_d^{(0)}\right] + \arg\det\left[\delta\mathcal{M}_u\delta\mathcal{M}_d\right]$$

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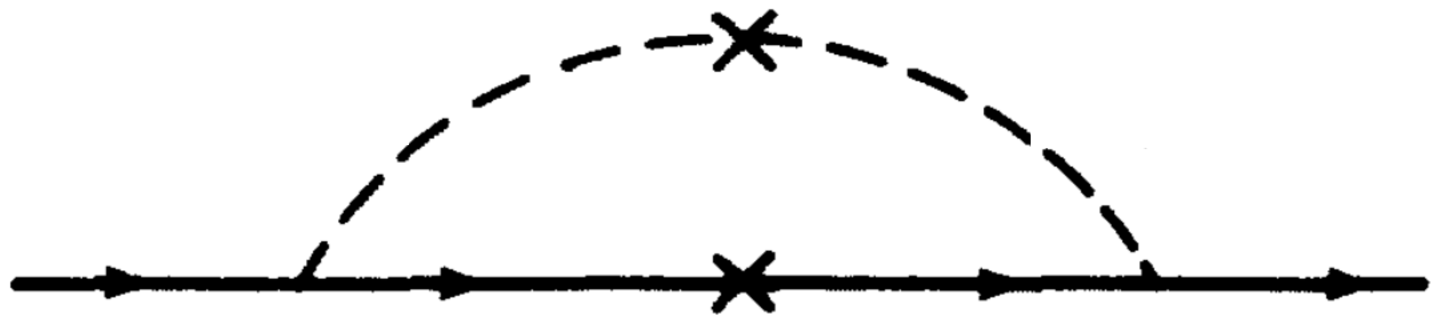
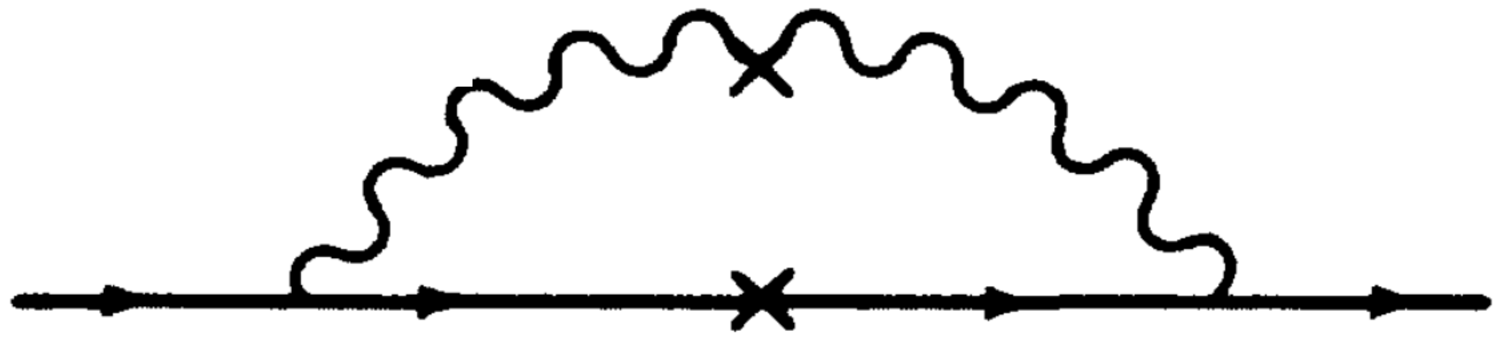


a loop of neutral NG bosons



a loop of gauge bosons

wrong points

- ☐ $\bar{\theta} = \theta + \arg\det\left[\mathcal{M}_u^{(0)}\mathcal{M}_d^{(0)}\right] + \arg\det\left[\delta\mathcal{M}_u\delta\mathcal{M}_d\right]$
- ☐ CP-even diagrams

K. S. Babu and R. N. Mohapatra , Phys. Rev. D 41 (1990), 1286

fermion

quark: $Q_L(2,1,1/3), Q_R(1,2,1/3)$,

lepton: $\Psi_L(1,1,-1), \Psi_R(1,2,-1)$,

VL up-type: $P_{L,R}(1,1,4/3)$,

VL down-type: $N_{L,R}(1,1,-2/3)$,

VL lepton: $E_{L,R}(1,1,-2)$.

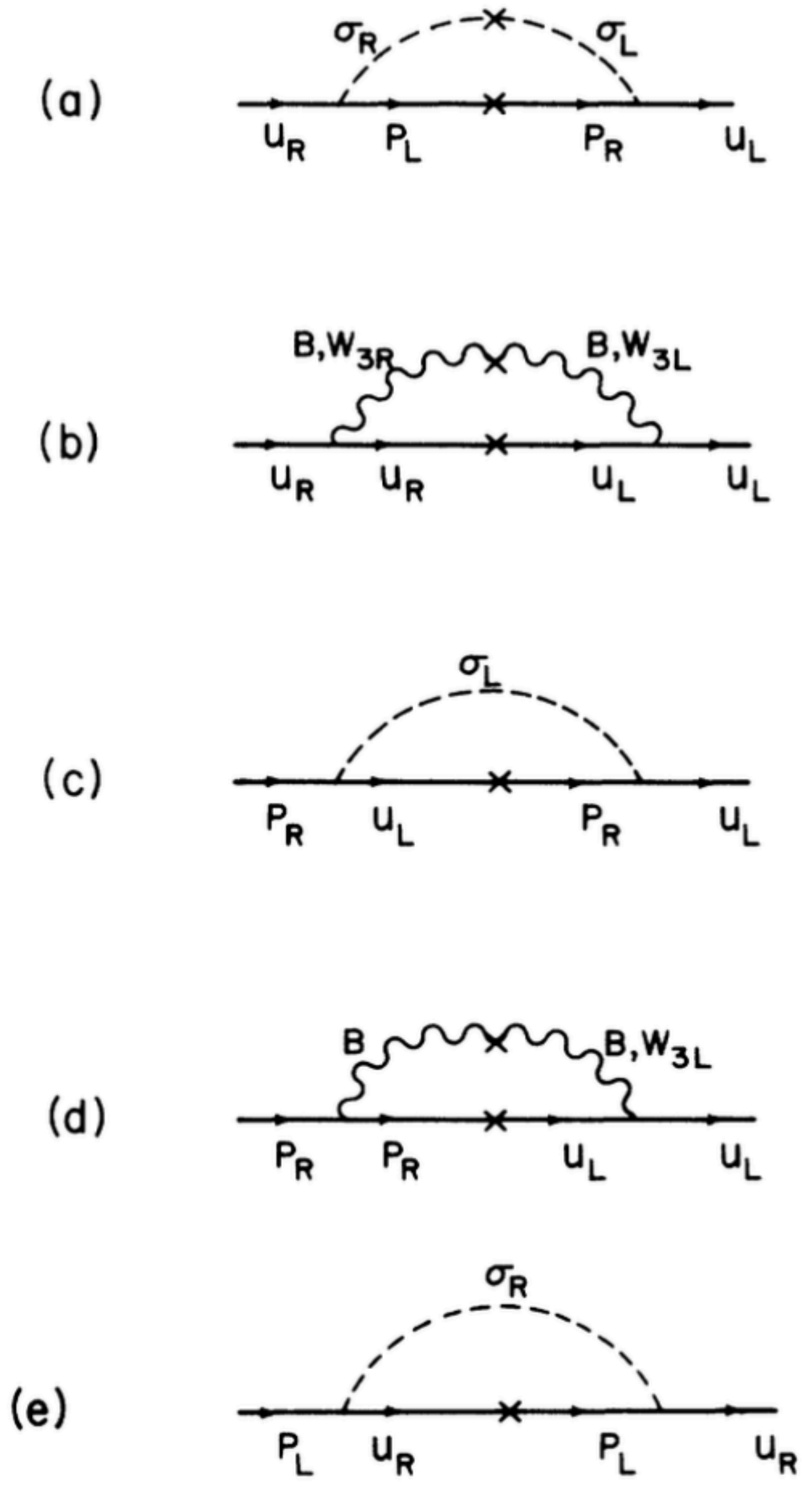
scalar

$SU(2)_L$ **doublet:** χ_L

$SU(2)_R$ **doublet:** χ_R

$$\sigma_L = \sqrt{2}\mathrm{Re}\left[\chi_L^0\right]$$

$$\sigma_R = \sqrt{3}\mathrm{Re}\left[\chi_R^0\right]$$



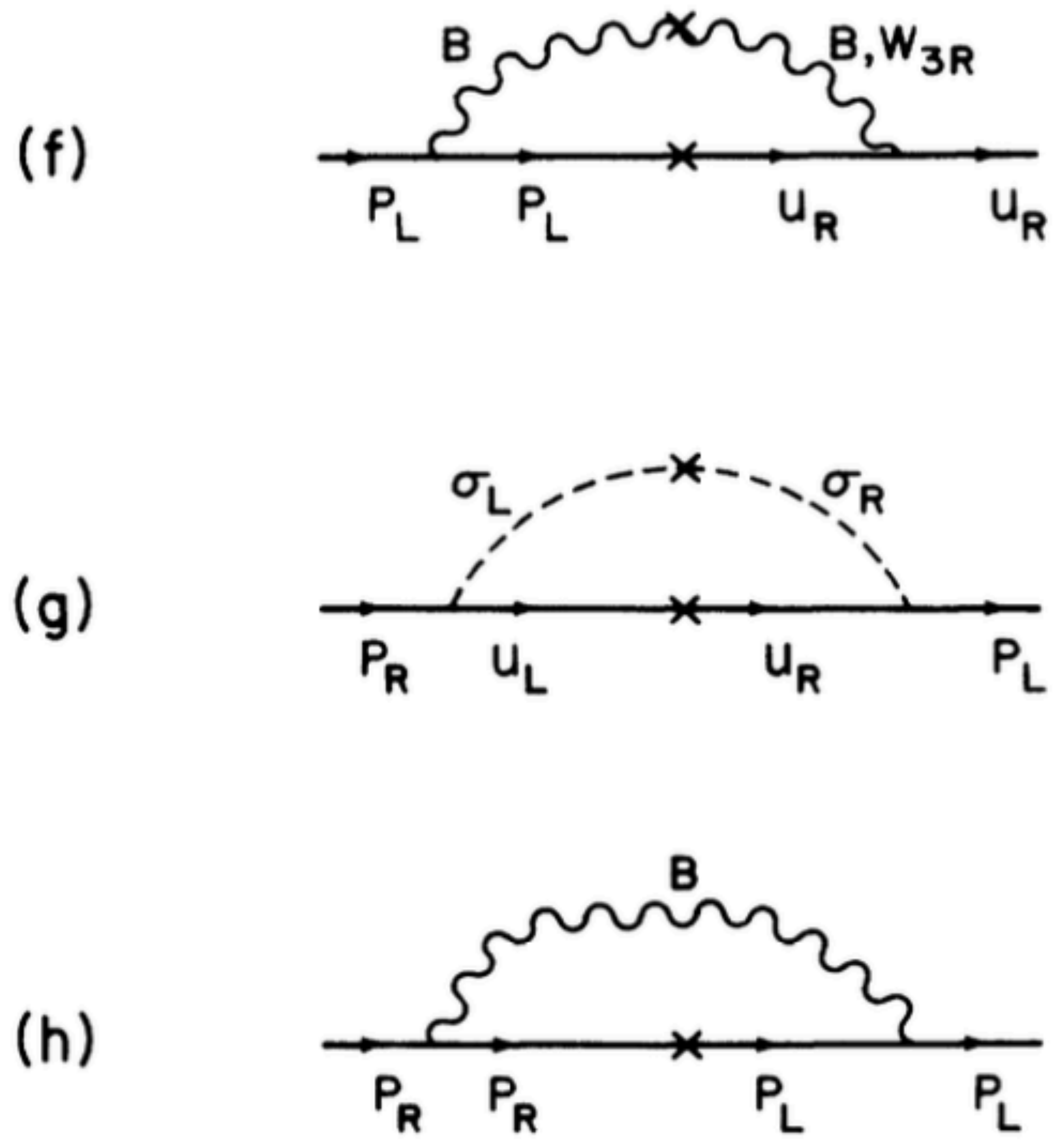
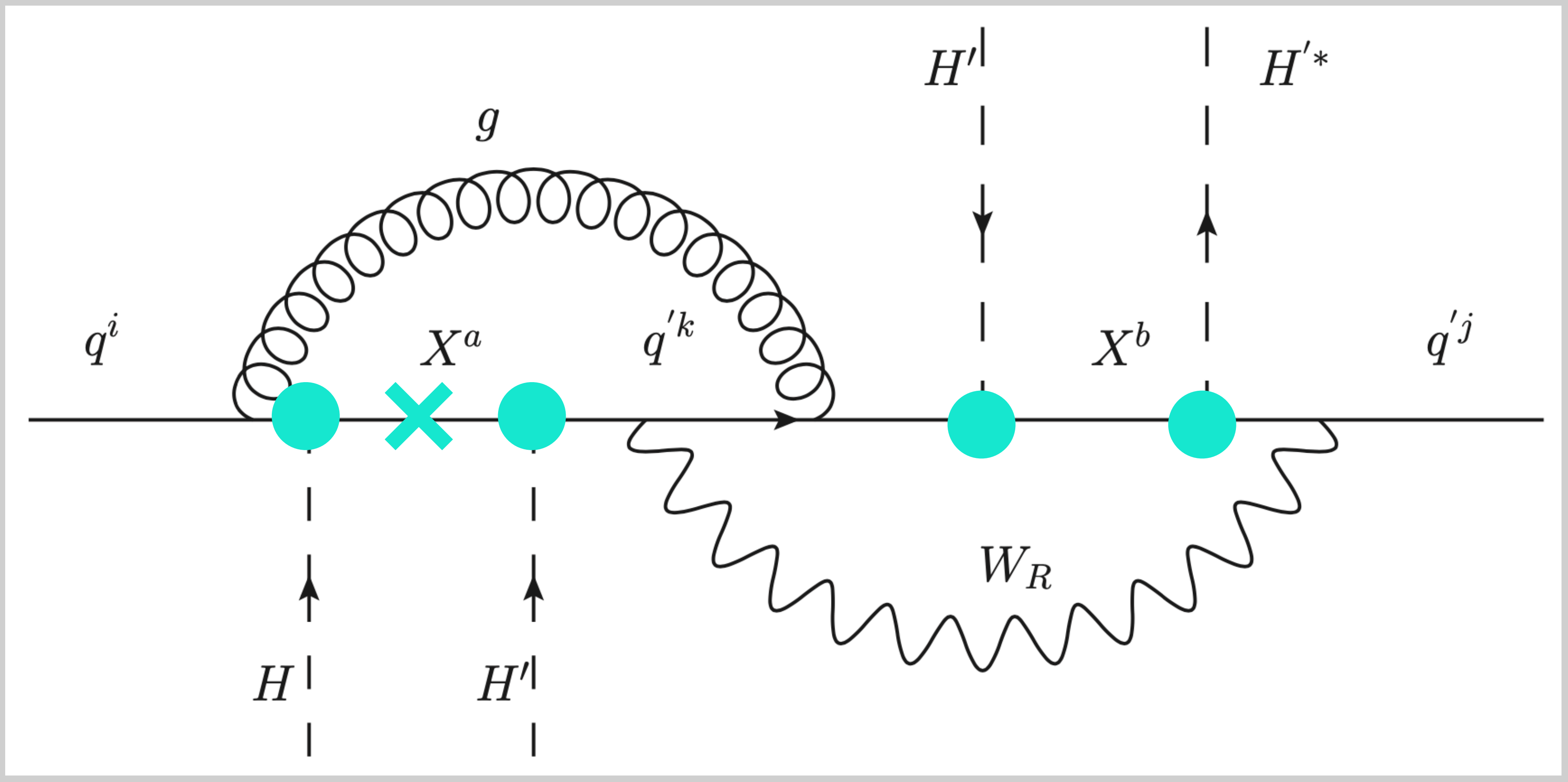


FIG. 1. One-loop radiative corrections to the up-quark mass matrix. The cross on the internal fermion line stands for all possible mass insertions.

Preceding studies about LR model

L. J. Hall and K. Harigaya, JHEP 10 (2018) 130

- They revealed the relationship between LR model and $SO(10)$ GUT.
- They estimated non-vanishing loop corrections to $\bar{\theta}$.



$$\propto x_u^{ia} M_u^a x_u^{\dagger ak} x_d^{kb} x_d^{\dagger bj}$$

non-hermite!

wrong point

☐ $\bar{\theta} = \theta + \arg \det \left[\mathcal{M}_u^{(0)} \mathcal{M}_d^{(0)}\right] + \arg \det \left[\delta\mathcal{M}_u \delta\mathcal{M}_d\right]$

Diagrammatic evaluation of $G\tilde{G}$

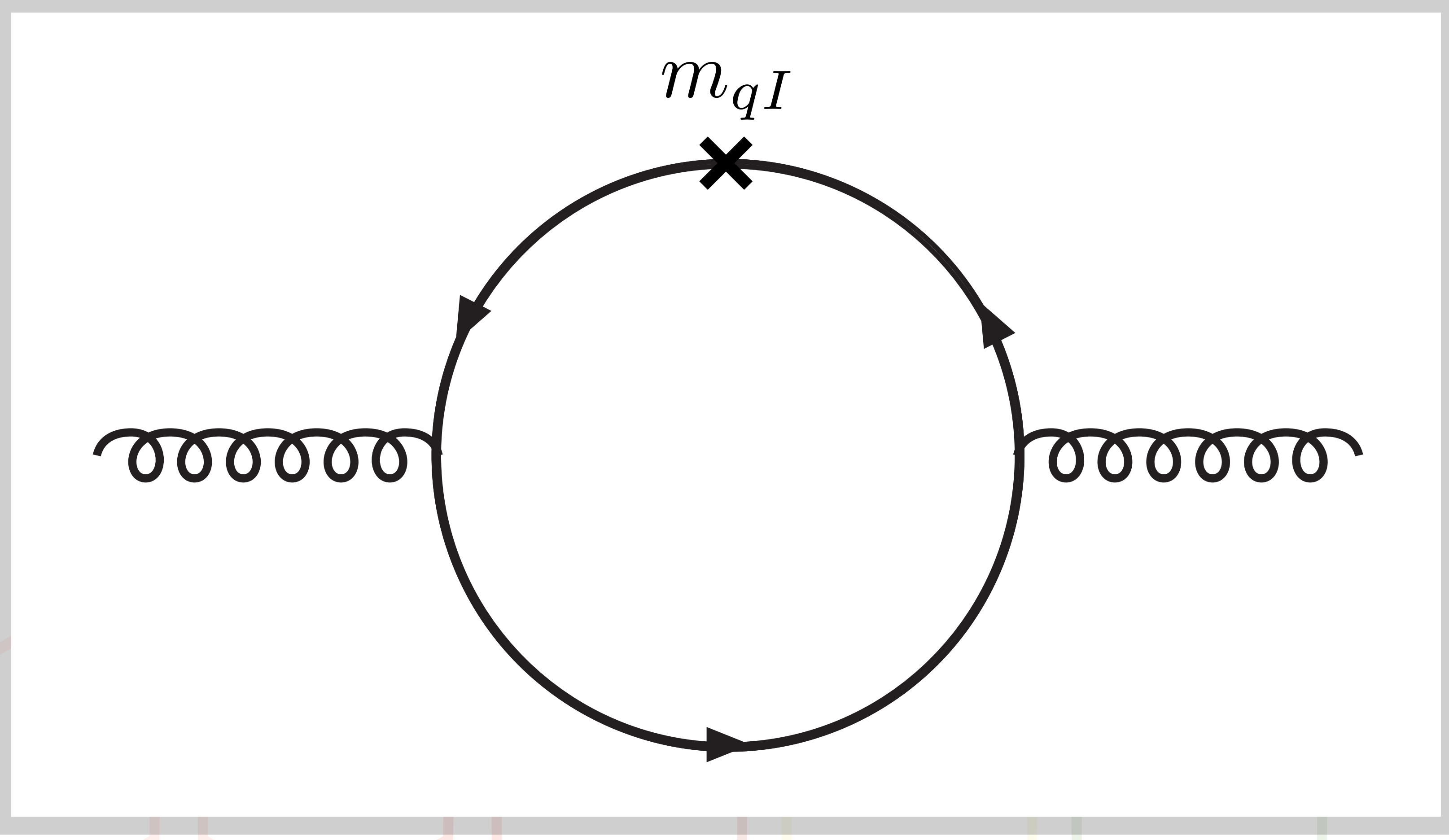
$$\mathcal{L}(\psi,\bar{\psi},A) = \bar{\psi} i \partial\!\!\!/ \psi - m_R \bar{\psi}\psi - m_I \bar{\psi} i\gamma_5 \psi + \mathcal{L}_{\text{gauge}}(A) \quad \textbf{(integrating } \psi\textbf{)}$$
$$= \mathcal{L}_{\text{EFT}}(G)$$

◆ **Fock-Schwinger gauge**

$x^\mu A^a_\mu(x) = 0$ **: violation of translation symmetry**

$$A^a_\mu(x) = \frac{1}{2} x^\nu G^a_{\mu\nu}(0) + \cdots$$

background field-strength



◆ **Operator Schwinger method**

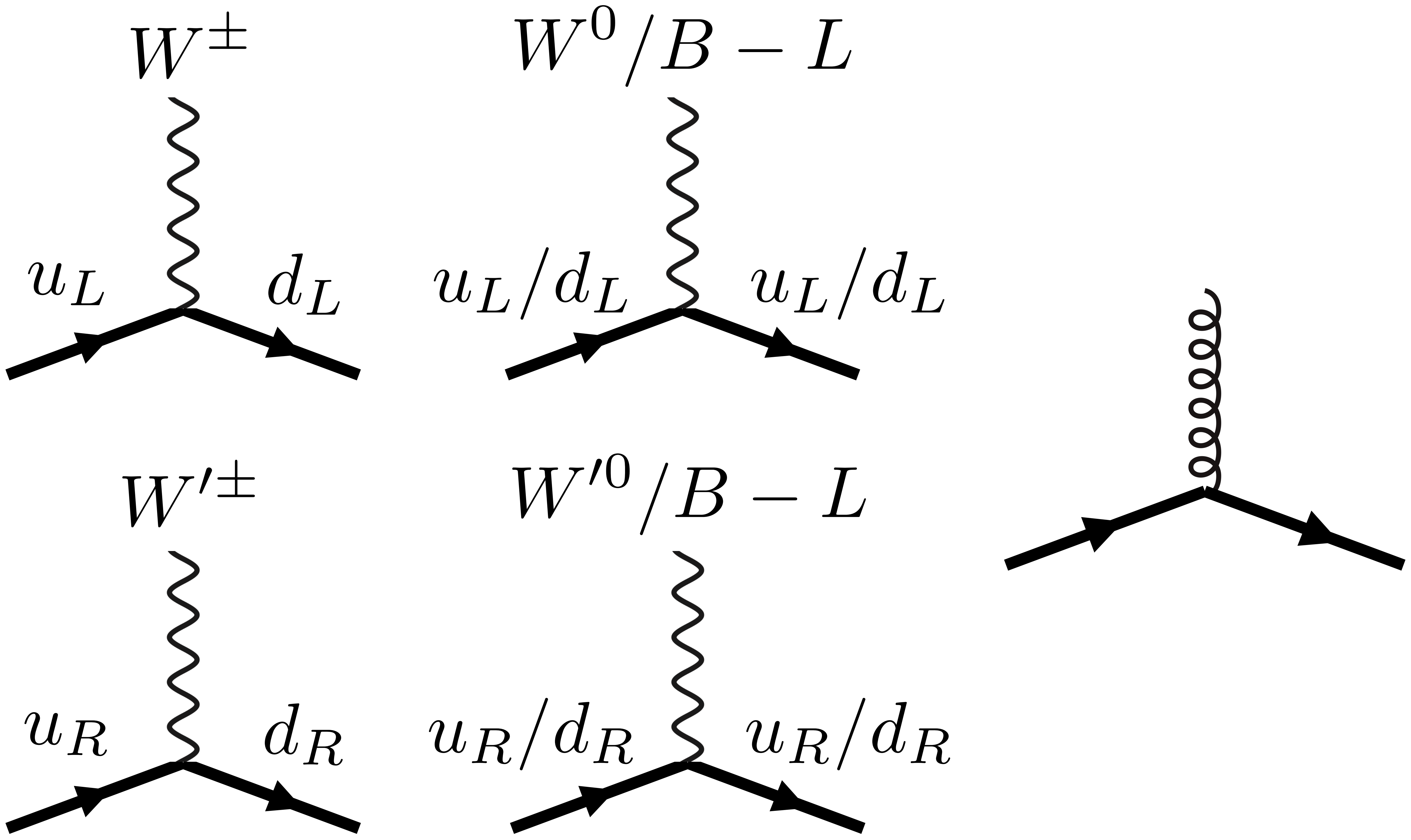
$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[i \int d^4x \bar{\psi}(iD\!\!\!\!/ - m)\psi \right] = \text{Det}\left[iD\!\!\!\!/ - m \right]$$

$$-\frac{d}{dm}\left(-i\text{Tr}\ln\left[iD\!\!\!\!/ - m\right]\right) = -i\text{Tr}\left[\frac{1}{(iD\!\!\!\!/ - m)(iD\!\!\!\!/ + m)}(iD\!\!\!\!/ + m)\right] = -i\text{Tr}\left[\frac{1}{(iD)^2 - m^2 + \frac{ig}{2}\sigma^{\mu\nu}G^a_{\mu\nu}t^a}\right]$$

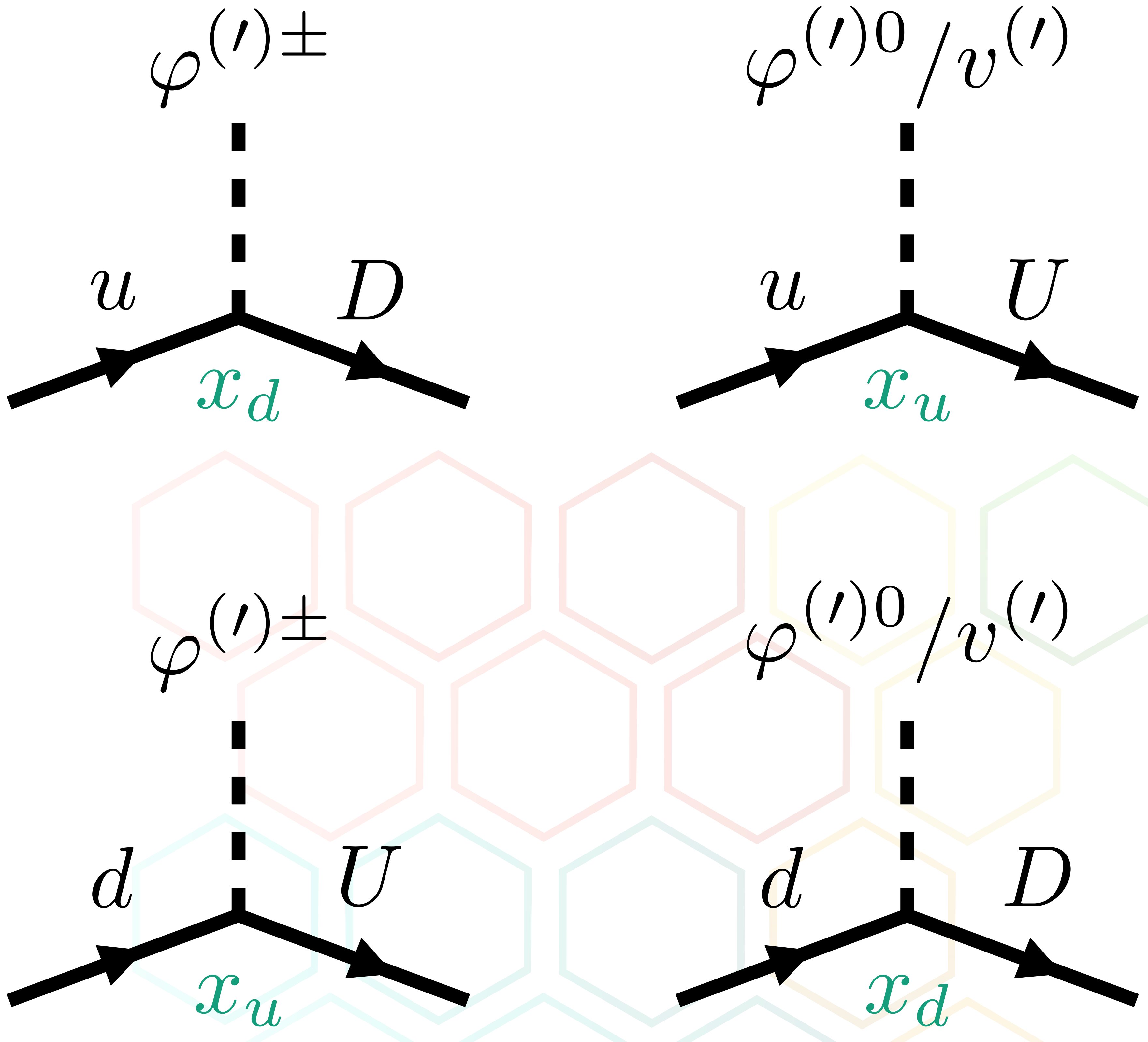
CP-odd fermion loops

Which interaction generate CP violation?

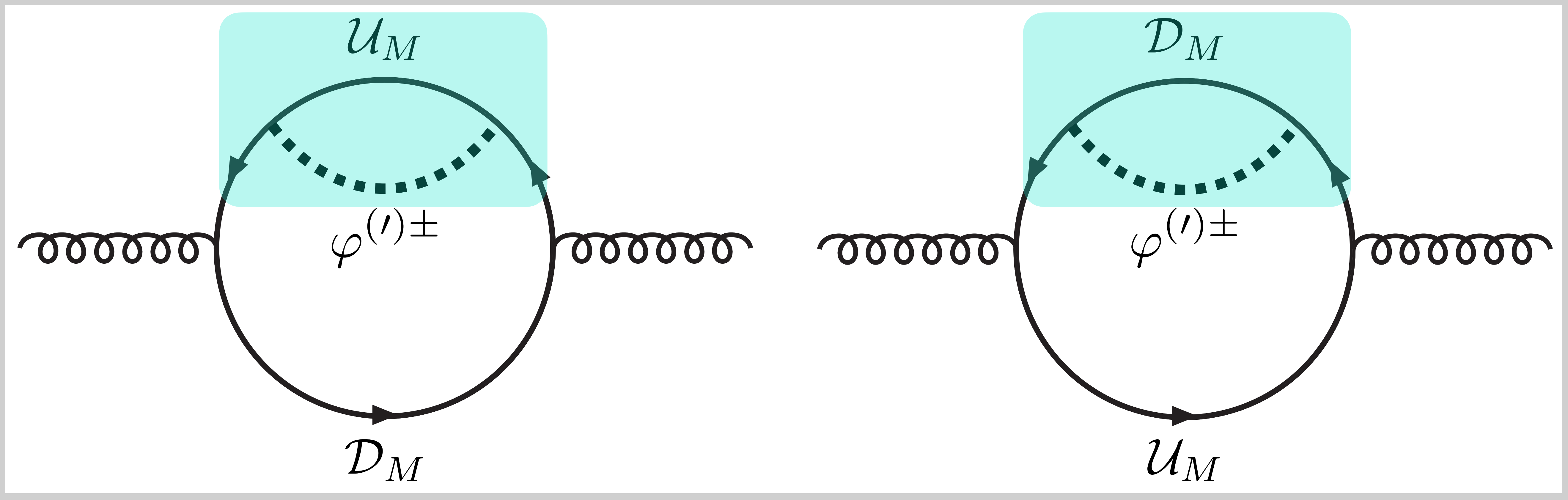
gauge interactions



Yukawa interactions



$$\mathrm{Im}\left[x_u^{ia}x_u^{\dagger aj}x_d^{jb}x_d^{\dagger bi}\right] f(M_u^a, M_d^b) = 0$$

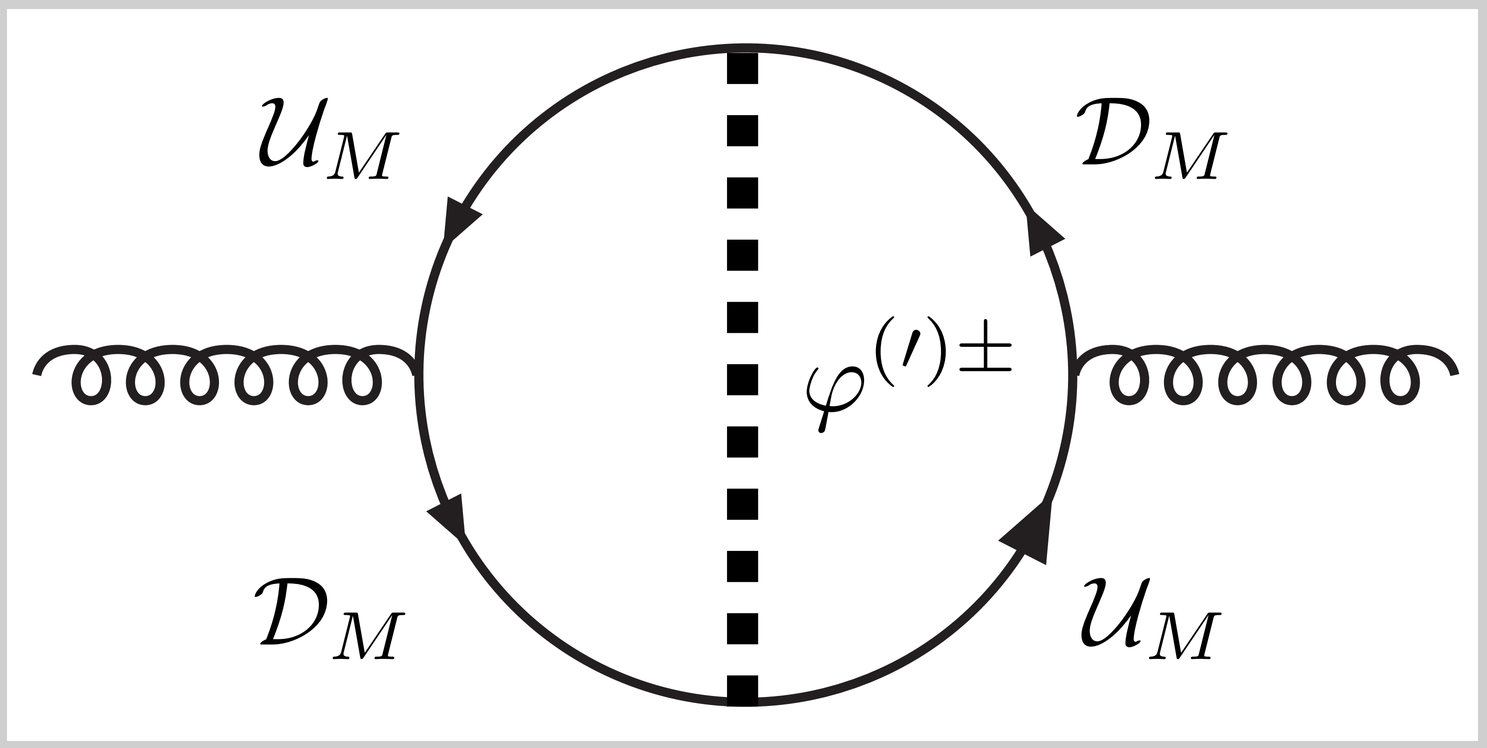


corrections to

$$\left(\overline{u}_L^i, \overline{U}_L^a\right)\begin{pmatrix} 0 & x_u^{ib}v \\ x_u^{\dagger aj}v' & M_u^a\delta^{ab}\end{pmatrix}\begin{pmatrix} u_R^j \\ U_R^b\end{pmatrix}$$

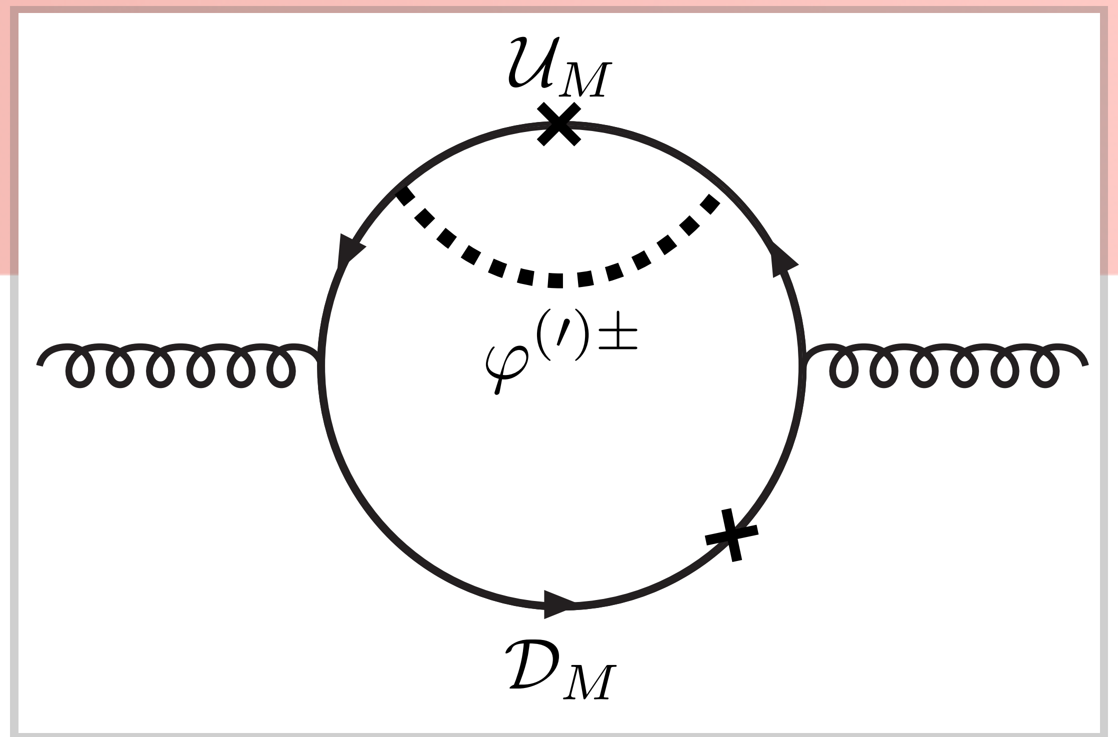
$$\left(\overline{d}_L^i, \overline{D}_L^a\right)\begin{pmatrix} 0 & x_d^{ib}v \\ x_d^{\dagger aj}v' & M_d^a\delta^{ab}\end{pmatrix}\begin{pmatrix} d_R^j \\ D_R^b\end{pmatrix}$$

in flavor basis



This diagram cannot be considered as the mass correction.

mass correction



$i\mathcal{M}_{A;\varphi'}$ $\quad (i, j \cdots :$ light flavor, $a, b \cdots :$ heavy flavor, $p, r :$ all flavor $)$

$$= -\frac{g_s^2}{2\left(16\pi^2\right)^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}(0)G^a_{\rho\sigma}(0)$$

$$\times \operatorname{Im}\left[\bar{M}^p_u\left(V^\dagger_{uL}\right)^{pa}x^{\dagger ai}_u\left(V_{dR}\right)^{ir}\bar{M}^r_d\left(V^\dagger_{dL}\right)^{rb}x^{\dagger bj}_d\left(V_{uR}\right)^{jp}J^1\left(\left(\bar{M}^r_d\right)^2,\left(\bar{M}^r_d\right)^2,\left(\bar{M}^r_d\right)^2,m^2_{W'},\left(\bar{M}^p_u\right)^2\right)\right]$$

$$= -\frac{g_s^2}{2\left(16\pi^2\right)^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}(0)G^a_{\rho\sigma}(0)$$

$$\times\left\{\operatorname{Im}\left[\bar{M}^p_u\left(V_{uL}\right)^{pa}x^{\dagger ai}_u\left(V^\dagger_{dR}\right)^{ic}\frac{1}{\bar{M}^c_d}\left(V_{dL}\right)^{cb}x^{\dagger bj}_d\left(V^\dagger_{uR}\right)^{jp}\left(\bar{M}^c_d\right)^2\bar{J}^1\left(\left(\bar{M}^c_d\right)^2,m^2_{W'},\left(\bar{M}^p_u\right)^2\right)\right]\right.$$

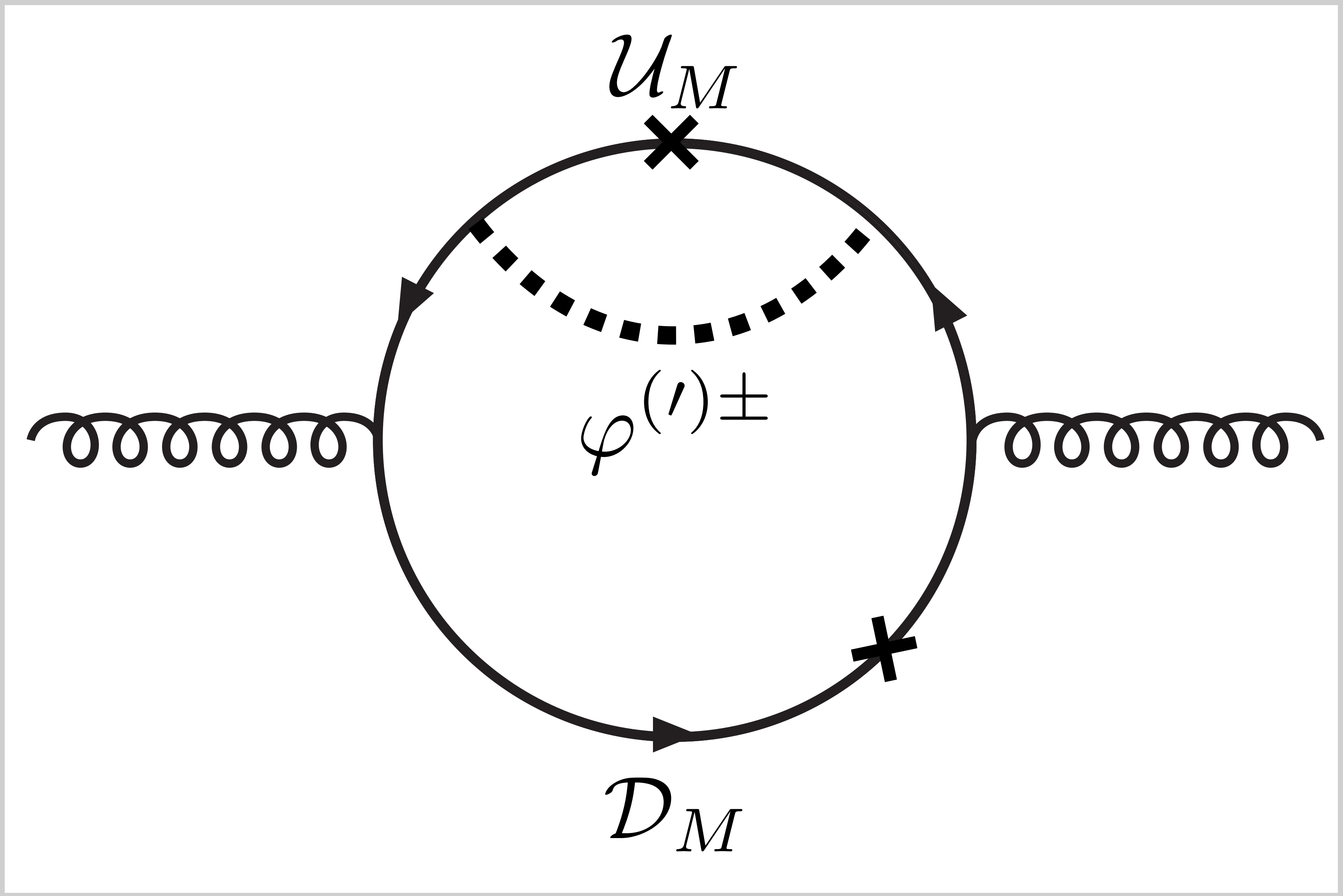
$$\left.-\operatorname{Im}\left[\bar{M}^p_u\left(V_{uL}\right)^{pa}x^{\dagger ai}_u\boxed{\left(V^\dagger_{dR}\right)^{ik}\frac{1}{\bar{M}^k_d}\left(V_{dL}\right)^{kb}}x^{\dagger bj}_d\left(V^\dagger_{uR}\right)^{jp}\frac{1}{8}F_0\left(0,m^2_{W'},\left(\bar{M}^p_u\right)^2\right)\right]\right\}$$

IR sensitive

$$\left(V^\dagger_{dR}\right)^{ik}\frac{1}{\bar{M}^k_d}\left(V_{dL}\right)^{kb} = \left(\mathcal{M}^{(0)-1}_d\right)^{ib} - \underbrace{\left(V^\dagger_{dR}\right)^{ic}\frac{1}{\bar{M}^c_d}\left(V_{dL}\right)^{cb}}_{v'/M}$$

$$\bar{\theta} = \sum_q \arg\operatorname{Det}[\mathcal{M}^{(0)}_q + \delta\mathcal{M}_q]$$
$$= \sum_q \arg\operatorname{Det}\left([\mathcal{M}^{(0)}_q] + [1 + \mathcal{M}^{(0)-1}_q\delta\mathcal{M}_q]\right)$$
$$\simeq \sum_q \operatorname{Im}\operatorname{Tr}[\boxed{\mathcal{M}^{(0)-1}_q}\delta\mathcal{M}_q]$$

Vanishment



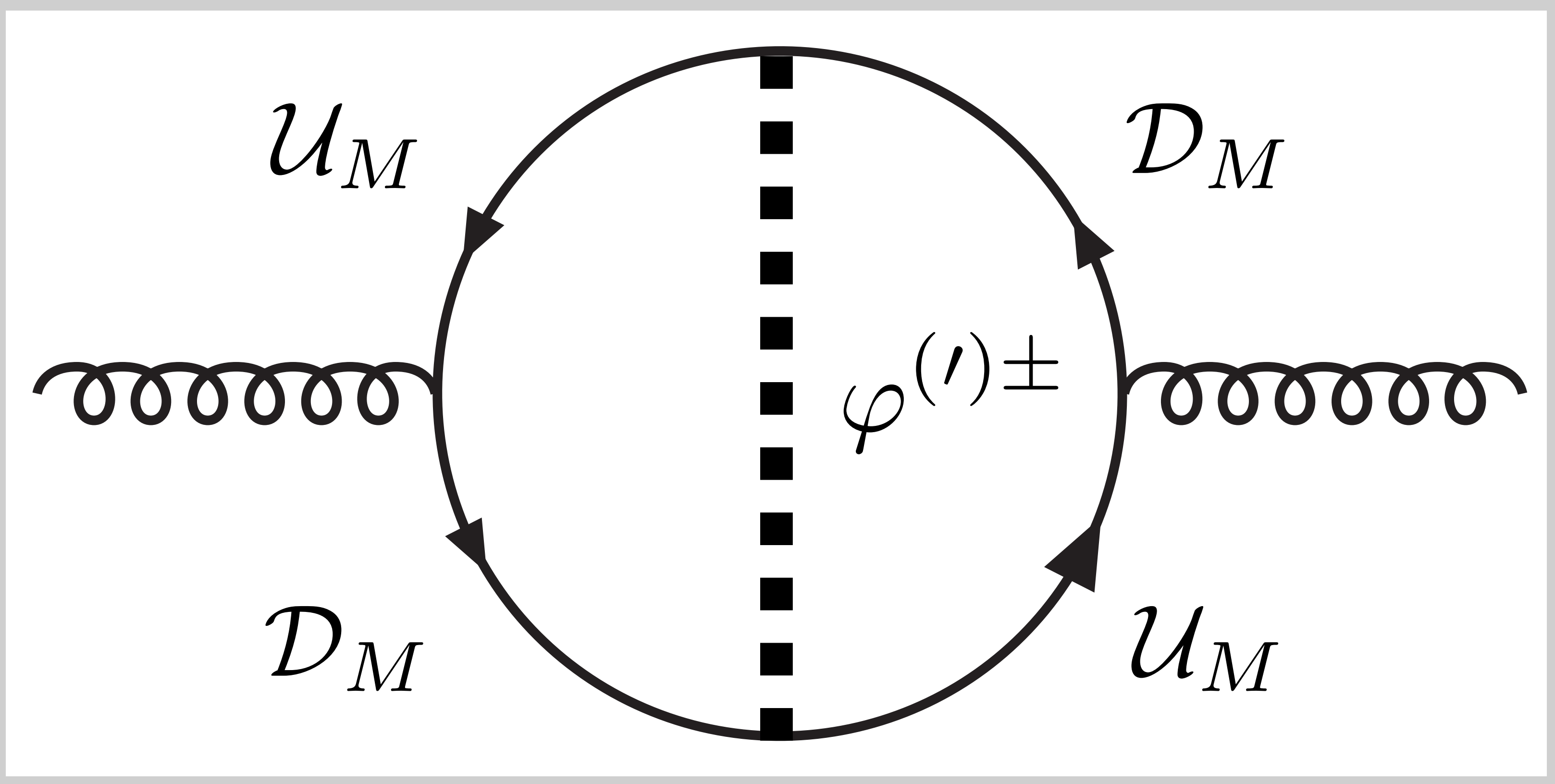
$$= -\frac{v'^2 g_s^2}{2(16\pi^2)^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}(0)G^a_{\rho\sigma}(0)\mathrm{Im}\left[x_u^{ja}x_u^{\dagger ai}x_d^{ib}x_d^{\dagger bj}\right] f\left((M_u^a)^2,(M_d^b)^2\right)$$

$$= 0$$

loop function

$$\mathrm{Im}\left[x_u^{ja}x_u^{\dagger ai}x_d^{ib}x_d^{\dagger bj}\right] f\left((M_u^a)^2,(M_d^b)^2\right)$$
$$= \frac{1}{2}\mathrm{Im}\left[x_u^{ja}x_u^{\dagger ai}x_d^{ib}x_d^{\dagger bj}\right] f\left((M_u^a)^2,(M_d^b)^2\right) - \frac{1}{2}\mathrm{Im}\left[x_d^{jb}x_d^{\dagger bi}x_u^{ia}x_u^{\dagger aj}\right] f\left((M_u^a)^2,(M_d^b)^2\right)$$
$$= \frac{1}{2}\mathrm{Im}\left[x_u^{ja}x_u^{\dagger ai}x_d^{ib}x_d^{\dagger bj} - x_u^{ia}x_u^{\dagger aj}x_d^{jb}x_d^{\dagger bi}\right] f\left((M_u^a)^2,(M_d^b)^2\right)$$
$$= 0$$

Vanishment



$$= -\frac{v'^2 g_s^2}{2(16\pi^2)^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}(0)G^a_{\rho\sigma}(0)\mathrm{Im}\left[x_u^{ja}x_u^{\dagger ai}x_d^{ib}x_d^{\dagger bj}\right] f\left((M_u^a)^2,(M_d^b)^2\right)$$

$$= 0$$

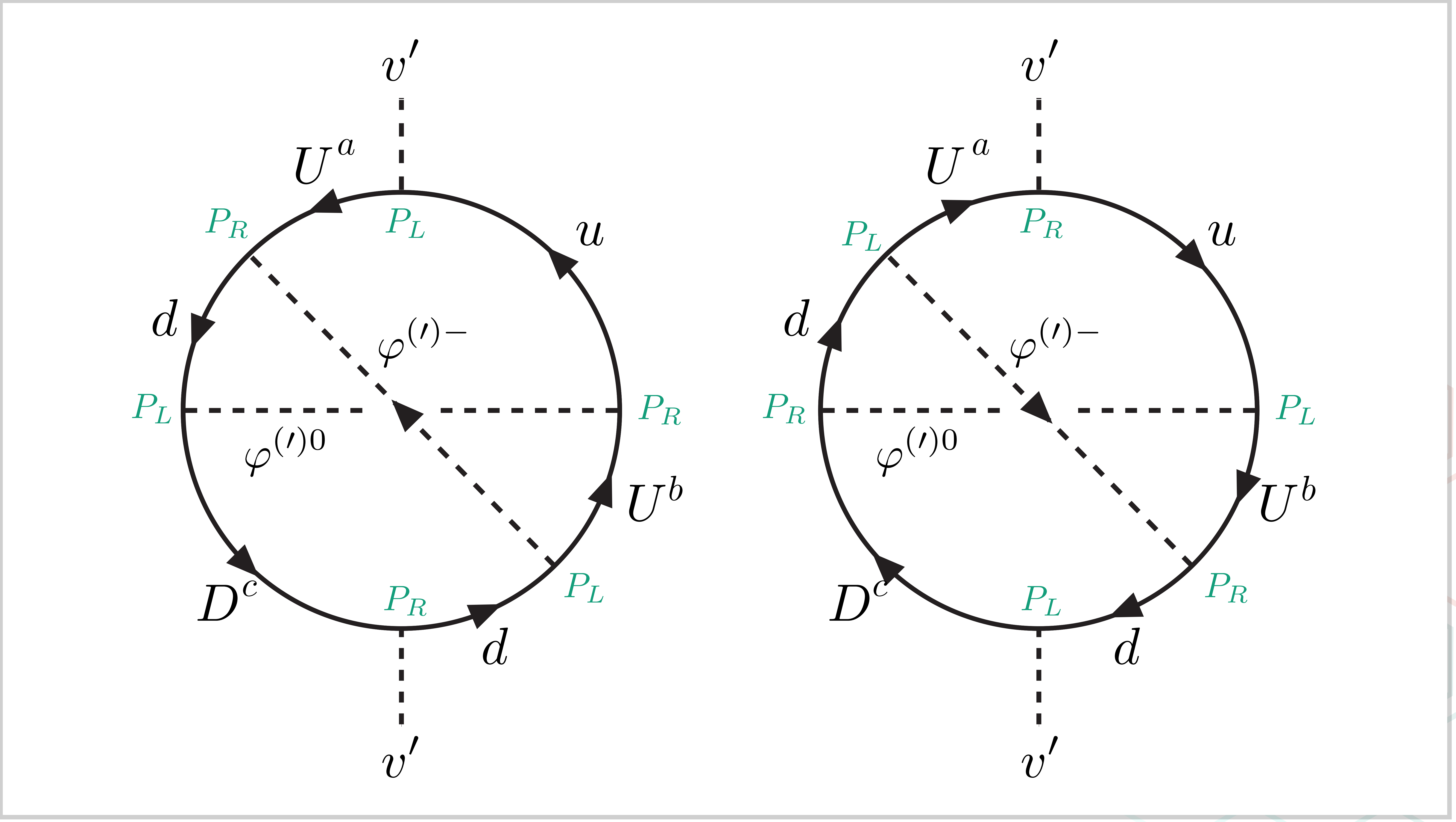
IR insensitive: $\cancel{\bar\theta = \sum_q \mathrm{Im}\ \mathrm{Tr}\left[\mathcal{M}_q^{(0)-1}\delta\mathcal{M}_q\right]}$

remaining problem(?):

Are there any other diagrams holding $\mathrm{Im}\ \mathrm{Tr}\left[x_u x_u^\dagger x_d x_d^\dagger\right]$**?**

Upper bound -duu-

$$\mathrm{Im}\,\mathrm{Tr}\left[\left(x_d^a x_d^{\dagger a}\right)\left[\left(x_u^b x_u^{\dagger b}\right),\left(x_u^c x_u^{\dagger c}\right)\right]\right] f(M_d^a, M_u^b, M_u^c)$$



Collider & Flavor constraints

- **ATLAS, charged lepton and missing** ⟶ **charged boson mass $m_{W'}$**

$$m_{W'} \gtrsim 6\mathrm{TeV}, \quad v' \gtrsim 18\mathrm{TeV}$$

- **Future Circular Collider (FCC), 100TeV pp collider**

$$m_{W'}, m_{Z'} \sim 40\mathrm{TeV}, \quad v' \gtrsim 120\mathrm{TeV}$$

: fine-tuning problem in the scalar potential

- **one-loop FCNCs, kaon mixing**

$$(\Delta m_K)_{u,c} \approx -6 \cdot 10^{-16}\mathrm{GeV}\left(\frac{6\mathrm{TeV}}{m_{W'}}\right)^2, \quad |\epsilon_K|_{u,c} \approx 7 \cdot 10^{-5}\left(\frac{6\mathrm{TeV}}{m_{W'}}\right)^2$$

an order of magnitude below the theoretical error in the SM prediction

N. Craig, I. Garcia Garcia, G. Koszegi, and A. McCune, JHEP 09 (2021) 130

B anomaly in the LR model

$R(D), R(D^*)$ **anomaly**

$$R(D)=\frac{\Gamma(B\to D\tau\nu)}{\Gamma(B\to D\ell\nu)},\quad R\left(D^*\right)=\frac{\Gamma\left(B\to D^*\tau\nu\right)}{\Gamma\left(B\to D^*\ell\nu\right)}$$

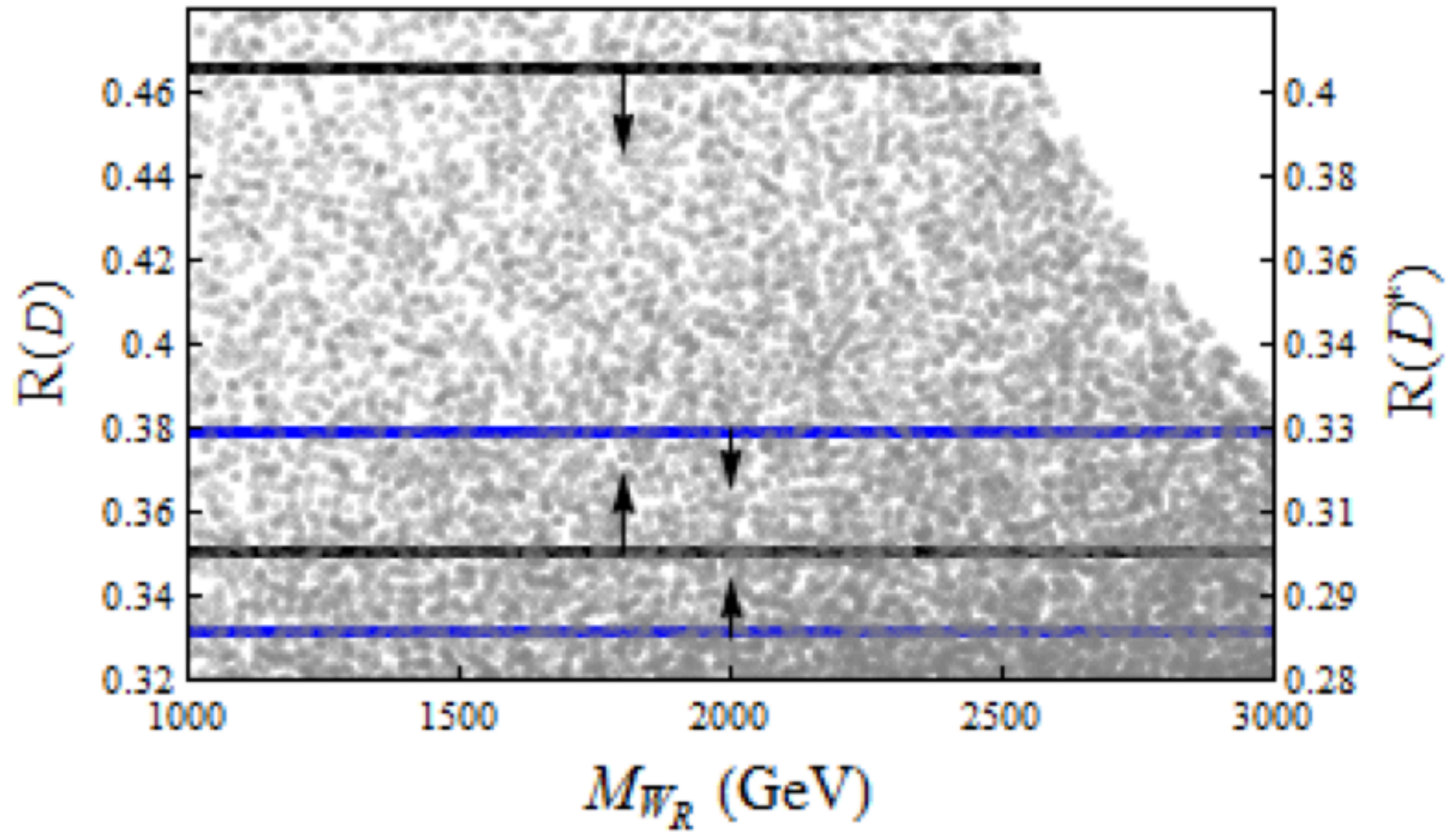


Figure 4: $R(D,D^*)$ scatter-plot is shown by varying g_R and M_{W_R}. The boundaries of $R(D)$ and $R(D^*)$ anomalies are shown by black and blue lines respectively. We show 1 σ allowed regions.

K. S. Babu, B. Dutta and R. N. Mohapatra, JHEP 01 (2019), 168