Lecture 2

Historical remarks

Sakharov realized that CP violation is one of the necessary conditions of the excess of matter over antimatter in the Universe	JETP Lett. 6, 21 1967
Kobayashi, Maskawa suggested method to introduce the CP violation in the Standard Model	Prog.Theor.Phys. 49, 652 1973
Carter and Sanda suggested way to search for CP violation in B meson decays	PRL 45, 952 1980

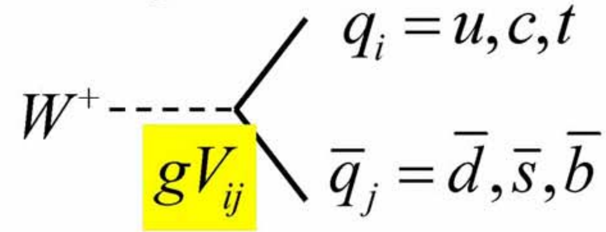
The CKM paradigm in the SM

(1973) M.Kobayashi and T.Maskawa

- CP violation $\Rightarrow$ third generation of quarks

u	c	t
d	s	b

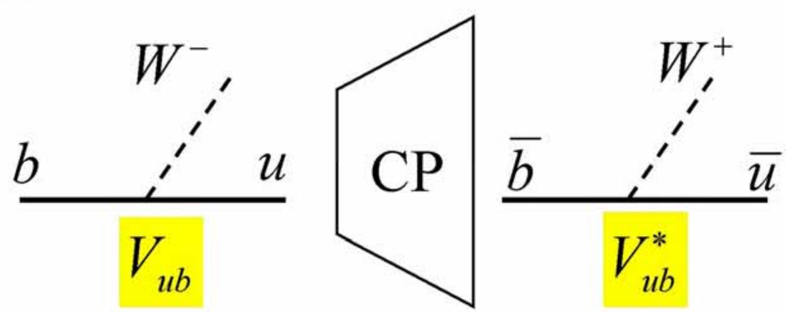
quark doublets



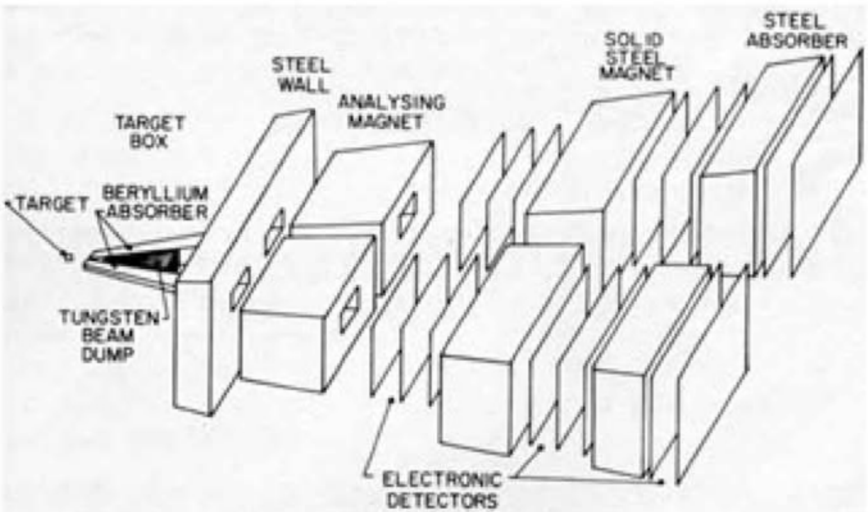
Cabibbo-Kobayashi-Maskawa matrix V

- couples quark charged currents to $W^{\pm}$
- mixes the left-handed (q_j=d,s,b) quark mass eigenstates to give weak eigenstates;
- unitary, with 4 independent parameters (e.g., 3 angles and 1 phase)
- complex elements: phase changes sign under CP
- interfering amplitudes can give observable CP-violating rate asymmetries

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Discovery of b quarks



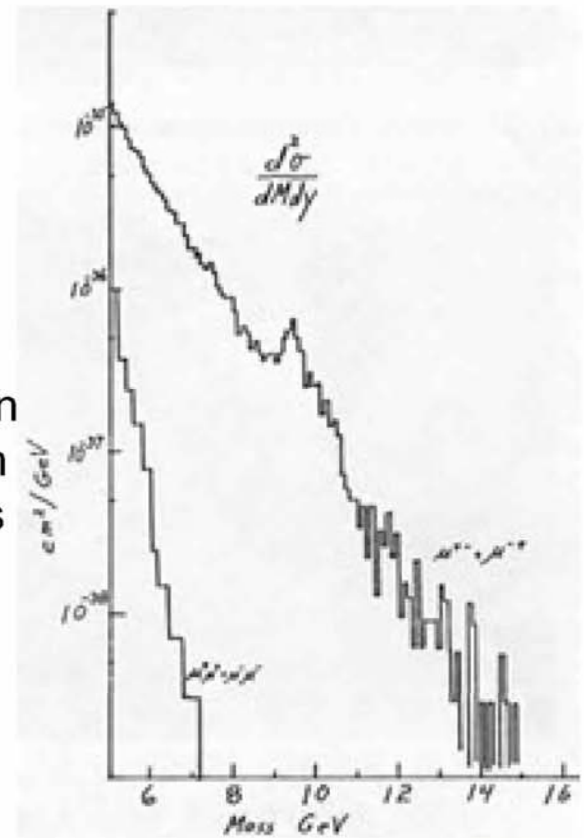
Summer 1977 at FNAL:

Discovery of $\Upsilon(9.46) \rightarrow \mu^+\mu^-$

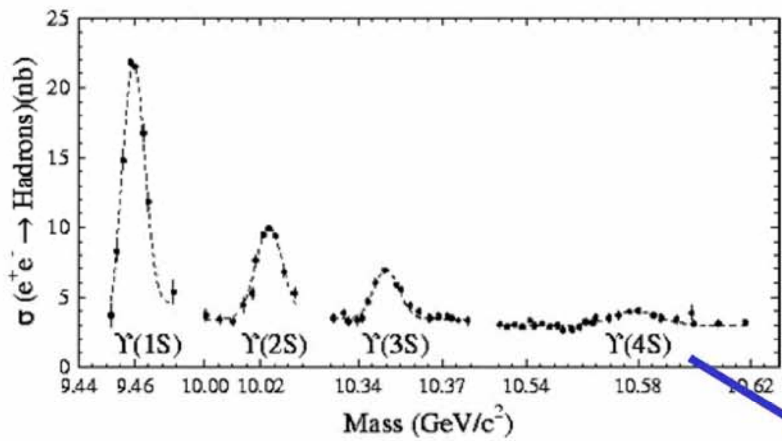
interpreted as 1^3S_1 $b\bar{b}$

"Observation of a Dimuon Resonance at 9.5 GeV in 400 GeV Proton-Nucleus Collisions,"

PRL 39, p. 252, (1977)



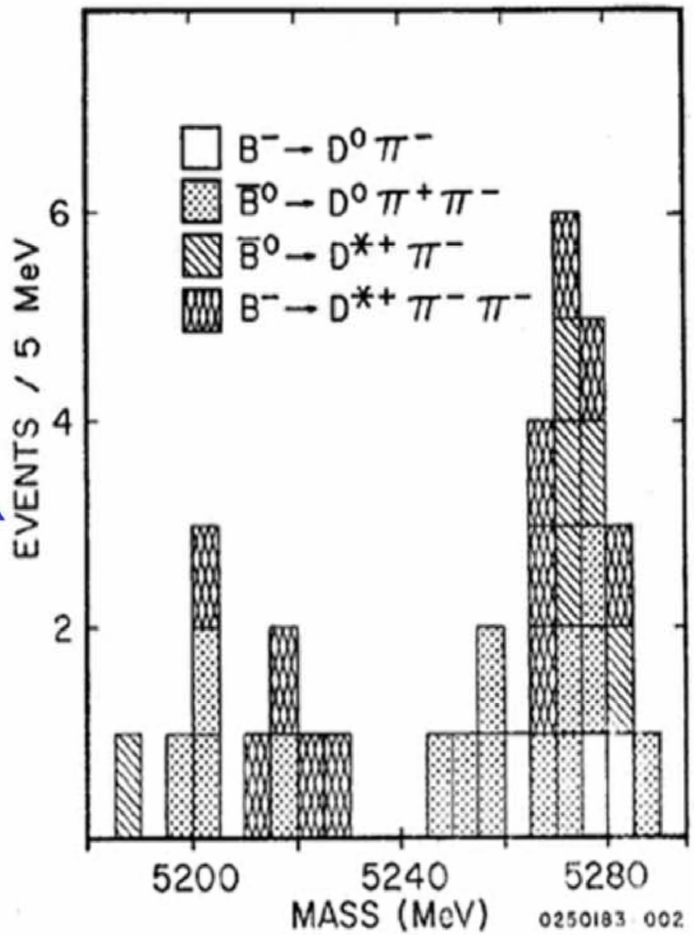
B meson production at e^+e^- colliders



CESR at Cornell:
"naked beauty"

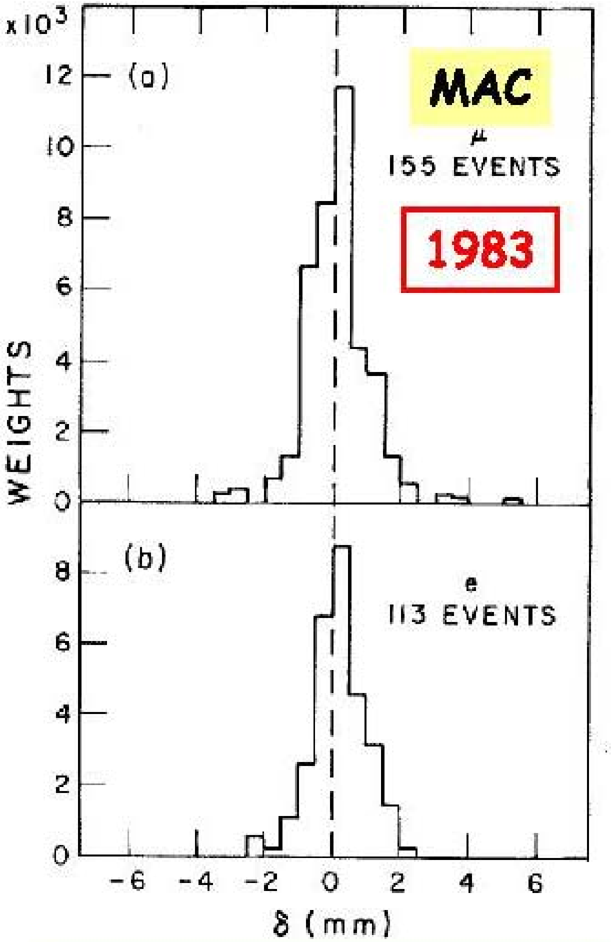
$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\overline{B}$

CLEO Collaboration, "Observation of Exclusive Decay Modes of b-Flavored Mesons", *PRL* 50, p. 881 (1983)



b lifetimes

- *Isolate samples of high-p_T leptons (155 muons, 113 electrons) wrt thrust axis*
  - Measure impact parameter δ wrt interaction point
  - Signed by taking thrust axis of *b*-jet as the *B* hadron direction
- *Lifetime implies V_{cb} small*
  - MAC: (1.8±0.6 ±0.4) ps
  - Mark II: (1.2±0.4 ±0.3) ps
- *Integrated luminosity at 29 GeV:*
  - 109 (92) pb^{-1} ~ 3,500 $b\bar{b}$ pairs



MAC, PRL **51**, 1022 (1983)
MARK II, PRL **51**, 1316 (1983)

CKM matrix and Unitarity Triangle

Wolfenstein parameterization:

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3[1-(1-\frac{1}{2}\lambda^2)(\rho+i\eta)] & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\rho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} \begin{matrix} u \\ c \\ t \end{matrix}$$

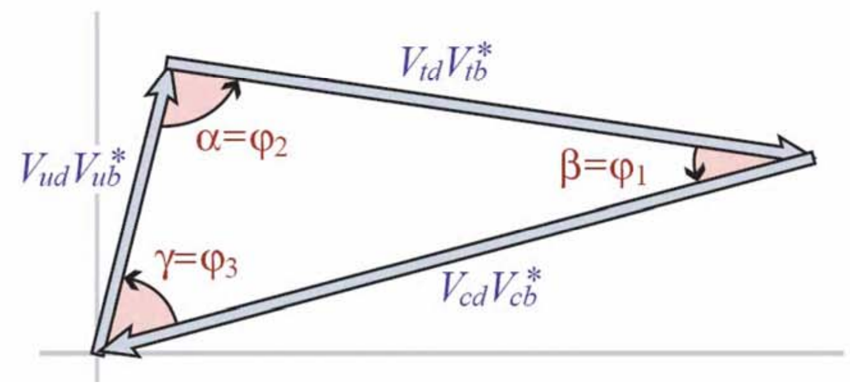
$$\begin{matrix} d & & s & & b \end{matrix}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\ .$$

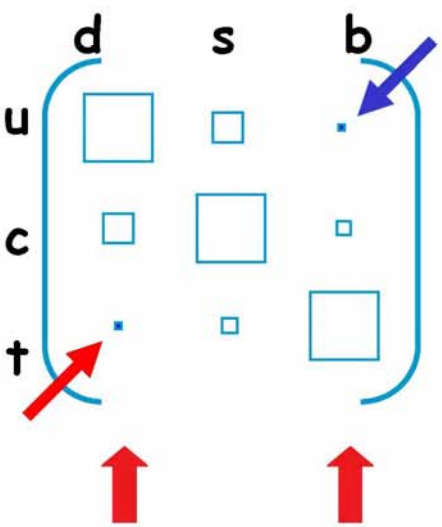
$$\alpha \equiv \varphi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right),$$

$$\beta \equiv \varphi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$$

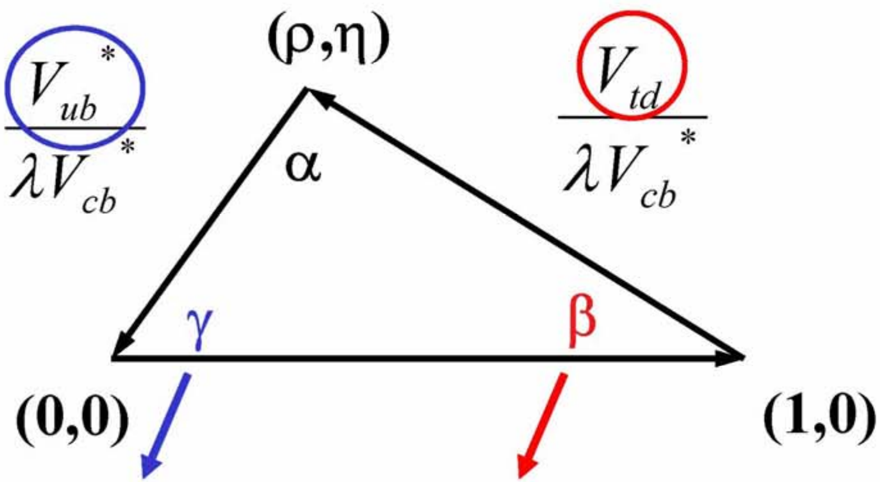
$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right),$$



The "normalized" Unitarity Triangle



apply unitarity constraint to these two columns



$$\approx \frac{V_{ub}^*}{\lambda V_{cb}^*} \qquad \frac{V_{td}}{\lambda V_{cb}^*}$$

$$\gamma \approx \arg V_{ub}^* \quad \beta \approx -\arg V_{td} \quad \alpha = \pi - \beta - \gamma$$

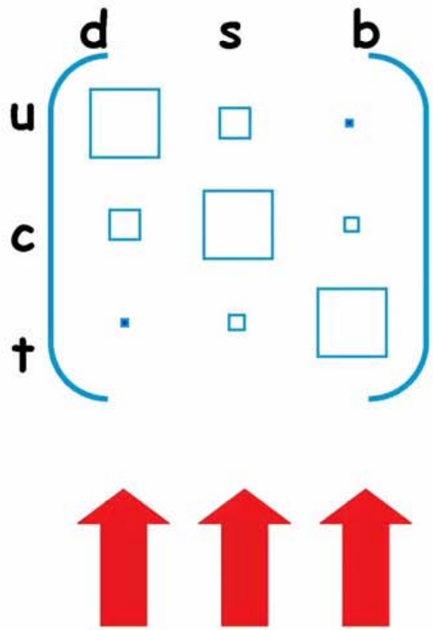
Orders of magnitude for Wolfenstein parameters:

$$\lambda \approx 0.22\,, \quad A \approx 0.8\,, \quad \sqrt{\rho^2 + \eta^2} \approx 0.4$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

$$V_{cd} = \lambda\,, \quad V_{ud} \approx V_{tb} \approx 1$$

The Unitarity Triangles



apply unitarity constraint to pairs of columns

$d \cdot s^* = 0$ (K system)

$s \cdot b^* = 0$ (B_s system)

$d \cdot b^* = 0$ (B_d system)

These three triangles (and the three triangles corresponding to the rows) all have the same area. A nonzero area is a measure of CP violation and is an invariant of the CKM matrix.

Mixing for $P = K, D, B$

Effective Hamiltonian approximation:

"dispersive" ↓ "absorptive"

$$i\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}; \quad P^0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \overline{P}^0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad H_{ij} = M_{ij} - i\Gamma_{ij}/2$$

From flavor to mass eigenstates (P_L, P_H) ≈ CP eigenstates (P_1, P_2):

$$\left|P_L^0\right\rangle = p\left|P^0\right\rangle + q\left|\overline{P}^0\right\rangle = \frac{1}{\sqrt{1+|\tilde{\varepsilon}|^2}}\left(\tilde{\varepsilon}\left|P_1\right\rangle + \left|P_2\right\rangle\right) \qquad \tilde{\varepsilon} = \frac{p-q}{p+q}$$

$$\left|P_H^0\right\rangle = p\left|P^0\right\rangle - q\left|\overline{P}^0\right\rangle = \frac{1}{\sqrt{1+|\tilde{\varepsilon}|^2}}\left(\left|P_1\right\rangle + \tilde{\varepsilon}\left|P_2\right\rangle\right) \qquad |q|^2 + |p|^2 = 1$$

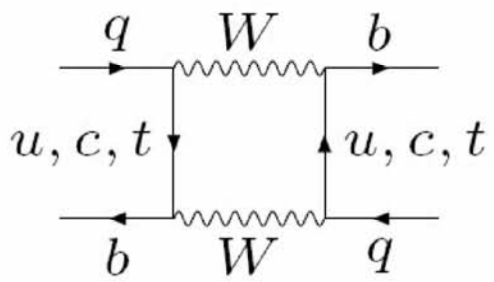
Solving the eigenvalue equations and defining: $\Delta m = m_H - m_L \quad \Delta\Gamma = \Gamma_H - \Gamma_L$

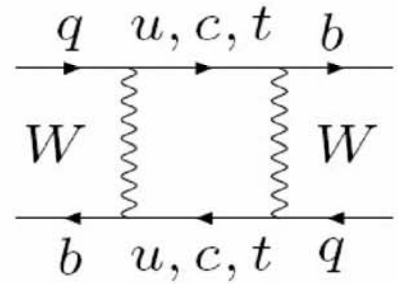
$$\Delta m^2 - 1/4\,\Delta\Gamma^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2$$

$$\Delta m \Delta\Gamma = 4\Re\mathrm{e}\left(M_{12}\Gamma_{12}^*\right)$$

q, p, Δm and $\Delta\Gamma$ for B_d and B_s

$B_d^0 = (\bar{b}d)$
$B_s^0 = (\bar{b}s)$





$\bar{B}_d^0 = (b\bar{d})$
$\bar{B}_s^0 = (b\bar{s})$

In the SM for B mesons:

M_{12} dominated by the top quark

Γ_{12} few common on-shell states

$$\Gamma_{12}/M_{12} << 1$$

$$\Rightarrow \Delta m \approx 2|M_{12}| \qquad \Delta\Gamma \approx \frac{2\Re e(M_{12}\Gamma_{12}^*)}{|M_{12}|} << \Delta m \qquad \frac{q}{p} = -\frac{\Delta m - i/2\Delta\Gamma}{2M_{12} - i\Gamma_{12}} \approx -\frac{|M_{12}|}{M_{12}}$$

CP-violating parameter: $\delta = |p|^2 - |q|^2 = \langle P_H | P_L \rangle = \dfrac{2\Im m(M_{12}^*\Gamma_{12})}{(\Delta m)^2 + |\Gamma_{12}|^2} \approx 10^{-3}$

Time evolution of neutral *B* mesons - 1

(assuming CPT as a good symmetry, for simplicity)…

Time evolution of mass eigenstates:

$$\left|B_L^0(t)\right\rangle = e^{-t\Gamma_B/2}\ e^{-itM_B}\ e^{+it\Delta m_B/2}\left|B_L^0(0)\right\rangle$$

$$\left|B_H^0(t)\right\rangle = e^{-t\Gamma_B/2}\ e^{-itM_B}\ e^{-it\Delta m_B/2}\left|B_H^0(0)\right\rangle$$

Time evolution of initially (t=0) pure flavour eigenstates:

$$\left|B_{phys}^0(t)\right\rangle = h_+(t)\left|B^0\right\rangle + \frac{q}{p}h_-(t)\left|\overline{B}^0\right\rangle$$

$$\left|\overline{B}_{phys}^0(t)\right\rangle = \frac{p}{q}h_-(t)\left|B^0\right\rangle + h_+(t)\left|\overline{B}^0\right\rangle$$

$$h_+(t) = e^{-t\Gamma_B/2}\ e^{-itM_B}\ \cos(t\,\Delta m_B/2)$$

$$h_-(t) = i\left[e^{-t\Gamma_B/2}\ e^{-itM_B}\ \sin(t\,\Delta m_B/2)\right]$$

Time evolution of neutral *B* mesons - 2

Flavour oscillations: for initially pure $B^0(t=0)$,
probability for finding $B^0\left(\overline{B}^0\right)$ at time t, assuming $|q/p|=1$

$$|h_\pm(t)|^2 = \frac{1}{2} e^{-t\Gamma_B}\left[1 \pm \cos(t\,\Delta m_B)\right] \;\Rightarrow\; a_{mix}(t) = \cos(t\,\Delta m) = \cos(x\Gamma t)$$

Time-integrated ratio and time-integrated oscillation probability:

$$r = \frac{N\left(\overline{B}^0\right)}{N\left(B^0\right)} = \frac{\int_0^\infty dt\,|h_-(t)|^2}{\int_0^\infty dt\,|h_+(t)|^2} = \frac{x^2}{2+x^2}, \quad \chi = \frac{r}{1+r} = P\left(B^0 \to \overline{B}^0\right), \quad x \equiv \frac{\Delta m}{\Gamma}$$

Observable by looking at self-flavour tagging semileptonic or hadronic decays! For example:

$$B^0 \to D^{*-} l^+ \nu \qquad \overline{B}^0 \to D^{*+} l^- \overline{\nu}$$

$$B^0 \to D^- \pi^+ \qquad \overline{B}^0 \to D^+ \pi^-$$

$$B_s^0 \to D_s^- l^+ \nu \qquad \overline{B}_s^0 \to D_s^+ l^- \overline{\nu}$$

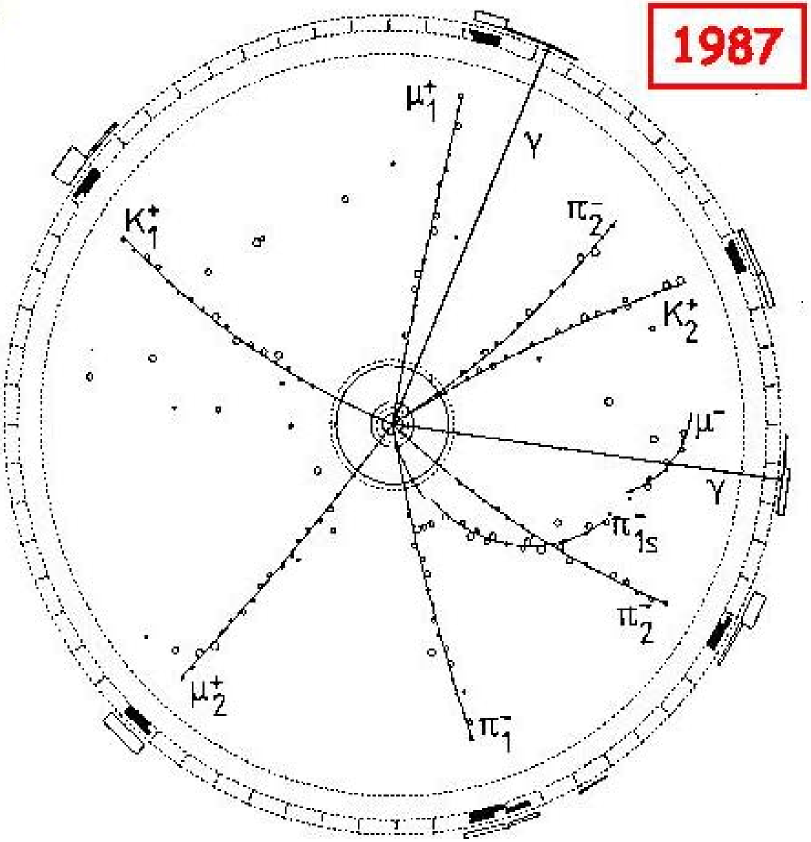
BB oscillations

- *Reconstructed $\Upsilon(4S)$ event*

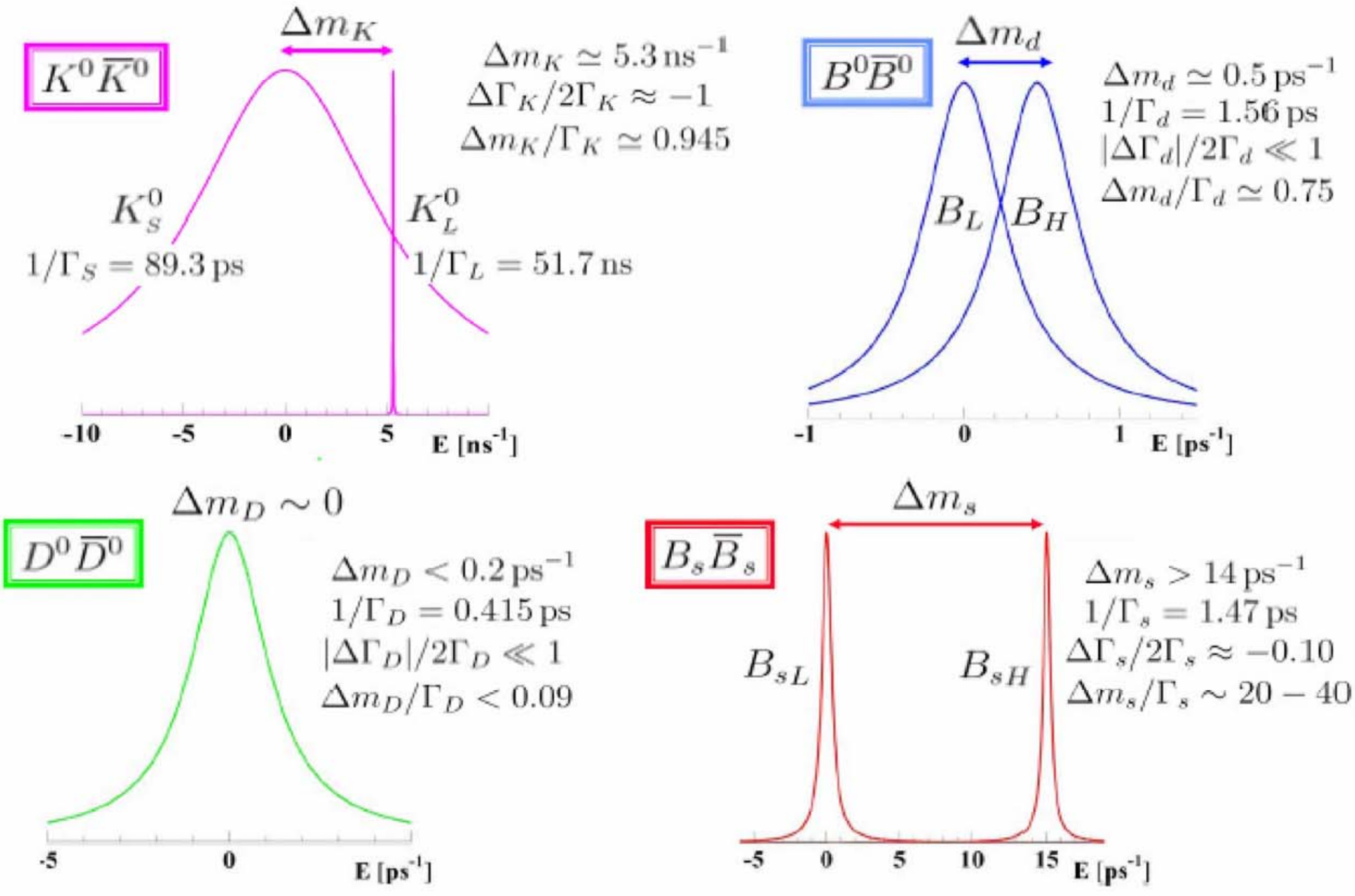
$$\Upsilon(4S) \to B^0 \bar{B}^0 \to B_1^0 B_2^0$$
$$B_1^0 \to D_1^{*-} \mu_1^+ \nu_1,\ D_1^{*-} \to \bar{D}^0 \pi_1^-$$
$$B_2^0 \to D_2^{*-} \mu_2^+ \nu_2,\ D_1^{*-} \to D^- \pi^0$$

- *Time-integrated 21% mixing rate*
  - 25 (270) like (opposite) sign dilepton events
  - 4.1 lepton-tagged semileptonic B decays
- *Integrated $\Upsilon(4S)$ luminosity 1983-87:*
  - 103 pb^{-1} ~ 110,000 B pairs

1987



ARGUS, PL B **192**, 245 (1987)

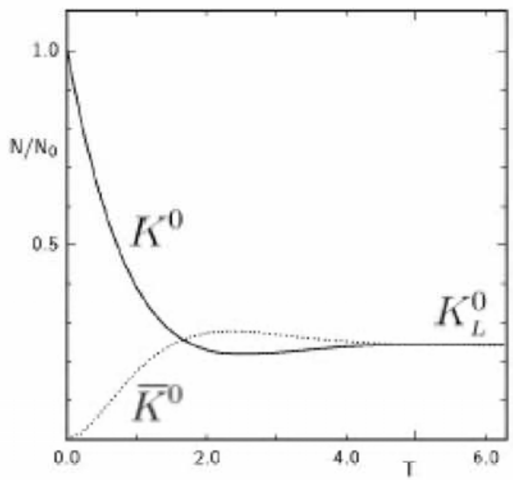


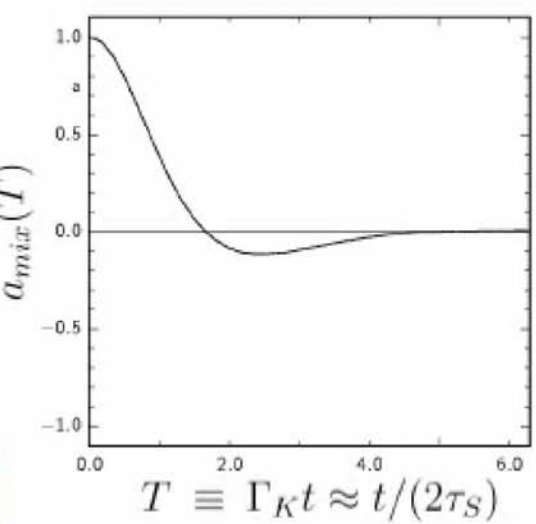
$K^0\overline{K}^0$
Δm_K
$\Delta m_K \simeq 5.3\,\mathrm{ns}^{-1}$
$\Delta\Gamma_K/2\Gamma_K \approx -1$
$\Delta m_K/\Gamma_K \simeq 0.945$
K_S^0
$1/\Gamma_S = 89.3\,\mathrm{ps}$
K_L^0
$1/\Gamma_L = 51.7\,\mathrm{ns}$
-10 -5 0 5 E [ns^{-1}]
$B^0\overline{B}^0$
Δm_d
$\Delta m_d \simeq 0.5\,\mathrm{ps}^{-1}$
$1/\Gamma_d = 1.56\,\mathrm{ps}$
$|\Delta\Gamma_d|/2\Gamma_d \ll 1$
$\Delta m_d/\Gamma_d \simeq 0.75$
B_L B_H
-1 0 1 E [ps^{-1}]
$D^0\overline{D}^0$
$\Delta m_D \sim 0$
$\Delta m_D < 0.2\,\mathrm{ps}^{-1}$
$1/\Gamma_D = 0.415\,\mathrm{ps}$
$|\Delta\Gamma_D|/2\Gamma_D \ll 1$
$\Delta m_D/\Gamma_D < 0.09$
-5 0 E [ps^{-1}]
$B_s\overline{B}_s$
Δm_s
$\Delta m_s > 14\,\mathrm{ps}^{-1}$
$1/\Gamma_s = 1.47\,\mathrm{ps}$
$\Delta\Gamma_s/2\Gamma_s \approx -0.10$
$\Delta m_s/\Gamma_s \sim 20-40$
B_{sL} B_{sH}
-5 0 5 10 15 E [ps^{-1}]

mixing

K⁰ system

CPV





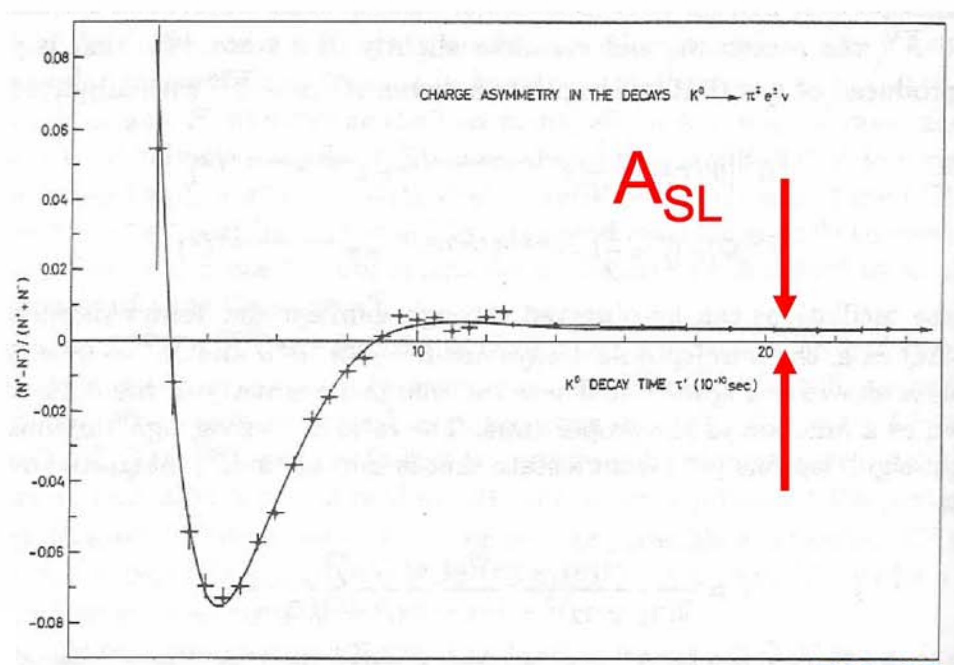
$$x_K \cong 0.95$$

$$y_K \cong -0.996$$

Both of order unity!
Only K^o_L is left after
≈ one oscillation

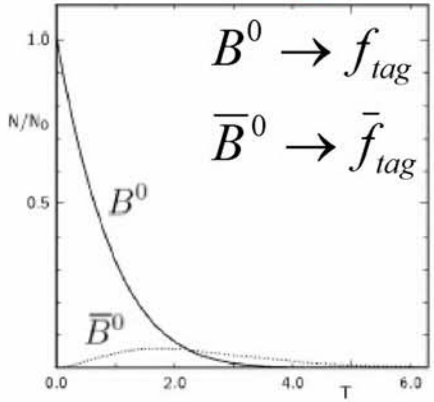
CPV is small…

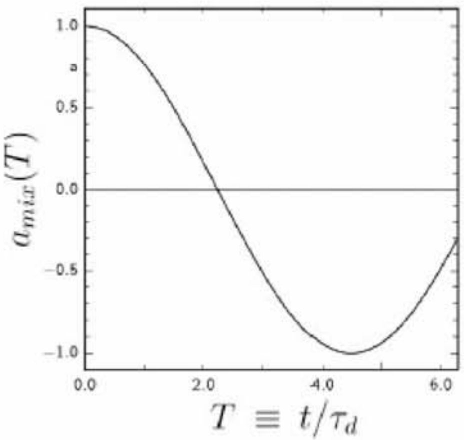
$$\delta_K = \frac{2\,\mathrm{Re}(\varepsilon_K)}{1+|\varepsilon_K|^2} \cong 3\times 10^{-3}$$



B_d system

mixing





$$x_d = 0.72 \pm 0.03$$

$$\chi_d = \frac{x_d^2}{2(1+x_d^2)} \cong 18\%$$

To a very good approx., equal decay widths and no CPV in mixing

$$y_d \approx 0$$

$$\delta_d \approx 0$$

$$a_{mix}(t) = \cos(\Delta m\, t) = \cos(x_d\, t/\tau_d)$$

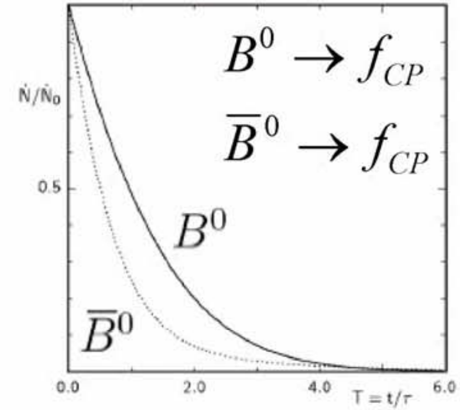
In the simplest case, time-dependent CP asymmetry:

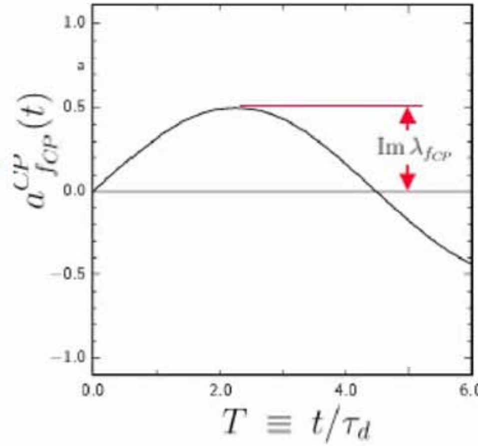
$$a_{f_{CP}}^{CP}(t) = \mathrm{Im}(\lambda_{f_{CP}})\sin(\Delta m\, t)$$

Time-integrated (incoherent!):

$$A_{f_{CP}}^{CP} = \frac{x_d}{1+x_d^2}\mathrm{Im}(\lambda_{f_{CP}})$$

CPV



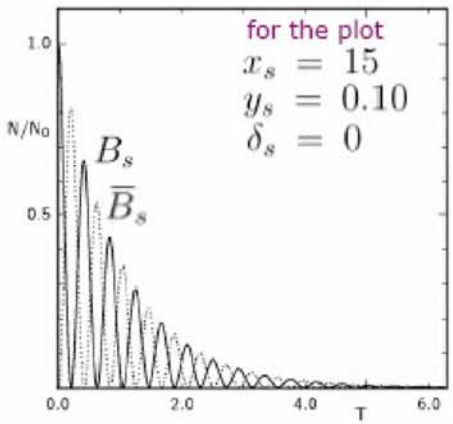


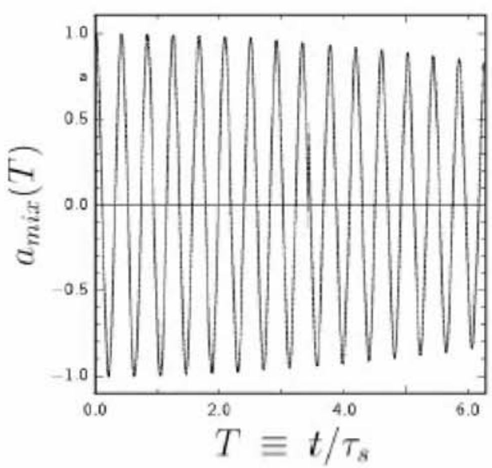
$$\frac{x_d}{1+x_d^2} \cong 0.47$$

mixing

B_s system

CPV





x_s is very large

y_s small, perhaps not negligible

$$x_s > 21 \quad (95\%\, CL)$$

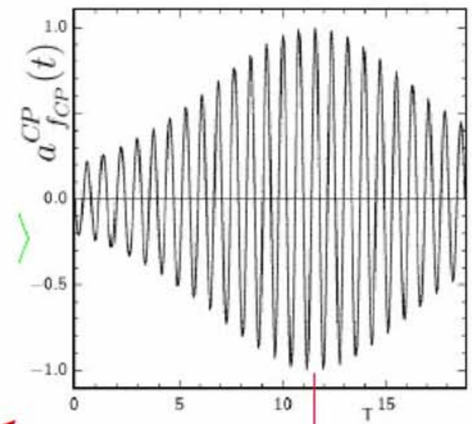
$$2y_s < 0.46 \quad (95\%\, CL)$$

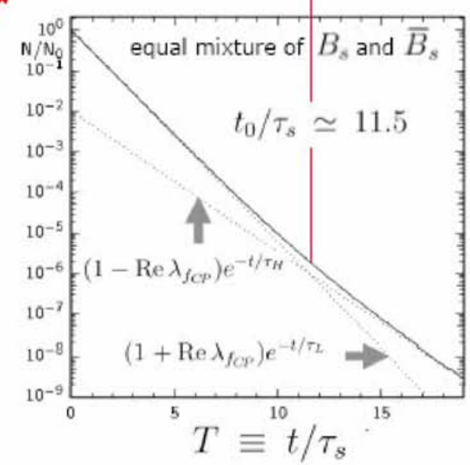
Mixing probability close to 50%

$$\chi_s = \frac{x_s^2 + y_s^2}{2(1+x_s^2)} > 0.4988$$

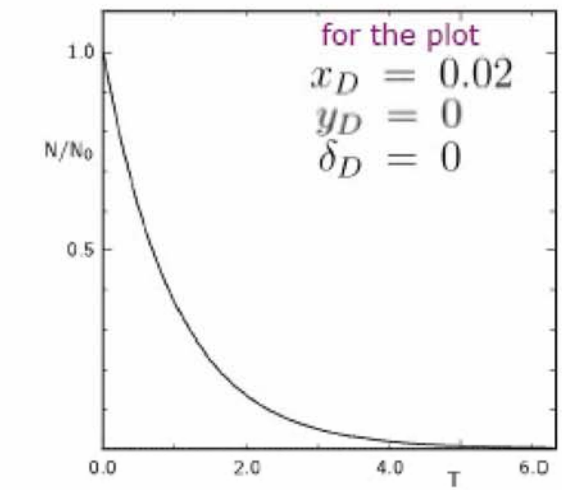
Time-dependent CP-asymmetry:
sinusoidal function, modulated by
a function f(t); 100% at the max.!

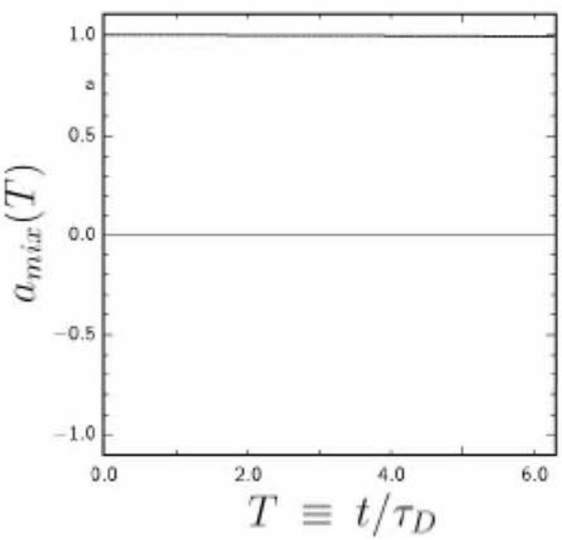
$$a_{f_{CP}}^{CP}(t) = \mathrm{Im}(\lambda_{f_{CP}})\sin(\Delta m\, t) f(t)$$





The D System





In the D^0-$\overline{D}^0$ system, both are very small

- y_D very small: only few common states

$$CP = +1 \quad \pi\pi,\ K\overline{K}, K_L^0\pi^0$$
$$CP = -1 \quad K_S^0\pi^0,\ K_S^0\omega$$

- x_D very small: strongly CKM suppressed

Mixing probability extremely small $\chi = \dfrac{x_D^2}{2(1+x_D^2)}$

interesting system to look for new physics

$$a_{mix}(t) \approx 1 - \frac{x_D^2 + y_D^2}{2} (t/\tau)^2$$