Numerical simulations of causal relativistic viscous hydrodynamics for high-energy heavy-ion collisions

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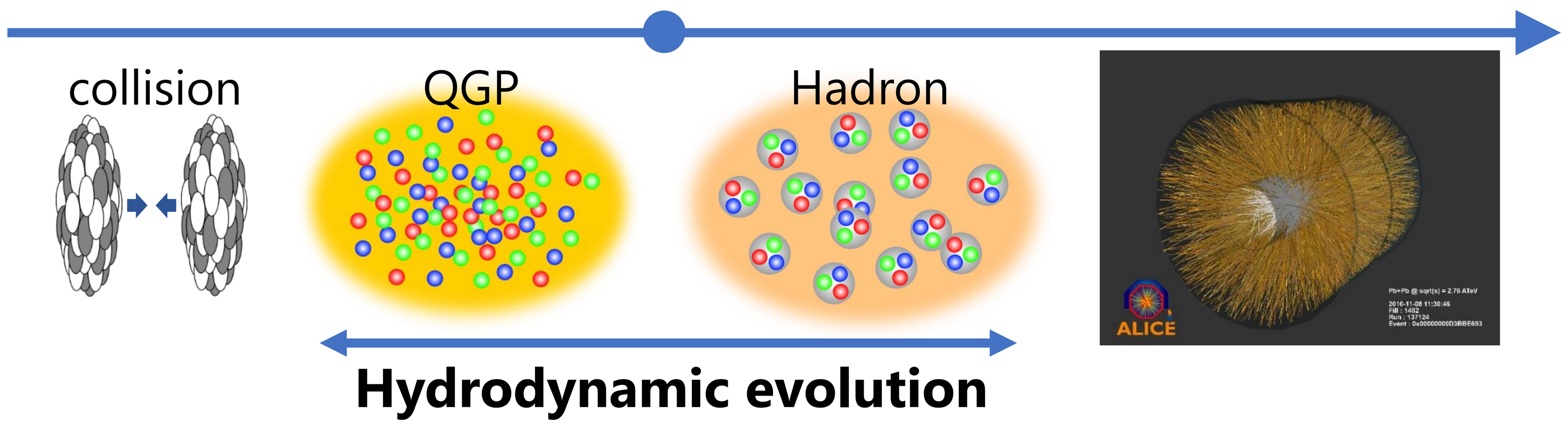
High-energy heavy-ion collisions

QGP formation is achieved at RHIC and LHC.



Comparison between experimental results and hydrodynamic simulations

➡ **Transport properties and the equation of state of QGP**

The propagation of initial fluctuations is sensitive to the property of QGP.

Towards quantitative understanding of QGP property, sophisticated viscous hydrodynamic simulations are needed.

Causal relativistic viscous hydrodynamics

Conservation equation

$$T^{\mu\nu}{}_{;\mu} = 0 \qquad T^{\mu\nu} = eu^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Simple relativistic extension of Navier-Stokes theory is acausal and unstable.

➡ **Second-order hydrodynamics**

Israel-Stewart equation

$$(\partial_\tau + v^i\partial_i)\pi^{\mu\nu} = -\frac{1}{\gamma\tau_\eta}(\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}) - I^{\mu\nu}$$

Relaxation to Navier-Stokes value

We have to treat additional equations, variables, and transport coefficients.

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu, \quad \nabla_\alpha A^{\mu_1\cdots\mu_n} \equiv \Delta^\beta_\alpha A^{\mu_1\cdots\mu_n}_{;\beta}, \quad \theta \equiv u^\mu{}_{;\mu}$$

$$\pi^{\mu\nu}_{\rm NS} = \eta\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\theta\right)$$

$I^{\mu\nu}$: Second-order terms + geometric source terms

Numerical approach

- We split the equations to an Ideal part and a viscous part. The Ideal part is solved by **Riemann Solver**. Akamatsu, Inutsuka, Nonaka, Takamoto, JCP256, 34(2014)
- When the relaxation time $\boldsymbol{\tau_\eta}$ is much shorter than the fluid timescale, the Israel-Stewart equation demands high numerical costs.
  ➡ **Peacewise Exact Solution(PES) method** Takamoto, Inutsuka,JCP230,7002(2011)
- Milne coordinates (τ, x, y, η) which are suitable to describe the longitudinal expansion of QGP are used. Okamoto, Akamatsu, Nonaka, EPJC76,579(2016)

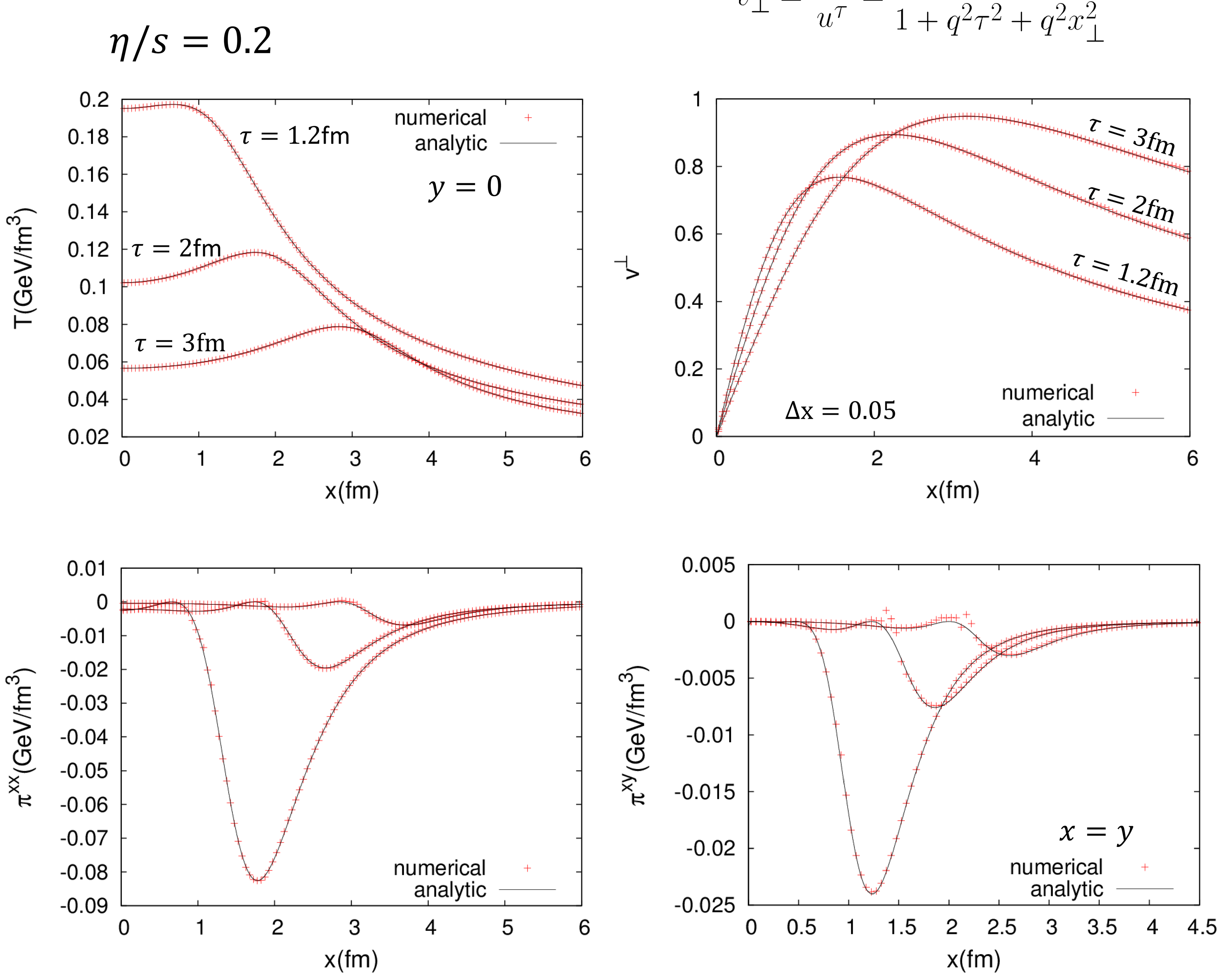
Numerical tests

Viscous Gubser flow

Marrochio et al., PRC91,014903(2015)

- Analytic solution of (3+1)D Israel-Stewart theory
- Radial expansion in transvers plane and boost-invariant expansion in longitudinal direction

$$v_\perp = \frac{u^\perp}{u^\tau} = \frac{2q^2\tau x_\perp}{1+q^2\tau^2+q^2x_\perp^2}$$



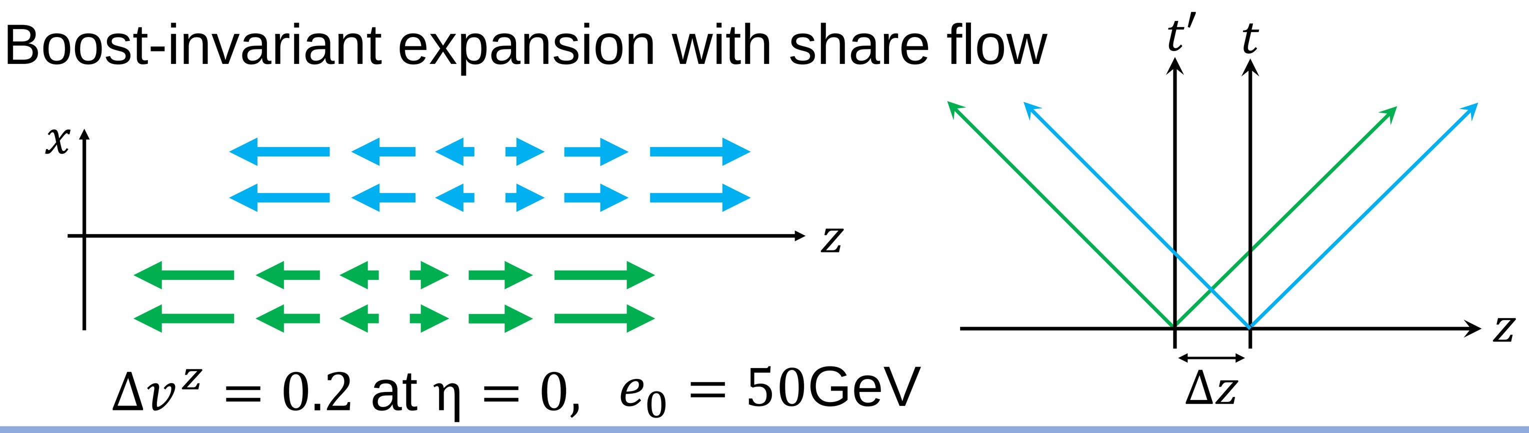
- The Israel-Stewart equation is solved with the MC limiter.
- Good agreement between analytic solutions and numerical calculations.

Kelvin-Helmholtz instability

We consider the possible existence of Kelvin-Helmholtz instability in heavy-ion collisions.

Initial condition

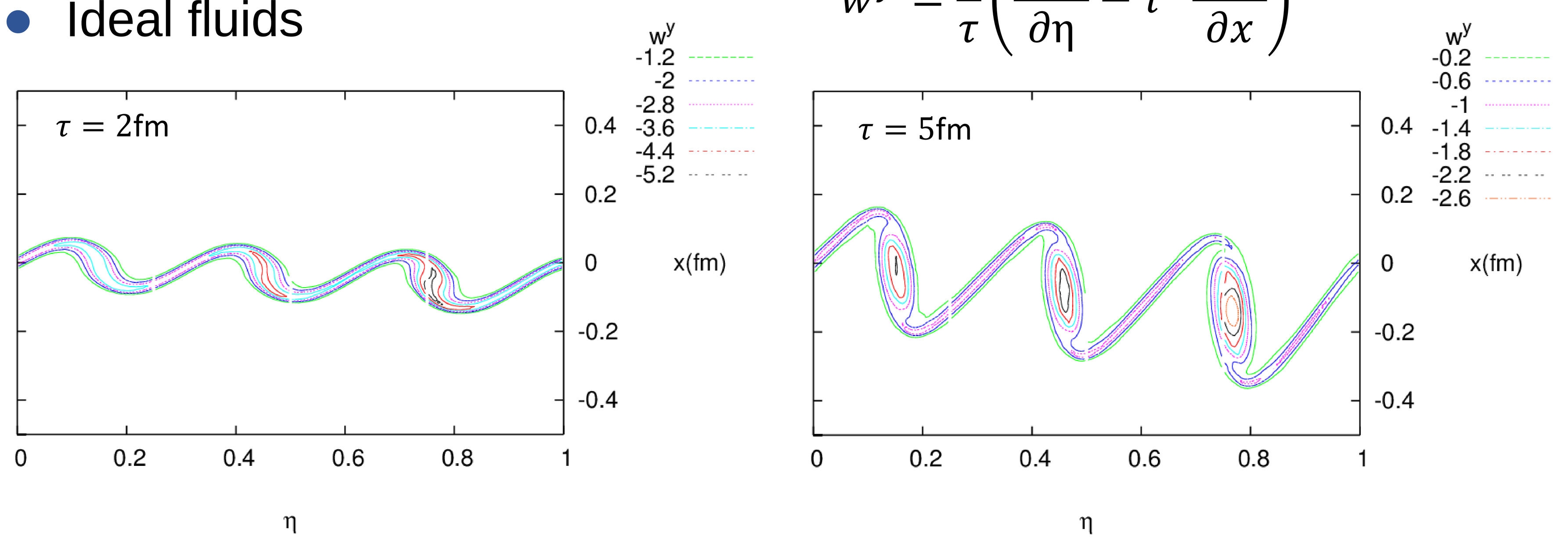
Boost-invariant expansion with share flow



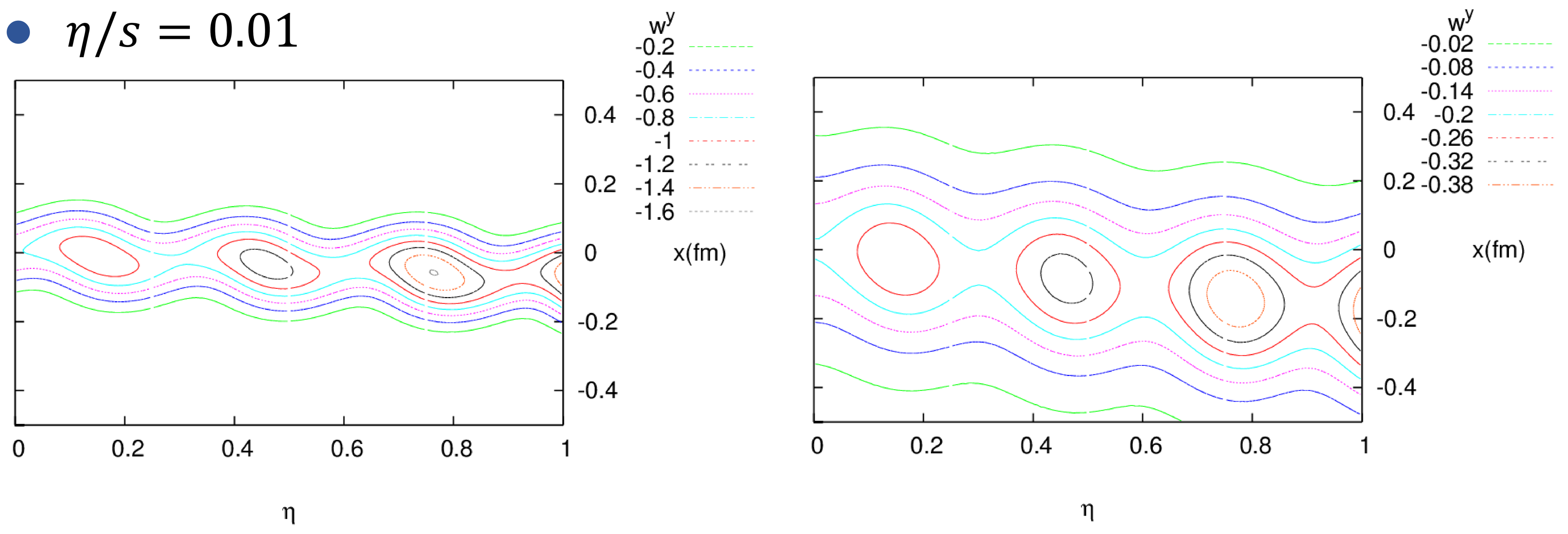
$\Delta v^z = 0.2$ at $\eta = 0$, $e_0 = 50$GeV

- Ideal fluids

$$w^y = \frac{1}{\tau}\left(\frac{\partial u^x}{\partial\eta} - \tau^2\frac{\partial u^\eta}{\partial x}\right)$$



- $\eta/s = 0.01$



The expansion and viscosity effects smear the vortices at later times.

Summary

- New code for relativistic viscous hydrodynamics in heavy-ion collisions.
- Our code can reproduce the analytic solutions with good accuracy.
- Kelvin-Helmholtz instability in heavy-ion collisions.

Future work

Phenomenological study of heavy-ion collisions

Higher-flow harmonics, event plane correlations…