# Modelling non-perturbative corrections in semi-inclusive $B$ decays


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**University of Rome "La Sapienza"**

## The work presented in this talk is based on the following papers:

*U. Aglietti, G. Ferrera and G. Ricciardi, Phys. Rev. D74 (2006), [hep-ph/0507285].*

*U. Aglietti, G. Ferrera and G. Ricciardi, Phys. Rev. D74 (2006), [hep-ph/0509095].*

*U. Aglietti, G. Ferrera and G. Ricciardi, Phys. Rev. D74 (2006), [hep-ph/0509271].*

*U. Aglietti, G. Ferrera and G. Ricciardi, [hep-ph/0608047].*

*U. Aglietti, G. Corcella and G. Ferrera, [hep-ph/0610035].*

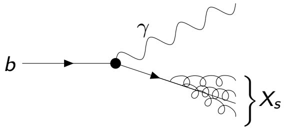
*U. Aglietti, L. Di Giustino, G. Ferrera and L. Trentadue, [hep-ph/0612073].*

# Outline

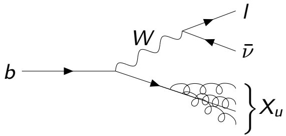
1. Semi-inclusive $B$ decays
2. Analytic QCD coupling
3. Phenomenological Analysis
4. Conclusions and Perspectives

# Semi-inclusive $B$ decays

$$b \to X_q + \langle \text{ non QCD partons } \rangle, \quad q \equiv u, s, d$$




Radiative decay:
$b \to X_s + \gamma$




Semileptonic charmless decay:
$b \to X_u + l + \bar{\nu}$

- Energy scales: $m_b \geq E_X \geq m_X, \quad Q = E_X + \sqrt{E_X^2 - m_X^2}$
- Threshold region: $m_X \ll E_X, \quad Q = 2\,E_X$

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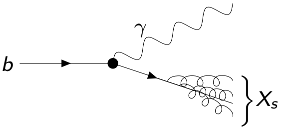


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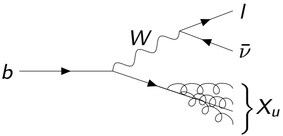


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- Threshold region: $m_X \ll E_X, \quad Q = 2\,E_X$

- The LO probability for a light quark to evolve into a jet with an invariant mass smaller than $m_X$ is:

$$P_<(m_X) = 1 + \alpha_S \frac{C_F}{\pi} \int_0^1 \frac{d\omega}{\omega} \int_0^1 \frac{d\theta^2}{\theta^2} \left[ \Theta\left( \frac{m_X^2}{Q^2} - \omega\theta^2 \right) - 1 \right] + O(\alpha_S^2)$$
$$= 1 - \alpha_S \frac{C_F}{2\pi} \ln^2\left( \frac{Q^2}{m_X^2} \right) + O(\alpha_S^2).$$

- Multiple gluon emission gives rise to the double logarithmic series, i.e. to powers of the term $\alpha_S \ln^2 \left(Q^2/m_X^2\right)$.

$$\sigma(y) = \sum_{n=1}^{\infty} \sum_{k=1}^{2n} c_{n,k}\, \alpha_S^n(Q) \ln^k y \,, \qquad y = \frac{m_X^2}{Q^2} \,.$$

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# Radiative and Semileptonic charmless decays: factorization

- Resummation formula for radiative decays [Aglietti ('01)]:

$$\frac{1}{\Gamma_r}\frac{d\Gamma_r}{dt} = C_r[\alpha_S(Q)]\,\sigma[t;\alpha_S(Q)] + d_r[t;\,\alpha_S(Q)]\ ,$$

where $t \equiv m_X^2/m_b^2$ and $Q = 2\,E_X$.
$Q = m_b(1 + m_X^2/m_b^2) \simeq m_b,\quad \alpha_S(Q) \simeq \alpha_S(m_b) \simeq 0.22$ .

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$$\frac{1}{\Gamma_s}\frac{d^3\Gamma_s}{dxdudw} = C_s\,[x,w;\alpha_S(Q)]\;\sigma\,[u;\alpha_S(Q)] + d_s\,[x,u,w;\alpha_S(Q)]$$

where $x \equiv \frac{2E_l}{m_b}$ , $w \equiv \frac{2E_X}{m_b}$ , $u \equiv \frac{1-\sqrt{1-4y}}{1+\sqrt{1-4y}}$ , $y \equiv \frac{m_X^2}{4E_X^2}$.
$Q = m_b(1 + m_X^2/m_b^2 - q^2/m_b^2)$ ; $q^2$ is the dilepton invariant mass.
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- In the Mellin space the threshold resummed form factor reads [Sterman ('87), Catani & Trentadue ('89)]:

$$\ln \sigma_N = \int_0^1 dy \frac{(1-y)^{N-1}-1}{y} \left\{ \int_{Q^2y^2}^{Q^2y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] + B[\alpha_S(Q^2y)] + D[\alpha_S(Q^2y^2)] \right\}$$

$$= \sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \alpha_S^n(Q) \ln^m N \,, \qquad A(\alpha_S) = \textstyle\sum_{n=1}^{\infty} A_n \alpha_S^n, \quad B(\alpha_S) = \cdots \,, \quad D(\alpha_S) = \cdots \,.$$

- The new expansion parameter is $\alpha_S L$ (where $L \equiv \ln N$):
  LL: $\alpha_S^n L^{n+1}$; NLL: $\alpha_S^n L^n$; NNLL: $\alpha_S^n L^{n-1}$; ... and so on.
- When $y = \frac{m_X^2}{Q^2} \to 0$ the integration in $k_\perp^2$ involves $\alpha_S$ at the Landau pole: it is necessary a prescription, e. g. the Minimal Prescription [Catani, Mangano, Nason & Trentadue ('96)].
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# Analytic QCD coupling

- Standard QCD coupling: physical cut at $\mu^2 < 0$ and unphysical pole at $\mu^2 = \Lambda^2_{QCD}$:

$$\alpha_S^{lo}(\mu^2) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2_{QCD}}} \, .$$

- Analytic QCD coupling: same discontinuity along the cut but analytic elsewhere in the complex plane [Shirkov & Solovtsov ('97)]:

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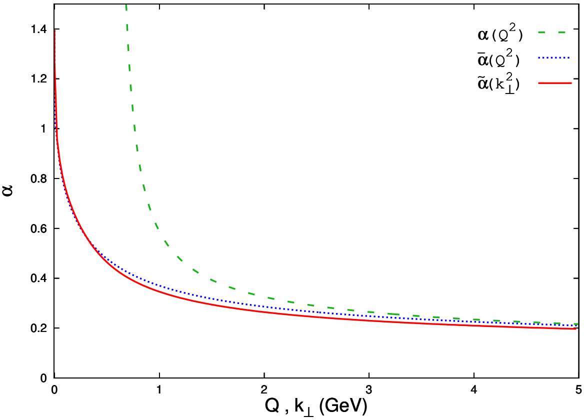


Figure 1: Time-like and space-like analytic couplings compared with the standard one.

# Phenomenological Analysis

*b*-quark fragmentation: $e^+e^- \to Z^0 \to B + X$, $x_b = \frac{2E_b}{m_Z}$

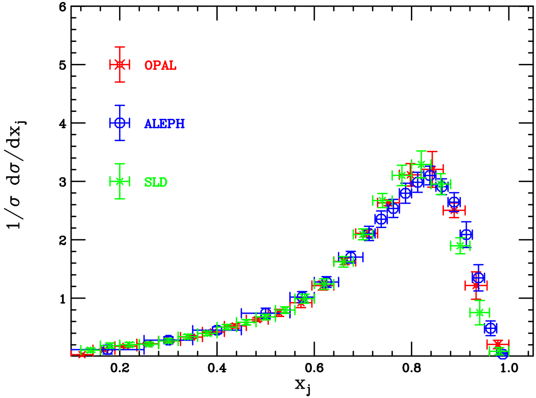


Figure 2: *B*-hadron spectrum in $e^+e^-$ annihilation at $Z^0$ peak: prevision of the model compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

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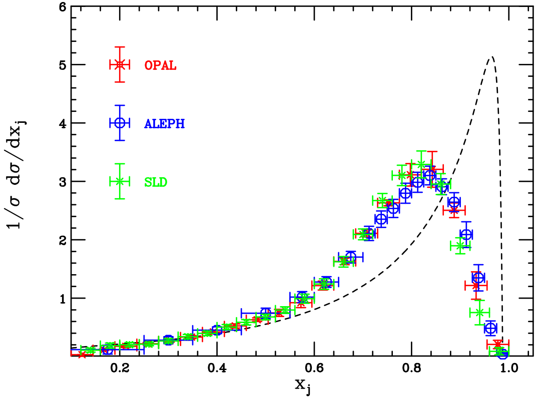


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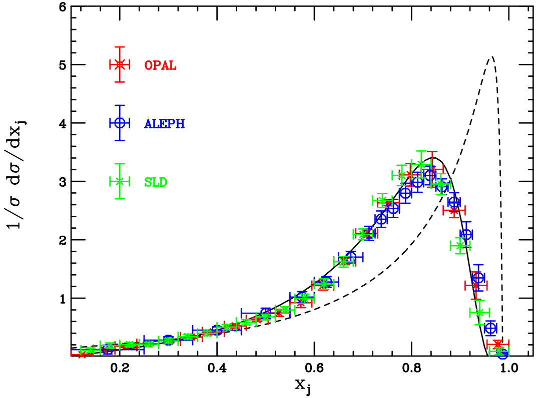


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## Radiative decay: hadron mass distribition

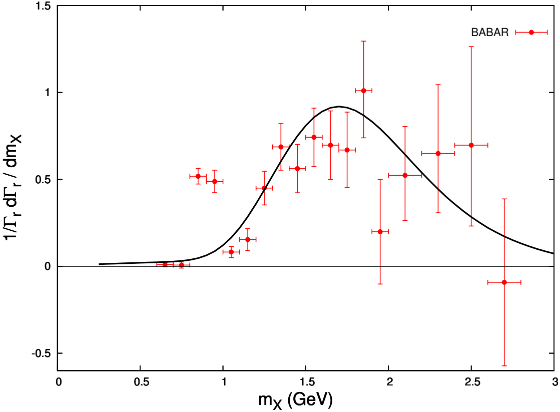


Figure 3: Invariant hadron mass distribution in the radiative decay: prevision of the model compared with the experimental data [BaBar ('05)].The $K^*$ peak cannot clearly be accounted for in a perturbative $QCD$ framework.

## Radiative decay: photon energy distribution

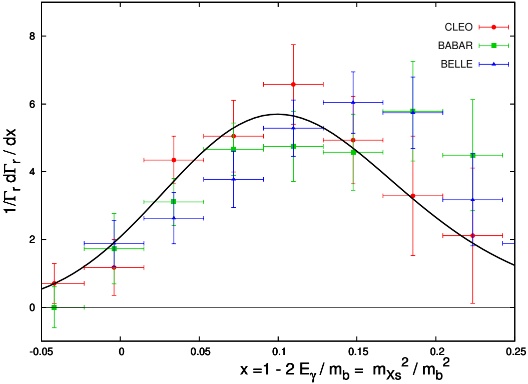


Figure 4: Photon energy spectrum in the radiative decay: prevision of the model compared with data [Cleo ('01), BaBar ('05), Belle ('05)]. To model the Doppler effect related to the motion of the $B$ mesons , we have convoluted the theoretical curve with a Gaussian distribution with $\sigma \sim 180$ MeV.

# Semileptonic decay: hadron mass distribution

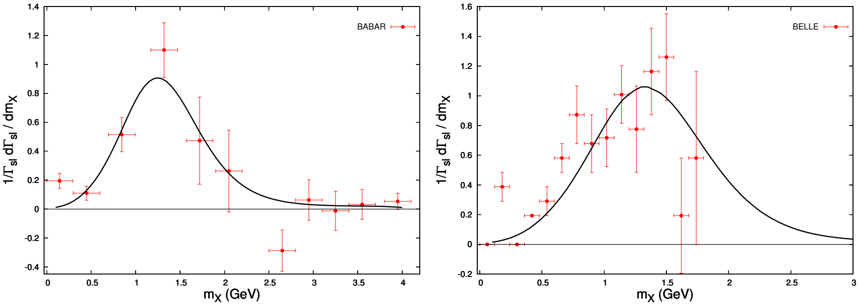


Figure 5: Invariant hadron mass distribution in the semileptonic decay: prevision of the model compared with the experimental [Belle ('04), BaBar ('05)]. Note the $\pi$ and the $\rho$ peaks at small hadron masses.

## Semileptonic decay: electron energy distribution

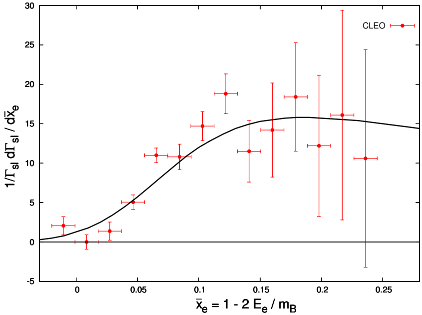


Figure 6: Inclusive charmless electron spectrum in the semileptonic decay: prevision of the model compared with data [Cleo ('01) ]. Doppler effect included convoluting with a Gaussian with $\sigma \sim 100$ MeV.

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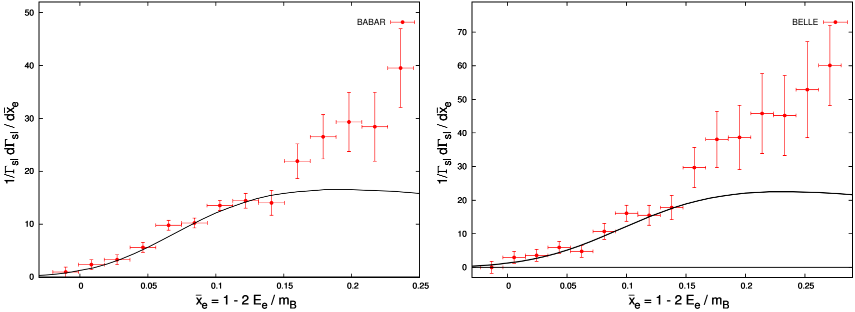


Figure 7: Inclusive charmless electron spectrum in the semileptonic decay: prevision of the model compared with data [Babar ('05) and Belle ('04)]. Doppler effect included convoluting with a Gaussian with $\sigma \sim 100$ MeV. We do not known whether the discrepancy is related to a deficiency of our model or to an under-estimate charm background.

# Extraction of $\alpha_S(m_Z)$ and $V_{ub}$ from the data

$b \to s\,\gamma$

$$\alpha_S(m_Z) = 0.117 \pm 0.004 \quad (E_\gamma : CLEO,\ \sigma_\gamma = 150\ MeV)$$
$$\alpha_S(m_Z) = 0.129 \pm 0.005 \quad (E_\gamma : BABAR,\ \sigma_\gamma = 200\ MeV)$$
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# A possible new measure

## Hadronic energy distribution in the semileptonic decay

- The QCD form factor can be experimentally measured from the $m_X$ or the $E_\gamma$ distribution of the radiative decay:

$$\frac{1}{\Gamma_r}\frac{d\Gamma_r}{dt} = C_r(\alpha_S)\,\sigma[t;\alpha_S(m_b)] + d_r(t;\,\alpha_S)\ ,$$

unfortunately the data are not sufficiently accurate to do this.

- The only single differential distribution in the semileptonic decay which permits the direct extraction of the QCD form factor is the hadronic energy distribution for $w \equiv 2E_X/m_b > 1$ :

$$\frac{1}{\Gamma_s}\frac{d\Gamma_s}{dw} = C_{w_1}(\alpha_S)\Big\{1 - C_{w_2}(\alpha_S)\Sigma[w-1;\alpha_S(m_b)] + H(w;\alpha_S)\Big\} \quad (w>1)$$

where $\Sigma[u;\alpha_S] = \int_0^u du'\sigma(u';\alpha_S)$.

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- The only single differential distribution in the semileptonic decay which permits the direct extraction of the QCD form factor is the hadronic energy distribution for $w \equiv 2E_X/m_b > 1$ :

$$\frac{1}{\Gamma_s}\frac{d\Gamma_s}{dw} = C_{w_1}(\alpha_S)\Big\{1 - C_{w_2}(\alpha_S)\Sigma[w-1;\alpha_S(m_b)] + H(w;\alpha_S)\Big\} \quad (\mathrm{w}>1)$$

where $\Sigma[u;\alpha_S] = \int_0^u du'\sigma(u';\alpha_S)$.

# Semileptonic decay: hadronic energy distribution

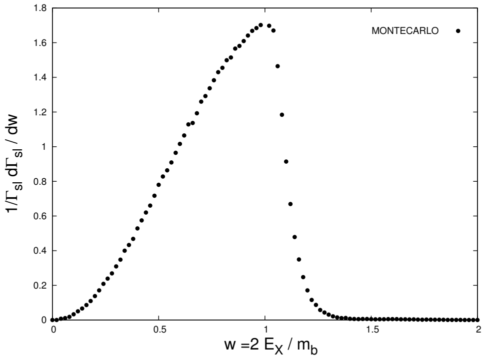


Figure 8: Hadronic energy distribution in the semileptonic decay generated by a montecarlo based on the model.

# Semi-leptonic charmed decay and $V_{cb}$

- To describe the process $B \to X_c \; l \; \nu$ we need a new formalism to take in account the non-vanishing charm mass $m_c$: [Aglietti, Di Giustino, G.F. & Trentadue ('06)]

$$\sigma_N(Q^2, m^2) = \sigma_N(Q^2)\,\delta_N(Q^2, m^2), \qquad r \equiv \frac{m^2}{Q^2} \simeq 0.1$$

$$\ln\delta_N = \int_0^1 dy \frac{(1-y)^{r(N-1)} - 1}{y} \left\{ -\int_{m^2 y^2}^{m^2 y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] - B[\alpha_S(m^2 y)] + D[\alpha_S(m^2 y^2)] \right\}$$

- Using this formula, the full $O(\alpha_S)$ triple differential distribution [Trott ('04), Aquila, Gambino, Ridolfi & Uraltsev ('05)] and the model previously described we are confident that we can provide quantitative description of the data.

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# Conclusions and Perspectives

- Through the analytic QCD coupling and NNLL threshold resummation we have developed a model that includes the large non-perturbative effects in the semi-inclusive $B$ decays: it describes with good accuracy the measured spectra without introducing any ad hoc non-perturbative component.
- We have a disagreement with the electron spectrum in the semileptonic decay (BaBar and Belle). It could be a deficiency of our model or an underestimate of the charm background.
- It would be interesting if experimentalist made a statistical analysis of the compatibility of our model with the data to extract $V_{ub}$.
- We propose to measure the $E_X$ spectrum of the semileptonic charmless decay: it gives direct information on the QCD form factor.
- Using a new resummation formalism recently developed together with available fixed order calculation we will carry out a phenomenological analysis of semileptonic charmed decays soon.

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# Back-up Slides

# Semileptonic decay: electron energy distribution

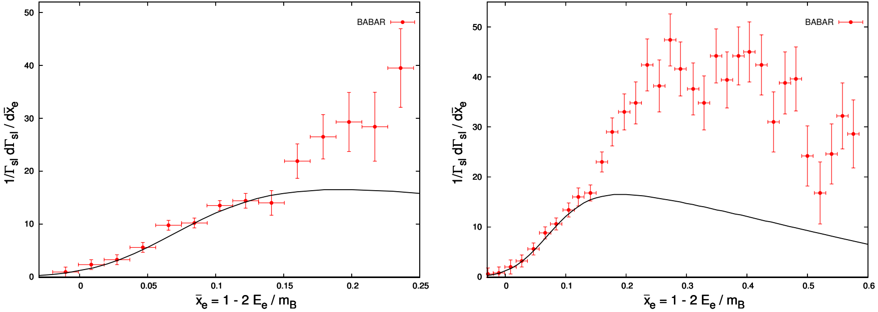


Figure 7: Inclusive charmless electron spectrum in the semileptonic decay: prevision of the model compared with data [Babar ('05)]. Doppler effect included convoluting with a Gaussian with $\sigma \sim 100$ MeV. We do not known whether the discrepancy is related to a deficiency of our model or to an under-estimate charm background.

# Radiative and Semileptonic charmless decays

- Resummation formula for radiative decays [Aglietti ('01)]:

$$\frac{1}{\Gamma_r}\frac{d\Gamma_r}{dt} = C_r[\alpha_S(Q)]\,\sigma[t;\alpha_S(Q)] + d_r[t;\,\alpha_S(Q)]\ ,$$

where $t \equiv m_X^2/m_b^2$ and $Q = 2\,E_X$.
$Q = m_b(1 + m_X^2/m_b^2) \simeq m_b, \quad \alpha_S(Q) \simeq \alpha_S(m_b) \simeq 0.22$ .

- Resummation formula for semileptonic charmless decays [Aglietti ('01)]:

$$\frac{1}{\Gamma_s}\frac{d^3\Gamma_s}{dxdudw} = C_s\,[x,w;\alpha_S(Q)]\ \sigma\,[u;\alpha_S(Q)] + d_s\,[x,u,w;\alpha_S(Q)]$$

where $x \equiv \frac{2E_l}{m_b}$ , $w \equiv \frac{2E_X}{m_b}$ , $u \equiv \frac{1-\sqrt{1-4y}}{1+\sqrt{1-4y}}$ , $y \equiv \frac{m_X^2}{4E_X^2}$.
$Q = m_b(1 + m_X^2/m_b^2 - q^2/m_b^2)$ ; $q^2$ is the dilepton invariant mass.
We can not put $\alpha_S(Q) \simeq \alpha_S(m_b)$ in the form factor $\sigma$:
$\alpha_S(Q) = \alpha_S(wm_b)$.

# Analytic QCD coupling

- Standard QCD coupling: physical cut at $\mu^2 < 0$ and unphysical pole at $\mu^2 = \Lambda^2_{QCD}$:

$$\alpha_S^{lo}(\mu^2) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2_{QCD}}} \,.$$

- Analytic QCD coupling: same discontinuity along the cut but analytic elsewhere in the complex plane [Shirkov & Solovtsov ('97)]:

$$\bar{\alpha}_S(Q^2) = \frac{1}{2\pi i} \int_0^\infty \frac{ds}{s + Q^2}\, Disc_s\, \alpha_S(-s), \quad \text{space-like.}$$

- The infrared pole is subtracted without modify high energy behaviour

$$\bar{\alpha}_S^{lo}(Q^2) = \frac{1}{\beta_0} \left[ \frac{1}{\ln Q^2/\Lambda^2_{QCD}} - \frac{\Lambda^2_{QCD}}{Q^2 - \Lambda^2_{QCD}} \right] \,,$$

$$\lim_{Q^2 \to 0} \bar{\alpha}_S(Q^2) = \frac{1}{\beta_0} \,, \qquad \lim_{Q^2 \to \infty} \bar{\alpha}_S(Q^2) = \lim_{Q^2 \to \infty} \alpha_S(Q^2) \,.$$

- Semi-inclusive $B$ decays are time-like processes:

$$\tilde{\alpha}_S(k_\perp^2) = \frac{i}{2\pi}\int_0^{k_\perp^2} ds\, Disc_s\, \frac{\bar{\alpha}_S(-s)}{s}, \quad \text{time-like.}$$

- At leading order we have:

$$\tilde{\alpha}_S^{lo}(k_\perp^2) = \frac{1}{\beta_0}\left(\frac{1}{2} - \frac{1}{\pi}\arctan\frac{\ln\frac{k_\perp^2}{\Lambda_{QCD}^2}}{\pi}\right),$$

$$\lim_{k_\perp^2\to 0}\tilde{\alpha}_S(k_\perp^2) = \frac{1}{\beta_0}, \qquad \lim_{k_\perp^2\to\infty}\tilde{\alpha}_S(k_\perp^2) = \lim_{k_\perp^2\to\infty}\alpha_S(k_\perp^2).$$

- The well defined quantity

$$\alpha_0 = \frac{1}{\mu_I}\int_0^{\mu_I}\tilde{\alpha}_S(k_\perp^2)\, dk_\perp \simeq 0.44,$$

is similar to the fitted value from shape variables data in the DMW model ($\alpha_0 \simeq 0.45$) [Dokshitzer, Marchesini & Webber ('95)].

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# Threshold Resummation

In the Mellin space the threshold resummed form factor reads [Sterman ('87), Catani & Trentadue ('89)]:

$$\ln \sigma_N = \int_0^1 dy \frac{(1-y)^{N-1}-1}{y} \left\{ \int_{Q^2 y^2}^{Q^2 y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] + B[\alpha_S(Q^2 y)] + D[\alpha_S(Q^2 y^2)] \right\}$$

$$= \sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm}\, \alpha_S^n(Q)\, L^m = L\, g_1(\lambda) + g_2(\lambda) + \alpha_S g_3(\lambda) + \cdots$$

where $A(\alpha_S) = \sum_{n=1}^{\infty} A_n\, \alpha_S^n, \quad B(\alpha_S) = \sum_{n=1}^{\infty} B_n\, \alpha_S^n, \quad D(\alpha_S) = \sum_{n=1}^{\infty} D_n\, \alpha_S^n$ .

and $g_i(\lambda) = \sum_{n=0}^{\infty} g_{i,n} \lambda^n, \quad \lambda \equiv \beta_0\, \alpha_S\, L, \quad L \equiv \ln N$

# Radiative $B$ decay: $B \to X_s \, \gamma$

- The resummation formula for the invariant mass distribution reads [Aglietti ('01)]:

$$\frac{1}{\Gamma_r}\frac{d\Gamma_r}{dt} = C_r[\alpha_S(Q)]\,\sigma[t;\alpha_S(Q)] + d_r[t;\,\alpha_S(Q)]\ ,$$

where $t \equiv m_X^2/m_b^2$ and $Q = 2\,E_X$.

$$Q = m_b(1 + m_X^2/m_b^2) \simeq m_b, \quad \alpha_S(Q) \simeq \alpha_S(m_b) \simeq 0.22\ .$$

- $C_r(\alpha_S) = C_r^{(0)} + \alpha_S\, C_r^{(1)} + \cdots$
  short-distance (process dependent) hard factor.
- $\Sigma(t;\alpha_S) = \int_0^t \sigma(t';\alpha_S)dt' = \sum_{n=0}^{\infty}\sum_{k=0}^{2n} \Sigma_{n,k}\, \alpha_S^n \ln^k(1/t)$
  long-distance dominated (universal) QCD form factor.
- $d_r(t;\alpha_S) = d_r^{(0)}(t) + \alpha_S\, d_r^{(1)}(t) + \cdots$
  short-distance (process dependent) remainder function, to have good approximation also in the region $m_X \le E_X$: $\lim_{t\to 0}\int_0^t d_r(t';\alpha_S)dt' = 0$ .

# Semileptonic $B$ decay: $B \to X_u \, l \, \nu$

- The resummation formula for the triple differential distribution reads [Aglietti ('01)]:

$$\frac{1}{\Gamma_s}\frac{d^3\Gamma_s}{dx du dw} = C_s\left[x, w; \alpha_S(Q)\right] \sigma\left[u; \alpha_S(Q)\right] + d_s\left[x, u, w; \alpha_S(Q)\right]$$

where $x \equiv \frac{2E_l}{m_b}$ , $w \equiv \frac{2E_X}{m_b}$ , $u \equiv \frac{1-\sqrt{1-4y}}{1+\sqrt{1-4y}}$ , $y \equiv \frac{m_X^2}{4E_X^2}$ and,

$Q = m_b(1 + m_X^2/m_b^2 - q^2/m_b^2)$ ; $q^2$ is the dilepton invariant mass. In this case we can not put $\alpha_S(Q) \simeq \alpha_S(m_b)$ in the resummed form factor where $\alpha_S(Q) = \alpha_S(w\, m_b)$.

- $C_s(x, w;\, \alpha_S) = C_s^{(0)}(x, w) + \alpha_S\, C_s^{(1)}(x, w) + \cdots$
short-distance (process dependent) hard factor.
- $d_s(x, u, w; \alpha_S) = d_s^{(0)}(x, u, w) + \alpha_S\, d_s^{(1)}(x, u, w) + \cdots$
short-distance (process dependent) remainder function.

# Non universality effects

- Universality of long-distance effects studied by comparing the logarithmic structure of different spectra.
- Spectra not involving integration over hadron energy: same infrared structure of the hadron invariant mass distribution of the radiative decay i.e pure short-distance relation [Aglietti, Ricciardi & G.F. ('05)].

$$\Sigma(u;\alpha_S) = \int_0^u \sigma(u';\alpha_S)du' = \sum_{n=0}^{\infty}\sum_{k=0}^{2n} \Sigma_{n,k}\,\alpha_S^n \ln^k \frac{1}{u}$$

- Spectra involving integration over hadron energy: different infrared structure from each other and from the hadron invariant mass distribution of the radiative decay i.e not pure short-distance relation [Aglietti, Ricciardi & G.F. ('05)].

$$\Sigma_U(u;\alpha_S) = \frac{\int_0^1\int_0^w C(x,w;\alpha_S)dx\Sigma(u;\alpha_S(wm_b))dw}{\int_0^1\int_0^w C(x,w;\alpha_S)\,dx\,dw} = \sum_{n=0}^{\infty}\sum_{k=0}^{2n}\Sigma_{U\,n,k}\alpha_S^n \ln^k \frac{1}{u}$$

# Model for soft gluon effects

- Non perturbative effects represent substantial contributions to the spectra that cannot be neglected both in semi-inclusive $B$ decay and in $b$-quark fragmentation.
- The usual approach is to factorize non perturbative contributions in a universal distribution, depending on free parameters fitted to experimental data, and convolute it with perturbative spectra.
- By the use of a regular low energy QCD coupling our model includes non perturbative effects. Without introducing an additional ad hoc non perturbative component we are able to compare our predictions directly with data [Aglietti, G.F. & Ricciardi ('06)].

# Comparison with DMW model

- Since the time-like coupling is regular for any value of $k_\perp$, we can compute the average of the coupling

$$\alpha_0 = \frac{1}{\mu_I}\int_0^{\mu_I} \tilde{\alpha}_S(k_\perp^2)\, dk_\perp ,$$

which is a free parameter to be determined with a fit to experimental data [Dokshitzer, Marchesini & Webber ('95)].

- Assuming $\alpha_S(m_b) = 0.22$, $n_f = 3$ and $\mu_I = 2\ GeV$, we obtain at leading order with the time-like coupling:

$$\alpha_0 \approx 0.44.$$

- The fitted value from shape variable in $e^+e^-$ data is around 0.45, not distant from our estimate.

# Improved threshold resummation

- The improved threshold resummation formula therefore reads

$$\ln \sigma_N = \int_0^1 dy \, \frac{(1-y)^{N-1}-1}{y} \left\{ \int_{Q^2 y^2}^{Q^2 y} \frac{dk_\perp^2}{k_\perp^2} \tilde{A}[\tilde{\alpha}_S(k_\perp^2)] + \tilde{B}[\tilde{\alpha}_S(Q^2 y)] + \tilde{D}[\tilde{\alpha}_S(Q^2 y^2)] \right\}$$

- The coefficients for the time-like coupling are obtained by imposing the equality:

$$A(\alpha_S) = \tilde{A}(\tilde{\alpha}_S),$$

where

$$\tilde{A}(\tilde{\alpha}_S) = \sum_{n=1}^{\infty} \tilde{A}_n \, \tilde{\alpha}_S^n = \tilde{A}_1 \, \tilde{\alpha}_S + \tilde{A}_2 \, \tilde{\alpha}_S^2 + \tilde{A}_3 \, \tilde{\alpha}_S^3 + \cdots .$$

- Expressing the time-like coupling in terms of the standard one, we obtain:

$$\tilde{A}_1 = A_1; \quad \tilde{A}_2 = A_2; \quad \tilde{A}_3 = A_3 + \frac{(\pi \beta_0)^2}{3} A_1 \simeq 0.31 + 0.72 \simeq 1;$$

analogous relations hold for $\tilde{B}_i$ and $\tilde{D}_i$.

# Phenomenological Analysis

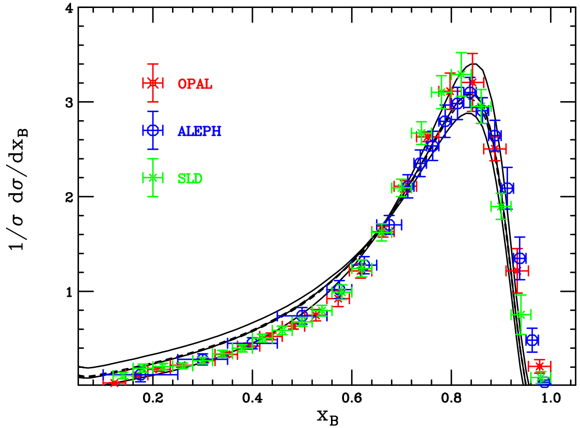


Figure 2: Model dependence on the factorizations scales of $B$-hadron spectrum in $e^+e^-$ annihilation at $Z^0$ peak. Solid lines: $\mu_{0F} = m_b/2$, $m_b$ and $2m_b$; dashed lines: $\mu_F = m_Z/2$, $m_Z$ and $2m_Z$.

# Extraction of $\alpha_S(m_Z)$ and $V_{ub}$ from the data

- Since our model not contain free paramenter to be fitted to the data we have been able to extract, for the first time, the value of $\alpha_S(m_Z)$ from the experimetal data of Babar, Belle, and Cleo Collaborations.
- For each spectrum (excluding the electron energy spectrum of BaBar and Belle) the extracted values of $\alpha_S(m_Z)$ are in agreement with the current world average within at most two standard deviation [PDG ('06)].
- Our statistical analisys is preliminary, a complete analisys requires the exact knowledge of the experimental resolution functions.
- With such detailed statistical analisys it is also possible to excract the value $V_{ub}$ of the CKM matrix.

$$g_1(\lambda) = -\frac{A_1}{2\beta_0\lambda}\left[(1-2\lambda)\ln(1-2\lambda) - 2(1-\lambda)\ln(1-\lambda)\right];$$

$$\begin{aligned}
g_2(\lambda) &= \frac{D_1}{2\beta_0}\ln(1-2\lambda) + \frac{B_1}{\beta_0}\ln(1-\lambda) + \frac{A_2}{2{\beta_0}^2}\left[\ln(1-2\lambda) - 2\ln(1-\lambda)\right] \\
&- \frac{A_1\beta_1}{4{\beta_0}^3}\left[2\ln(1-2\lambda) + \ln^2(1-2\lambda) - 4\ln(1-\lambda) - 2\ln^2(1-\lambda)\right] \\
&+ \frac{A_1\gamma_E}{\beta_0}\left[\ln(1-2\lambda) - \ln(1-\lambda)\right];
\end{aligned}$$

$$\begin{aligned}
g_3(\lambda) &= -\frac{D_2\lambda}{\beta_0(1-2\lambda)} - \frac{2D_1\gamma_E\lambda}{1-2\lambda} + \frac{D_1\beta_1}{2{\beta_0}^2}\left(\frac{2\lambda}{1-2\lambda} + \frac{\ln(1-2\lambda)}{1-2\lambda}\right) - \frac{B_2\lambda}{\beta_0(1-\lambda)} - \frac{B_1\gamma_E\lambda}{1-\lambda} \\
&+ \frac{B_1}{{\beta_0}^2}\beta_1\left(\frac{\lambda}{1-\lambda} + \frac{\ln(1-\lambda)}{1-\lambda}\right) - \frac{A_3}{2{\beta_0}^2}\left(\frac{\lambda}{1-2\lambda} - \frac{\lambda}{1-\lambda}\right) - \frac{A_2\gamma_E}{\beta_0}\left(\frac{1}{1-2\lambda} - \frac{1}{1-\lambda}\right) \\
&+ \frac{A_2\beta_1}{2{\beta_0}^3}\left(\frac{3\lambda}{1-2\lambda} - \frac{3\lambda}{1-\lambda} + \frac{\ln(1-2\lambda)}{1-2\lambda} - \frac{2\ln(1-\lambda)}{1-\lambda}\right) - \frac{A_1{\gamma_E}^2}{2}\left(\frac{4\lambda}{1-2\lambda} - \frac{\lambda}{1-\lambda}\right) \\
&- \frac{A_1\pi^2}{12}\left(\frac{4\lambda}{1-2\lambda} - \frac{\lambda}{1-\lambda}\right) - \frac{A_1\beta_2}{4{\beta_0}^3}\left(\frac{2\lambda}{1-2\lambda} - \frac{2\lambda}{1-\lambda} + 2\ln(1-2\lambda) - 4\ln(1-\lambda)\right) \\
&+ \frac{A_1\beta_1\gamma_E}{{\beta_0}^2}\left(\frac{1}{1-2\lambda} - \frac{1}{1-\lambda} + \frac{\ln(1-2\lambda)}{1-2\lambda} - \frac{\ln(1-\lambda)}{1-\lambda}\right) - \frac{A_1{\beta_1}^2}{2{\beta_0}^4}\left(\frac{\lambda}{1-2\lambda} - \frac{\lambda}{1-\lambda}\right. \\
&\left. - \ln(1-2\lambda) + \frac{\ln(1-2\lambda)}{1-2\lambda} + \frac{\ln(1-2\lambda)^2}{2(1-2\lambda)} + 2\ln(1-\lambda) - \frac{2\ln(1-\lambda)}{1-\lambda} - \frac{\ln(1-\lambda)^2}{1-\lambda}\right).
\end{aligned}$$

# Infrared Logarithm Resummation

- The resummation of large logarithms contributions is possible for those observable which exponentiate i.e. the matrix element and the phase space can be factorized by expressing the emission of $n$ infrared (soft or collinear) gluons as the product of $n$ single gluon emission.
- With general approximation can be demonstrated that the matrix elements of the gluon emissions exponentiate up a given logarithmic accuracy.
- In general the factorization of the phase space is not true, it depends by the particular process.
- In the case of the threshold resummation the exponentiation of the phase space can be demonstrated in the Mellin space:

$$f_N \equiv \int_0^1 f(z)\, z^{N-1}\, dz$$

# Inverse Mellin transform

- Integration over $y$ in $G_N$ is performed exactly in numerical way; this is possible because the time-like coupling does not have the Landau pole and is regular for any value of $N$.
- The inverse transform from $N$-space to $x$-space is also made exactly in numerical way by the formula

$$\sigma_N(\tilde{\alpha}_S) \equiv \int_0^1 (1-y)^{N-1} \sigma(y, \tilde{\alpha}_S)\, dy,$$

$$\sigma(y; \tilde{\alpha}_S) = \int_{C-i\infty}^{C+i\infty} \frac{dN}{2\pi i} (1-y)^{-N} \sigma_N(\tilde{\alpha}_S),$$

where the constant $C$ is chosen so that the integration contour in the $N$-plane lies to the right of all the singularities of $\sigma_N(\alpha_S)$.

- Most previous analyses predicted $B$-hadron production convoluting the parton-level spectrum with a non-perturbative fragmentation function which contains few parameters which are to be fitted to experimental data.
- Instead of fitting the parameters of a hadronization model we model non-perturbative effects by the use of an analytic effective coupling constant which does not contain any free parameter, extending the analysis carried out in the framework of heavy-flavour decays.
- We shall then be able to compare our predictions directly with data, without using any extra hadronization model.

# Perspective

The work presented in this talk permit a wide range of improvements and developes. Some of theme are in progress as well:

- Analyze the case of $B \to X_c l\nu$ decays similarly to the $B \to X_u l\nu$ decays. The theoretical complication in this case is given by the not negligible mass of the $c$-quark. However the experimental data for such processes are much more accurate.
- There are some other spectra in $B \to X_u l\nu$ decays which have phenomenological interest which can be computed with our formalism.
- In the $b$-quark fragmentation case it is possible to perform a complete (NNLO+NNLL) resummation. After this we are confident that the reduced theoretical uncertainties will permit us to extract $\alpha_S(m_Z)$ as we have done with the decay spectra.
- Other possible extensions is to apply our formalism to other process and observables as the $B$ production in top and Higgs decays or charm production in $e^+e^-$ annihilations at the $Z^0$ peak ($m_Z \sim 90\ GeV$) or even much below at $\Upsilon(4s)$ peak ($m_\Upsilon \sim 10\ GeV$).