

# Development of $B \rightarrow D(^*)\tau\nu$ MC Event Generator with New Physics Effects

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Mini-workshop on  $D(^*)\tau\nu$  and related topics

# New physics effect for $B \rightarrow D(^*)\tau\nu$ analysis

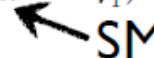
- $R(D)$ ,  $R(D^*)$  are changed
  - Decay properties are also changed, such as
    - $q^2$
    - $D^*$  and tau polarization
      - ← sensitive to the NP contribution
- Need to include these effects in the event generator

## Model-independent approach

Effective Lagrangian for  $b \rightarrow c\tau\bar{\nu}$

all possible 4-fermi operators with LH neutrinos

$$-\mathcal{L}_{\text{eff}} = 2\sqrt{2}G_F V_{cb} \sum_{l=e,\mu,\tau} [(\delta_{l\tau} + C_{V_1}^l)\mathcal{O}_{V_1}^l + C_{V_2}^l\mathcal{O}_{V_2}^l + C_{S_1}^l\mathcal{O}_{S_1}^l + C_{S_2}^l\mathcal{O}_{S_2}^l + C_T^l\mathcal{O}_T^l]$$


← SM

$$\mathcal{O}_{V_1}^l = \bar{c}_L \gamma^\mu b_L \bar{\tau}_L \gamma_\mu \nu_{Ll}, \quad \text{V-A} \quad \text{SM-like}$$

$$\mathcal{O}_{V_2}^l = \bar{c}_R \gamma^\mu b_R \bar{\tau}_L \gamma_\mu \nu_{Ll}, \quad \text{V+A} \quad \text{RH current}$$

$$\mathcal{O}_{S_1}^l = \bar{c}_L b_R \bar{\tau}_R \nu_{Ll}, \quad \text{S+P} \quad \text{charged Higgs (II)}$$

$$\mathcal{O}_{S_2}^l = \bar{c}_R b_L \bar{\tau}_R \nu_{Ll}, \quad \text{S-P} \quad \text{charged Higgs}$$

$$\mathcal{O}_T^l = \bar{c}_R \sigma^{\mu\nu} b_L \bar{\tau}_R \sigma_{\mu\nu} \nu_{Ll} \quad \text{Tensor} \quad \text{GUT?}$$

M. Tanaka and R. Watanabe,  
PhysRevD87, 034028 (2013)  
(hep-ph/1212.1878)

$C_{V_1}, C_{V_2}, C_{S_1}, C_{S_2}, C_T$  denotes Wilson coefficients

– Ex) 2HDM Type2:  $C_{S_1} = -\frac{m_b m_\tau}{m_{H^+}^2} \tan^2 \beta$      $C_{S_2} = -\frac{m_c m_\tau}{m_{H^+}^2}$      $C_{V_1} = C_{V_2} = C_T = 0$

→ Implement this model-independent calculation as  
the **EvtGen decay model**

# How to implement $B \rightarrow D(^*)\tau\nu$ EvtGen model

- PhysRevD.87.034028 gives helicity dependent amplitudes

The helicity amplitude of  $\bar{B} \rightarrow M\tau\bar{\nu}$  ( $M = D, D^*$ ) is written as

$$\mathcal{M}_l^{\lambda_\tau, \lambda_M} = \delta_{l\tau} \mathcal{M}_{SM}^{\lambda_\tau, \lambda_M} + \mathcal{M}_{V_1, l}^{\lambda_\tau, \lambda_M} + \mathcal{M}_{V_2, l}^{\lambda_\tau, \lambda_M} + \mathcal{M}_{S_1, l}^{\lambda_\tau, \lambda_M} + \mathcal{M}_{S_2, l}^{\lambda_\tau, \lambda_M} + \mathcal{M}_{T, l}^{\lambda_\tau, \lambda_M},$$

The helicity amplitude is a function of  
Wilson coefficients  $C_{V_1}, C_{V_2}, C_{S_1}, C_{S_2}, C_T$

(details are written in the  
appendix of the paper)

$\lambda_\tau$  : helicity of the tau in the (l+nu) rest frame

$\lambda_M$  : meson ( $D, D^*$ ) helicity in the B rest frame

$w$ : velocity transfer  $w = p_B \cdot p_M / (m_B m_M) \rightarrow q^2 = (m_B^2 + m_D^2 - 2m_B m_D^* w)$

$\cos\theta_\tau$  : angle between momenta of  $\tau$  and the meson in the (l+nu) rest frame

- M. Tanaka and R. Watanabe kindly provided their Mathematica codes of the helicity amplitude
- Well structured as a function library  
→ ported to C++ class → EvtGen decay model

# Implementation by EvtGen Model codes

Spin dependent complex amplitude is calculated

$$A = \sum_{\lambda_{D^*} \lambda_{\tau}} A_{\lambda_{D^*} \lambda_{\tau}}^{B \rightarrow D^* \tau \nu} \times A_{\lambda_{D^*}}^{D^* \rightarrow D \pi} \times A_{\lambda_{\tau}}^{\tau \rightarrow \pi \nu},$$

$\lambda_{D^*}, \lambda_{\tau}$  : spin state at  $D^*, \tau$  rest frame

BSTD model

```
class EvtBSemiTauonic;  
( we are developing )
```

VSS model (vector to 2 scalars)

```
class EvtVSS;  
(Already exist in EvtGen)
```

TAUSCALARNU model

```
class EvtTauScalarnu;  
(Already exist in EvtGen)
```

In each decay() function,

1. Randomly generate the kinematics of daughter particles based on the phase space
2. Calculate decay amplitudes for each spin state
3. Set amplitude (as complex variables)  
by function vertex(mother\_spinstate, daughter\_spinstate1,...)

Calculation of the probability values and the spin density matrices are taken cared by EvtGen core

# EvtGen Spin states and Helicity states

Need to translate between

- EvtGen Spin states in each rest frame
- Helicity states of BSTD model
  - $\lambda_\tau$  : helicity of the tau in the (l+nu) rest frame
  - $\lambda_M$ : meson (D, D\*) helicity in the B rest frame

- Prepared the rotation matrix in the boosted frame

EvtSpinDensity **EvtBSemiTauonicAmplitude::RotateToHelicityBasisInBoostedFrame**(const EvtParticle\* p, EvtVector4R p4boost)

- Utilize EvtDiracSpinor class of EvtGen
- Copied and added the boost to “EvtSpinDensity EvtDiracParticle::rotateToHelicityBasis()” in EvtDiracParticle
- Prepared by me. It seems still has a bug? (explained later)

# Decay.dec Parameters

# Br children Model\_Name parameters...

```
Decay MyB0
0.5 D*- tau+ nu_tau   BSTD   rhoA12 R11 R21 aR3 m_b m_c MagCV1 ArgCV1 MagCV2 ArgCV2 MagCS1 ArgCS1 MagCS2 ArgCS2 MagCT ArgCT;
0.5 D- tau+ nu_tau   BSTD   rho12 aS1 m_b m_c MagCV1 ArgCV1 MagCV2 ArgCV2 MagCS1 ArgCS1 MagCS2 ArgCS2 MagCT ArgCT;

#0.5 D*- tau+ nu_tau   BSTD_2HDMTYPE2 rhoA12 R11 R21 aR3 m_b m_c tanBeta/m_H+;
#0.5 D- tau+ nu_tau   BSTD_2HDMTYPE2 rho12 aS1 m_b m_c tanBeta/m_H+;

Enddecay
```

**rhoA12, R11, R21** : HQET Form Factor parameters from  $B \rightarrow D^{(*)} \ell \nu$  data  
**m\_b, m\_c** : quark masses at  $m_b$  scale

**MagCXX, ArgCXX** : Wilson coefficients for New Physics contributions  
Each Wilson coefficient is calculated by  $CXX = \text{MagCXX} * \exp(\text{ArgCXX} * i)$   
All  $CXX = 0 \rightarrow \text{SM}$

**aR3, aS1** : Parameters related to the systematic errors for  $O(1/m_q)$  correction in the scalar form factors  
PRD87,034028 uses  $1 \pm 1$  for these values to take into account the theoretical error.

# Check of the BSTD Model

- Compare helicity dependent amplitude calculation
  - Original theory calculation and C++ functions used for the BSTD Model
    - the fraction of the longitudinally polarized  $D^*$ 

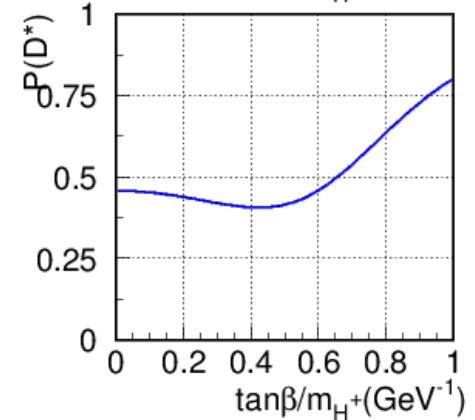
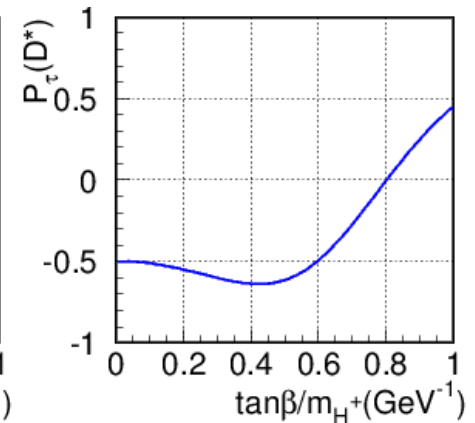
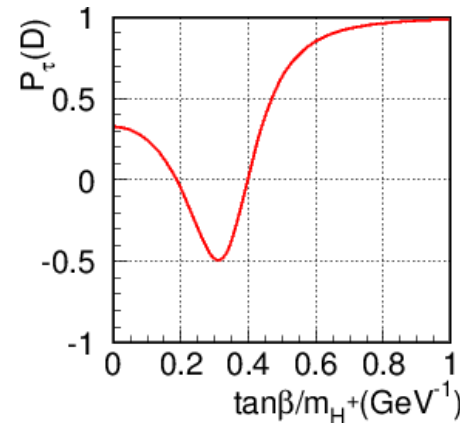
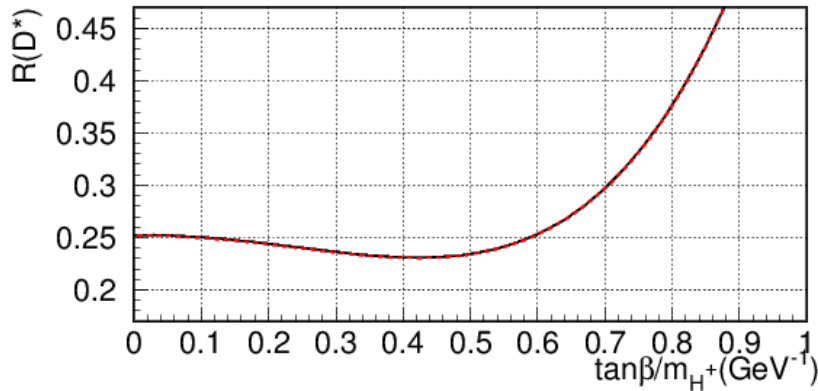
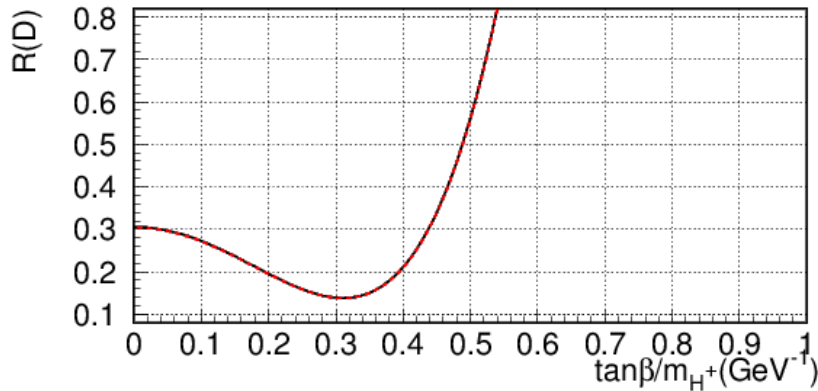
$$P_{D^*} = \frac{\Gamma(D_L^*)}{\Gamma(D_L^*) + \Gamma(D_T^*)},$$
    - the tau polarization
 
$$P_\tau(D) = \frac{\Gamma^+(D) - \Gamma^-(D)}{\Gamma^+(D) + \Gamma^-(D)},$$
- Compare distributions generated by EvtGen to the calculation
  - Velocity transfer  $w$ 

$$w = p_B \cdot p_M / (m_B m_M) \quad (\rightarrow q^2 = (m_B^2 + m_D^2 - 2m_B m_D^* w))$$
  - $\cos\theta_\tau$  angle between momenta of  $\tau$  and the meson in the  $(l+\nu)$  rest frame
- Check the  $D^*$  and tau polarization in decays generated by EvtGen
  - Check helicity angle distribution of  $D^* \rightarrow D\pi$  and  $\tau \rightarrow \pi\nu$  decays in the generated  $B \rightarrow D(^*)\tau\nu$ 
    - Compare to calculated values of  $P_{D^*}$  and  $P_\tau$



# R and polarizations for 2HDM Type2

## Check of helicity amplitude functions

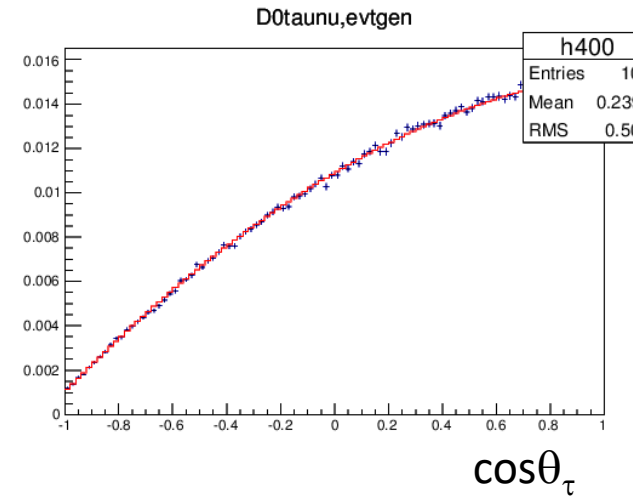
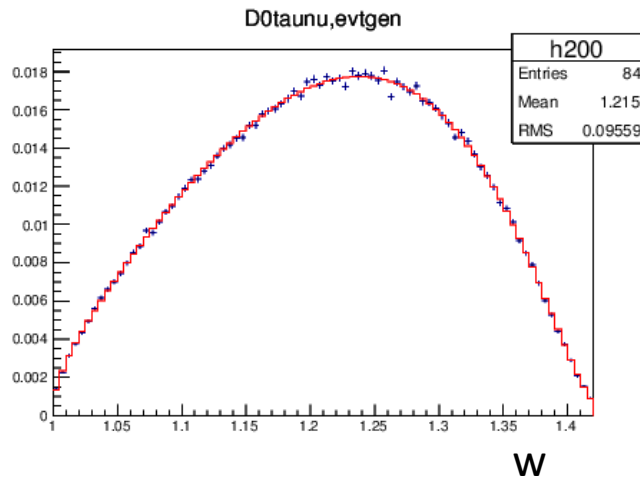


- Have been cross checked with the Tanaka-san's calculation

# $B^+ \rightarrow D^+ \tau^+ \nu_\tau$ , SM

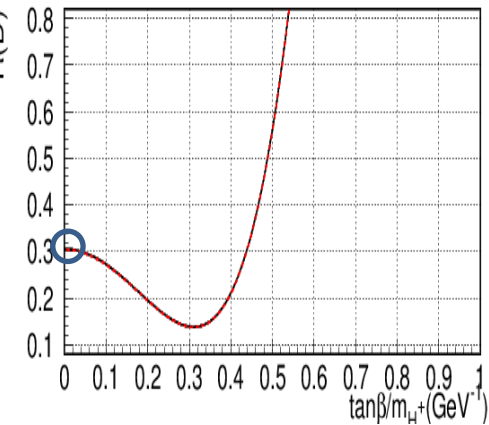
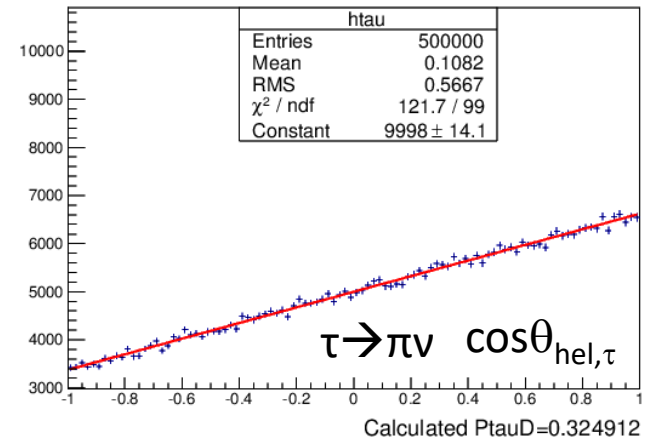
Data points:  
EvtGen events

Red line:  
Model Calculation

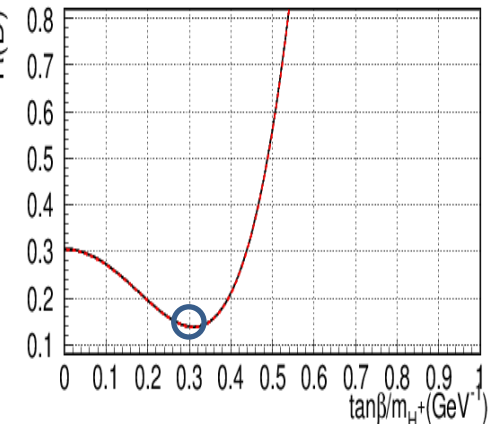
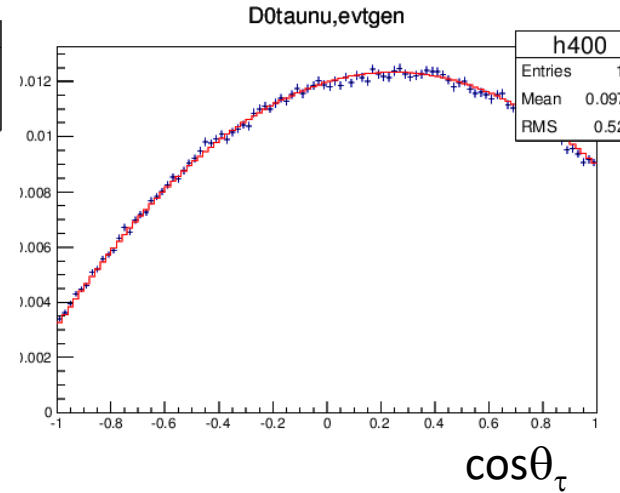
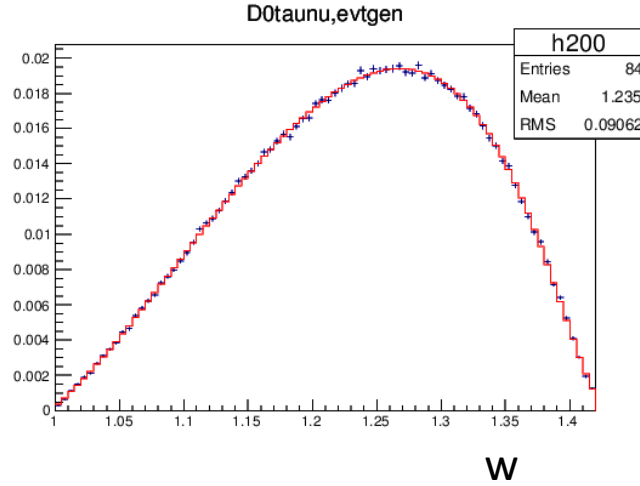


Calculated RGammaD=0.305911

tau coshel

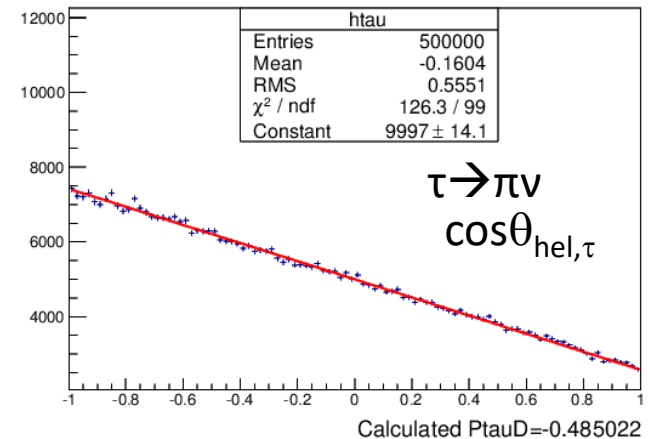


# $B \rightarrow D \tau \nu$ , $\tan(\beta)/m_H = 0.30$



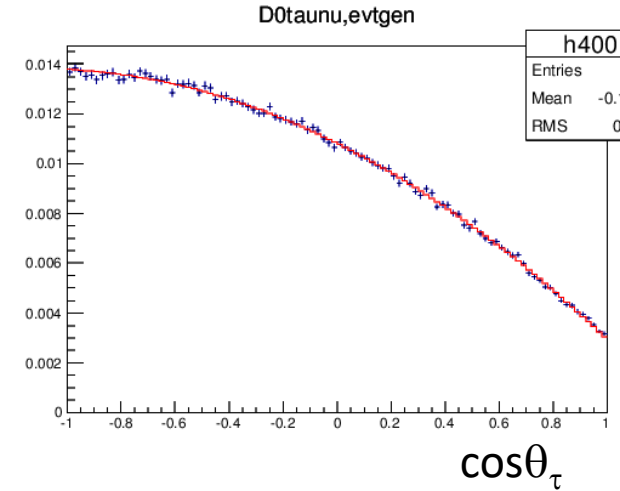
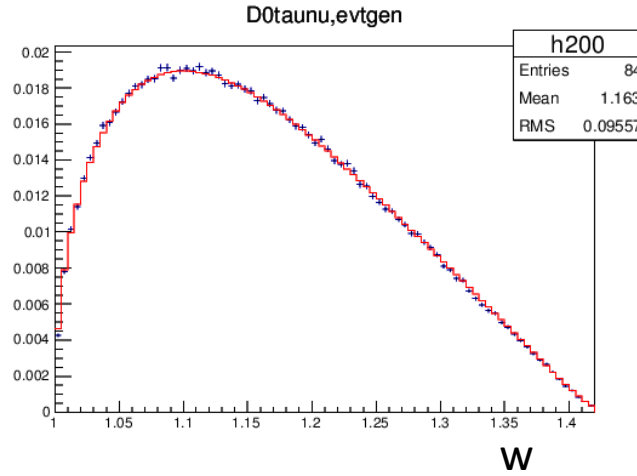
Calculated  $R_{\text{GamD}} = 0.139066$

tau coshel

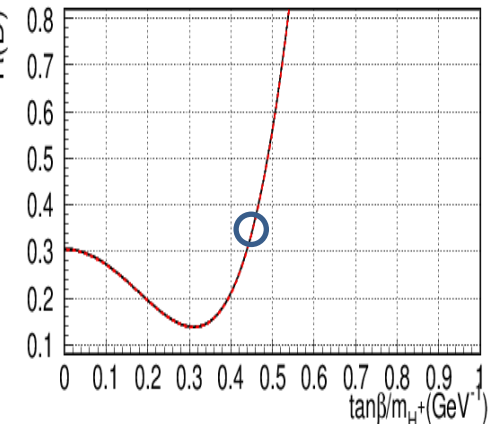


Calculated  $P_{\text{tauD}} = -0.485022$

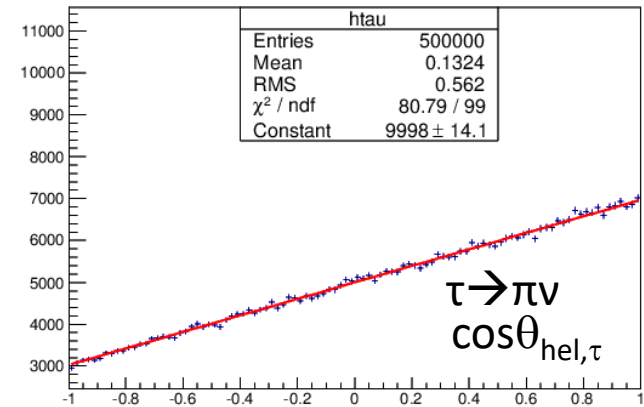
# $B \rightarrow D \tau \nu$ , $\tan(\beta)/m_H = 0.45$



Calculated RGamD=0.340971



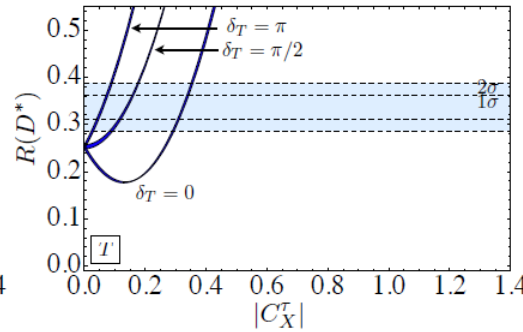
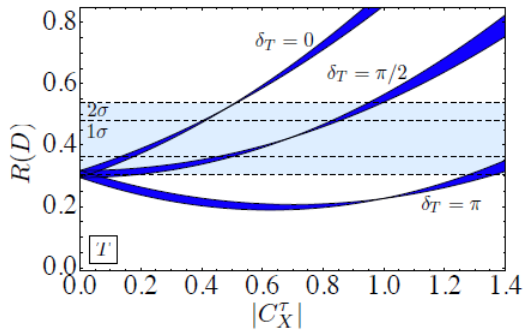
tau coshel



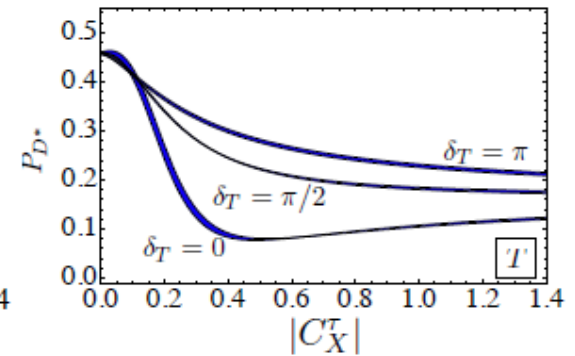
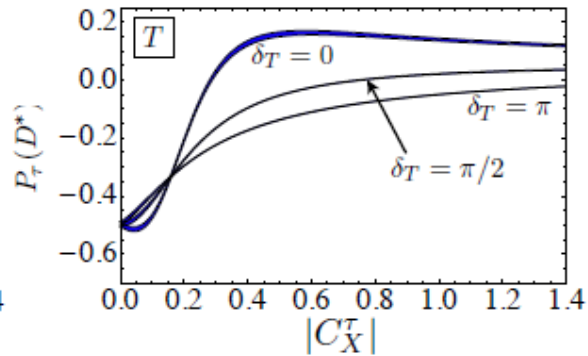
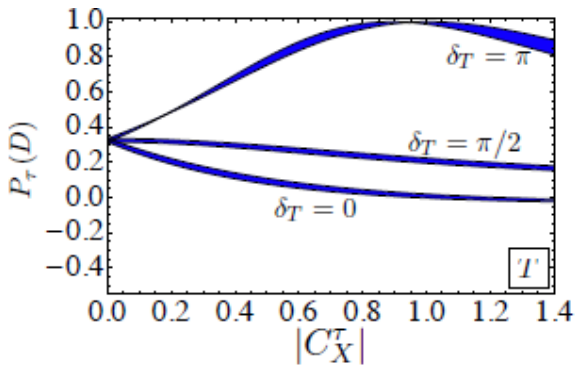
Calculated PtauD=0.394329

# SM + T case: Original Theory Calculation

Set others (CS1,CS2, CV1, CV2) to be 0

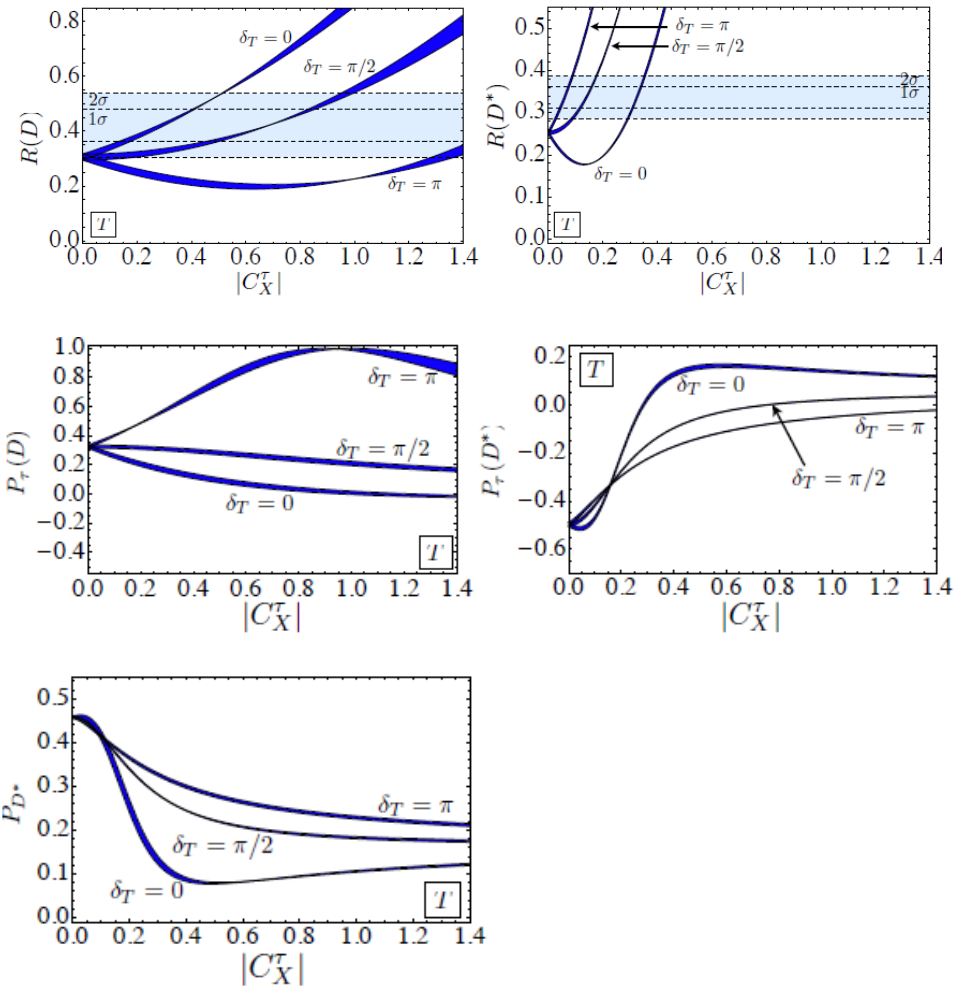


Plots from  
M. Tanaka and R. Watanabe,  
PhysRevD87, 034028 (2013)

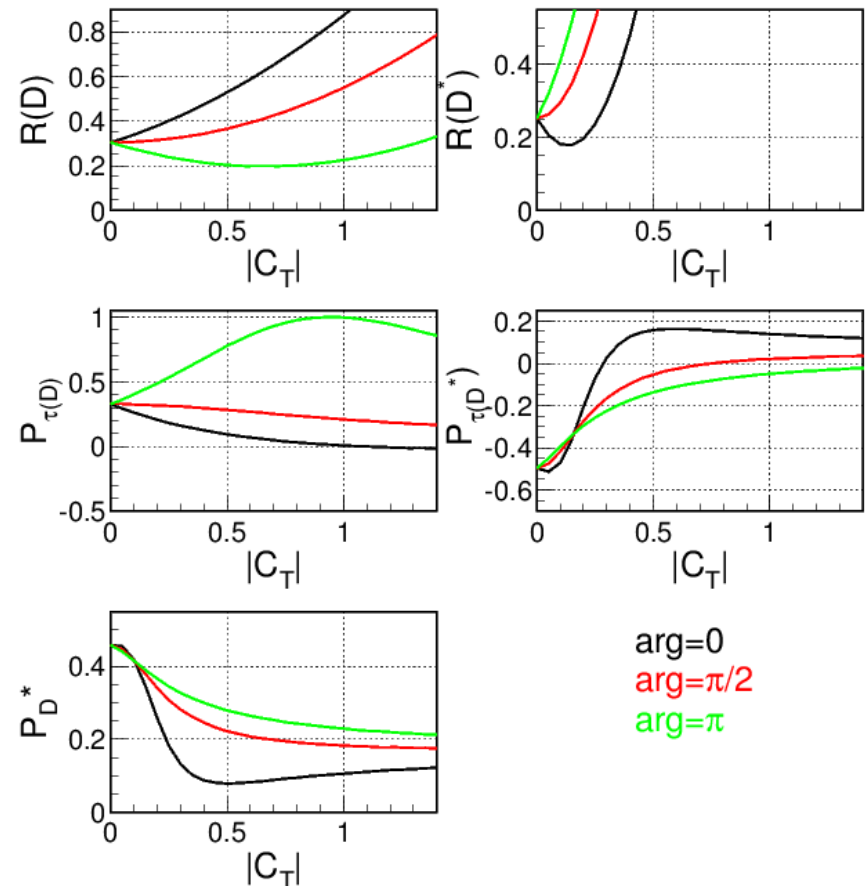


# Cross check of R, P of SM + T case

Plots from PhysRevD87, 034028 (2013)

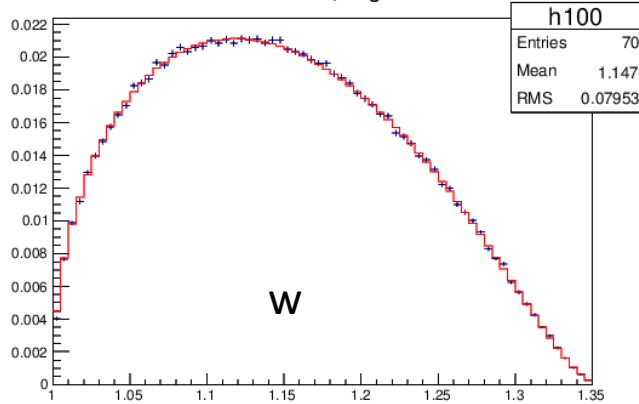


Plots by C++ helicity amplitude functions used in the BSTD model

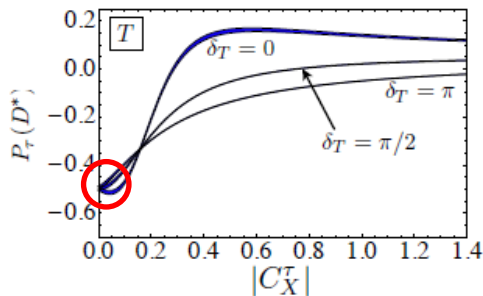
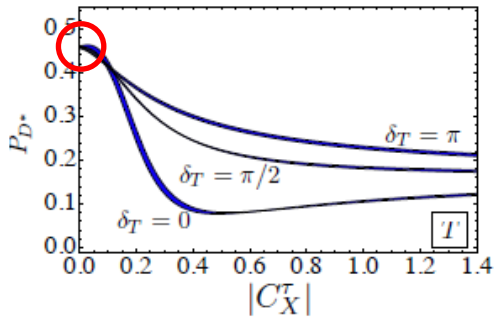
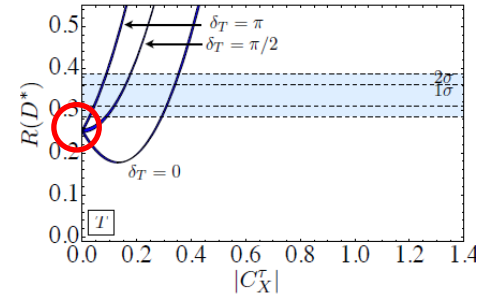
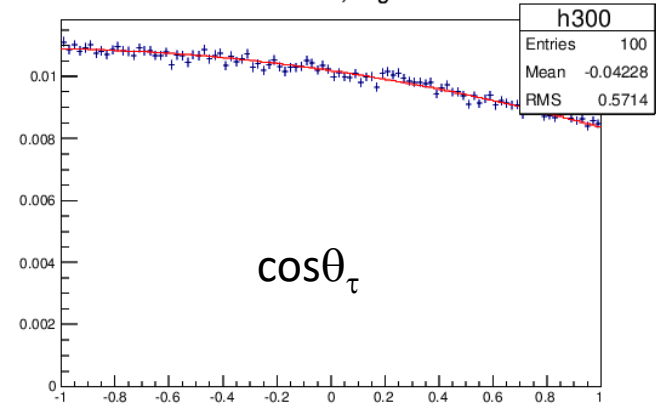


# $B^+ \rightarrow D^* \tau \nu$ , SM

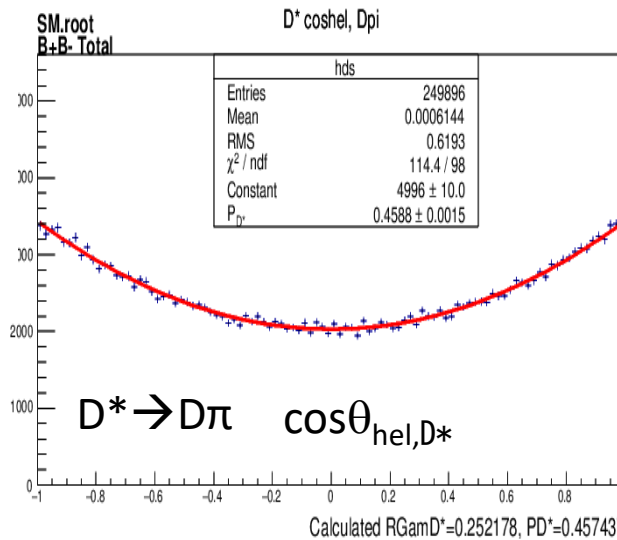
D\*0tau,evtgen



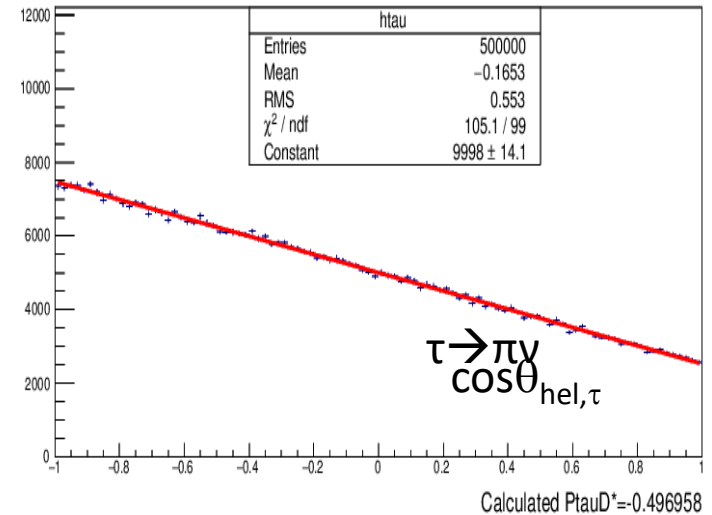
D\*0tau,evtgen



SM.root  
B+B- Total



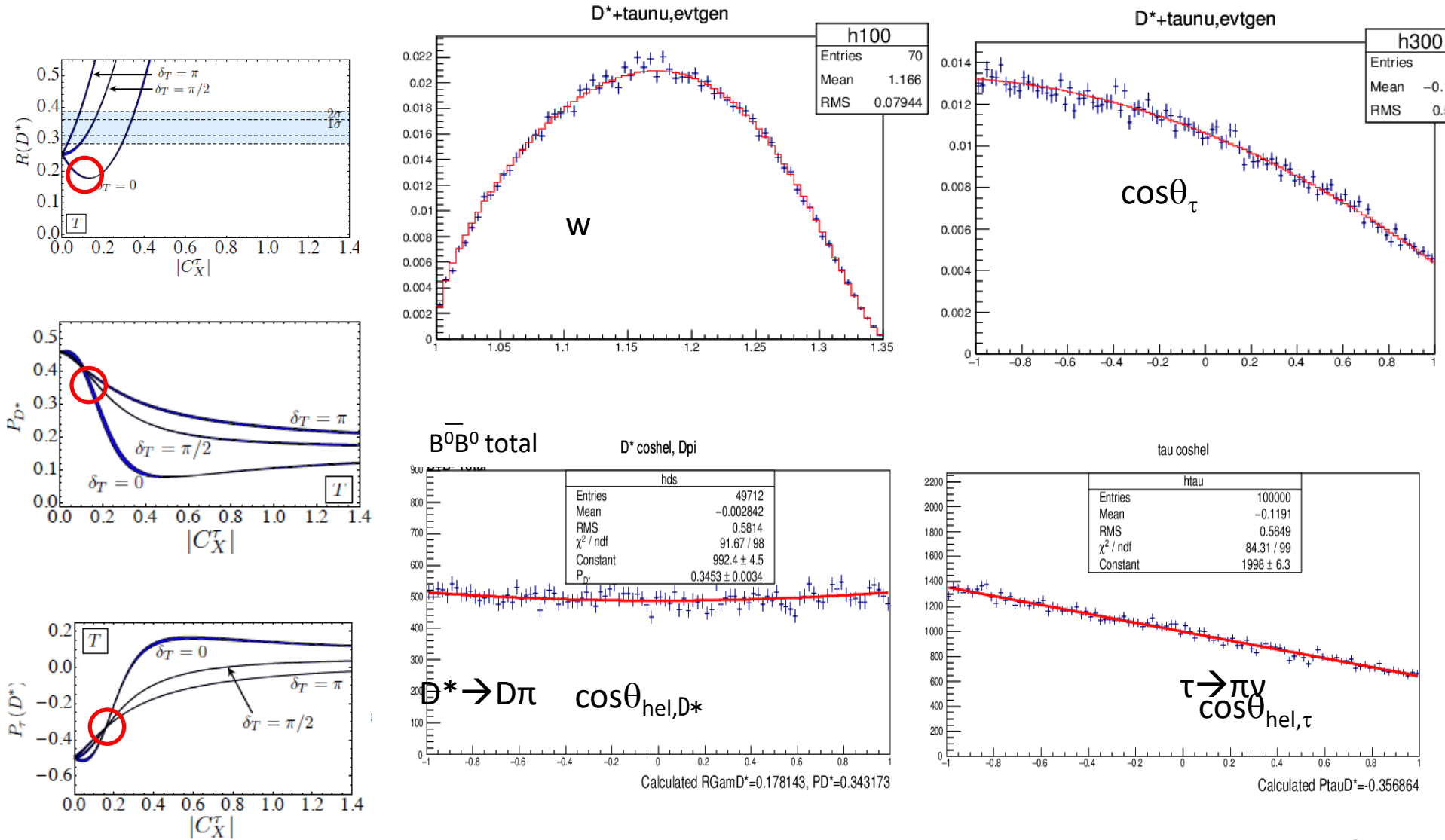
tau coshel



Calculated  $R_{\text{Gam}D^*} = 0.252178$ ,  $P_{D^*} = 0.457437$

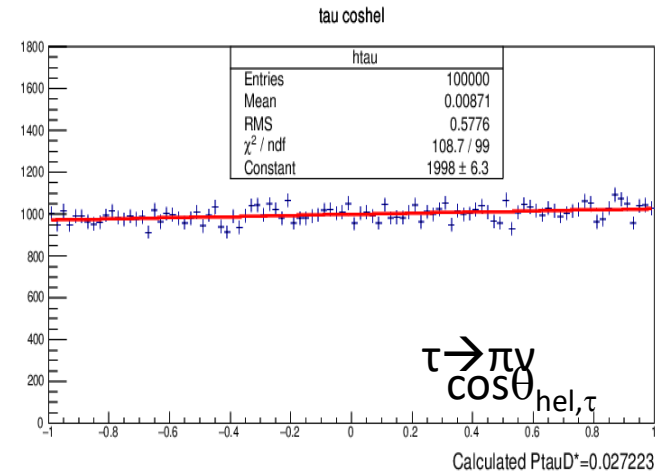
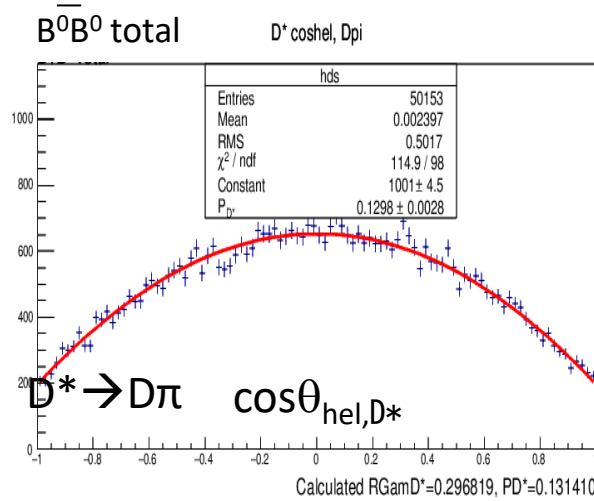
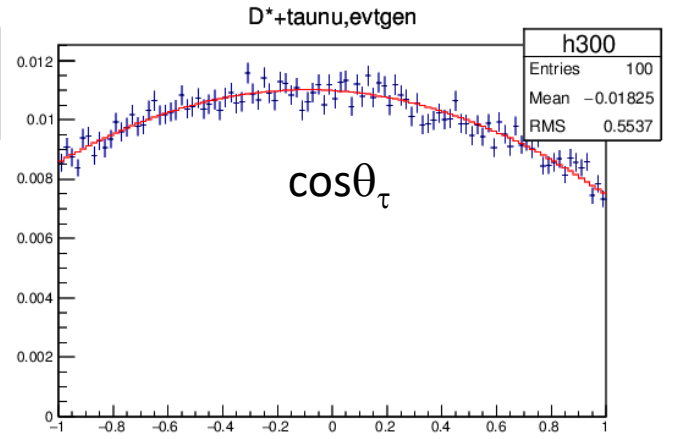
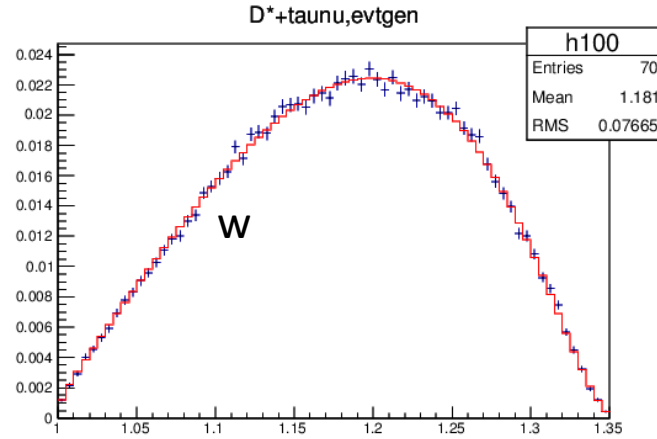
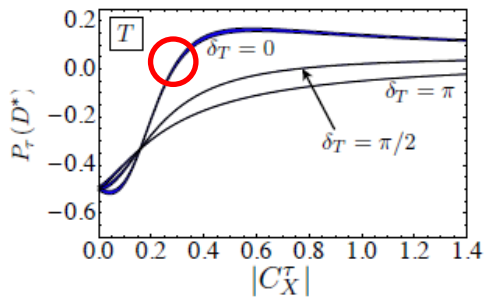
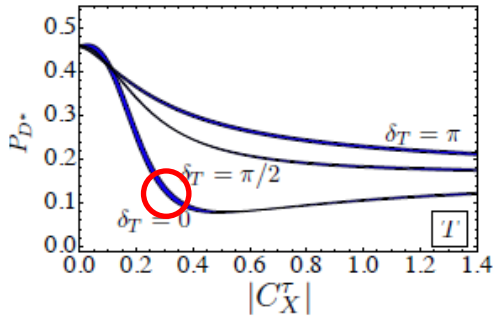
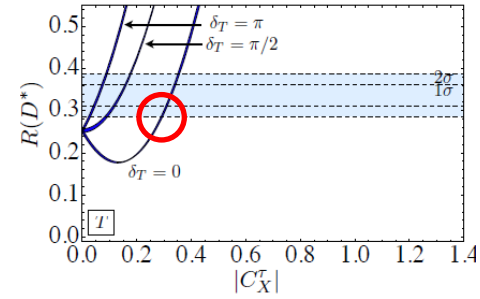
Calculated  $P_{\text{tau}D^*} = -0.496958$

# SM + 0.15 T

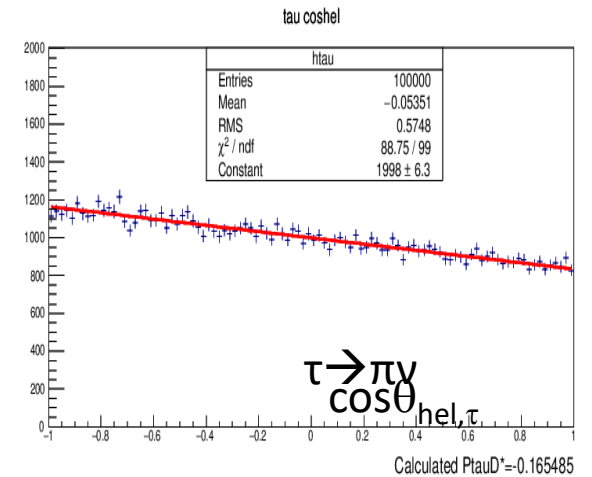
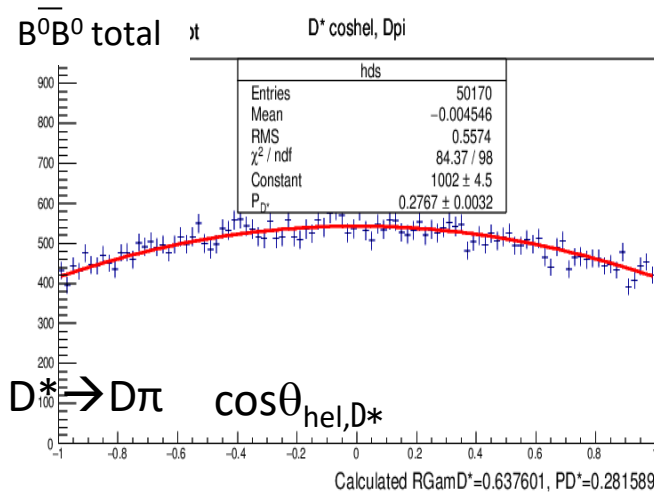
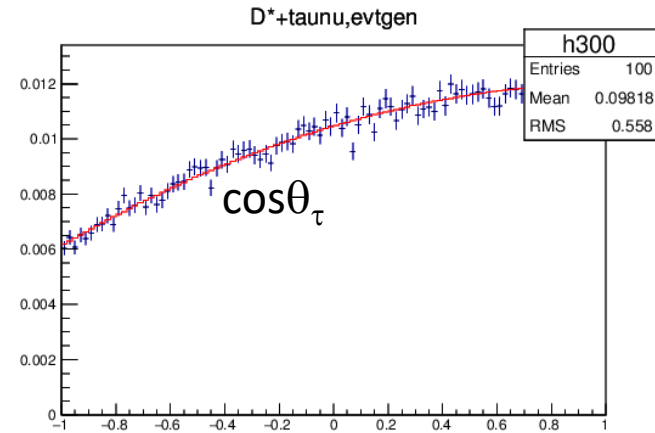
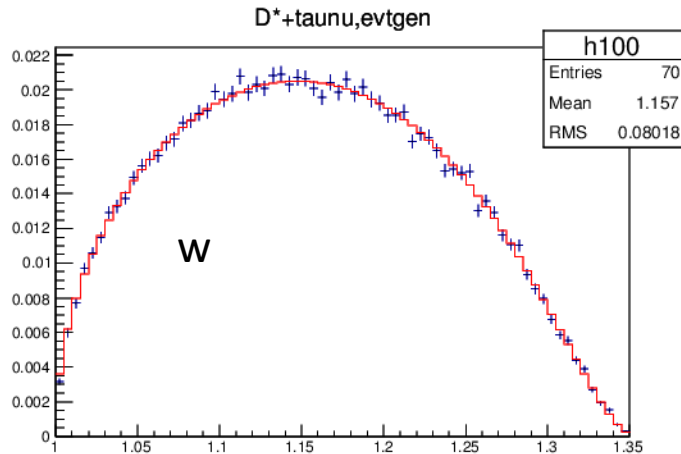
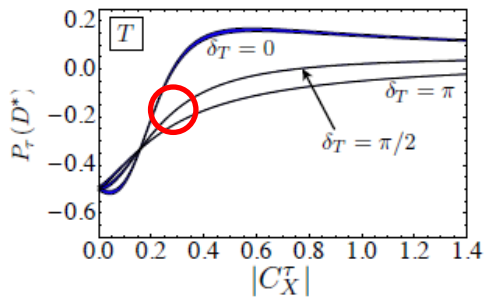
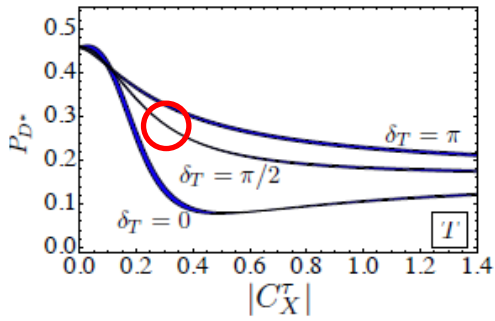
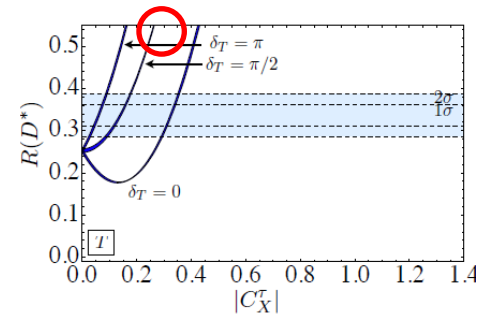




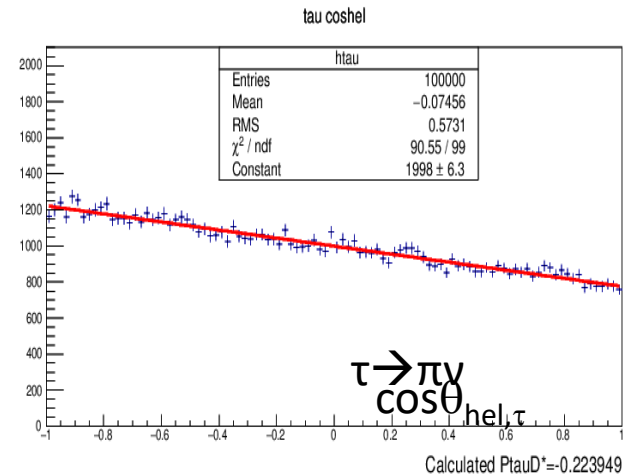
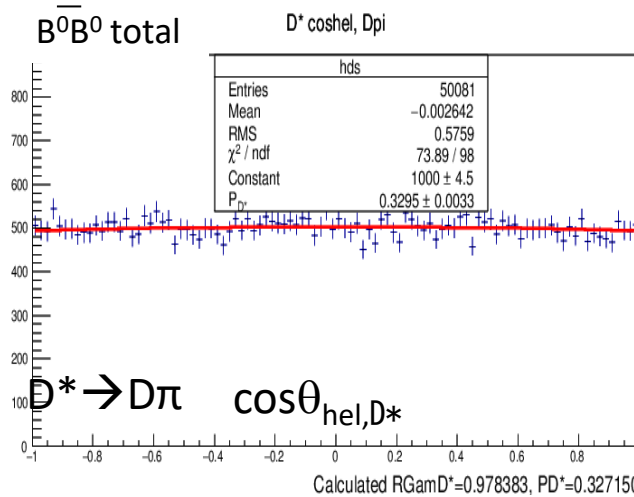
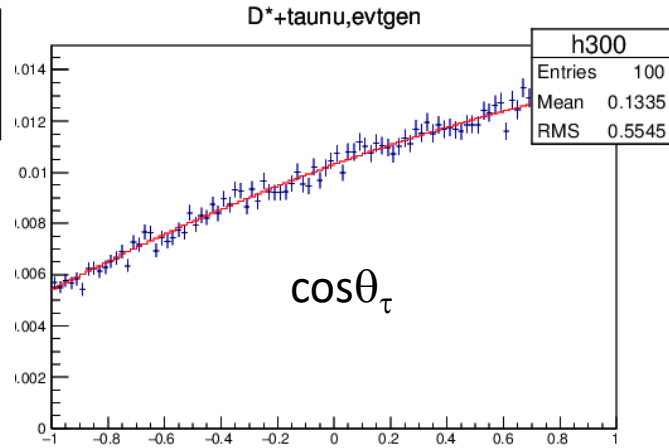
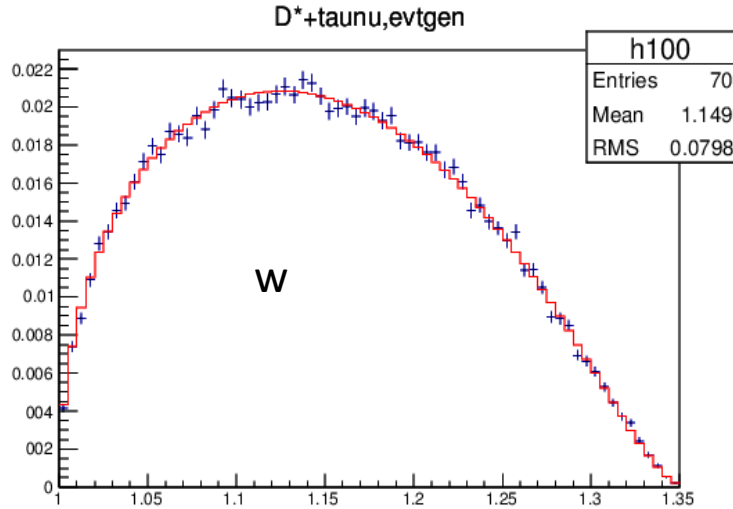
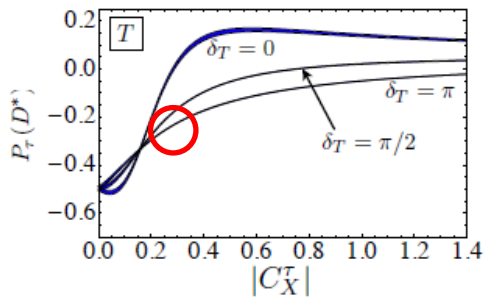
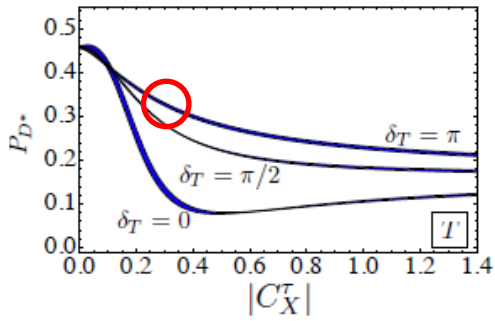
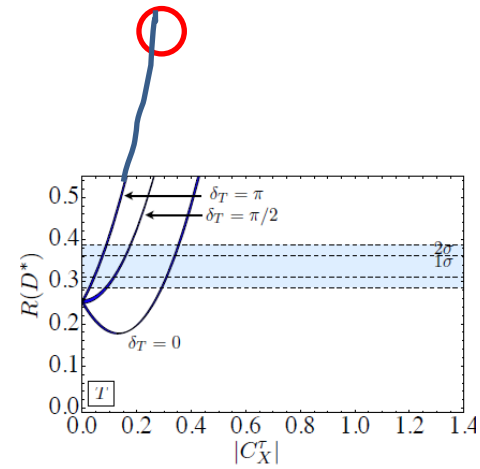
# SM + 0.3 T



# SM + $0.3 e^{i\pi/2} \tau$



# SM + 0.3 $e^{i\pi} T$



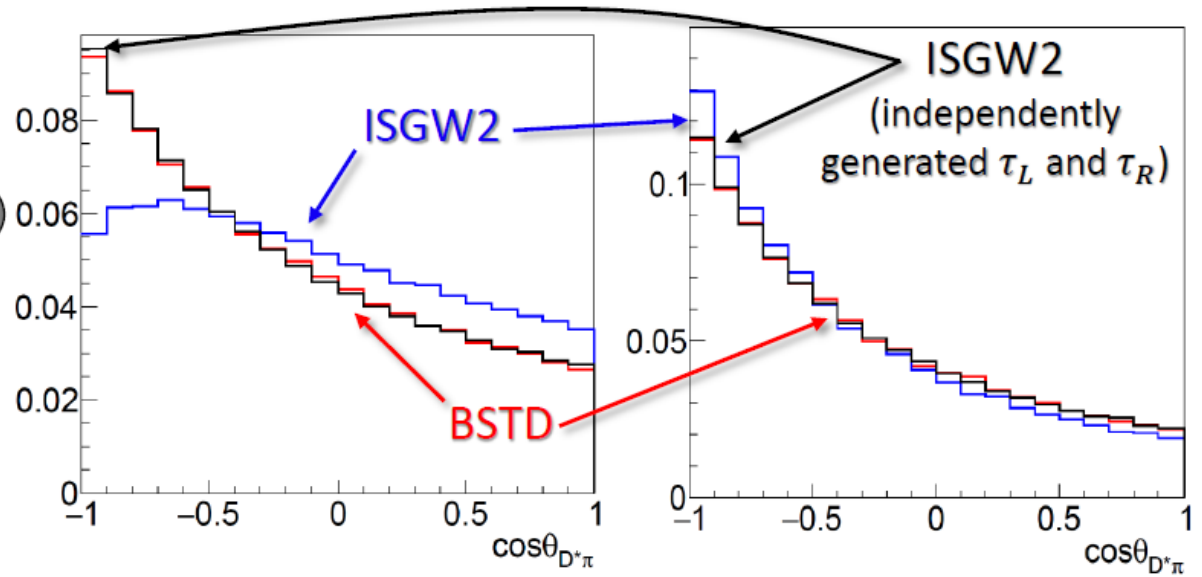
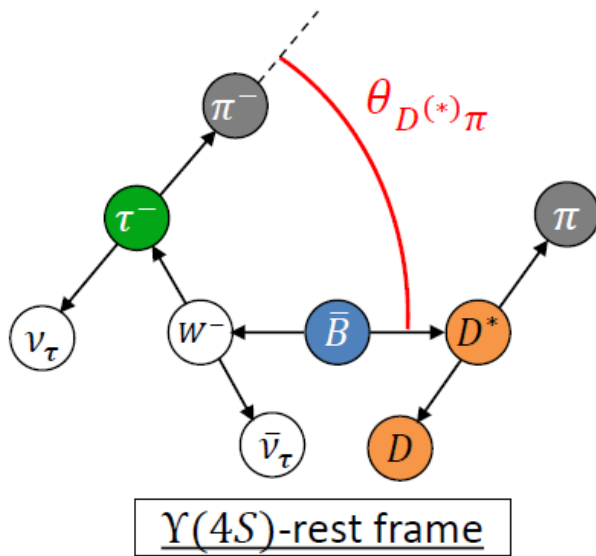
# So far It looks good, but..

- C++ Helicity amplitude functions are compared with the original theory calculation → identical results
- Polarization of  $D^*$  and tau checked by the helicity angle distribution generated by EvtGen → seems good
- Developed BSTD model is installed in the Belle and Belle II library
  - Being checked in Belle group analyses
  - Several bugs found and fixed
  - Remaining problem: no or wrong interference between +/- tau helicities

# Discrepancy between BSTD and ISGW2

- K. Adamczyk and M. Rozanska (Krakow) reported that one angular distribution is different between BSTD and ISGW2
  - ISGW2 is the original decay model in EvtGen
- Difference is significant in the  $\tau^- \rightarrow \pi^- \nu_\tau$  mode

$$\bar{B} \rightarrow D^* \tau^- \bar{\nu}_\tau, \tau^- \rightarrow \pi^- \nu_\tau \quad \bar{B} \rightarrow D^* \tau^- \bar{\nu}_\tau, \tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau$$



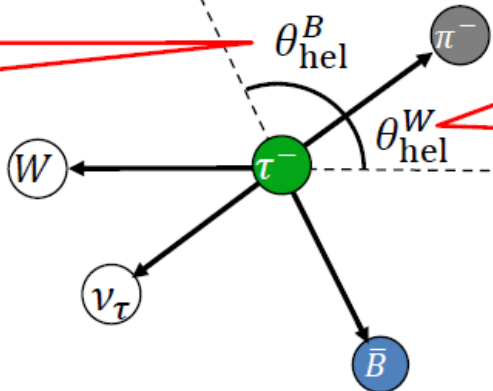
No interference between two  $\tau$  helicities in BSTD  
 (This problem also happens in  $\bar{B} \rightarrow D \tau^- \bar{\nu}_\tau$ )

# ■ $P_\tau(D^*)$ in B-rest frame

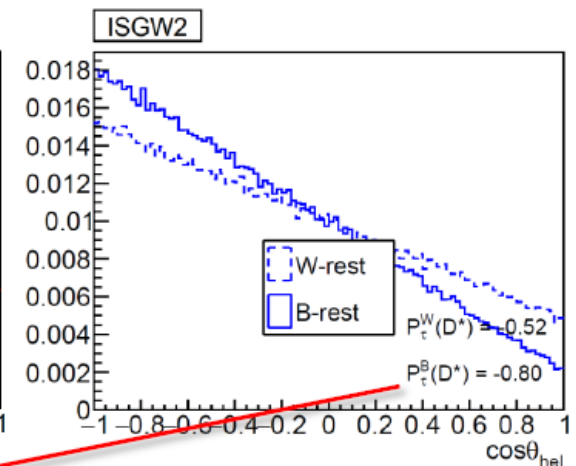
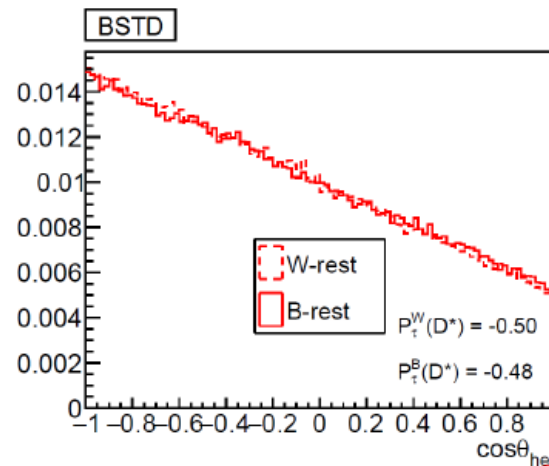
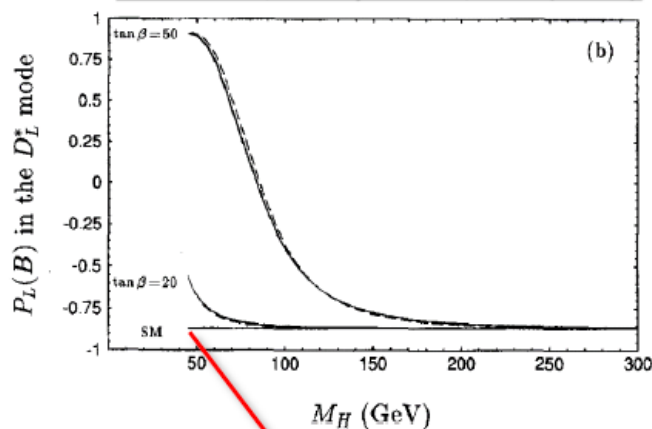
- Cross-check suggested by M. Tanaka (author of PRD 87, 034028): calculate  $P_\tau(D^*)$  in B-rest frame

$P_\tau(D^*)$  can be also defined in B-rest frame

Usually we discuss  $P_\tau(D^*)$  in W-rest frame



*M. Tanaka, Z. Phys. C 67, 321 (1995)*



Polarization in ISGW2 is correct → Obviously there is a problem in BSTD

# +/- tau helicity interference problem

- It should be OK up to the helicity amplitude calculation
- Suspicious part is the conversion between helicity amplitudes to the spin amplitudes

$\lambda_\tau$  : helicity of the tau in the (l+nu) rest frame  
 $\lambda_M$  : meson (D, D\*) helicity in the B rest frame

Conversion function made by me



$\lambda_{D^*}, \lambda_\tau$  : spin state at D\*,  $\tau$  rest frame

- Still working to fix the problem

# Summary

- Developing the EvtGen model of  $B \rightarrow D(^*)\tau\nu$  including New Physics effects
  - Based on M. Tanaka and R. Watanabe PRD87, 034028 (2013)
  - Including Vector, Scalar, Tensor NP contributions, controlled by complex Wilson Coefficients
- Debugging on-going in Belle/Belle II library
- Interference problem remains  $\rightarrow$  working on the fix
- When it gets fixed, will release to public to be a model in the official EvtGen





# Particle Spin in EvtGen Decay

[<http://evtgen.warwick.ac.uk/static/docs/EvtGenGuide.pdf>]

## 1. Generate B decay

Decay Amplitude  $A = \sum_{\lambda_{D^*} \lambda_{\tau}} A_{\lambda_{D^*} \lambda_{\tau}}^{B \rightarrow D^* \tau \nu} \times A_{\lambda_{D^*}}^{D^* \rightarrow D \pi} \times A_{\lambda_{\tau}}^{\tau \rightarrow \pi \nu}$ ,  
 $\lambda$  is the spin state of  $D^*$  and  $\tau$

(Re)generate B decay kinematics randomly based on the phase space and

$$P_B = \sum_{\lambda_{D^*} \lambda_{\tau}} |A_{\lambda_{D^*} \lambda_{\tau}}^{B \rightarrow D^* \tau \nu}|^2.$$

take the event passes the accept-reject algorithm for  $P_B$

## 2. $D^*$ spin density matrix $\rightarrow D^*$ decay

Calculate the spin density matrix for  $D^*$  after summing over  $\tau$  spins

$$\rho_{\lambda_{D^*} \lambda'_{D^*}}^{D^*} = \sum_{\lambda_{\tau}} A_{\lambda_{D^*} \lambda_{\tau}}^{B \rightarrow D^* \tau \nu} [A_{\lambda'_{D^*} \lambda_{\tau}}^{B \rightarrow D^* \tau \nu}]^*,$$

The decay probability of  $D^*$  is given by

$$P_{D^*} = \frac{1}{\text{Tr} \rho^{D^*}} \sum_{\lambda_{D^*} \lambda'_{D^*}} \rho_{\lambda_{D^*} \lambda'_{D^*}}^{D^*} A_{\lambda_{D^*}}^{D^* \rightarrow D \pi} [A_{\lambda'_{D^*}}^{D^* \rightarrow D \pi}]^*,$$

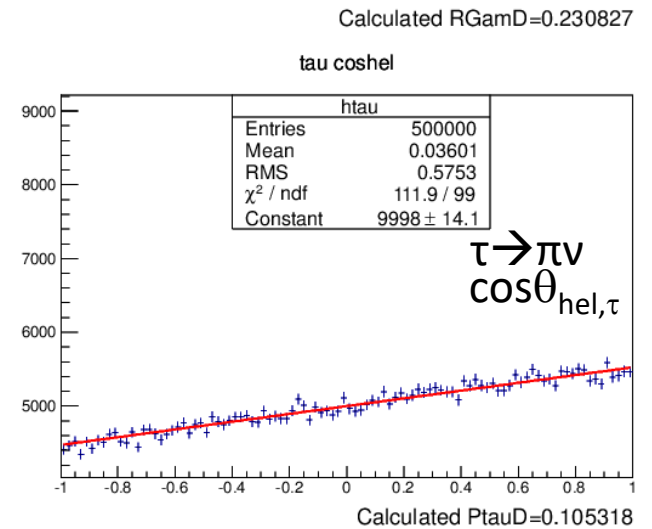
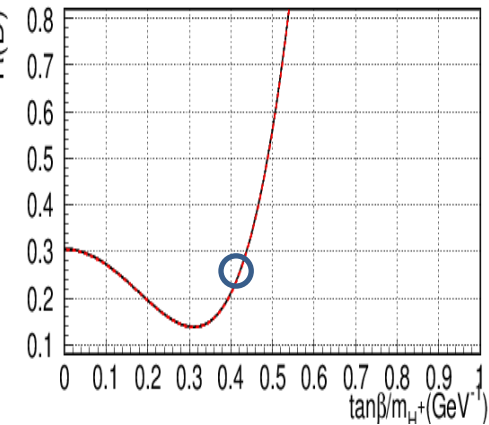
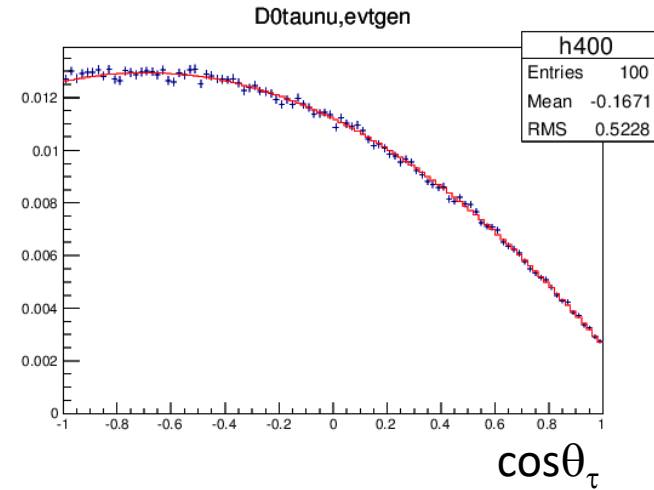
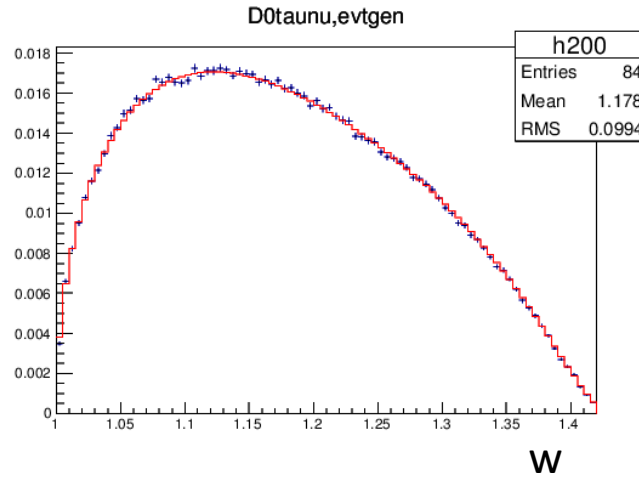
## 3. Tau spin density matrix $\rightarrow$ tau decay

Calculate the spin density matrix for  $\tau$  including kinematic correlation with  $D^*$

$$\tilde{\rho}_{\lambda_{D^*} \lambda'_{D^*}}^{D^*} = A_{\lambda_{D^*}}^{D^* \rightarrow D \pi} [A_{\lambda'_{D^*}}^{D^* \rightarrow D \pi}]^*,$$

$$\rho_{\lambda_{\tau} \lambda'_{\tau}}^{\tau} = \sum_{\lambda_{D^*} \lambda'_{D^*}} \tilde{\rho}_{\lambda_{D^*} \lambda'_{D^*}}^{D^*} A_{\lambda_{D^*} \lambda_{\tau}}^{B \rightarrow D^* \tau \nu} [A_{\lambda'_{D^*} \lambda'_{\tau}}^{B \rightarrow D^* \tau \nu}]^*.$$

# $B \rightarrow D \tau \nu$ , $\tan(\beta)/mH=0.41$



# Example of imported C++ codes

*// Total amplitude*

```
EvtComplex EvtBSemiTauonicHelicityAmplitudeCalculator::helAmp(const EvtComplex& CV1, const  
EvtComplex& CV2, const EvtComplex& CS1, const EvtComplex& CS2, const EvtComplex& CT$  
    double mtau, int tauhel, int Dhel, double w, double costau) const  
{  
    // sanity check  
    assert(chktauhel(tauhel) && chkDhel(Dhel));  
  
    return //(GF/sqrt(2))*Vcb* // <-- constants which does not affect the distribution omitted  
    (1.*helampSM(mtau, tauhel, Dhel, w, costau)  
    + CV1 * helampV1(mtau, tauhel, Dhel, w, costau)  
    + CV2 * helampV2(mtau, tauhel, Dhel, w, costau)  
    + CS1 * helampS1(mtau, tauhel, Dhel, w, costau)  
    + CS2 * helampS2(mtau, tauhel, Dhel, w, costau)  
    + CT * helampT(mtau, tauhel, Dhel, w, costau));  
}
```

*// Helicity Amplitudes*

*// SM*

```
double EvtBSemiTauonicHelicityAmplitudeCalculator::helampSM(double mtau, int tauhel, int
Dhel, double w, double costau) const
{
    double amp(0.);
    for (int whel = -1; whel <= 2; whel++) {
        amp += eta(whel) * Lep(mtau, tauhel, whel, q2(Dhel, w), costau)
            * HadV1(Dhel, whel, w);
    }
    return amp;
}
```

```

// Hadronic Amplitudes //
// V-A
double EvtBSemiTauonicHelicityAmplitudeCalculator::HadV1(int Dhel, int whel, double w) const
{
    // sanity check
    assert(chkDhel(Dhel) && chkwhel(whel));

    const double r0 = r(Dhel);
    if (Dhel == 2 && whel == 0) {
        return m_mB * sqrt(r0 * (w * w - 1.) / qh2(2, w)) * ((1 + r0) * hp(w) - (1 - r0) * hm(w));
    }
    if (Dhel == 2 && whel == 2) {
        return m_mB * sqrt(r0 / qh2(2, w)) * ((1 - r0) * (w + 1) * hp(w) - (1 + r0) * (w - 1) * hm(w));
    }
    if (Dhel == +1 && whel == +1) {
        return m_mB * sqrt(r0) * ((w + 1) * hA1(w) - sqrt(w * w - 1) * hV(w));
    }
    if (Dhel == -1 && whel == -1) {
        return m_mB * sqrt(r0) * ((w + 1) * hA1(w) + sqrt(w * w - 1) * hV(w));
    }
    if (Dhel == 0 && whel == 0) {
        return m_mB * sqrt(r0 / qh2(0, w)) * (w + 1) *
            (-(w - r0) * hA1(w) + (w - 1) * (r0 * hA2(w) + hA3(w)));
    }
    if (Dhel == 0 && whel == 2) {
        return m_mB * sqrt(r0 * (w * w - 1) / qh2(0, w)) * (-(w + 1) * hA1(w) + (1 - r0 * w) * hA2(w) + (w - r0) * hA3(w));
    }

    // other cases
    return 0.;
}

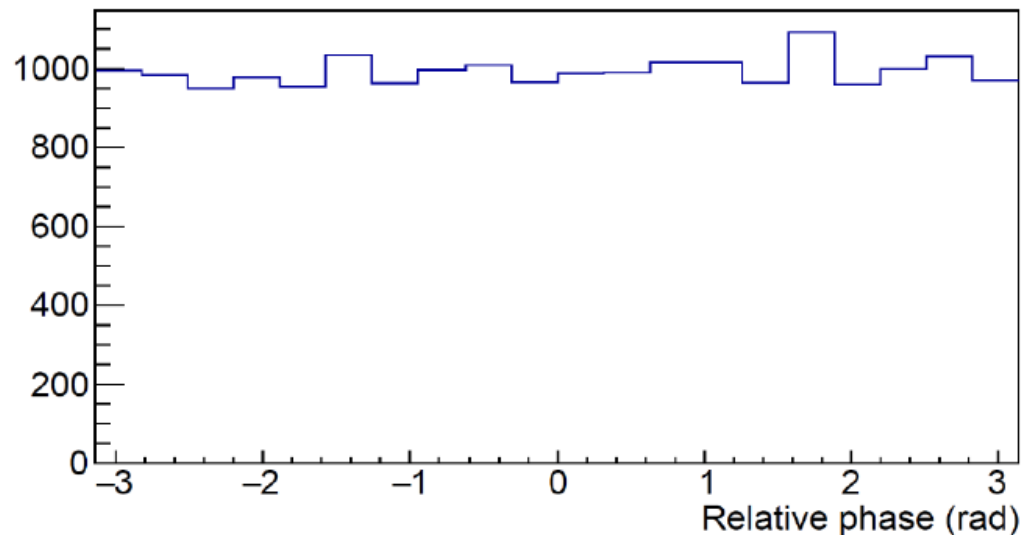
```

# Misc. function added EvtGen model

- Parameter initialization
  - Wilson coefficients  $CV1$ ,  $CV2$ ,  $CS1$ ,  $CS2$ ,  $CT$
  - Quark masses
  - HQET FF parameters
- Set maximum probability value properly for the input parameters
- Proper CP-transformation between B and B-bar
  - Model calculation given for B-bar
- Checked in the Belle / Belle II library

# ■ Relative Phase

- Checked relative phase between  $\tau_L$  and  $\tau_R$  amplitudes
  - If relative phase is 90 degrees, interference doesn't happen
- Phase difference is not always 90 degrees
  - Interference must be considered in BSTD code, but it disappears at some point...
- Interference occurs after the  $\tau$  decay
  - $\tau_L \rightarrow \pi\nu$  and  $\tau_R \rightarrow \pi\nu$  interferes
  - Helicity information is lost somewhere in EvtGen base functions?





```

# example decay file of BSTD model, based on M. Tanaka and R. Watanabe PRD87,034028(2013)
# author: Koji Hara (koji.hara@kek.jp)

# Wilson coefficients for New Physics contributions
# Each Wilson coefficient is calculated by
# CXX = MagCXX * exp( ArgCXX * i )
# Negative value of MagCXX is allowed.
# all CXX = 0 --> Standard Model
Define MagCV1      0
Define ArgCV1      0
Define MagCV2      0
Define ArgCV2      0
Define MagCS1      0
Define ArgCS1      0
Define MagCS2      0
Define ArgCS2      0
Define MagCT       0
Define ArgCT       0

# quark masses at the m_b scale
# taken from PRD77, 113016 (2008)
# m_b = 4.20 +/- 0.07 GeV/c^2
# m_c = 0.901 +/- 0.111 - 0.113 GeV/c^2
Define m_b         4.20
Define m_c         0.901

# HQET Form Factor parameters from B->D(*) lnu data
# taken from HFAG End Of Year 2011
# rho_1^2 = 1.186 +/- 0.036 +/- 0.041
# rho_A1^2 = 1.207 +/- 0.015 +/- 0.021
# R_1(1) = 1.403 +/- 0.033
# R_2(1) = 0.854 +/- 0.020
Define rho12       1.186
Define rhoA12      1.207
Define R11         1.403
Define R21         0.854

# Parameters related to the O(1/m_q) correction in the scalar form factors
# The authors of PRD87,034028 use 1 +/- 1 for these values to take into account the theoretical error.
Define aS1         1.0
Define aR3         1.0

```