Physics of τ Lepton Decays into Hadrons

Simon Eidelman

Budker Institute of Nuclear Physics,
Novosibirsk, Russia
and
Nagoya University,
Nagoya, Japan

Outline

1. General properties of τ lepton
2. Cabibbo-favored decays. CVC
3. Decays with η mesons
4. Decays with kaons
5. Conclusions

Our Goal or Six "What"?

- What hadronic decays exist (or should exist)?
- What is their experimental status?
- What are the branching fractions?
- What are their mechanisms?
- What are theoretical predictions?
- What should we focus on at Belle?

Today we mainly discuss Cabibbo-favored decays,
in particular decays into pion final states

Leptons And Quarks

Particle	e^-	u	μ^-	c	τ^-	t
Q. number	+1	+1/2	+1	+1	+1	+1
Flavor	Electron	Isospin	Muon	Charm	τ	Topness
Particle	ν_e	d	ν_μ	s	ν_τ	b
Q. number	+1	-1/2	+1	-1	+1	-1
Flavor	Electron	Isospin	Muon	Strangeness	τ	Bottomness

$$LF(e^-) = LF(\nu_e) \quad LF(\mu^-) = LF(\nu_\mu) \quad LF(\tau^-) = LF(\nu_\tau)$$

$$LF(e^-) \neq LF(\mu^-) \neq LF(\tau^-), \quad LF(\nu_e) \neq LF(\nu_\mu) \neq LF(\nu_\tau).$$

General

- τ lepton and its neutrino ν_τ are two of the six fundamental leptons: e^-, ν_e, μ^-, ν_μ, τ^-, ν_τ
- As the heaviest lepton, τ may decay into both leptons and hadrons: PDG-2006 lists more than 200 different τ decays
- We can study all interactions allowed in the Standard Model and search for effects of New Physics
- It is a very clean laboratory with no hadrons in the initial and only a few in the final state

τ Lepton Factories

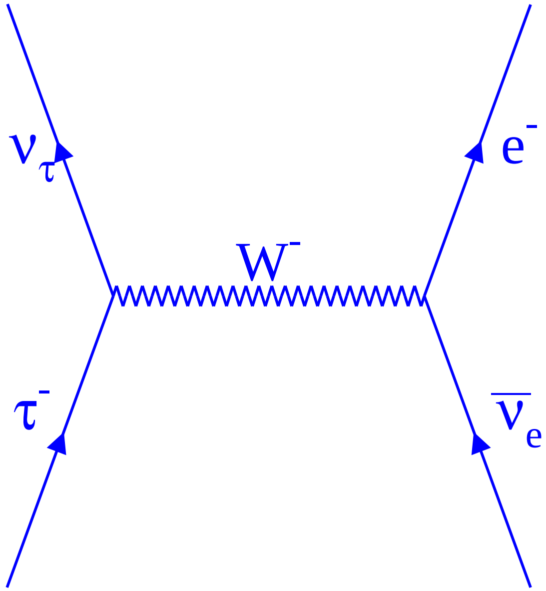
Group	$\int L\, dt$, fb^{-1}	$N_{\tau\tau}, 10^6$
LEP (Z-peak)	0.34	0.33
CLEO (10.6 GeV)	13.8	12.6
BaBar (10.6 GeV)	221	197
Belle (10.6 GeV)	535	477
τ-c (4.2 GeV)	10	32
SuperB	50k	45k

BaBar ($\sim$ 400 fb^{-1}) and Belle ($\sim$ 700 fb^{-1}) collected together about 1.1 ab^{-1}

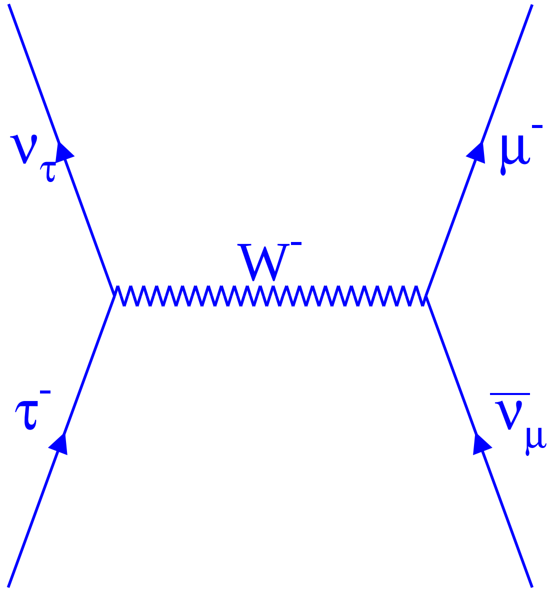
B-factory is also a τ factory producing $0.9 \cdot 10^6$ $\tau^+\tau^-$ pairs per each fb^{-1}!!

Leptonic Decays of τ in the Standard Model

$\tau^- \to e^- \bar{\nu}_e \nu_\tau$



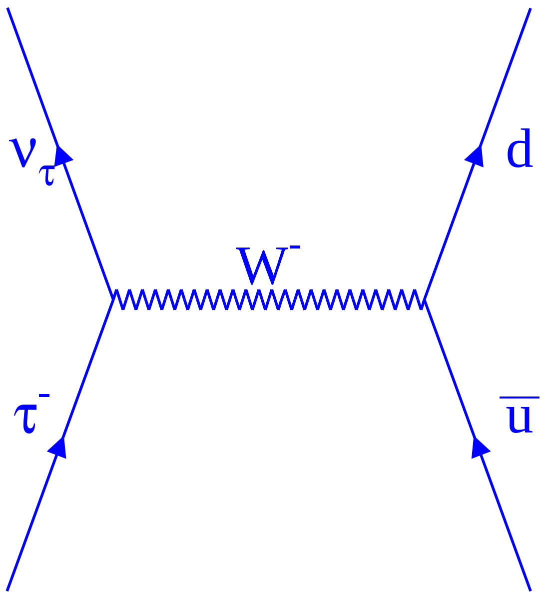
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$



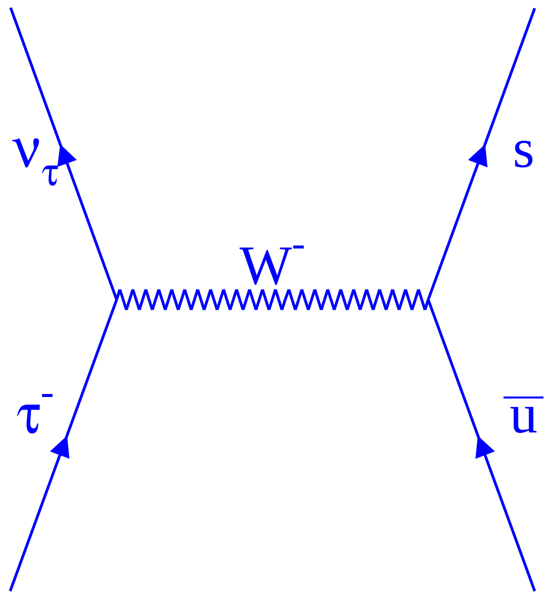
Each $W l \nu_l$ vertex includes the Fermi constant $G_F = 1.16637 \cdot 10^{-5}$ GeV^{-2}

Hadronic Decays of τ in the Standard Model

Cabibbo-favored decays



Cabibbo-suppressed decays

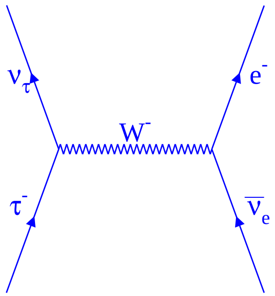


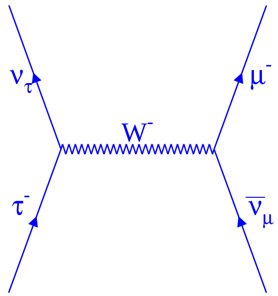
$\mathcal{B}$ of Cabibbo-favored decays $\propto cos\theta_C$, while for Cabibbo-suppressed $\propto cos\theta_C$.

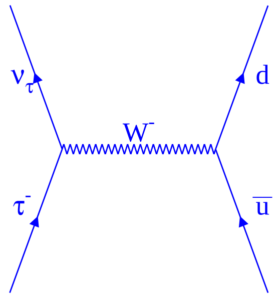
$cos\theta_C \approx |V_{ud}| = 0.97377 \pm 0.00027$, so the suppression $\propto \; tan^2\theta_C \; \approx 18$.

Each Wud (Wus) vertex includes the CKM m.e. $V_{ud(us)}$

How Large is $\mathcal{B}(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$?







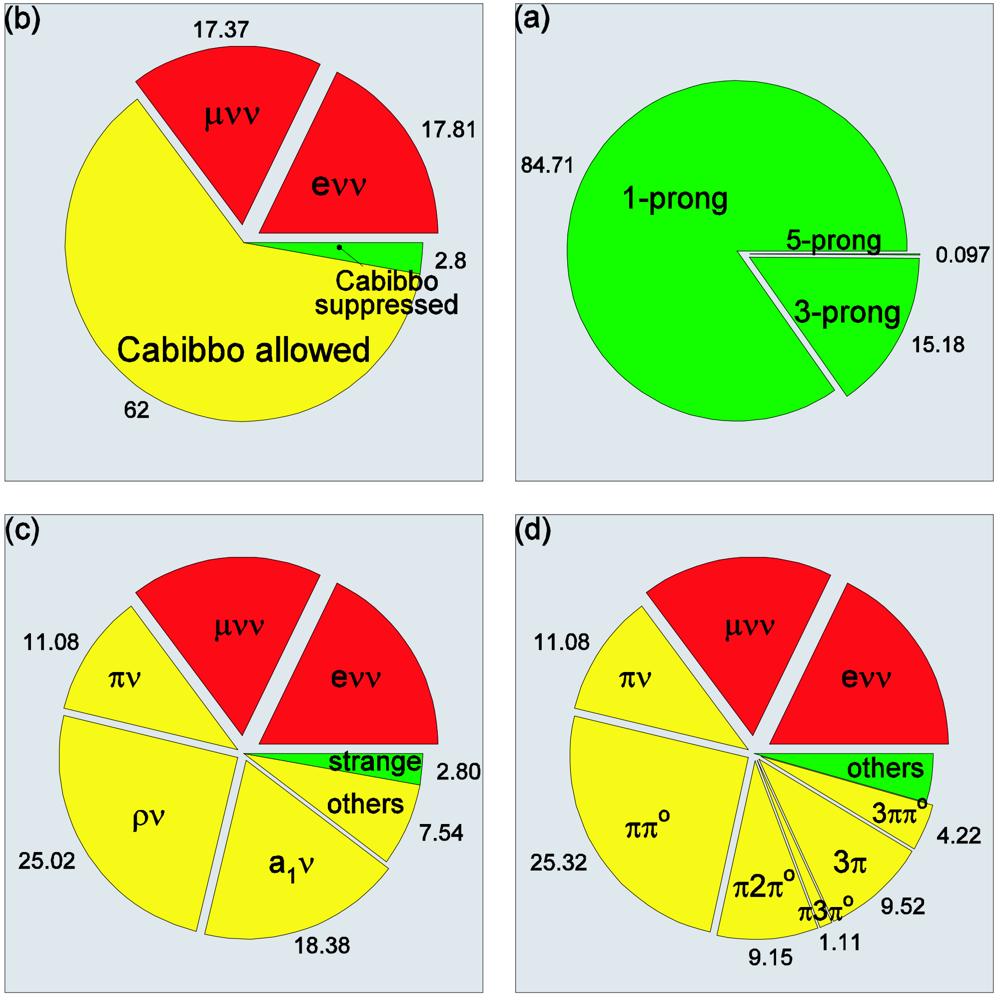
$R_\tau = \frac{\Gamma(\tau^- \to \text{hadrons})}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}$, In asymptotics ($m_\tau \to \infty$) $R_\tau = N_c = 3$,

QCD and EW ($\alpha_s(m_\tau) \sim 0.35$:

$$R_\tau = 3.058[1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.2\frac{\alpha_s(m_\tau)}{\pi}^2 + 26.4\frac{\alpha_s(m_\tau)}{\pi}^3 + \ldots + n.p.t.]$$

Decay mode	$\mathcal{B}$, % (No QCD)	$\mathcal{B}$, % (QCD)	$\mathcal{B}$, % (Exp-nt)
$e^-\bar{\nu}_e\nu_\tau$	20	17.6	17.84 ± 0.06
$\mu^-\bar{\nu}_e\nu_\tau$	20	17.6	17.36 ± 0.06
Hadrons + ν_τ	60	64.8	~ 65

τ Lepton Decays



τ–μ Universality In Hadronic Decays

$$J_W \ = \ cos\theta_C J_{\bar{u}d} \ + \ sin\theta_C J_{\bar{u}s}$$

$$\Gamma(\tau^- \to \pi^- \nu_\tau) = \frac{G_F^2 f_\pi^2 cos^2\theta_C}{16\pi} m_\tau^3 (1 \ - \ m_\pi^2/m_\tau^2)^2,$$

$$\Gamma(\pi^- \to \mu^- \bar{\nu_\mu}) = \frac{G_F^2 f_\pi^2 cos^2\theta_C}{8\pi} m_\pi \ m_\mu^2 (1 \ - \ m_\mu^2/m_\pi^2)^2,$$

$$\frac{\mathcal{B}(\tau^- \to \pi^- \nu_\tau)}{\mathcal{B}(\pi^- \to \mu^- \bar{\nu_\mu})} = \frac{m_\tau^3 (1 \ - \ m_\pi^2/m_\tau^2)^2}{2m_\pi \ m_\mu^2 (1 \ - \ m_\mu^2/m_\pi^2)^2} \frac{\tau_\tau}{\tau_\pi}.$$

For $\Gamma(\tau^- \to K^- \nu_\tau)$ similarly, but $f_\pi \to f_{\rm K}$ and $cos\theta_C \to sin\theta_C$:

$$\frac{\mathcal{B}(\tau^- \to K^- \nu_\tau)}{\mathcal{B}(K^- \to \mu^- \bar{\nu_\mu})} = \frac{m_\tau^3 (1 \ - \ m_{\rm K}^2/m_\tau^2)^2}{2m_{\rm K} \ m_\mu^2 (1 \ - \ m_\mu^2/m_{\rm K}^2)^2} \frac{\tau_\tau}{\tau_{\rm K}}$$

Mode	$\mathcal{B}^{(\rm th)}$	$\mathcal{B}^{(\rm exp)}$
$\tau^- \to \pi^- \nu_\tau$	$(10.87 \pm 0.05)\%$	$(10.90 \pm 0.07)\%$
$\tau^- \to K^- \nu_\tau$	$(7.08 \pm 0.04) \cdot 10^{-3}$	$(6.91 \pm 0.23) \cdot 10^{-3}$

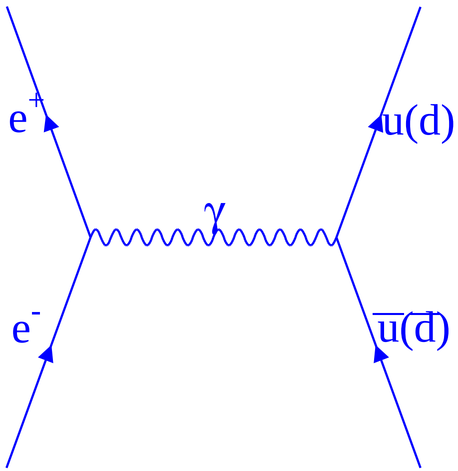
Spectral Functions

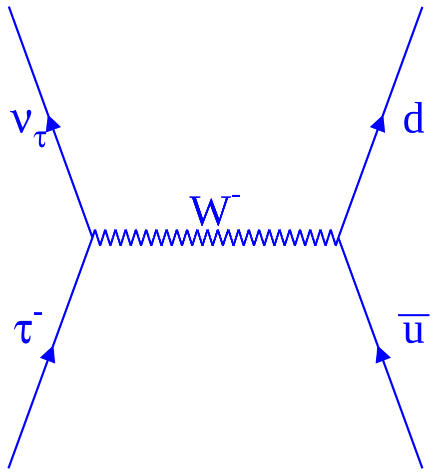
S.f. are obtained from the normalized hadronic mass distribution $(1/N_{V/A})(dN_{V/A}/ds)$ and defined separately for various quantum numbers and flavor of the hadronic system:

$$v_1(s)/a_1(s) = \frac{m_\tau^2}{6|V_{ud}|^2 S_{\rm EW}} \frac{\mathcal{B}(\tau^- \to V^-/A^- \nu_\tau)}{\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \times \frac{dN_{V/A}}{N_{V/A}\, ds} \left[\left(1-\frac{s}{m_\tau^2}\right)^2 \left(1+\frac{2s}{m_\tau^2}\right)\right]^{-1},$$

$$a_0(s) = \frac{m_\tau^2}{6|V_{ud}|^2 S_{\rm EW}} \frac{\mathcal{B}(\tau^- \to \pi^-(K^-)\, \nu_\tau)}{\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \times \frac{dN_A}{N_A\, ds} \left(1-\frac{s}{m_\tau^2}\right)^{-2}$$

CVC. $e^+e^- \to V^0$ and $\tau^- \to \nu_\tau V^-$





Allowed $I^G J^P = 1^+1^-$: $V^- = \pi^-\pi^0, (4\pi)^-, \omega\pi^-, \eta\pi^-\pi^0, K^-K^0, (6\pi)^-$, $\mathcal{B}(V^-\nu_\tau) \sim 32\%$

CVC tests showed good agreement of the τ branchings predicted from e^+e^- with τ data (N.Kawamoto, A.Sanda, 1978; F.Gilman, D.Miller, 1978; SE, V.Ivanchenko, 1991, 1997).

The very first application of τ data to $a_\mu^{\rm had,LO}$ improved the accuracy by a factor of 1.5 (R.Alemany, M.Davier, A.Höcker, 1998)!

Specific CVC predictions

For some modes there is one final state only in both τ and e^+e^-:

$\pi^-\pi^0 \Longleftrightarrow \pi^+\pi^-$, $\omega\pi^- \Longleftrightarrow \omega\pi^0$, $\eta\pi^-\pi^0 \Longleftrightarrow \eta\pi^+\pi^-$, $K^-K^0 \Longleftrightarrow K^+K^-$:

$v_{\pi^-\pi^0} = \frac{s}{4\pi\alpha^2}\sigma(e^+e^- \to \pi^+\pi^-)$, $\quad v_{\omega\pi^-} = \frac{s}{4\pi\alpha^2}\sigma(e^+e^- \to \omega\pi^0)$

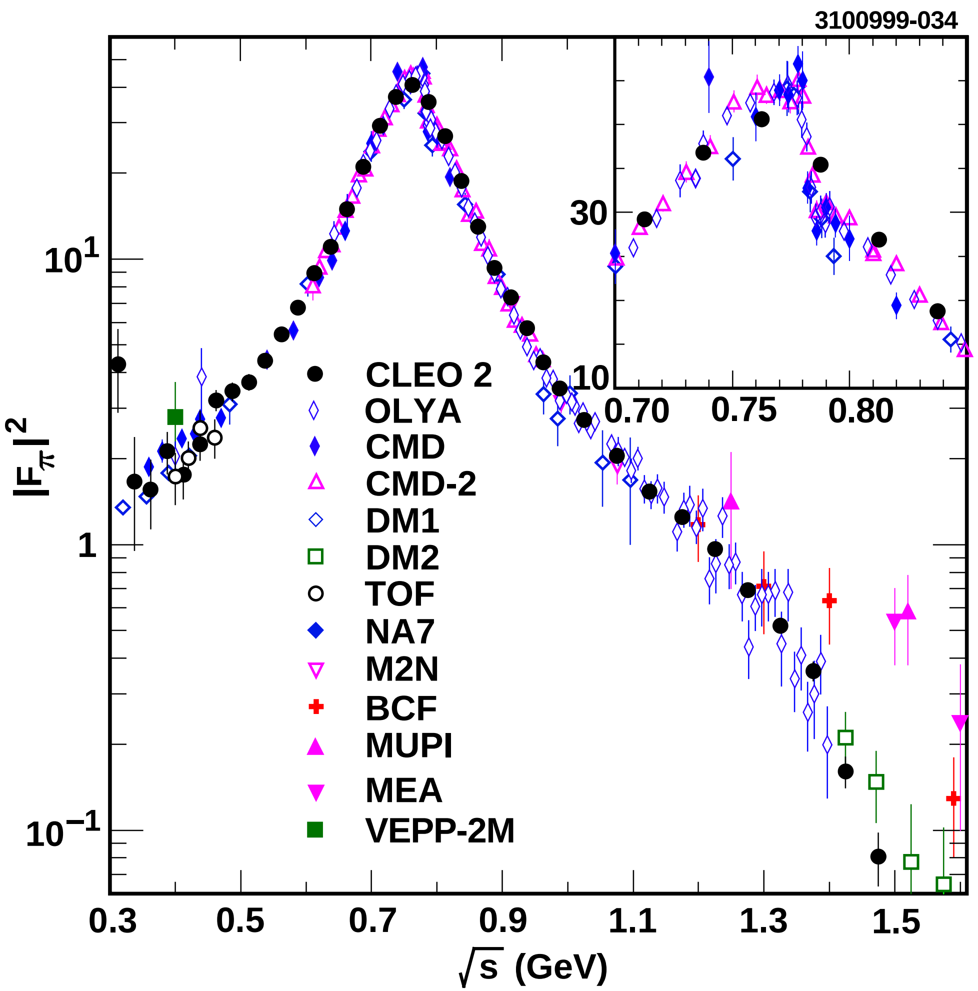
For the 4π channel there are two modes both in τ decay ($\pi^-3\pi^0$and $2\pi^-\pi^+\pi^0$) and e^+e^- ($2\pi^+2\pi^-$ and $\pi^+\pi^-2\pi^0$):

$v_{\pi^-3\pi^0} = \frac{s}{8\pi\alpha^2}\sigma(e^+e^- \to 2\pi^+2\pi^-)$

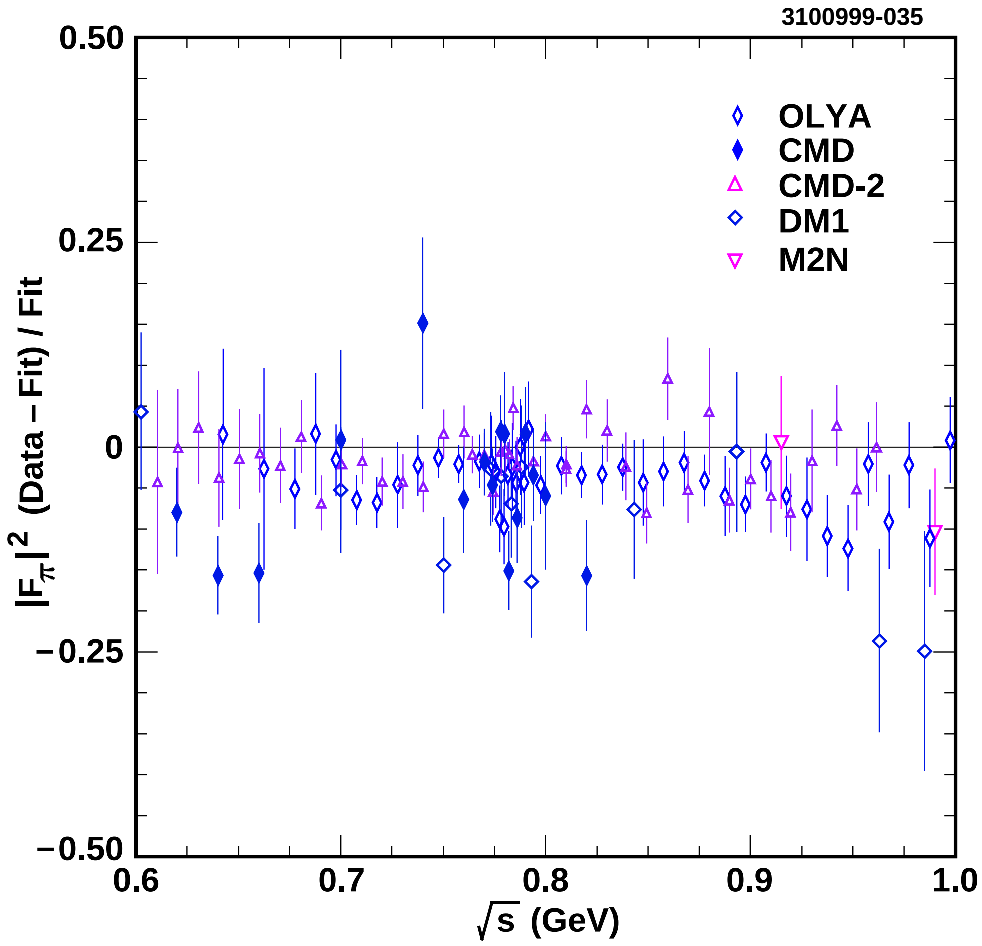
$v_{2\pi^-\pi^+\pi^0} = \frac{s}{8\pi\alpha^2}(\sigma(e^+e^- \to 2\pi^+2\pi^-) + \sigma(e^+e^- \to \pi^+\pi^-2\pi^0))$.

For six-pion modes there are three final states in both $\tau$and $e^+e^-$and only inequalities are possible.

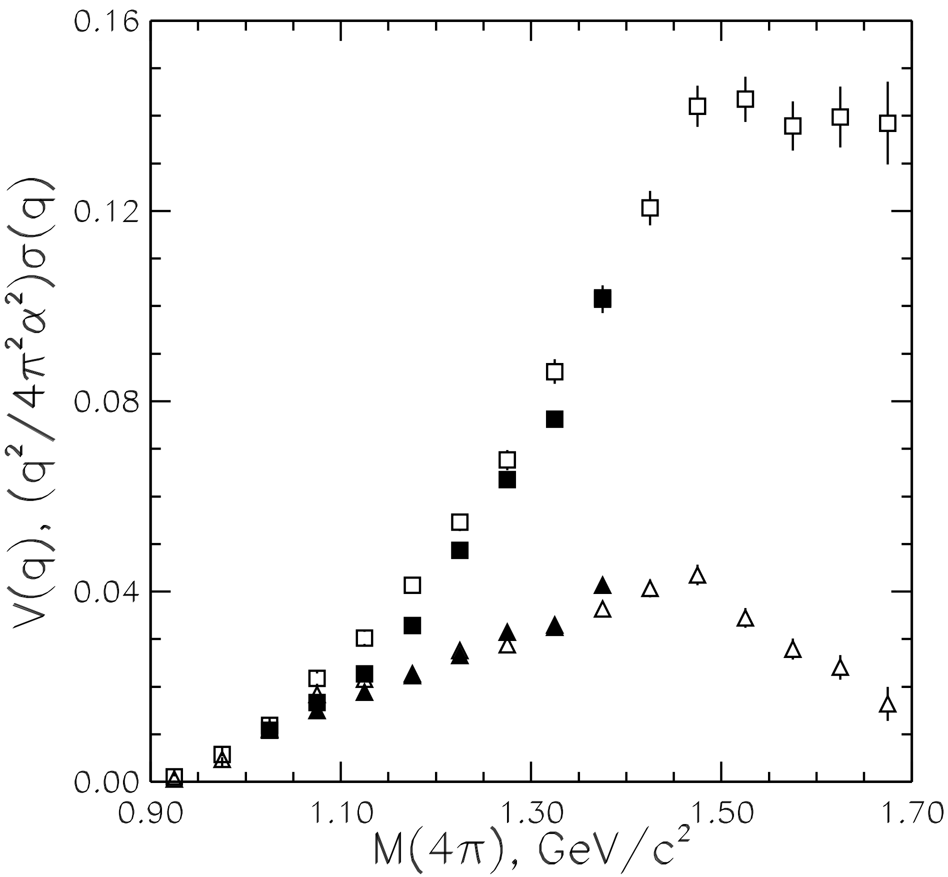
CLEO Test of CVC in $\tau^- \to \pi^-\pi^0\nu_\tau$ – I



CLEO Test of CVC in $\tau^- \to \pi^-\pi^0\nu_\tau$ – II



CLEO Test of CVC in $\tau^- \to (4\pi)^- \nu_\tau$



Branchings of $\tau^- \to X^- \nu_\tau$ Decay,%

Hadronic State X	Experiment, 2002	CVC Prediction	$\mathcal{B}_{\rm exp} - \mathcal{B}_{\rm CVC}$
$\pi^-\pi^0$	25.31 ± 0.18	24.76 ± 0.25	0.55 ± 0.31
$\pi^- 3\pi^0$	1.08 ± 0.10	1.07 ± 0.05	0.01 ± 0.11
$2\pi^-\pi^+\pi^0$	4.19 ± 0.23	3.84 ± 0.17	0.35 ± 0.29
$\omega\pi^-$	1.94 ± 0.07	1.82 ± 0.07	0.12 ± 0.10
$\eta\pi^-\pi^0$	0.174 ± 0.024	0.13 ± 0.02	0.044 ± 0.030
$K^- K^0$	0.154 ± 0.016	0.12 ± 0.03	0.034 ± 0.016
$\phi\pi^-$	< 0.02	< 0.06	-
Total	31.59 ± 0.31	30.28 ± 0.34	1.31 ± 0.46

With more accurate data some deviations have been observed.

Muon $g-2$ and τ – I

$$\vec{\mu} = g\frac{e}{2m}\vec{s}, \qquad a = (g-2)/2.$$

a_μ is measured with a $5 \cdot 10^{-7}$ relative accuracy:

G.W. Bennett et al., 2004, 2006 $\quad a_\mu = (11659208.0 \pm 6.3) \cdot 10^{-10}$.

Any significant difference of $a_\mu^{\rm exp}$ from $a_\mu^{\rm th}$ indicates new physics beyond the Standard Model.

$$a_\mu^{\rm th} = a_\mu^{\rm SM} + a_\mu^{\rm non-SM}, \qquad a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}.$$

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm had,LBL}$$

Muon $g-2$ and τ – II

$$a_\mu^{\rm QED} = (116584718.09 \pm 0.14 \pm 0.08) \cdot 10^{-11}$$

$$a_\mu^{\rm EW} = (15.4 \pm 0.1 \pm 0.2) \cdot 10^{-10}$$

$$a_\mu^{\rm had,LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \, \frac{R(s)\,\hat{K}(s)}{s^2},$$

$$R(s) = \frac{\sigma(e^+e^- \to {\rm hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)},$$

$\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \to \infty$, $1/s^2$ emphasizes the role of low energies, particularly important is the reaction $e^+e^- \to \pi^+\pi^-$ with a large cross section below 1 GeV.

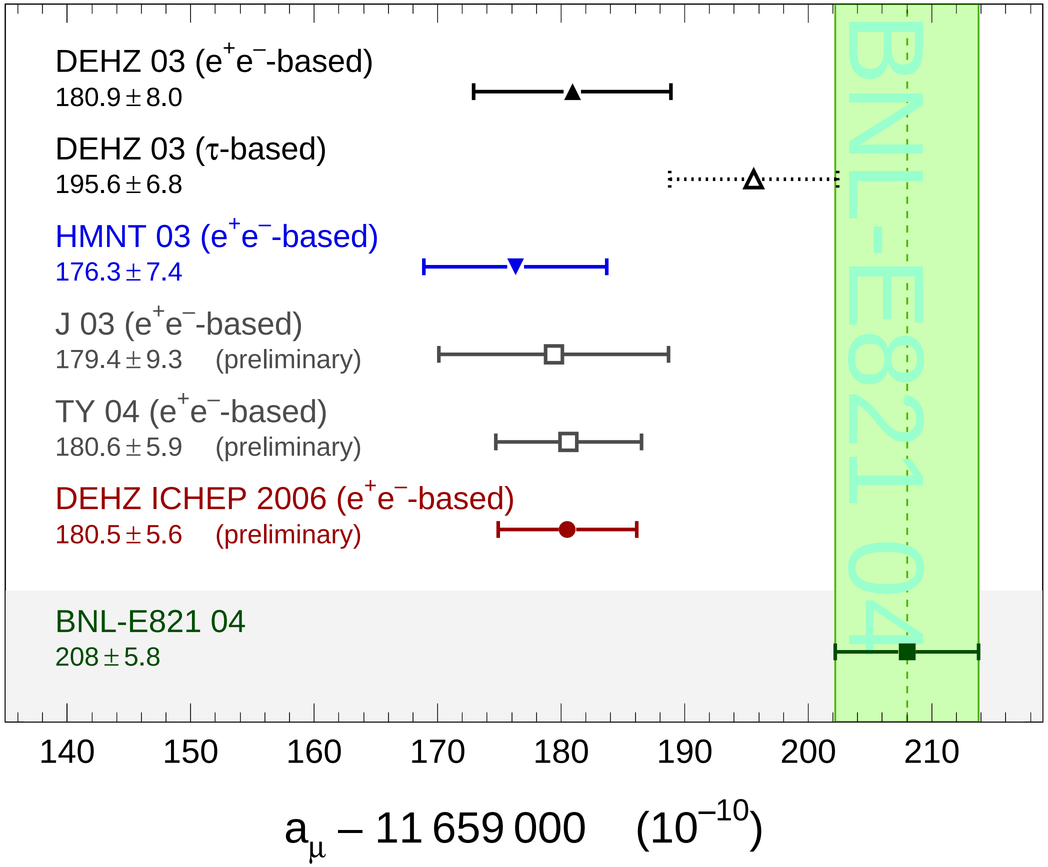
Muon $g-2$ and τ – III

Contribution	$a_\mu, 10^{-10}$
Experiment	11659208.0 ± 6.3
QED	11658471.8 ± 0.016
Electroweak	$15.4 \pm 0.1 \pm 0.2$
Hadronic	693.1 ± 5.6
Theory	11659180.3 ± 5.6
Exp.–Theory	$27.7 \pm 8.4\ (3.3\sigma)$

The difference between experiment and theory is 3.3σ!

(K.Hagiwara et al., 2006 claim even 3.4σ)

Muon $g-2$ and τ – IV

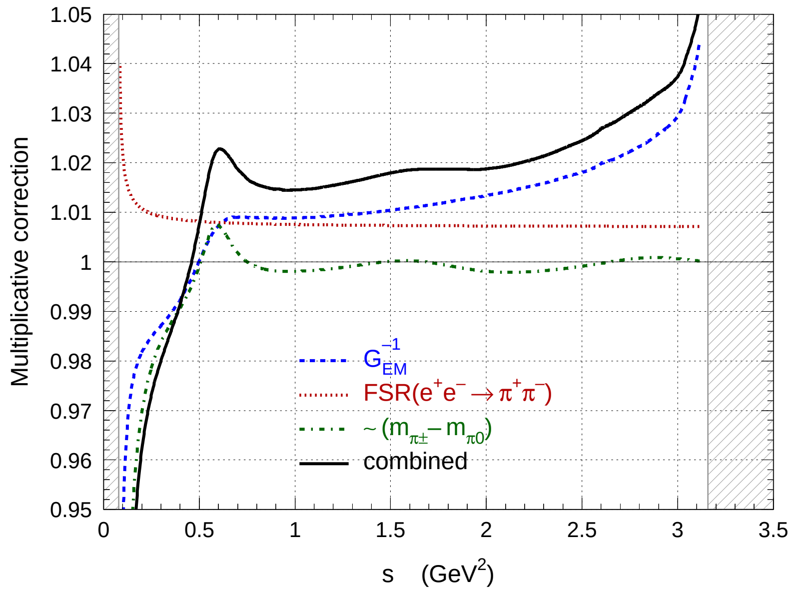


Corrections to the $\tau^- \to \pi^-\pi^0\nu_\tau$ Spectral Functions

- $S_{EW} = 1.0233 \pm 0.0006$
- Real photons, loops
- FSR
- $m_{\pi^\pm} \neq m_{\pi^0}$ (phase space, Γ_ρ)
- $m_{\rho^\pm} \neq m_{\rho^0}$
- $\rho - \omega$ interference
- Radiative decays ($\pi\pi\gamma, \pi(\eta)\gamma, l^+l^-$)
- $m_u \neq m_d$ and 2 class currents

V. Cirigliano, G. Ecker, H. Neufeld, 2002

M.Davier et al., 2002



$\tau^- \to \pi^-\pi^0\nu_\tau$ – I

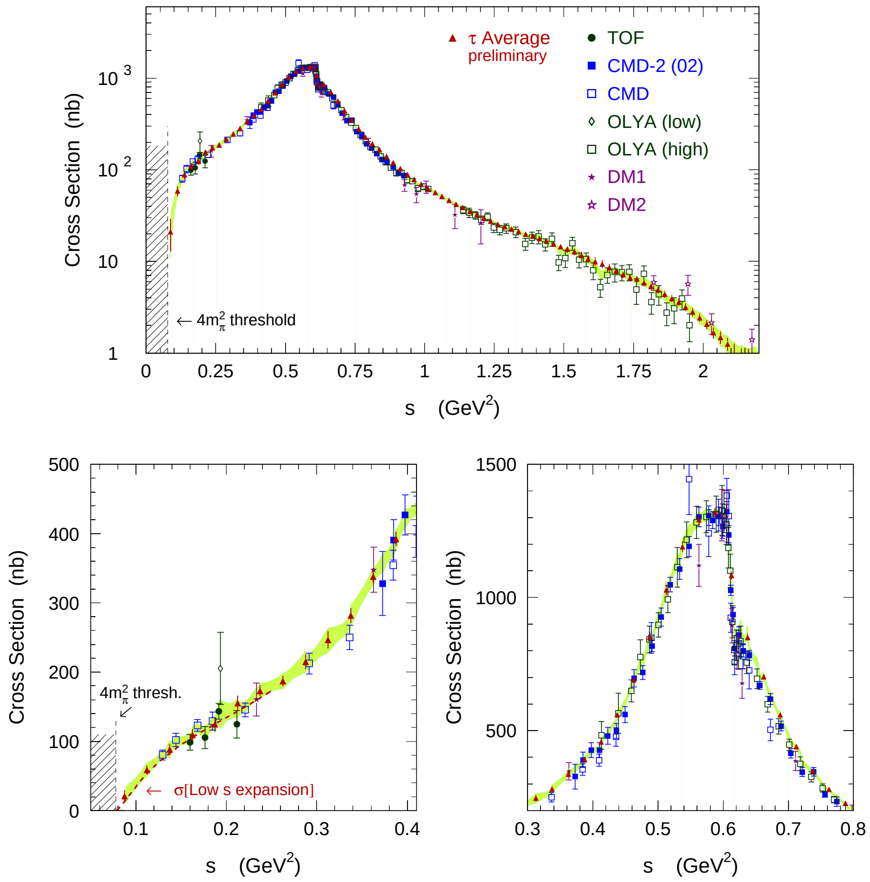
Measurements of $\mathcal{B}(\pi^-\pi^0)$

Mode	Group	$N_{\rm ev}, 10^3$	$\mathcal{B}$, %
$h^-\pi^0$	L3,1995	6.6	$25.05 \pm 0.35 \pm 0.50$
	OPAL,1998	40.5	$25.89 \pm 0.17 \pm 0.29$
	DELPHI,2006	35	$25.740 \pm 0.201 \pm 0.138$
$\pi^-\pi^0$	CLEO,1994	51	25.36 ± 0.44
	ALEPH,2005	81	$25.471 \pm 0.097 \pm 0.085$

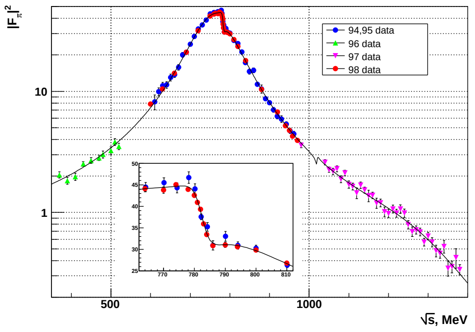
ALEPH systematic errors

Type	π^0	sel	bkg	pid	int	trk	mcs
δ,%	.063	.027	.019	.011	.045	.009	.027

$\tau^- \to \pi^-\pi^0\nu_\tau$ – II

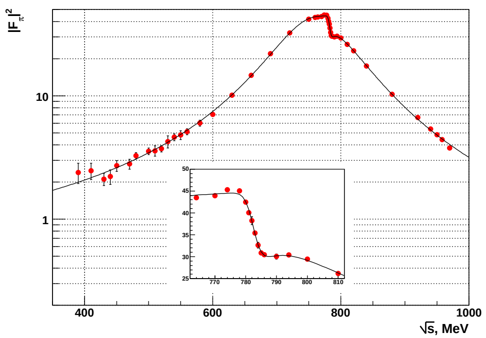


$e^+e^- \to \pi^+\pi^-$ (CMD-2, SND and KLOE)



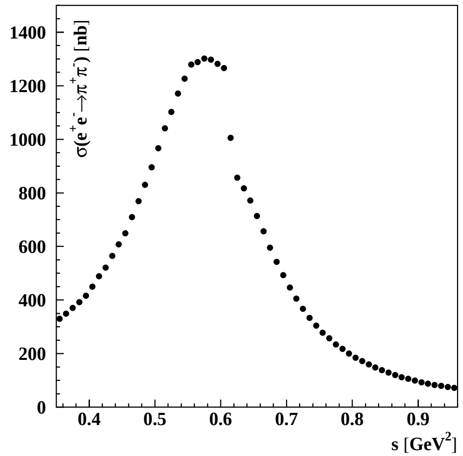
CMD-2: $\sim 9 \cdot 10^5$ ev.

2E, MeV	σ, %
370-520	0.7
600-970	0.6-0.8
1040-1380	1.3-4.2



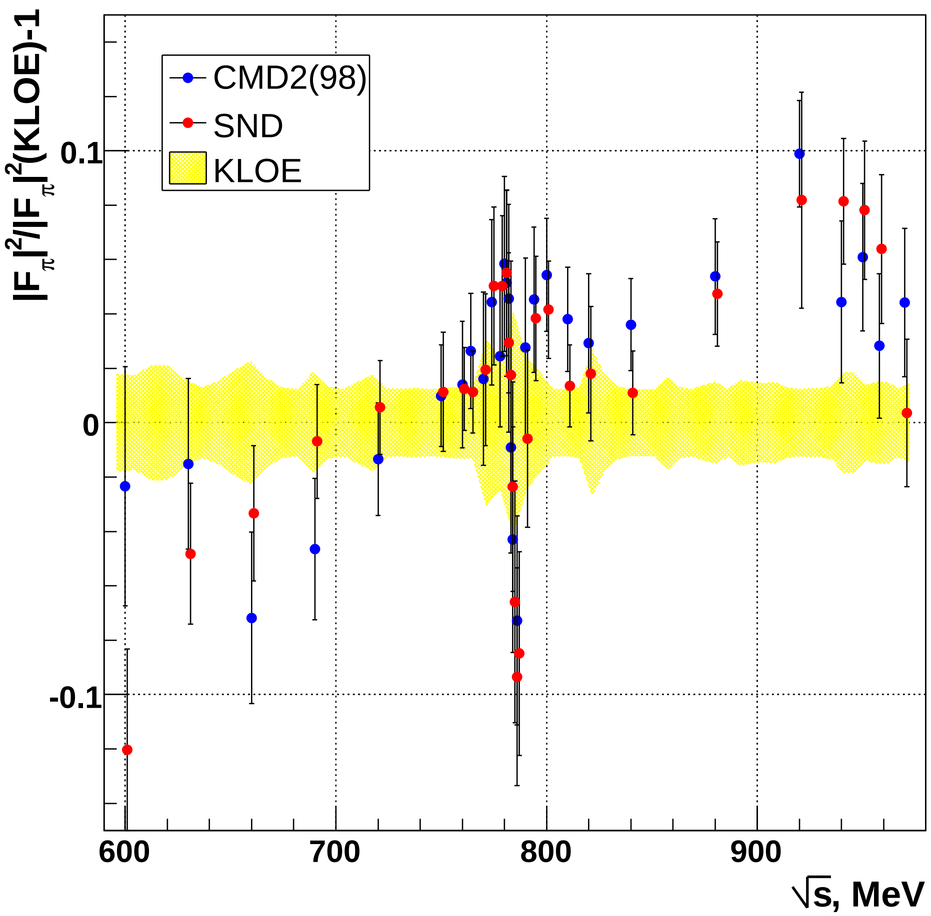
SND: $\sim 8 \cdot 10^5$ ev.

2E, MeV	σ, %
390-420	3.2
430-970	1.3

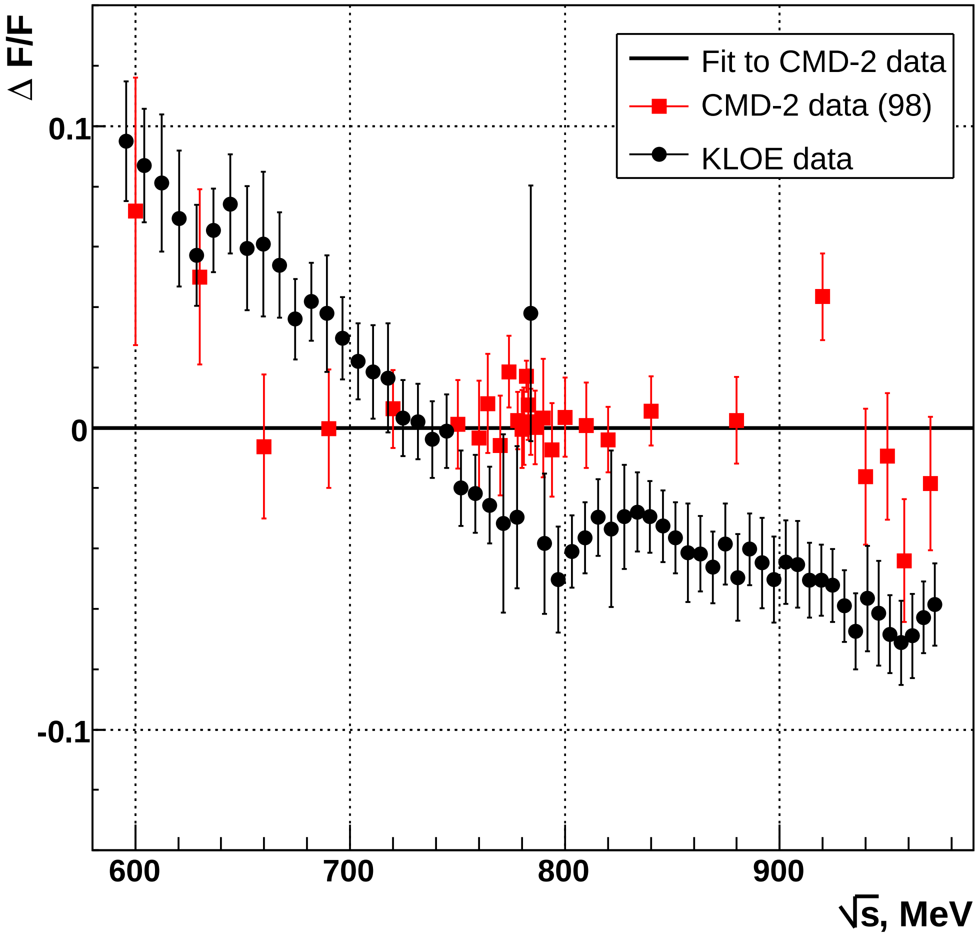


KLOE: $\sim 1.5 \cdot 10^6$ ev.
(590-970) MeV - 1.3%

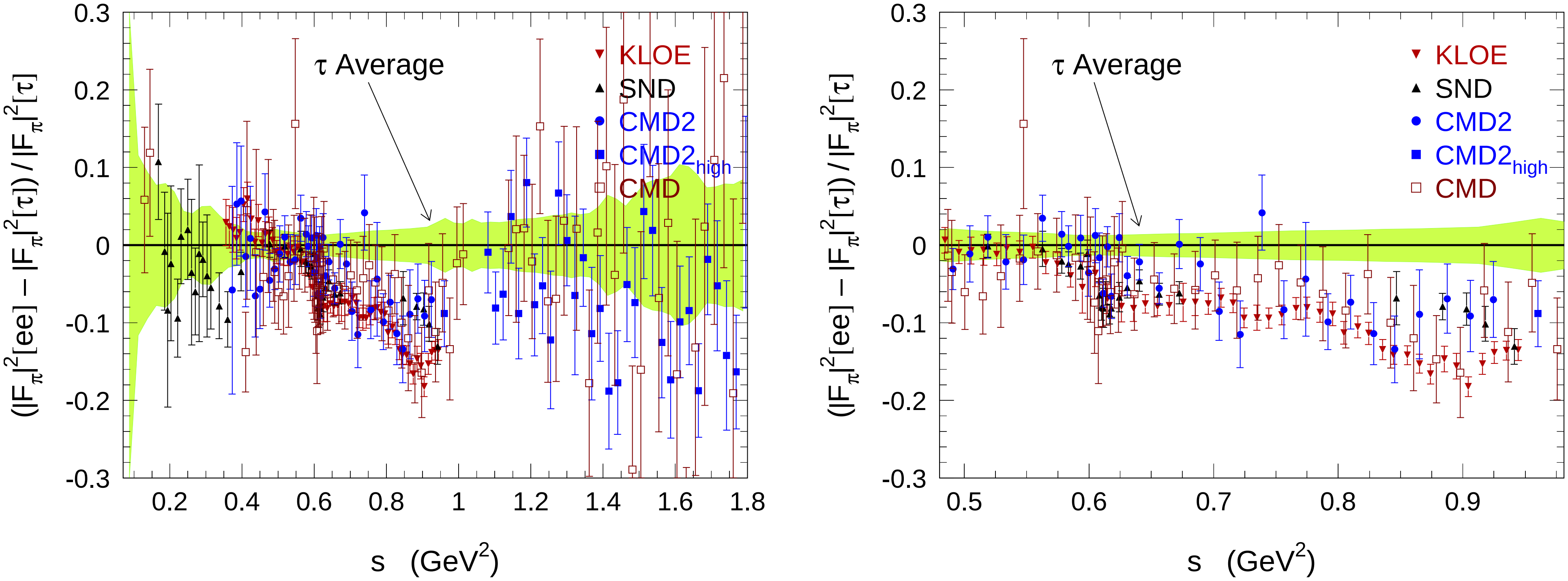
Comparison of CMD-2, SND and KLOE data



Comparison of CMD-2 and KLOE data

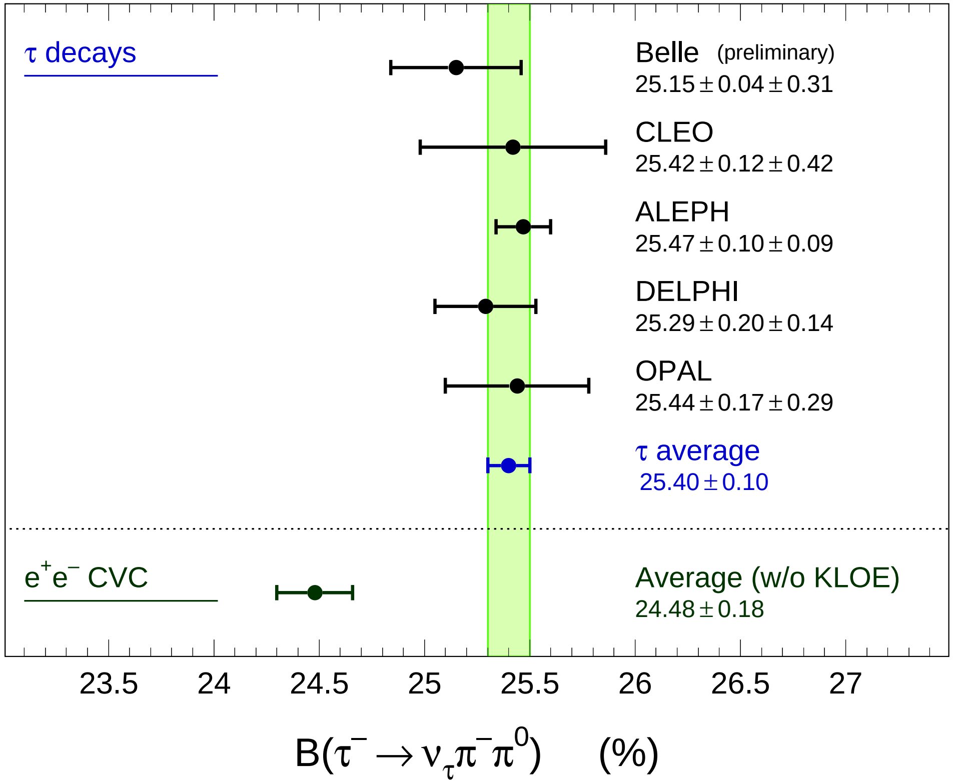


CVC in the 2π Channel. e^+e^- vs. τ (Spectra)



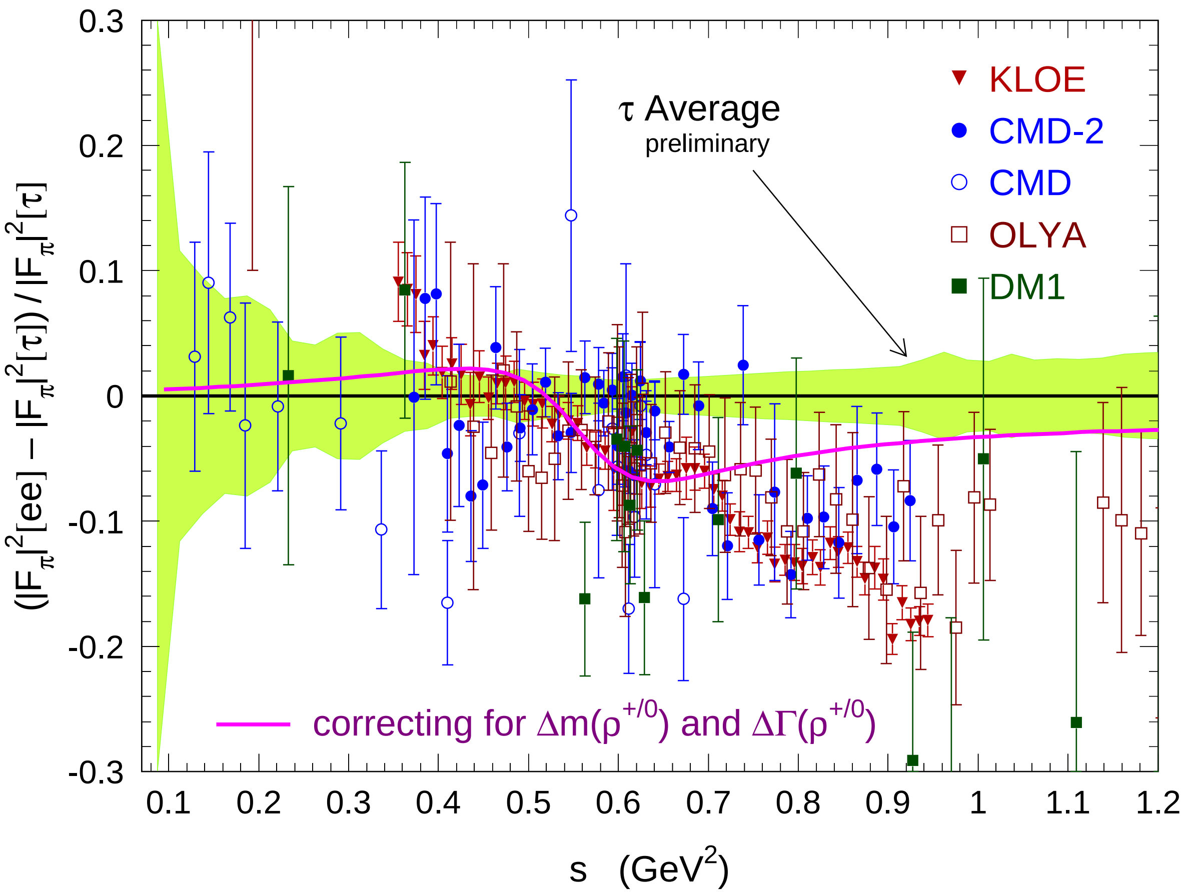
Above the ρ meson e^+e^- spectral functions are lower than in τ decays

CVC in the 2π Channel. e^+e^- vs. τ (Branchings)



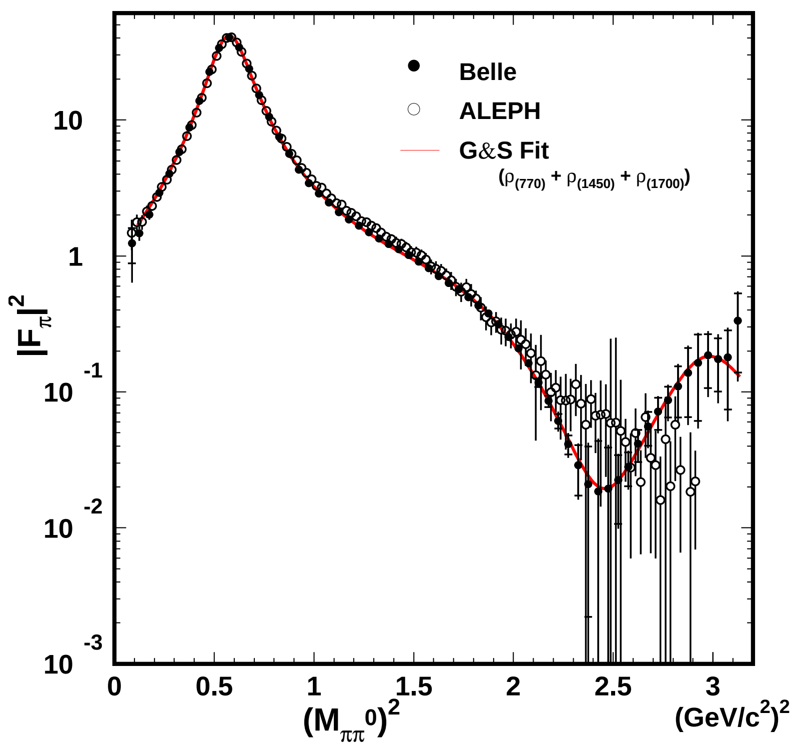
The branching from all groups is systematically higher than the CVC prediction: $\mathcal{B}_\tau - \mathcal{B}_{ee} = (0.92 \pm 0.21)\%$ or 4.5σ from 0. The discrepancy is a 3.6% effect, about twice the SU(2) correction.

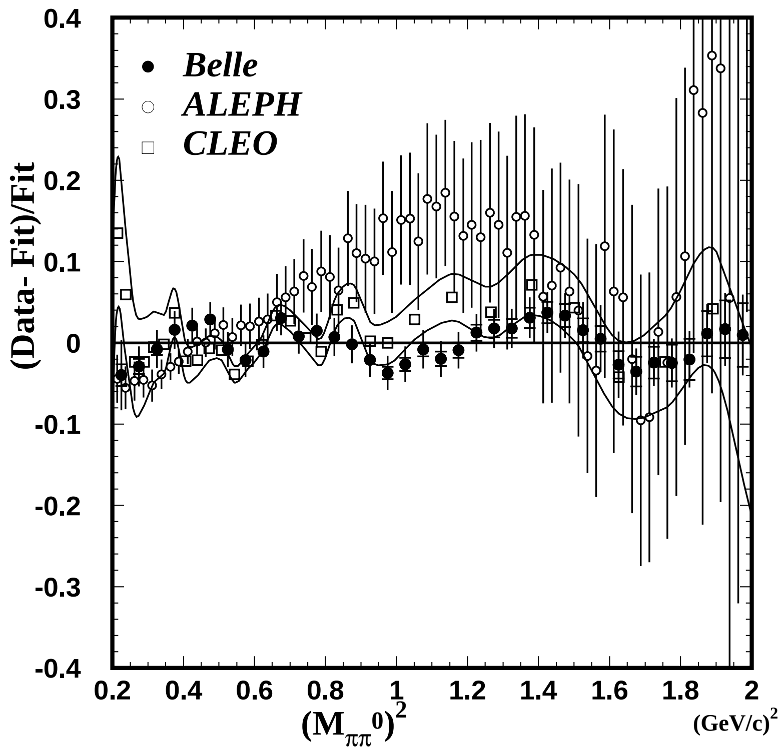
Influence of M_ρ, Γ_ρ



New data on $\tau^- \to \pi^-\pi^0\nu_\tau$ from Belle

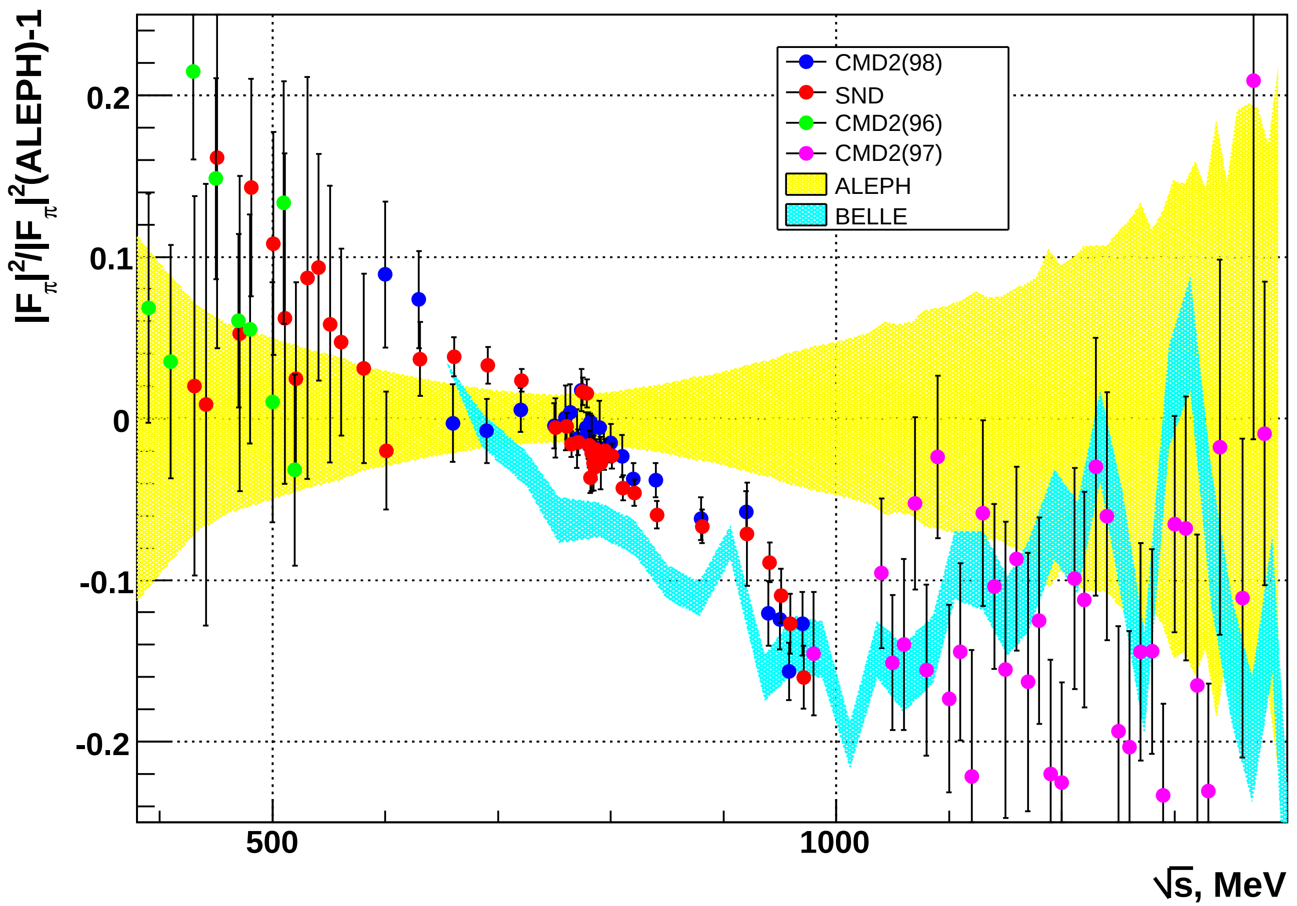
From 64M $\tau^+\tau^-$ pairs Belle selects 5.5M $\tau^- \to h^-\pi^0\nu_\tau$ events!





$$\mathcal{B}_{\rm Belle} = (25.15 \pm 0.04 \pm 0.31)\% \qquad \mathcal{B}_{\rm ALEPH} = (25.471 \pm 0.097 \pm 0.085)\%$$

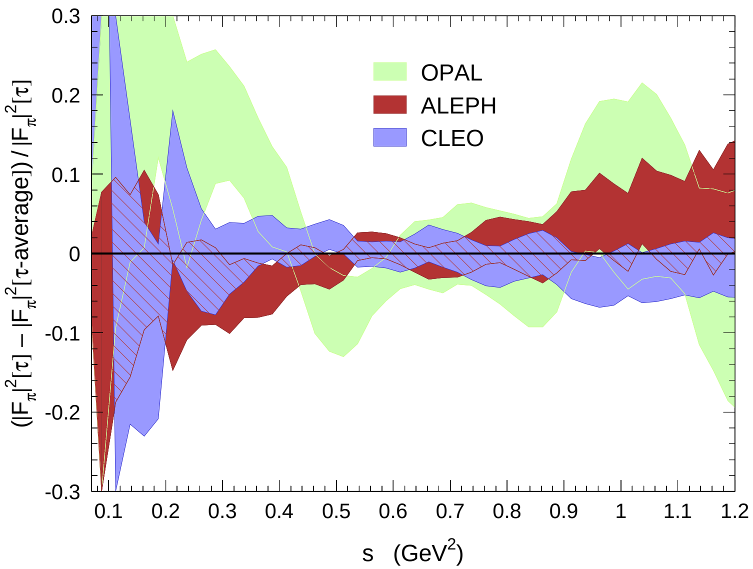
e^+e^- vs. τ after Belle

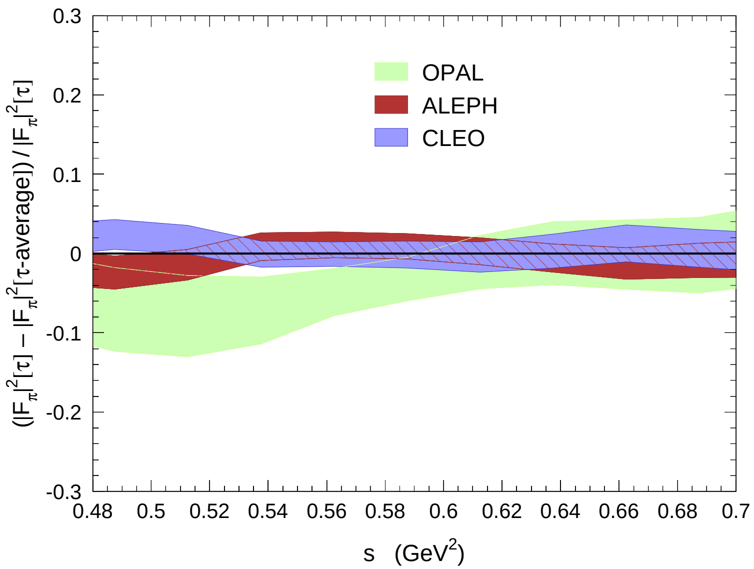


Why are e^+e^- and τ Spectral Functions Different?

- Problems with data: underestimated systematics, normalization, rad. corr.
- Problems with SU(2) breaking corrections; Is ChPT reliable? The uncertainty of corrections may be large (K.Maltman, 2005)
- Non (V-A) contribution to e/w interactions (M.Chizhov, 2003) inspired by problems in $\pi^+ \to e^+\nu_e\gamma$ (E.Frlez et al., 2003)
- Effect of charged Higgs propagator in τ decay
- $m_{\rho^\pm} > m_{\rho^0}$ by a few MeV (S.Ghozzi and F.Jegerlehner, 2003, M.Davier, 2003). Current experiments indicate equality within a few MeV.

Comparison of the $\tau \to 2\pi\nu$ Spectral Functions





Spectral functions for $\tau^- \to \pi^-\pi^0\nu_\tau$
from CLEO, ALEPH and OPAL are consistent

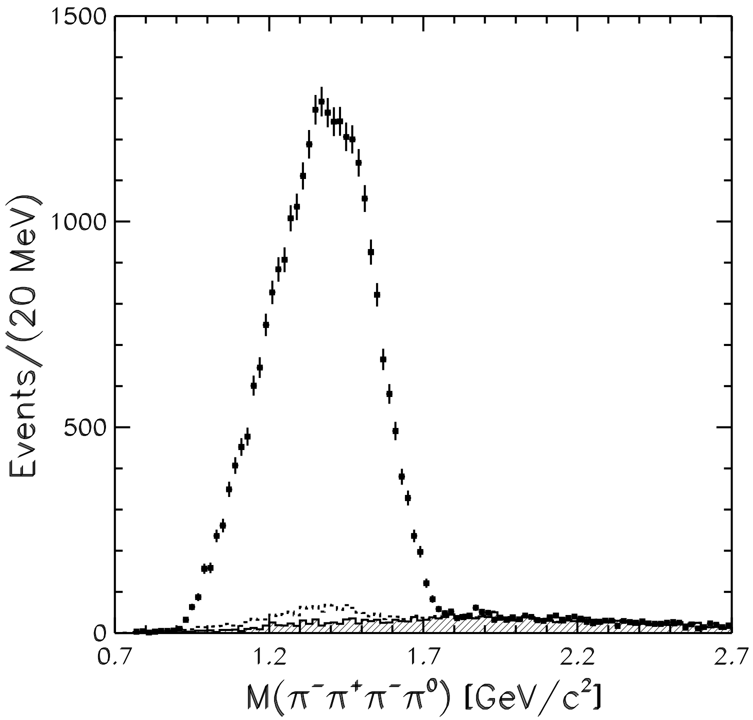
$\tau^- \to (4\pi)^- \nu_\tau$ – I

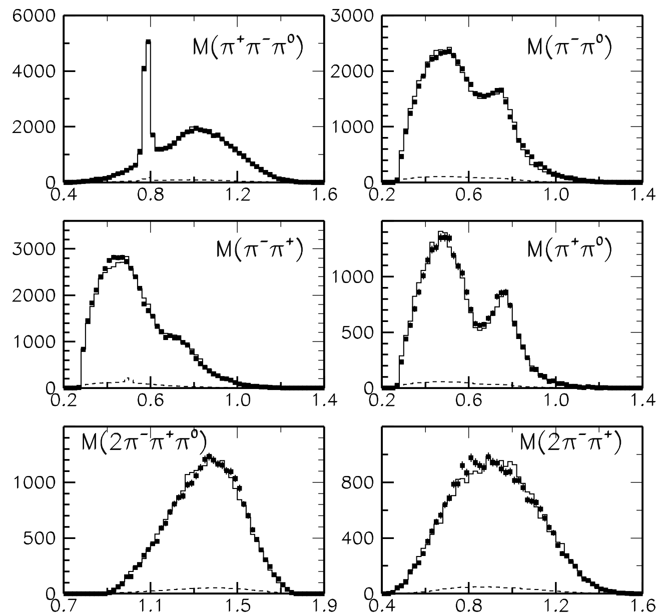
1. $\tau^- \to \pi^- 3\pi^0 \nu_\tau$
   - Because of the high cross-feed usually studied together with other modes having one charged and n π^0, where $1 \leq n \leq 4$
   - L3 and CLEO collected about 300 events each, with $\mathcal{B}(h^- 3\pi^0 \nu_\tau)$ of $(1.70 \pm 0.45)\%$ and $(1.15 \pm 0.15)\%$, ALEPH has $\sim 6 \cdot 10^3$ events and $\mathcal{B}(\pi^- 3\pi^0 \nu_\tau) = (0.977 \pm 0.069 \pm 0.058)\%$
   - PDG-2007: $\mathcal{B}(\pi^- 3\pi^0 \nu_\tau) = (1.04 \pm 0.08)\%$
2. $\tau^- \to 2\pi^- \pi^+ \pi^0 \nu_\tau$
   - There are several measurements of $\mathcal{B}(h^- h^- h^+ \pi^0 \nu_\tau)$ from LEP and CLEO
   - Two highest data sets of 16k and 24k belong to ALEPH and CLEO, the corresponding $\mathcal{B}(2\pi^- \pi^+ \pi^0 \nu_\tau)$ are $(4.598 \pm 0.057 \pm 0.064)\%$ and $(4.19 \pm 0.10 \pm 0.21)\%$
   - PDG-2007: $\mathcal{B}(2\pi^- \pi^+ \pi^0 \nu_\tau) = (4.59 \pm 0.07)\%$

$\tau^- \to (4\pi)^- \nu_\tau$ – II

CLEO, 2000: 4.68 fb^{-1} or $4.27 \cdot 10^6$ $\tau^+\tau^-$ pairs

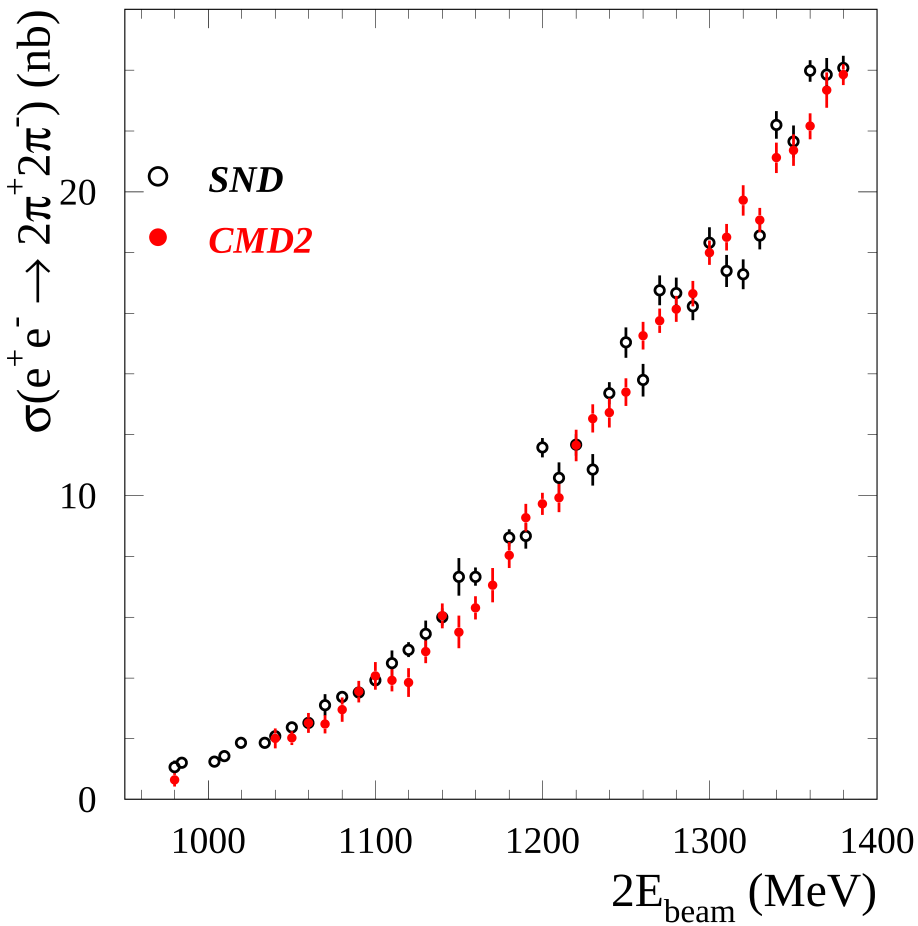
25374 $2\pi^-\pi^+\pi^0\nu_\tau$ events with a background of 1246 events.

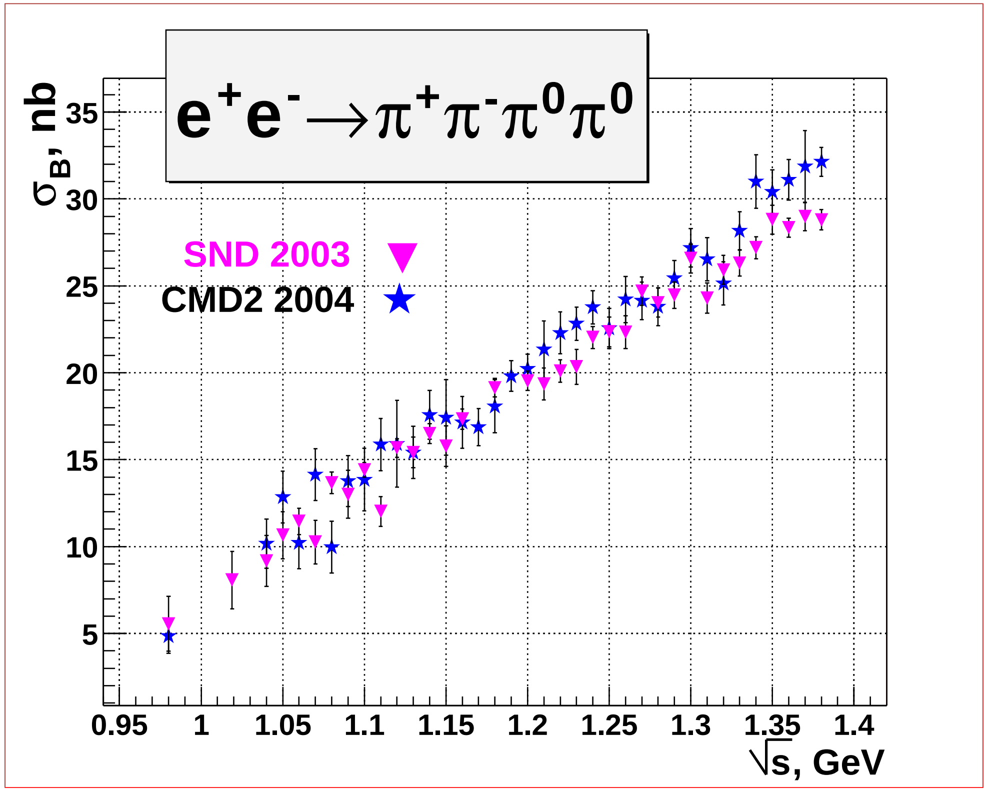




Spectra are consistent with a mixture of $\omega\pi$ and $a_1(1260)\pi$

Study of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ with CMD-2 and SND





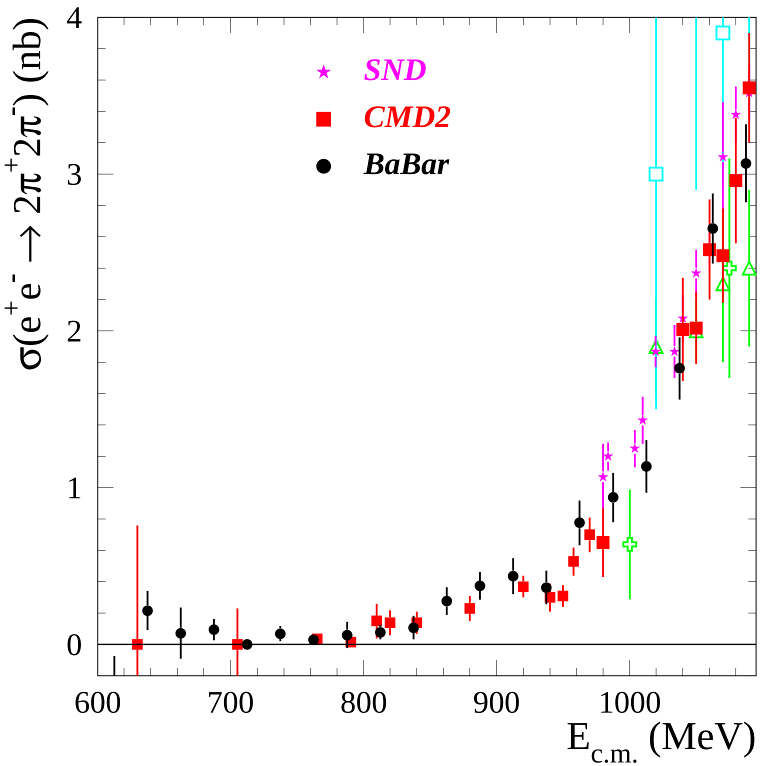
CMD-2: $38 \cdot 10^3$ ev., (5–7)% syst.

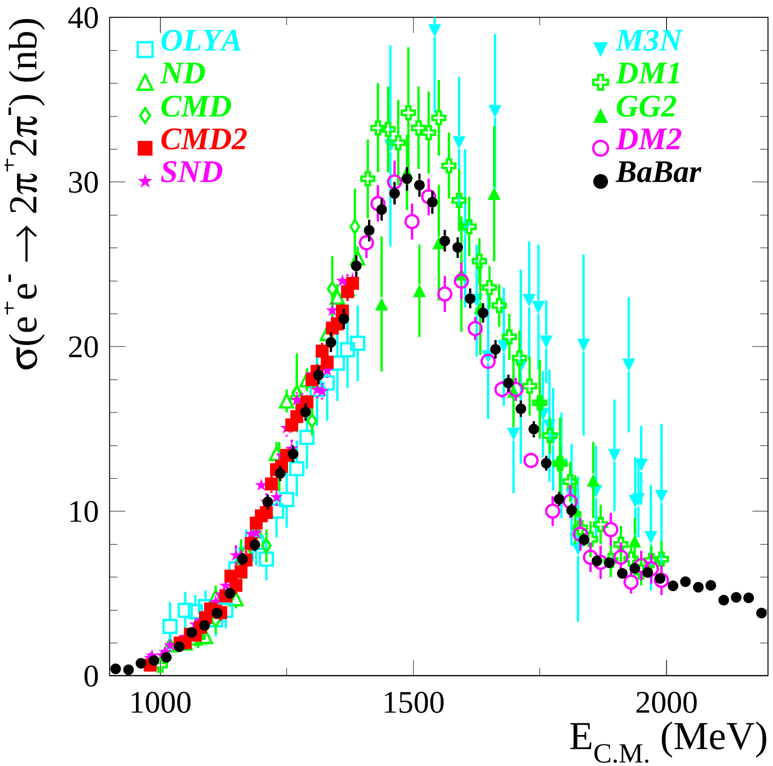
SND: $41 \cdot 10^3$ ev., 7% syst.

CMD-2: $10 \cdot 10^3$ ev.,, 6% syst.

SND: $54 \cdot 10^3$ ev., 8% syst.

$e^+e^- \to 2\pi^+2\pi^-$ – New Data





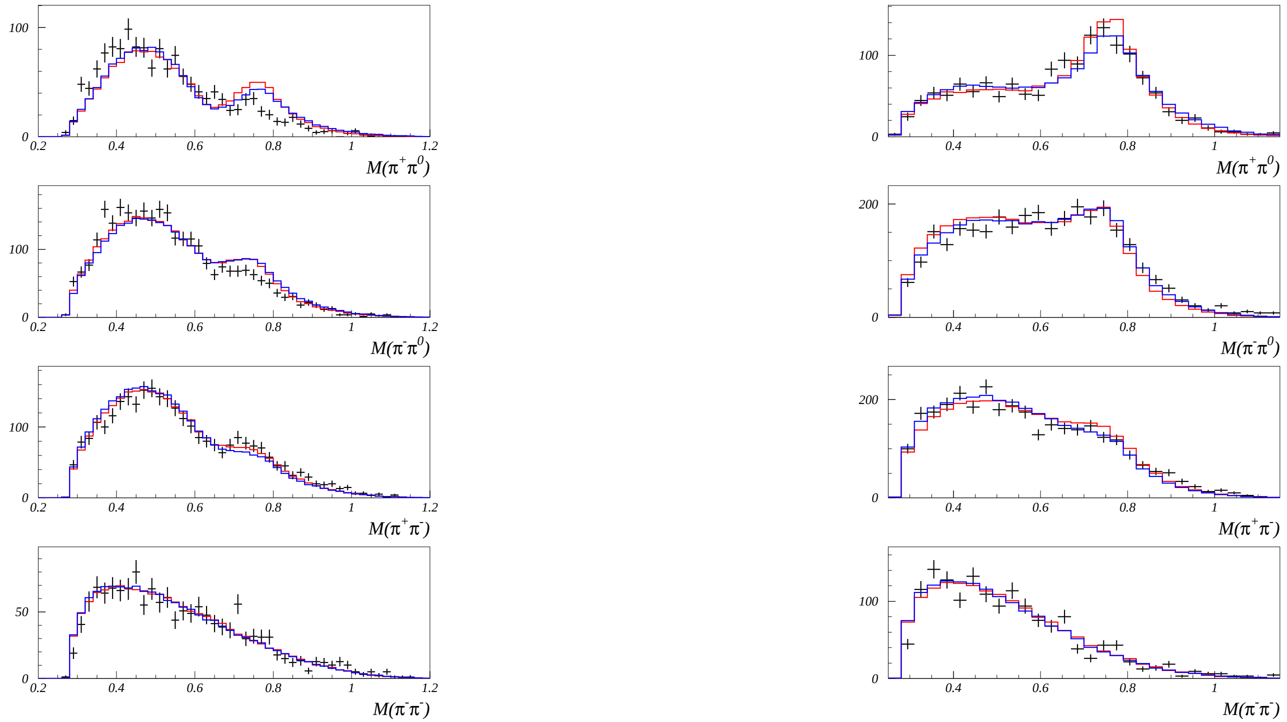
Good agreement between BaBar and CMD-2/SND.

Error of continuum below 2 GeV improved.

$2\pi^+2\pi^-$ dominated by $a_1(1260)\pi$, $\pi^+\pi^-2\pi^0$ – by $a_1(1260)\pi$ and $\omega\pi$

$\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$ Mechanisms – ARGUS and ALEPH

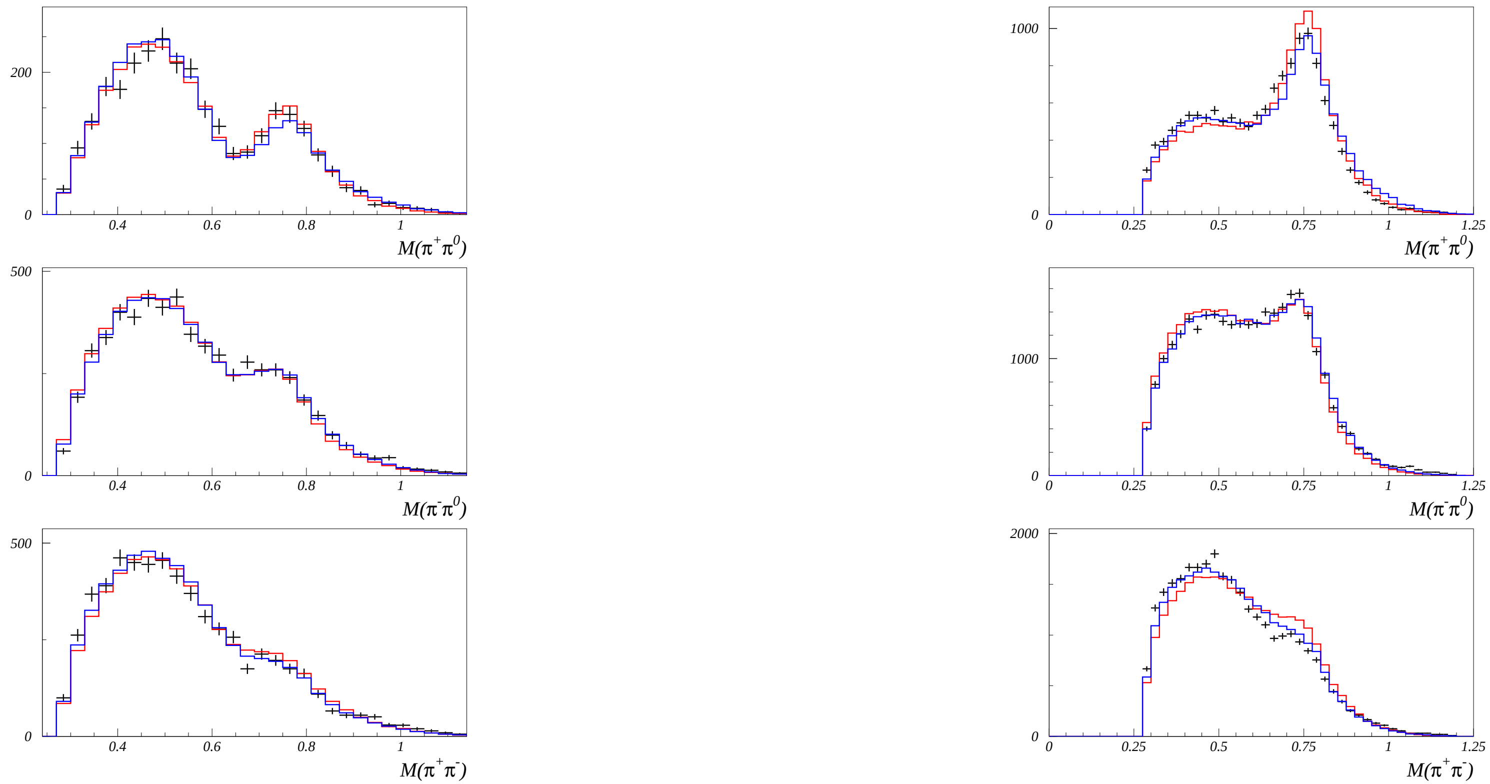
Using CMD-2 data on $e^+e^- \to 4\pi$ and CVC we constructed a model for the $\tau^- \to (4\pi)^-\nu_\tau$ decay. Red – $a_1(1260) \to \rho\pi$, blue – $a_1(1260) \to \rho\pi,\ f_0\pi$



This model: $\mathcal{B}(\tau^- \to a_1^-\nu_\tau)/\mathcal{B}(\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau) \approx 0.8$ consistent with CLEO, but in conflict with < 0.44 at 95% at ARGUS

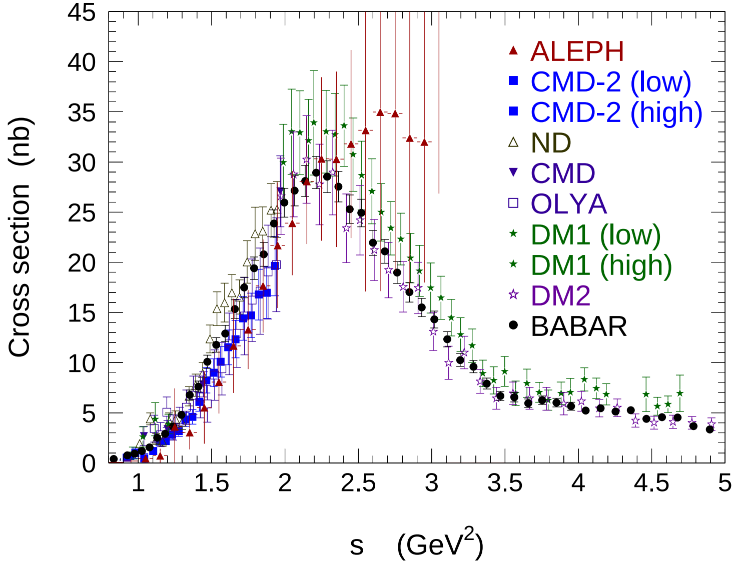
$\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$ Mechanisms – CLEO

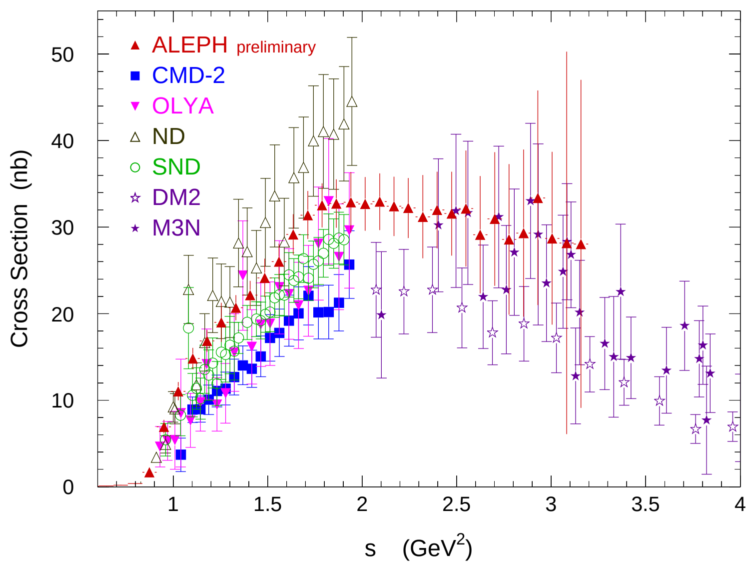
Comparison for CLEO with $\omega\pi$ not subtracted and subtracted. Red – $a_1(1260) \to \rho\pi$, blue – $a_1(1260) \to \rho\pi,\ f_0\pi$



This model is now used in TAUOLA

CVC for the 4π Channels





In the $\pi^-3\pi^0$ mode $\mathcal{B}_\tau = (1.04 \pm 0.08)\%$ vs. $\mathcal{B}_{\rm CVC} = (1.09 \pm 0.08)\%$.

In the $2\pi^-\pi^+\pi^0$ mode $\mathcal{B}_\tau = (4.59 \pm 0.07)\%$ vs. $\mathcal{B}_{\rm CVC} = (3.63 \pm 0.21)\%$.

The discrepancy is $(0.96 \pm 0.23)\%$ or 21%!!

2nd Class Currents in $\tau^- \to \omega\pi^-\nu_\tau$

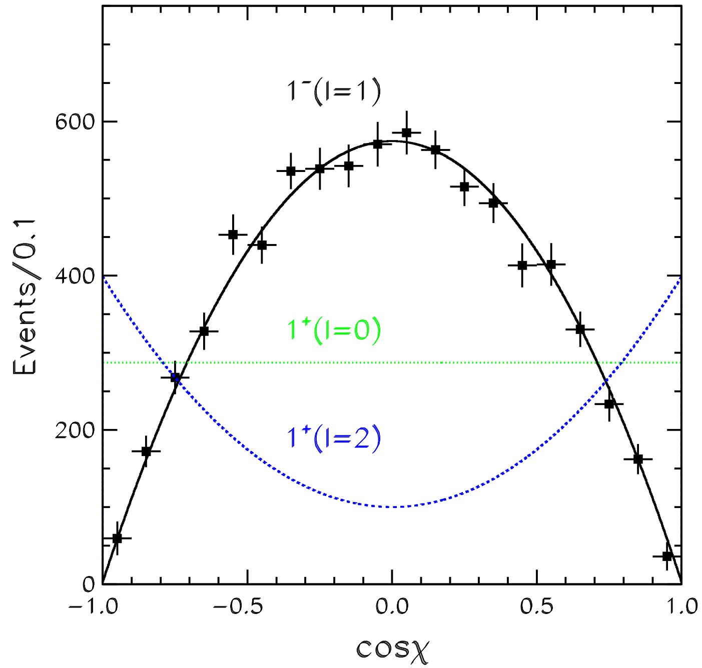
Hadronic currents in τ decay can be of two classes depending on their G-parity ($G = C(-1)^I$, e.g., $G(\rho) = +1,\ G(a_1) = -1$). For strong decays the even number of pions $\Rightarrow G = +1$ while the odd number of pions $\Rightarrow G = -1$.

Both currents possible:

1st class current $J^{PG} = 1^{-+}, l = 1, \mathcal{B} \sim 1.9\%$.

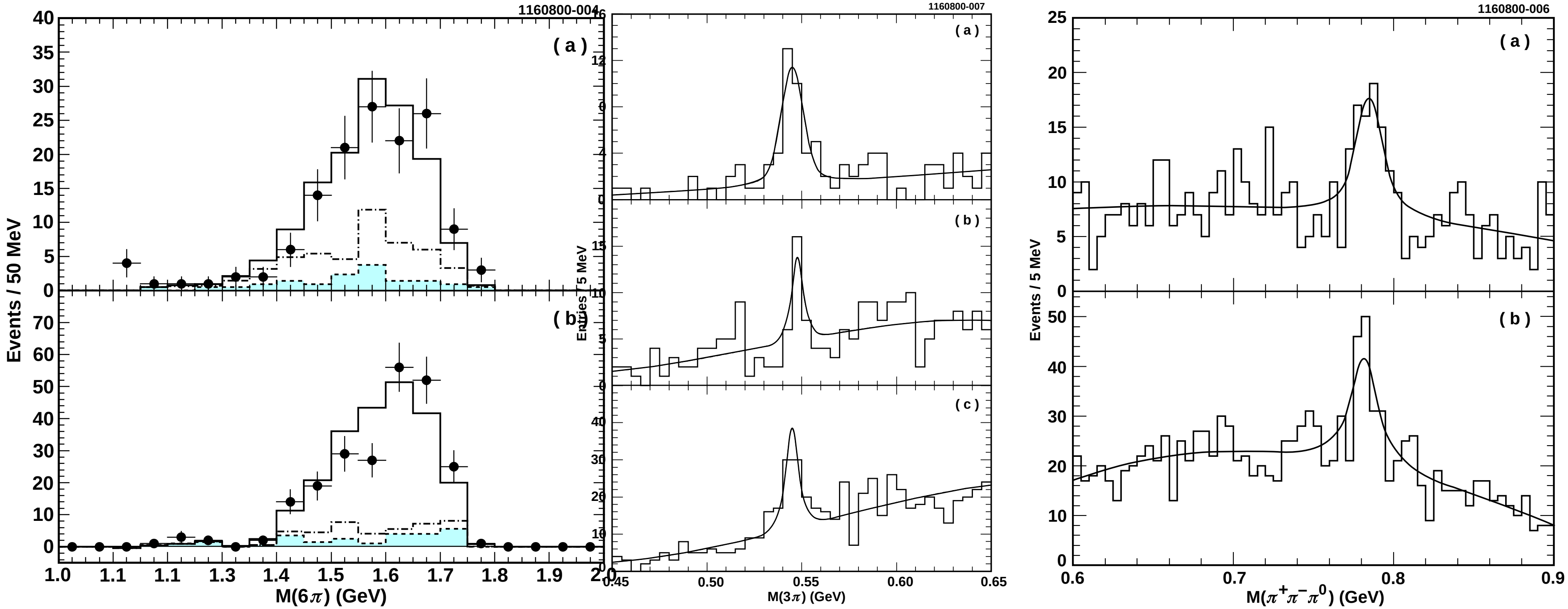
2nd class current $J^{PG} = 1^{++}, l = 0, 2$.

Group	$\mathcal{B}_{\not V}/\mathcal{B}_{\rm V}$ 95% CL
ARGUS, 1987	$<$ 0.5
ALEPH, 1997	$<$ 0.086
CLEO, 2000	$<$ 0.064



CLEO results on $\tau^- \to (6\pi)^- \nu_\tau$ – I

CLEO with 13.5 fb^{-1} or $12.3 \cdot 10^6$ $\tau^+\tau^-$:



Two final states $2\pi^-\pi^+3\pi^0\nu_\tau$ and $3\pi^-2\pi^+\pi^0\nu_\tau$ are observed

CLEO results on $\tau^- \to (6\pi)^- \nu_\tau$ – II

Mode	$2\pi^-\pi^+3\pi^0\nu_\tau$	$3\pi^-2\pi^+\pi^0\nu_\tau$
Yield	$139 \pm 12\ (50 \pm 5)$	$231 \pm 19\ (45 \pm 8)$
Eff.,%	1.65	4.45
$\mathcal{B},\ 10^{-4}$	$2.2 \pm 0.3 \pm 0.4$	$1.7 \pm 0.2 \pm 0.2$

- To obtain inclusive $\mathcal{B}$ they use MC in which τ decays as a mixture of $\eta 3\pi\nu_\tau$ and $\omega 3\pi\nu_\tau$
- They assume that $\eta 3\pi$ decays proceed via $a_1(1260) \to f_1(1285)\pi$ and $\omega 3\pi$ – via $\rho(1700)$
- After determining inclusive $\mathcal{B}$ they separate decays with η and ω

CLEO results on $\tau^- \to (6\pi)^- \nu_\tau$ – III

Mode	$2\pi^-\pi^+\eta\nu_\tau$ $\eta \to 3\pi^0$	$\pi^-2\pi^0\eta\nu_\tau$ $\eta \to \pi^+\pi^-\pi^0$	$2\pi^-\pi^+\eta\nu_\tau$ $\eta \to \pi^+\pi^-\pi^0$
Yield	$32 \pm 7(3 \pm 2)$	$15 \pm 5(2 \pm 2)$	$49 \pm 10(8 \pm 4)$
Eff.,%	1.28	1.48	4.18
$\mathcal{B}$, 10^{-4}	$2.9 \pm 0.7 \pm 0.5$	$1.5 \pm 0.6 \pm 0.3$	$1.7 \pm 0.4 \pm 0.3$

They also use their measurements with the $\eta \to 2\gamma$ mode:

$$\bar{\mathcal{B}}(2\pi^-\pi^+\eta\nu_\tau) = (2.3 \pm 0.5) \cdot 10^{-4},$$

$$\bar{\mathcal{B}}(\pi^-2\pi^0\eta\nu_\tau) = (1.5 \pm 0.5) \cdot 10^{-4}.$$

These decays come from the axial-vector current.

Chiral theory predictions are from $1.2 \cdot 10^{-6}$ to $1.9 \cdot 10^{-3}$.

CLEO results on $\tau^- \to (6\pi)^-\nu_\tau$ – IV

Mode	$\pi^- 2\pi^0 \omega \nu_\tau$	$2\pi^- \pi^+ \omega \nu_\tau$
Yield	$53 \pm 11(10 \pm 4)$	$110 \pm 19(6 \pm 6)$
Eff.,%	1.39	4.06
$\mathcal{B}$, 10^{-4}	$1.4 \pm 0.4 \pm 0.3$	$1.2 \pm 0.2 \pm 0.1$

J.Gao, B.A.Li, 2001 predict in effective large N_c QCD the same $\mathcal{B} \sim 2 \cdot 10^{-4}$ for both modes.

$$\mathcal{B}_V(2\pi^-\pi^+3\pi^0\nu_\tau) = (1.1 \pm 0.4) \cdot 10^{-4},$$
$$\mathcal{B}_V(3\pi^-2\pi^+\pi^0\nu_\tau) = (1.2 \pm 0.2) \cdot 10^{-4},$$

or $\sim 50\%$ and $\sim 70\%$ of the inclusive $\mathcal{B}$.

$\tau^- \to (6\pi)^-\nu_\tau$ and Isospin Invariance

There are three possible final states of the $(6\pi)^-$ in $\tau^- \to (6\pi)^-\nu_\tau$: $\pi^-5\pi^0$, $2\pi^-\pi^+3\pi^0$ and $3\pi^-2\pi^+\pi^0$.

Since six pions, each with I=1, have to add to the total I=1, inequalities arise for the possible combinations of their charges and their relative abundancy:

$$0 \le f_1 = \frac{\pi^-5\pi^0}{\mathrm{all}(6\pi)^-} \le \frac{9}{35},$$

$$\frac{1}{5} \le f_3 = \frac{2\pi^-\pi^+3\pi^0}{\mathrm{all}(6\pi)^-} \le \frac{4}{5},$$

$$\frac{1}{5} \le f_5 = \frac{3\pi^-2\pi^+\pi^0}{\mathrm{all}(6\pi)^-} \le \frac{4}{5},$$

$$f_1 \le \frac{9}{7}f_3, \qquad f_1 \le \frac{9}{7}f_5, \qquad f_1 \le \frac{9}{26}(f_3+f_5).$$

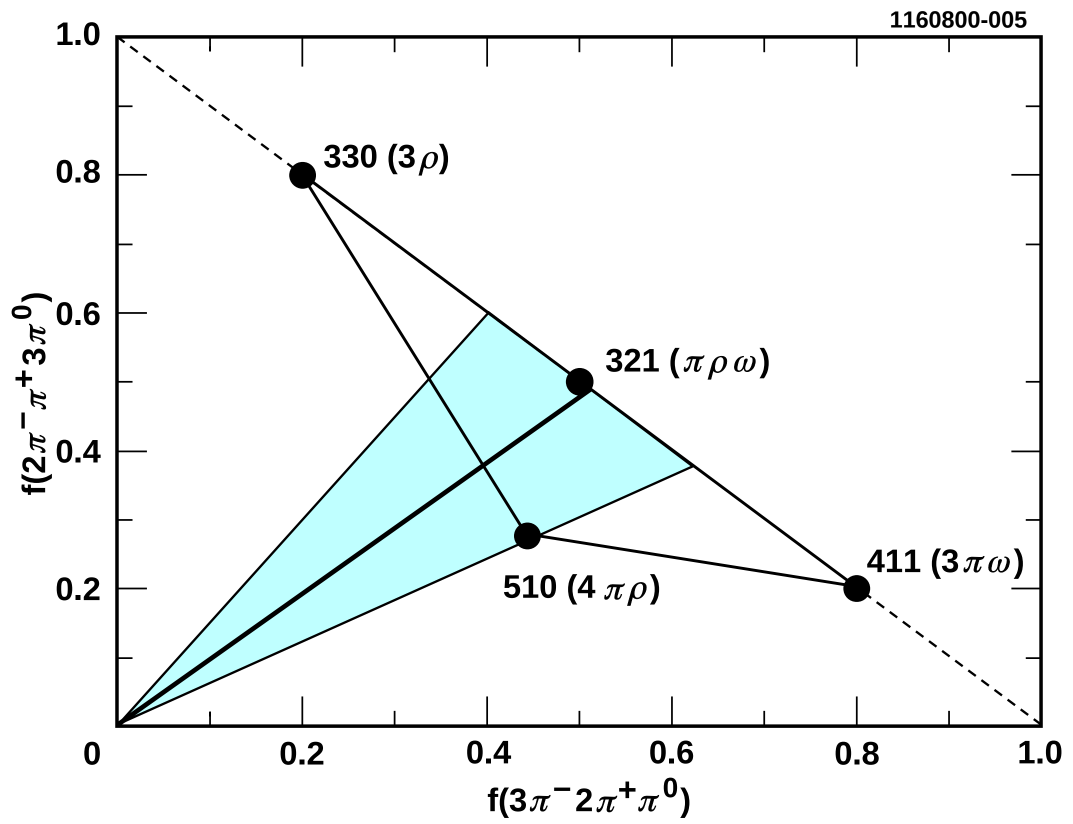
Similar inequalities can be written for the three possible final states of $e^+e^- \to 6\pi$: $3\pi^+3\pi^-,\ 2\pi^+2\pi^-2\pi^0,\ \pi^+\pi^-4\pi^0$ ($6\pi^0$ is forbidden by C-invariance)

Isospin partitions

In 1960 A.Pais studied how isospin conservation affects a system of n pions. Any system can be classified in terms of orthogonal states (partitions), each partition $\Rightarrow (n_1, n_2, n_3)$, where $n_1 + n_2 + n_3 = n$ and $n_1 \geq n_2 \geq n_3$. Internal pion symmetries: n_3 – the number of 3π states with I=0 (ω), $n_2 - n_3$ – the number of 2π systems with I=1 (ρ), $n_1 - n_2$ – the number of single pions.

Partition	State	$\pi^- 5\pi^0$	$\pi^+ 2\pi^- 3\pi^0$	$2\pi^+ 3\pi^- \pi^0$
510	$4\pi\rho$	9/35	2/7	16/35
411	$3\pi\omega$	0	1/5	4/5
330	3ρ	0	4/5	1/5
321	$\pi\rho\omega$	0	1/2	1/2

CLEO 6π Partitions



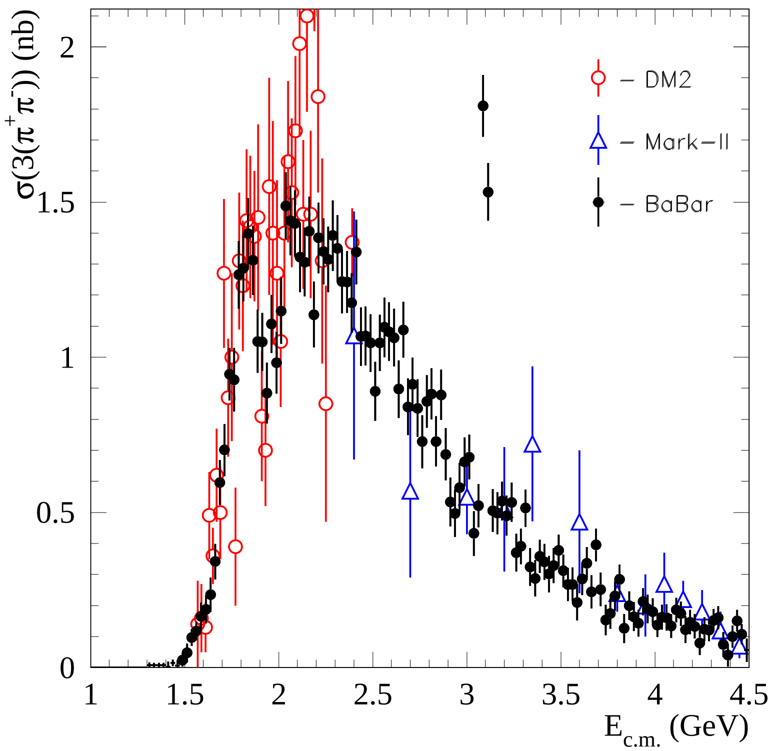
The 321 $(\pi\rho\omega)$ partition is dominant!

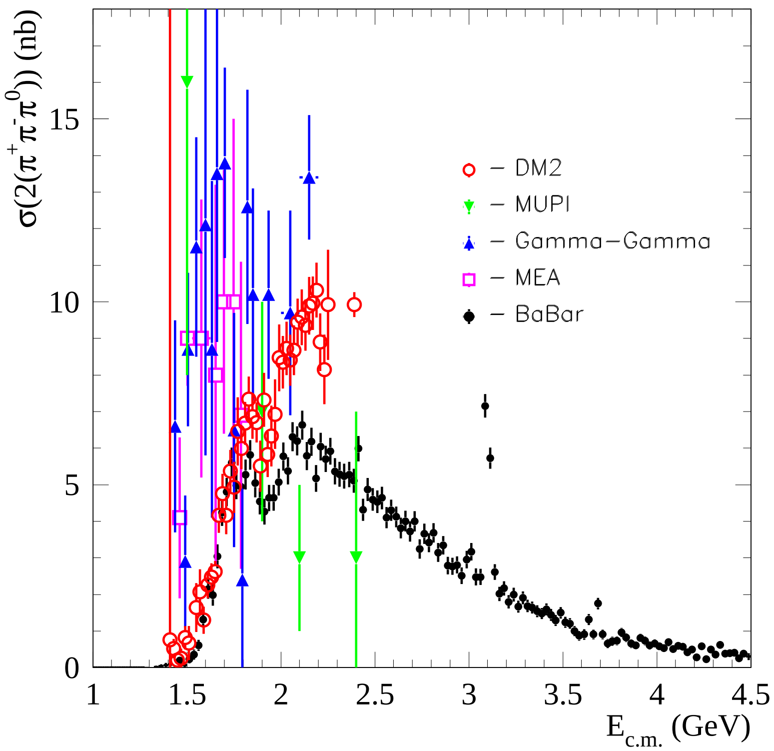
Comparison of CLEO Results with CVC

In 1996 from the sum of $\sigma(e^+e^- \to 2\pi^+2\pi^-2\pi^0)$ and $\sigma(e^+e^- \to 3\pi^+3\pi^-)$ we obtained using CVC:
$\mathcal{B}(6\pi) \geq (1.23 \pm 0.19) \cdot 10^{-3}$ and $\mathcal{B}_{5,1}(\mathcal{B}_{3,3}) \geq (0.025 \pm 0.004)\%$.
This is larger than the measured $\mathcal{B}_V$. This discrepancy is even larger if we compare the predicted inclusive $\mathcal{B}(6\pi) \geq (1.23 \pm 0.19) \cdot 10^{-3}$ with $\mathcal{B}_V \approx \mathcal{B}_{5,1} + \mathcal{B}_{3,3} \geq (2.3 \pm 0.5) \cdot 10^{-4}$ assuming that $\mathcal{B}_{1,5}$ is small. This is supported by the observed dominance of the (321) partition meaning also that (510) is small. In addition, CLEO in 1996 set $\mathcal{B}_{1,5} < 2 \cdot 10^{-4}$.

The way out is if $\sigma(e^+e^- \to 6\pi)$ is too high or has a sizable fraction of I=0.

$e^+e^- \longrightarrow 6\pi$

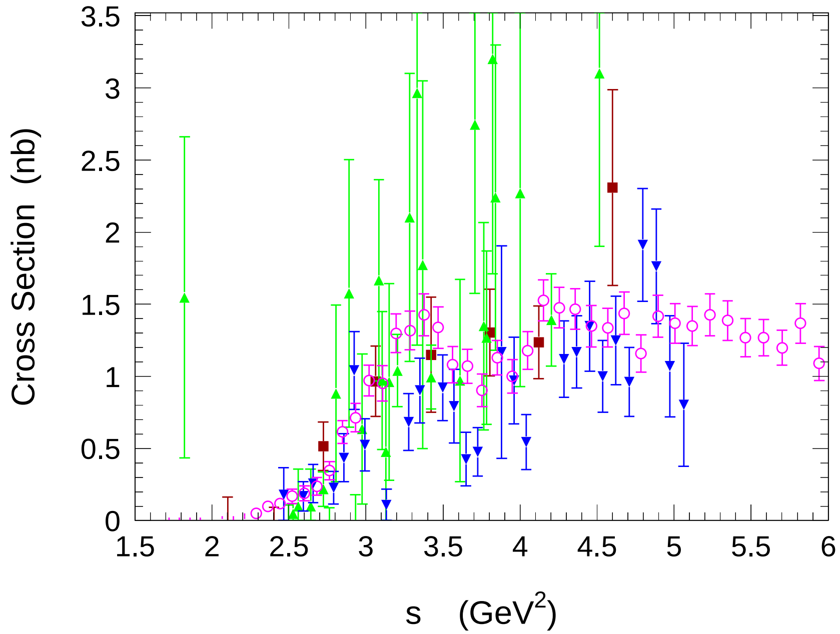


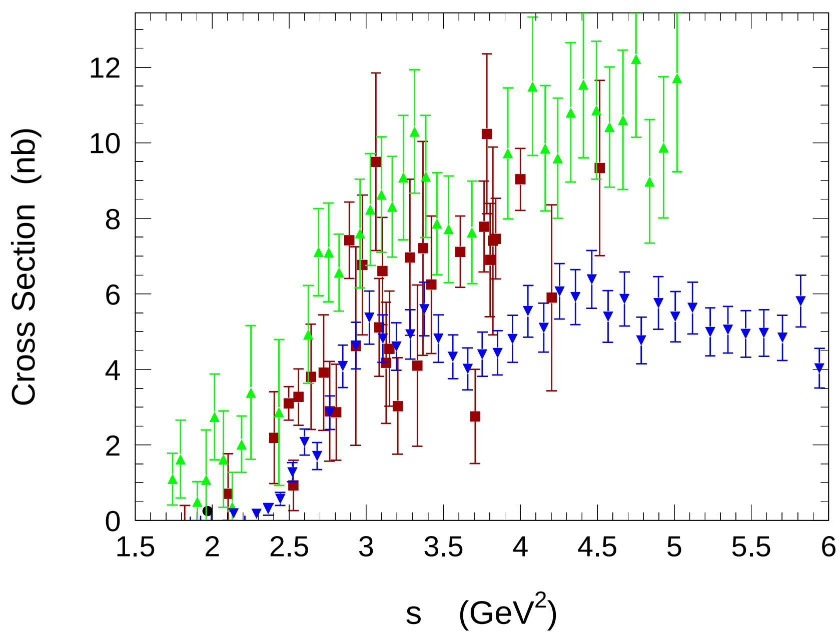


Significant improvement compared to the previous data!

The mode $e^+e^- \to \pi^+\pi^-4\pi^0$ is not measured.

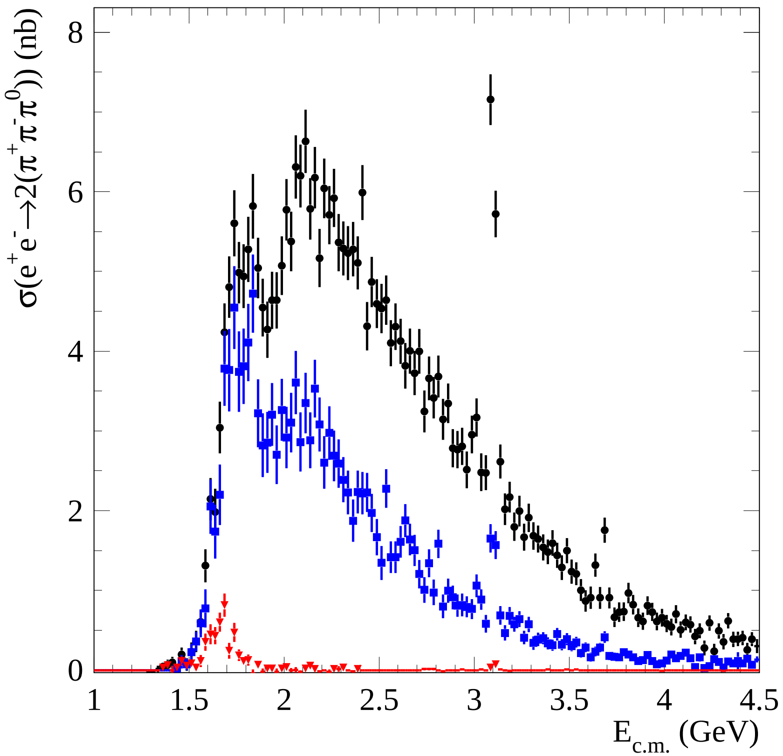
$e^+e^- \to 6\pi$, τ Energy Range





$\sigma(e^+e^- \to 2\pi^+2\pi^-2\pi^0)$ is smaller than previously!

$e^+e^- \to 2\pi^+2\pi^-2\pi^0$, I=0,1



BaBar: $\sigma(e^+e^- \to 2\pi^+2\pi^-2\pi^0)$ is saturated by the $\omega 3\pi$. There is some I=0 component $(\omega\eta)$, so CLEO assumptions seem to be correct.
New CVC calculation is awaited.

$\tau^- \to (3\pi)^- \nu_\tau$ – I

- This decay proceeds via the axial-vector current
- There are two possible final states: $2\pi^-\pi^+$ and $\pi^- 2\pi^0$
- Decay is expected to proceed via the $a_1(1260)^-$ meson, which decays with equal probabilities into $\rho^0\pi^-$ and $\rho^-\pi^0$
- τ decay is the best place to study properties of the $a_1(1260)$:
  PDG gives for its mass 1230 ± 40 MeV and for its width $250 - 600$ MeV
- DELPHI observed a_1' in τ decays, earlier also seen in $\pi^- p$ and $\bar{p}p$. It is now in PDG as $a_1(1640)$ with mass 1647 ± 22 MeV and width 254 ± 27 MeV.

$\tau^- \to (3\pi)^- \nu_\tau$ – II

Measurements of $\mathcal{B}(2\pi^-\pi^+\nu_\tau)$ (S=1.3)

Mode	Group	$N_{\rm ev}, 10^3$	$\mathcal{B}$, %
$h^-h^-h^+$	CLEO,1995	37.7	$9.51 \pm 0.07 \pm 0.20$
	OPAL,1995	6.4	$9.87 \pm 0.10 \pm 0.24$
	DELPHI,2006	12.2	$9.317 \pm 0.090 \pm 0.082$
$2\pi^-\pi^+$	CLEO,2003	43	$9.13 \pm 0.05 \pm 0.46$
	ALEPH,2005	29	$9.041 \pm 0.060 \pm 0.076$
	PDG fit,2007	–	9.33 ± 0.08

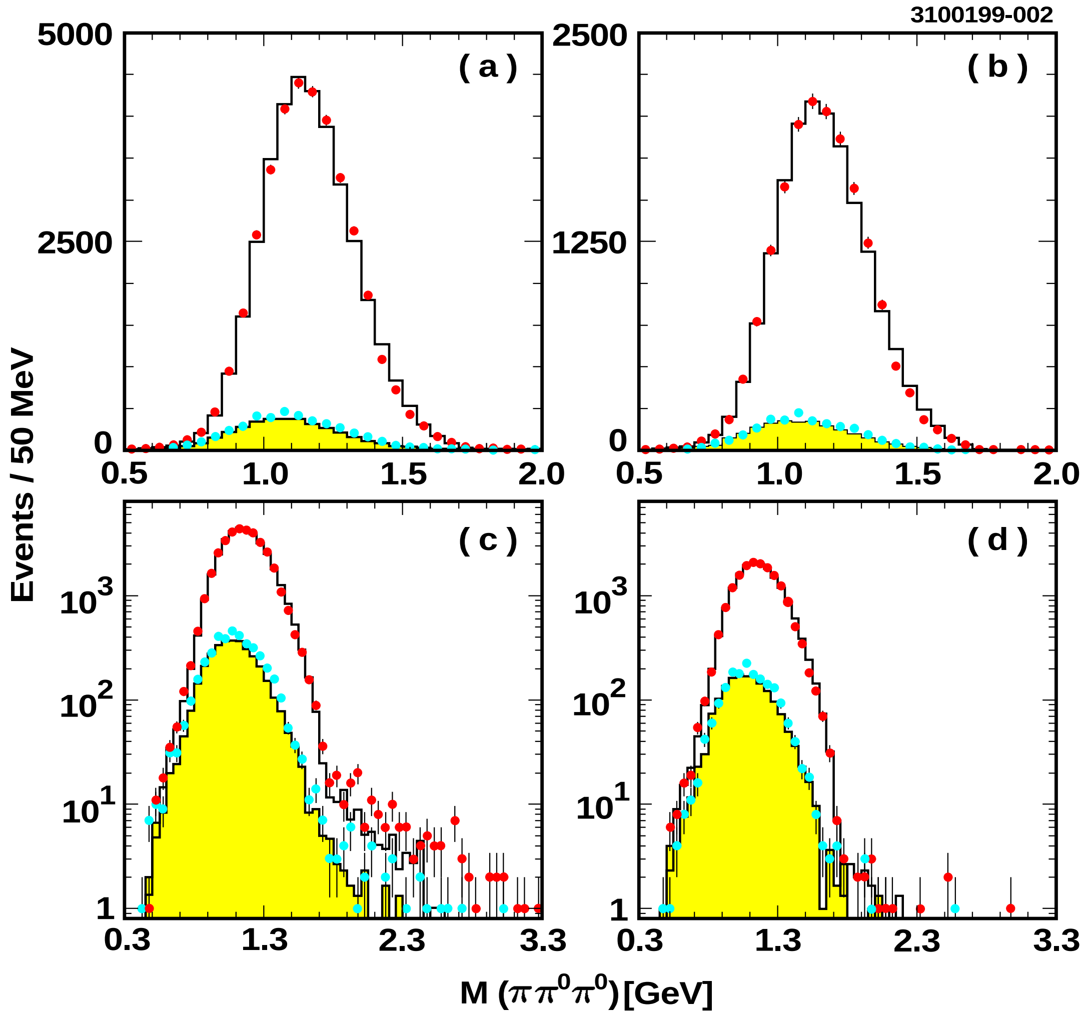
$\tau^- \to (3\pi)^- \nu_\tau$ – III

Measurements of $\mathcal{B}(\pi^- 2\pi^0 \nu_\tau)$ (S=1.3)

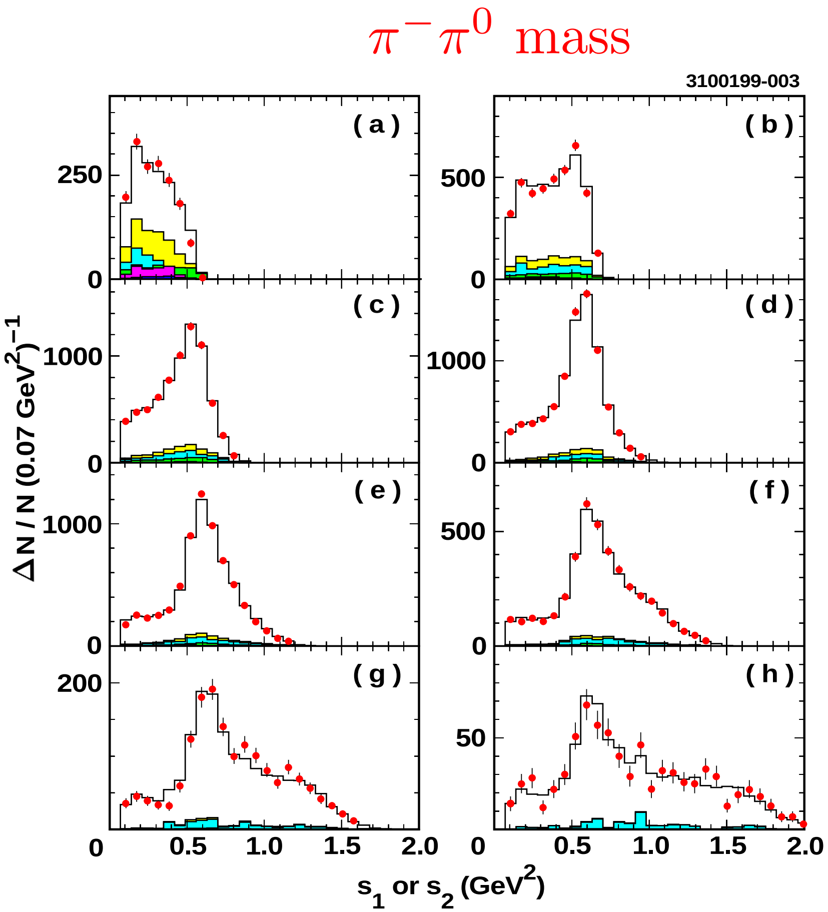
Mode	Group	$N_{\tau\tau}, 10^3$	$\mathcal{B}$, %
$h^- 2\pi^0$	CLEO,1993	4.6	$8.96 \pm 0.16 \pm 0.44$
	L3,1995	1.1	$8.88 \pm 0.37 \pm 0.42$
	DELPHI,2006	9.5	$9.498 \pm 0.320 \pm 0.275$
$\pi^- 2\pi^0$	ALEPH,2005	31	$9.239 \pm 0.086 \pm 0.090$
	PDG fit,2007	–	9.25 ± 0.12

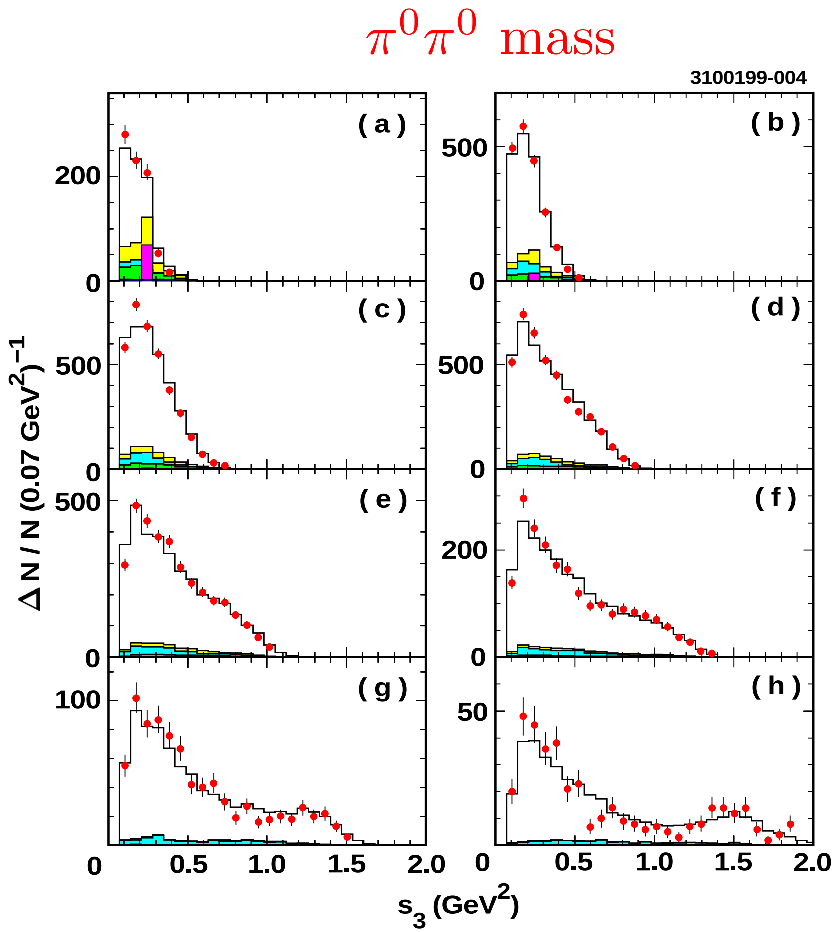
$\tau^- \to (3\pi)^- \nu_\tau$ – IV

51100 events from $N_{\tau\tau} = 4.3 \cdot 10^6$, CLEO, 1999



$\tau^- \to (3\pi)^- \nu_\tau$ – V





$\tau^- \to (3\pi)^- \nu_\tau$ – VI

Mode	Wave	$\mathcal{B}$, %
$\rho\pi$	s	60.19
$\rho(1450)\pi$	s	0.56 ± 0.84
$\rho\pi$	d	1.30 ± 0.60
$\rho(1450)\pi$	d	2.04 ± 1.20
$f_2(1270)\pi$	p	1.19 ± 0.49
$f_0(600)\pi$	p	18.76 ± 4.29
$f_0(1370)\pi$	p	7.40 ± 2.71
K^*K	s	3.3 ± 0.5

We learned from τ decays new modes of $a_1(1260)$ decays!

$\tau^- \to (5\pi)^- \nu_\tau$ – I

There is only one total inclusive measurement by CLEO:
$\mathcal{B}(\tau^- \to (5\pi)^- \nu_\tau) = (0.61 \pm 0.06 \pm 0.08)\%$

Mode	Group	$N_{\rm ev}$	$\mathcal{B}$, %
$h^-4\pi^0$	CLEO,1993	25	$0.16 \pm 0.05 \pm 0.05$
	ALEPH,2005	957	$0.112 \pm 0.037 \pm 0.035$
	PDG fit,2007	–	0.10 ± 0.04
Mode	Group	$N_{\rm ev}$	$\mathcal{B}$, %
$h^-h^-h^+2\pi^0$	ALEPH,2005	2.6k	$0.435 \pm 0.030 \pm 0.035$
	PDG fit,2007	–	0.492 ± 0.034

$\tau^- \to (5\pi)^- \nu_\tau$ – II

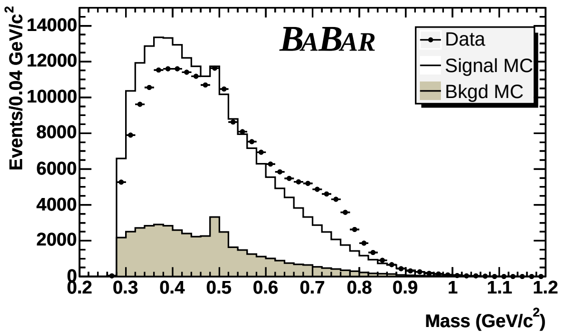
Measurements of $\mathcal{B}(\tau^- \to 3h^- 2h^+ \nu_\tau)$ (S=1.1)

Mode	Group	$N_{\rm ev}$	$\mathcal{B}, 10^{-4}$
$h^- h^- h^+$	CLEO,1994	295	$7.7 \pm 0.5 \pm 0.9$
	OPAL,1999	97	$9.1 \pm 1.4 \pm 0.6$
	ALEPH,2005	165	$7.2 \pm 0.9 \pm 1.2$
	BaBar, 2005	34k	$8.56 \pm 0.05 \pm 0.42$
	DELPHI,2006	96	$9.7 \pm 1.5 \pm 0.5$
	PDG fit,2007	–	8.38 ± 0.35

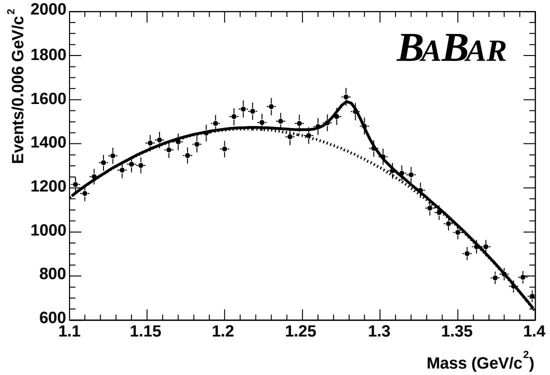
Other Cabibbo-favored Decays (BaBar)

$h^-h^+h^-\nu_\tau,\ 3\pi\pi^0\nu_\tau,\ 3\pi\eta\nu_\tau,\ 3h^-2h^+\nu_\tau,\ f_1(1285)\pi^-\nu_\tau,\ \geq 7\pi\nu_\tau$

$\tau^- \to 3h^-2h^+\nu_\tau$



$\tau^- \to f_1(1285)\pi^-\nu_\tau$



Mode	Group	$N_{\rm ev}$	$\mathcal{B}, 10^{-4}$
$3h^-2h^+\nu_\tau$	BaBar, 2005	34,000	$8.56 \pm 0.05 \pm 0.42$
	CLEO, 1994	295	$7.7 \pm 0.5 \pm 0.9$
$f_1(1285)\pi^-\nu_\tau$	BaBar, 2005	1369 ± 232	$3.9 \pm 0.7 \pm 0.5$
	CLEO, 1997	54	$5.8 \pm 1.4 \pm 1.8$

Search for $\tau^- \to (7\pi)^- \nu_\tau$

$\tau^- \to 4\pi^- 3\pi^+ (\pi^0) \nu_\tau$

Group	$N_{\tau\tau}$	ϵ,%	$\mathcal{B}$, 90%CL UL
HRS, 1987	$4.1 \cdot 10^4$	18	$1.9 \cdot 10^{-4}$
OPAL, 1997	$2.1 \cdot 10^5$	41	$1.4 \cdot 10^{-5}$
CLEO, 1997	$4.2 \cdot 10^6$	16	$2.4 \cdot 10^{-6}$
BaBar, 2005	$206.6 \cdot 10^6$	9	$3.0 \cdot 10^{-7}$

$\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_\tau$

Group	$N_{\tau\tau}$	ϵ,%	$\mathcal{B}$, 90%CL UL
CLEO, 1994	$1.6 \cdot 10^6$	0.9	$1.1 \cdot 10^{-4}$
BaBar, 2006	$206.5 \cdot 10^6$	0.66	$3.4 \cdot 10^{-6}$

Other multiparticle modes from BaBar

BaBar searched for some exclusive modes:

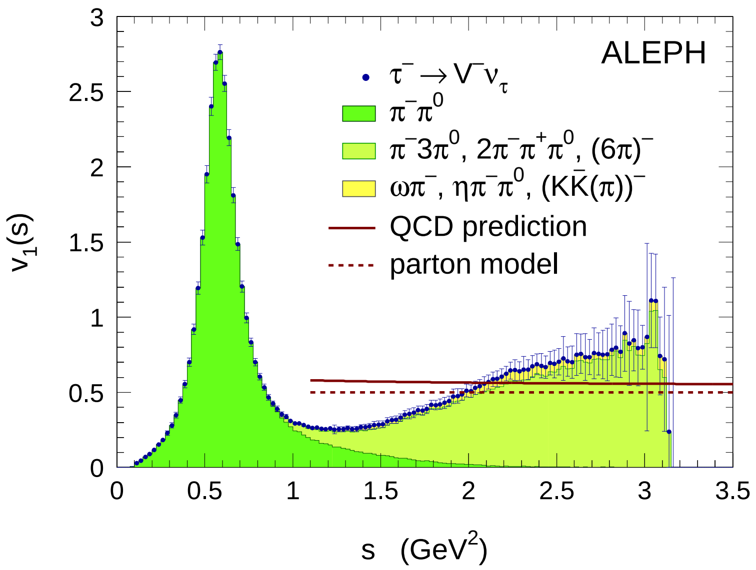
Mode	ϵ,%	$\mathcal{B}$, 90%CL UL
$4\pi^-3\pi^+\nu_\tau$	16	$4.3 \cdot 10^{-7}$
$4\pi^-3\pi^+\pi^0\nu_\tau$	16	$2.5 \cdot 10^{-7}$
$\omega\omega\pi^-\nu_\tau$	1.5	$5.4 \cdot 10^{-7}$

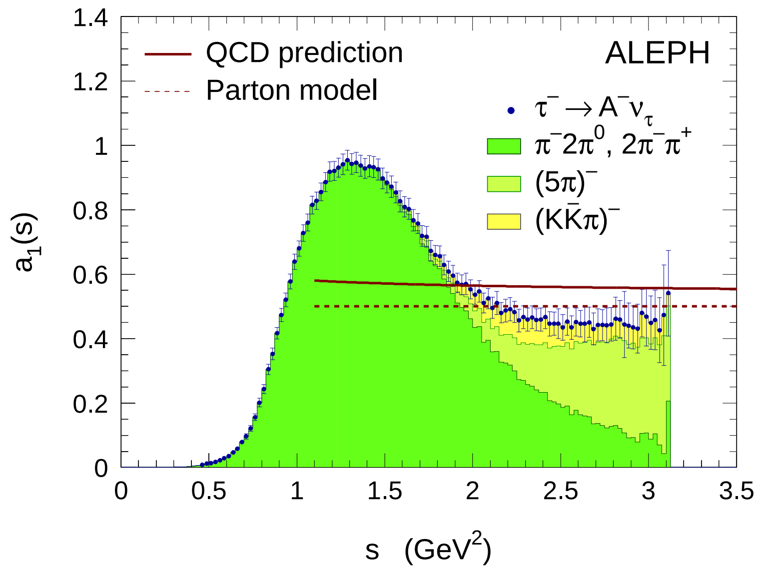
Theory: $\mathcal{B}(4\pi^-3\pi^+\nu_\tau) = 6 \cdot 10^{-11}$

There are two more unobserved modes with high π^0 multiplicity:

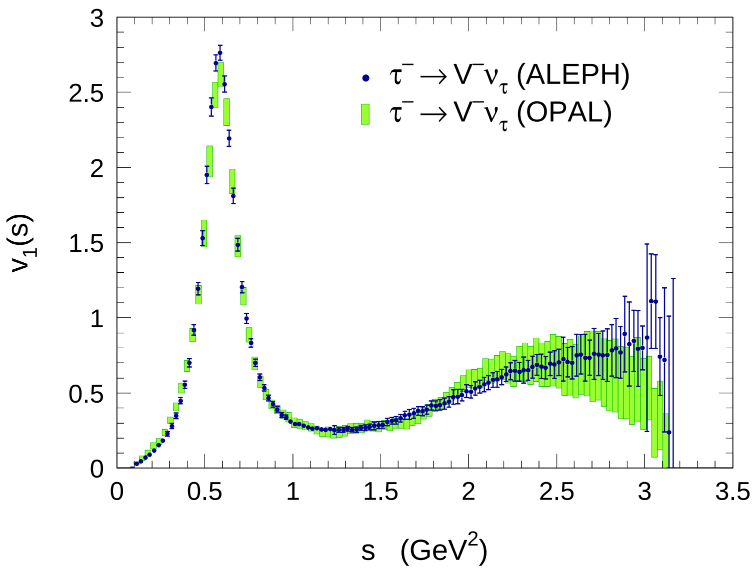
$2\pi^-\pi^+4\pi^0\nu_\tau$ and $\pi^-6\pi^0\nu_\tau$

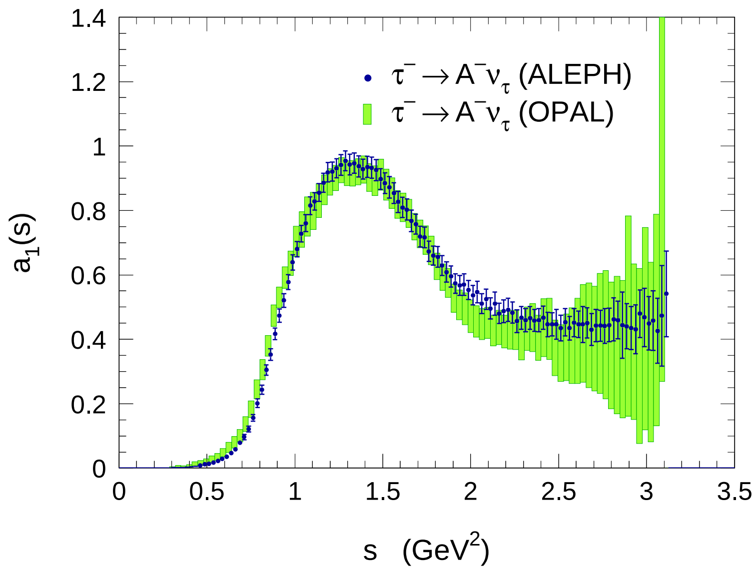
Vector and Axial-Vector Spectral Functions from ALEPH





Comparison of Spectral Functions from ALEPH and OPAL





ALEPH: $\alpha_s(m_\tau^2) = 0.340 \pm 0.005 \pm 0.014$ from 300k τ's

OPAL: $\alpha_s(m_\tau^2) = 0.348 \pm 0.009 \pm 0.019$ from 66k τ's

Summary on Decays with Even N_π

Mode	$\mathcal{B}_{\rm exp}$, %	$\mathcal{B}_{\rm th}$, %	Comments
$\pi^-\pi^0$	25.40 ± 0.10	24.48 ± 0.18	CVC
$\pi^-3\pi^0$	1.04 ± 0.08	1.09 ± 0.08	CVC
$2\pi^-\pi^+\pi^0$	4.59 ± 0.07	3.63 ± 0.21	CVC
$\omega\pi^-$	1.99 ± 0.08	1.82 ± 0.07	CVC
$(6\pi)^-$	$(2.3 \pm 0.5) \cdot 10^{-4}$	$\geq (1.2 \pm 0.2) \cdot 10^{-3}$	CVC

Conclusions

- We already know a lot after CLEO and LEP, Belle and Babar gaining speed
- Big advantages in statistical accuracy and searches for rare modes. Systematic effects may be a problem
- Problems with CVC still exist
- Interesting possibilities for QCD
- Clean laboratory for studies of light mesons
- An excellent testbench for low energy theoretical models
- B factories with 2 ab^{-1} are also unique τ factories with high potential for New Physics and precision studies in SM