Vanishing or non-vanishing rainbow?

Reduction formulas of electric dipole moment

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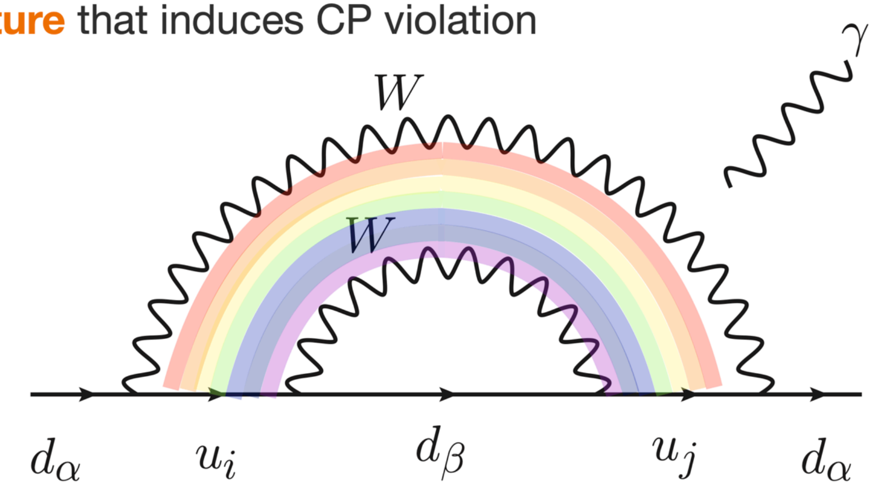
Based on **MF**, J. Hisano, C. Kanai, T. Toma, JHEP 04 (2021) 114 [arXiv:2012.14585]
MF, J. Hisano, T. Toma, [arXiv:2106.03384]

Today's Talk

We derive the **reduction** formulas for **Electric Dipole Moments (EDMs)**

Key1: Symmetry in QED (**Ward-Takahashi identity**)

Key2: **Flavor structure** that induces CP violation



Hint: the SM neutron EDM cancellation @2-loop

※Photon can couple to any charged particle line

- **Specify the the condition for “rainbow” diagram to induce nonzero EDMs**
- **Applicable formulas for the wider class of models**

Electric Dipole Moment (EDM)

What is EDM?

- Coupling btw Electromagnetic field and Spin of NR particles ($\boldsymbol{S}$)

$$H = \boxed{-d\boldsymbol{E}\cdot\frac{\boldsymbol{S}}{S}} - \mu\boldsymbol{B}\cdot\frac{\boldsymbol{S}}{S}.$$

$$\begin{bmatrix} \boldsymbol{S} \xrightarrow{T} -\boldsymbol{S}, & \boldsymbol{S} \xrightarrow{P} +\boldsymbol{S} \\ \boldsymbol{B} \xrightarrow{T} -\boldsymbol{B}, & \boldsymbol{B} \xrightarrow{P} +\boldsymbol{B} \\ \boldsymbol{E} \xrightarrow{T} +\boldsymbol{E}, & \boldsymbol{E} \xrightarrow{P} -\boldsymbol{E} \end{bmatrix}$$

EDM — CP violating

Magnetic moment — CP conserving

※ Assuming CPT theorem

- Experimentally probed in high precision
- EDM prediction in the SM is highly suppressed ← **Reason of suppression?**

➡ EDM is
- clean observables to probe New Physics w/ CP violation
- sensitive even for the New Physics that induce EDM at higher order

Motivation: **Drive EDM reduction formulas to probe New Physics!**

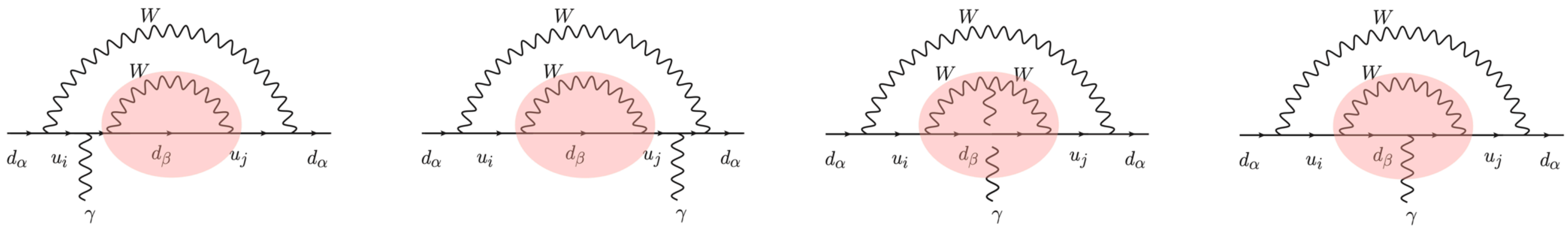
nEDM cancellation in the SM

Neutron EDM (nEDM) from W-coupling

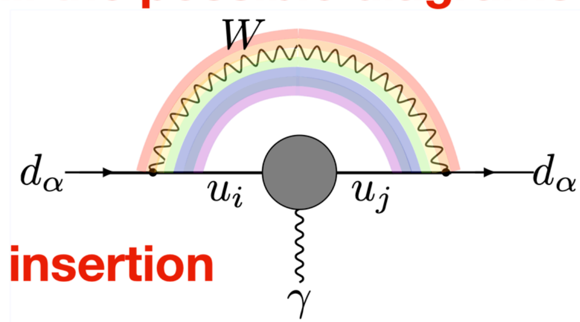
$$\mathcal{L} = -\left(\frac{g_2}{\sqrt{2}} V_{i\alpha} W^+_\mu \overline{u}_i \gamma^\mu P_L d_\alpha + \text{h.c.}\right) - eQ_u A_\mu \overline{u}_i \gamma^\mu u_i, \qquad (i, \alpha = 1, 2, 3)$$

@1-loop → No CP phase

@2-loop → LO contribution?



Add up all the possible diagrams



Photon insertion

Shabalin's proof [E. P. Shabalin (1978)]

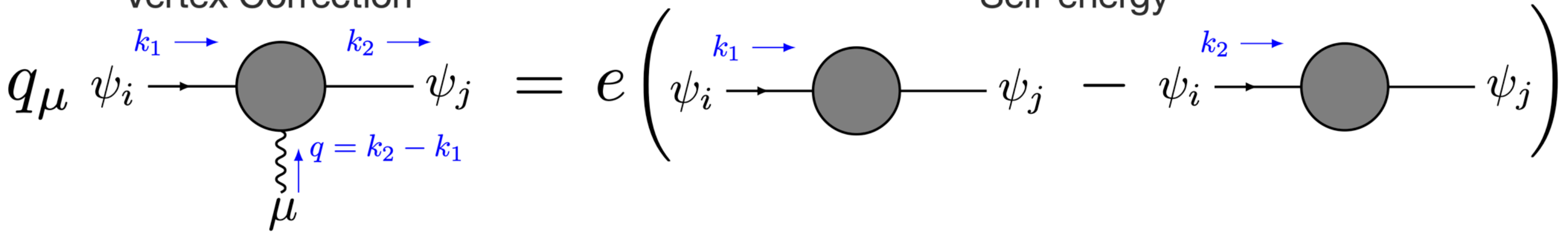
nEDM induced from W-exchange processes are totally cancelled @2-loop

Although each diagram induces nonzero EDM!

- Derived by explicit 2-loop cancellation
- Nontrivial EDM cancellation due to **the Ward-Takahashi identity** [J. C. Ward (1950)] [Y. Takahashi (1957)]

Vertex Correction

Self-energy



Lessons from the SM calculations

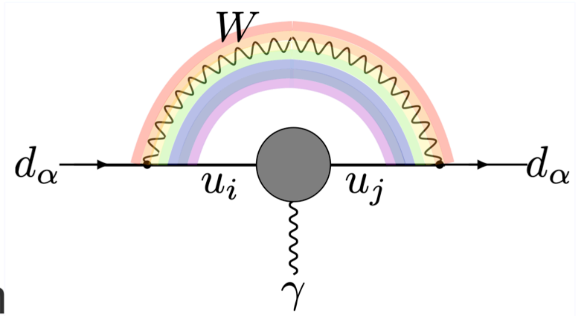
Key for cancellation

- **The Ward-Takahashi identity**
- Focusing on $\mathcal{O}(q)$ term
- **Flavor structure**: symmetric under $i \leftrightarrow j$

[q: momentum of photon
cf. EDM operator $\mathcal{L} \supset -d\frac{i}{2}\bar{\psi}\sigma^{\mu\nu}\gamma^5\psi F^{\mu\nu}$.]

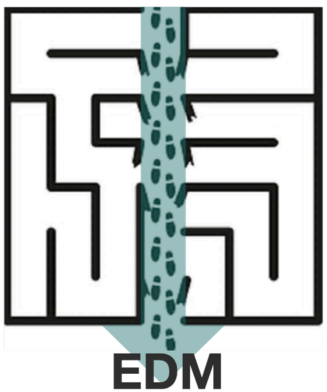
$$\mathrm{Im}\left[V_{j\alpha}^* V_{j\beta} V_{i\beta}^* V_{i\alpha}\right] \xrightarrow{i\leftrightarrow j} -\mathrm{Im}\left[V_{j\alpha}^* V_{j\beta} V_{i\beta}^* V_{i\alpha}\right]$$

⚠ **Shabalin's proof is model dependent**



EDM reduction formulas

- Apply these tips for more general setup (New Physics models)
- Conditions to obtain nonzero EDM from rainbow diagram
- Shortcut to obtain EDM



Drive EDM reduction formulas
using cancellation mechanism in the SM EDM!

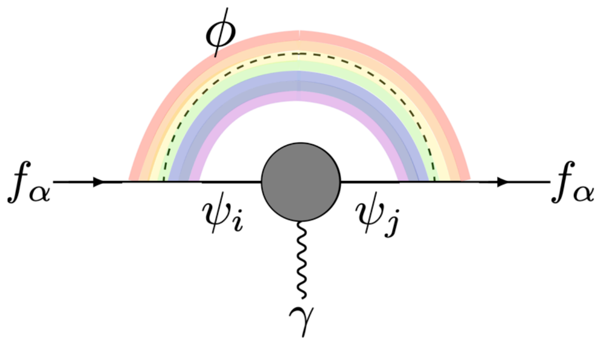
Setup for generalization (1/2)

	f_α	ψ_i	ϕ
Spin	$1/2$	$1/2$	0
electric charge	Q_f	Q_ψ	Q_ϕ

Generalized setup

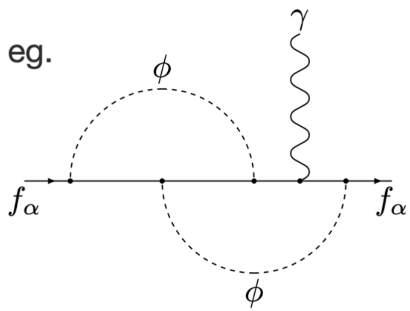
$$\mathcal{L}_{\text{eff}} = \bar{\psi}_i(i\partial\!\!\!/ - m_i)\psi_i + \bar{f}_\alpha(i\partial\!\!\!/ - m_\alpha)f_\alpha - \left(y_{i\alpha}\phi^*\overline{\psi_i}P_L f_\alpha + \text{H.c.}\right) - eQ_\psi A_\mu\overline{\psi_i}\gamma^\mu\psi_i + ieQ_\phi A_\mu(\phi\overleftrightarrow{\partial}\phi^*)$$
$$- eA_\mu\overline{\psi_i}\Lambda^\mu_{ij}\psi_j - \overline{\psi_i}\Sigma_{ij}\psi_j.$$

CP sources



- Simplified model with Scalar mediator
- Extension for Vector mediator case is straight-forward
- If LH & RH fermion couple to the same mediator, EDM is induced @1-loop level
  → Rainbow diagrams are sub-leading
- For electrically charged mediator ($Q_\phi \neq 0$), the LO contribution is rainbow diagram
  ※ Our setup has no Majorana mass
- For electrically neutral mediator ($Q_\phi = 0$),
  we also have diagrams w/o sub-diagram structures

eg.



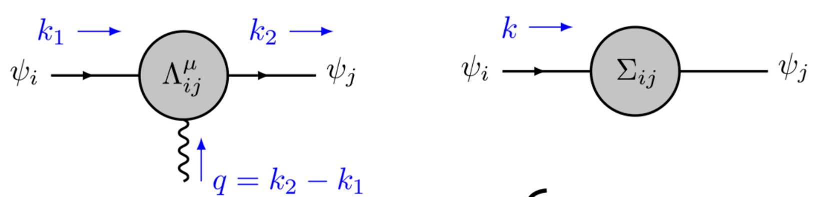
Setup for generalization (2/2)

Generalized setup

	f_α	ψ_i	ϕ
Spin	$1/2$	$1/2$	0
electric charge	Q_f	Q_ψ	Q_ϕ

$$\mathcal{L}_{\mathrm{eff}} = \bar{\psi}_i(i\partial\!\!\!/ - m_i)\psi_i + \bar{f}_\alpha(i\partial\!\!\!/ - m_\alpha)f_\alpha - \left(y_{i\alpha}\phi^*\overline{\psi_i}P_L f_\alpha + \mathrm{H.c.}\right) - eQ_\psi A_\mu \overline{\psi_i}\gamma^\mu\psi_i + ieQ_\phi A_\mu(\phi \overleftrightarrow{\partial} \phi^*)$$
$$- eA_\mu \overline{\psi_i}\Lambda^\mu_{ij}\psi_j - \overline{\psi_i}\Sigma_{ij}\psi_j .$$

CP sources



Read out from diagrams w/ {off-shell fermions, on-shell photon}

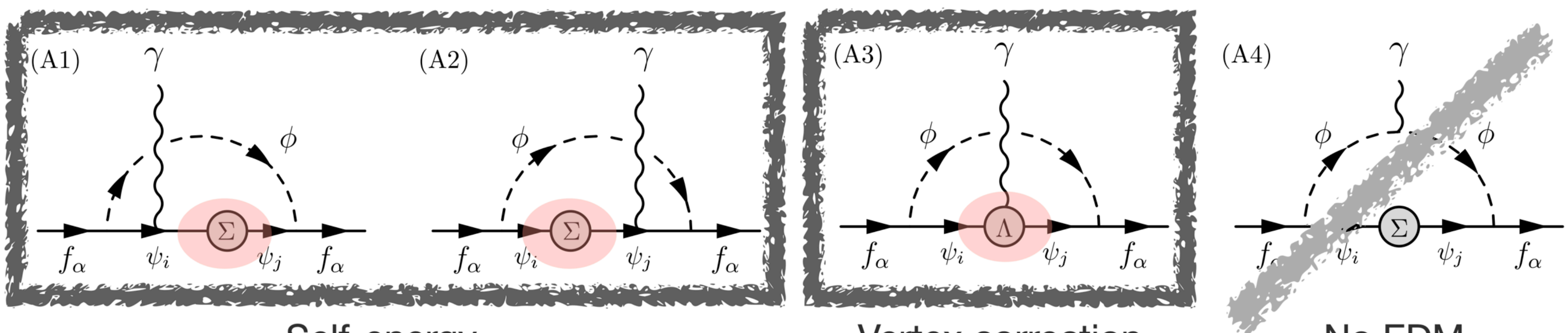
Assumption: Σ_{ij} is real

→ Hermiticity condition

$$\left(\Sigma^\dagger\right)_{ji}\gamma^0 = \gamma^0\Sigma_{ij}$$

※ General assumption on **Flavor Structure**

Diagrams



Self-energy

Vertex correction

No EDM

Ward-Takahashi density

Fermion off-shell vertices

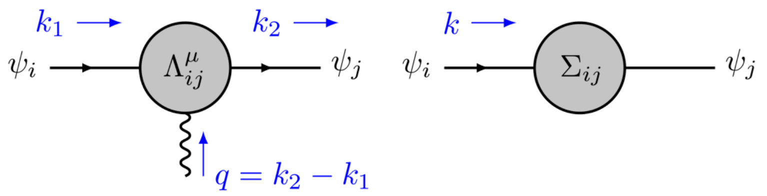
Self-energy

$$\Sigma_{ji}(\not{k}) = \underbrace{A^L_{ji}(k^2)\not{k}P_L + A^R_{ji}(k^2)\not{k}P_R}_{\text{No chirality flip}} + \underbrace{B^L_{ji}(k^2)P_L + B^R_{ji}(k^2)P_R}_{\text{Chirality flip}},$$

Hermiticity condition

$$\left(\Sigma^\dagger\right)_{ji}\gamma^0 = \gamma^0\Sigma_{ij} \quad \Leftrightarrow \quad A^L_{ji} = A^{L*}_{ij}, \quad A^R_{ji} = A^{R*}_{ij}, \quad B^L_{ji} = B^{R*}_{ij}(\equiv B_{ji})$$

Ward-Takahashi identity



$$q_\mu\Lambda^\mu(k_1, k_2) = \Sigma\left(\not{k}_1\right) - \Sigma\left(\not{k}_2\right).$$

decompose

(1) Longitudinal $\quad q_\mu\Lambda^\mu_{\mathrm{L}}(k_1, k_2) = \Sigma\left(\not{k}_1\right) - \Sigma\left(\not{k}_2\right)$

(2) Transverse $\quad q_\mu\Lambda^\mu_{\mathrm{T}}(k_1, k_2) = 0.$

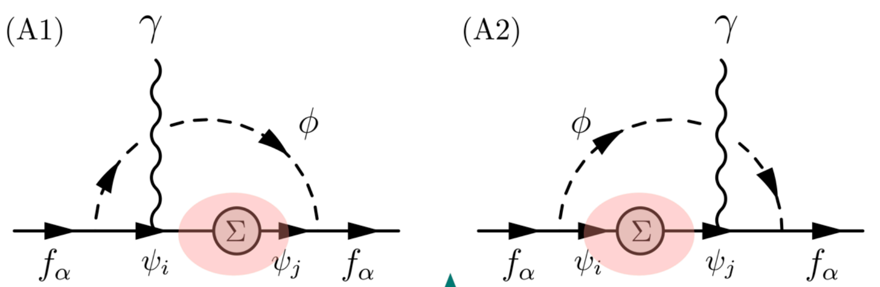
- Reducing general form of to use **W-T id.**
- Picking up $\mathcal{O}(q)$ to focus on EDM terms
- Using Hermiticity condition (**Flavor structure**)

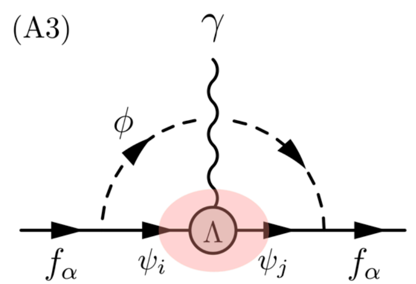
Specify the condition
to obtain nonzero EDM

Reduction using W-T identity

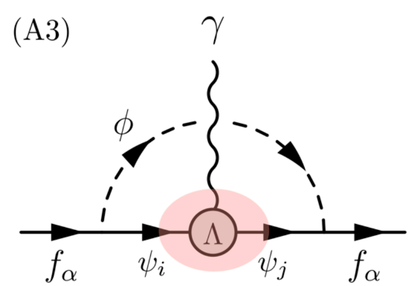
EDM contribution

Self-energy		Vertex correction





(Longitudinal)



(Transverse)

→ **Chirality Flipping B_{ji}**
induces EDM

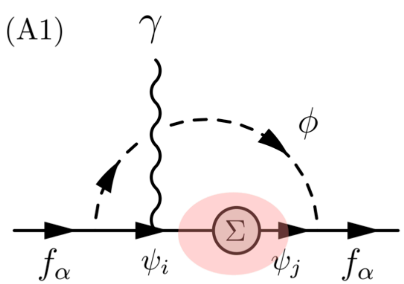
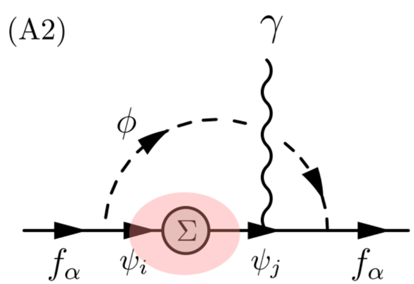
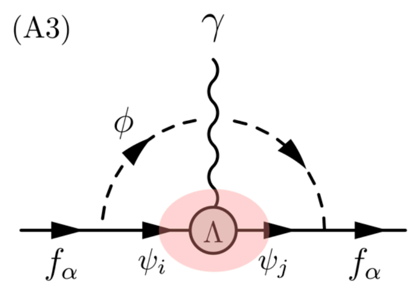
(1) Longitudinal $\underbrace{q_\mu \Lambda^\mu_{\mathrm{L}}(k_1, k_2)}_{\text{Vertex correction}} = \underbrace{\Sigma(\not{k}_1) - \Sigma(\not{k}_2)}_{\text{Self-energy}}$

→$A^{L/R}_{ji}$ cancels out totally

$$\Sigma_{ji}(\not{k}) = \underbrace{A^L_{ji}(k^2)\not{k}P_L + A^R_{ji}(k^2)\not{k}P_R}_{\text{No chirality flip}} + \underbrace{B^L_{ji}(k^2)P_L + B^R_{ji}(k^2)P_R}_{\text{Chirality flip}}$$

Reduction using W-T identity

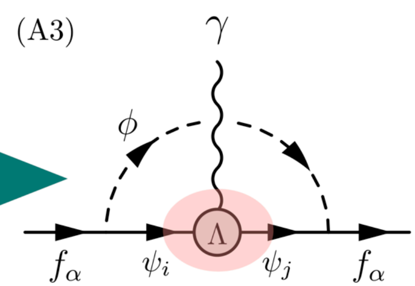
EDM contribution

Self-energy		Vertex correction
		 (Longitudinal)

(2) Transverse $q_\mu \Lambda^\mu_{\rm T}(k_1, k_2) = 0.$

$$\Lambda^\mu_T(k_1,k_2) \supset C^L_{ji} V^\mu_C P_L + D^L_{ji} V^\mu_D P_L$$

$$\left[\begin{array}{l} V^\mu_C = \not{k}[(k\cdot q)\gamma^\mu - \not{q} k^\mu] \\ V^\mu_D = i\sigma^{\mu\nu} q_\nu \end{array} \right.$$ Picking up CP phase
LH only survives



(Transverse)

→ **Chirality Flipping B_{ji}**
induces EDM

→ **Chirality Flipping $C^L{}_{ji}/D^L{}_{ji}$**
induce EDM

We need Chirality Flip in Σ_{ji} & Λ^μ_{ji} to induce EDM from rainbow!

EDM Reduction formulas

Amplitude

$$
i\mathcal{M}^{\mathrm{CP}}_{\mathrm{scalar}} = 2iem_i \int \frac{d^4k}{(2\pi)^4} \overline{u}(p_2) \left[(k\cdot q) \not{\epsilon} - (\epsilon\cdot k) \not{q} \right] P_L u(p_1) \frac{1}{(k-p)^2 - m_\phi^2} \frac{1}{k^2 - m_i^2} \frac{1}{k^2 - m_j^2}
$$
$$
\times \operatorname{Im}\left[Q_\psi \frac{d\tilde{B}_{ji}}{dk^2} + k^2 \tilde{C}_{ji}(k^2) + \tilde{D}_{ji}(k^2) \right] .
$$

$$
\frac{1}{(k-p)^2 - m_\phi^2} \approx \frac{1}{k^2 - m_\phi^2} + \underbrace{\frac{2\,(k\cdot p)}{\left(k^2 - m_\phi^2\right)^2}}_{\text{LO term}} .
$$

(Expand in p)

EDM coefficient

$$
\frac{d_\alpha}{e} \approx -\frac{i m_i m_\alpha}{2} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{\left(k^2 - m_\phi^2\right)^2} \frac{1}{k^2 - m_i^2} \frac{1}{k^2 - m_j^2} \operatorname{Im}\left[Q_\psi \frac{d\tilde{B}_{ji}}{dk^2} + k^2 \tilde{C}_{ji}(k^2) + \tilde{D}_{ji}(k^2) \right]
$$

(Scalar loop Ver.)

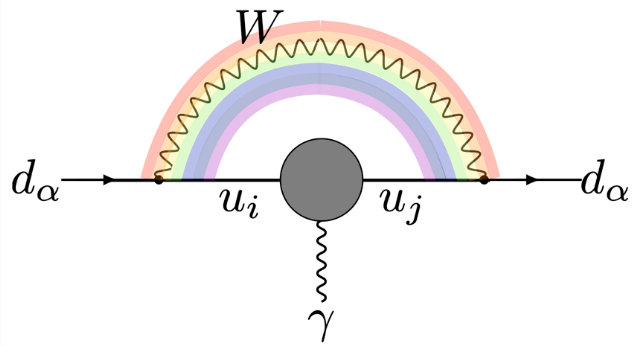
- EDM is induced by **internal Chirality Flipping effects**
- All we have to calculate is $\tilde{B}, \tilde{C}, \tilde{D}$
- Almost the same derivation for Vector loop Ver.

Application

SM nEDM (revisited)

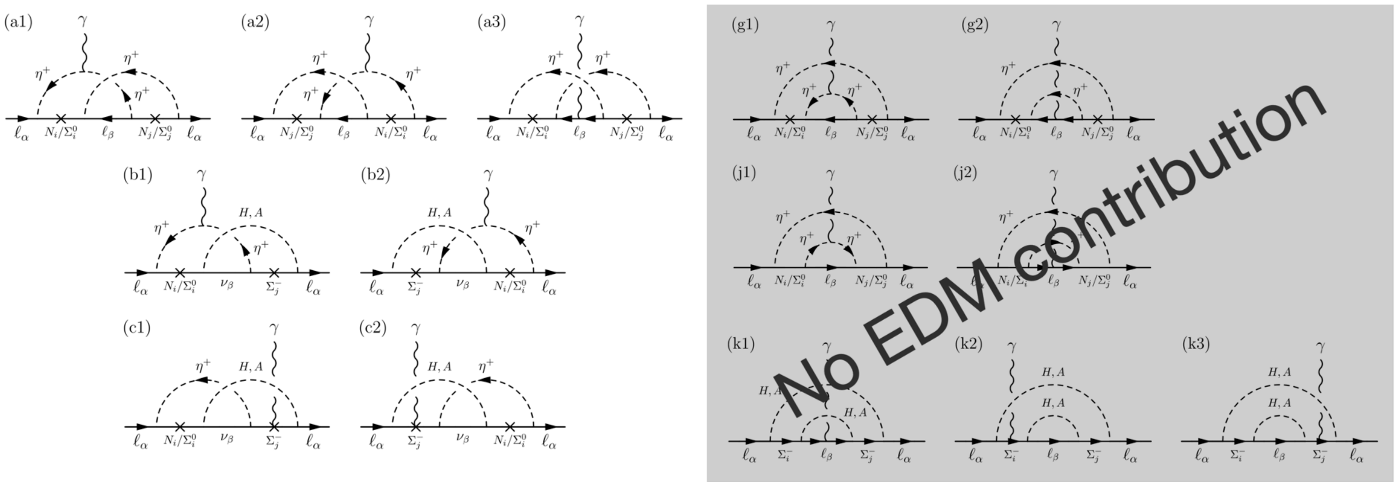
- No internal chirality flip → **No EDM**

  (Just by counting # of γ-matrices in amplitudes!)



Extended Scotogenic Model [MF, J. Hisano, C. Kanai, T. Toma (2021)]

- The same structure as the SM nEDM → **No EDM from rainbow diagrams**
- We can reduce # of diagrams to calculate by half!



Please make use of our reduction formulas in your EDM studies!

Summary

Key for reduction

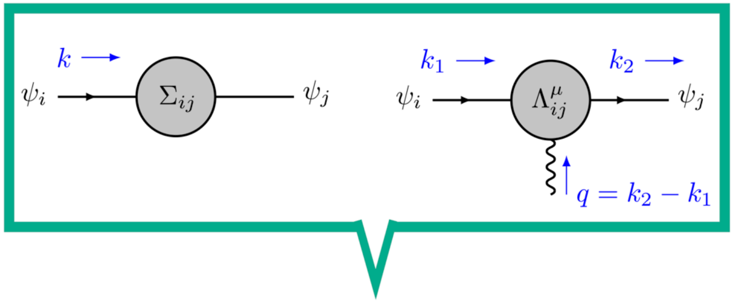
- **The Ward-Takahashi identity**
- Focusing on $\mathcal{O}(q)$ term
- **Flavor structure**: symmetric under

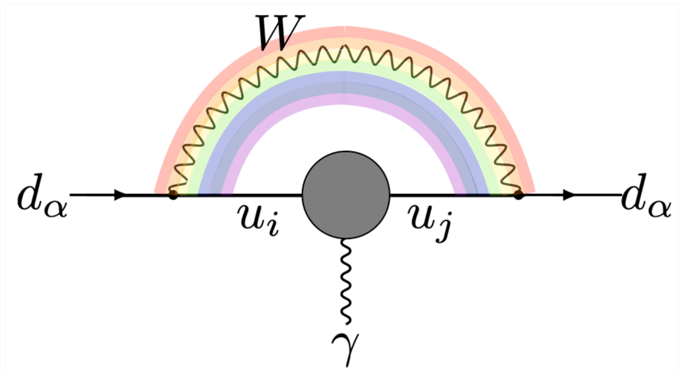
EDM reduction formulas

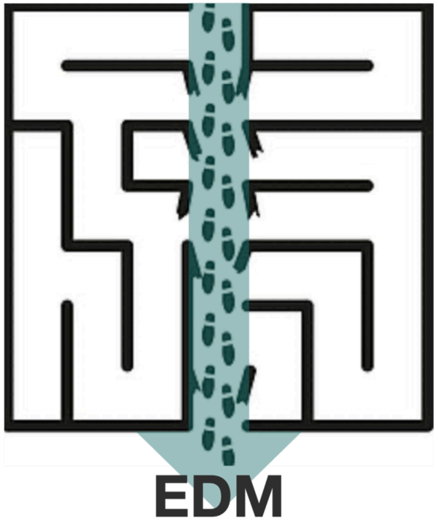
- Derive reduction formulas using off-shell vertices
- Specify the condition to obtain non-zero EDM

Chirality Flip in Σ_{ji} & Λ^{μ}_{ji} to induce EDM from rainbow!

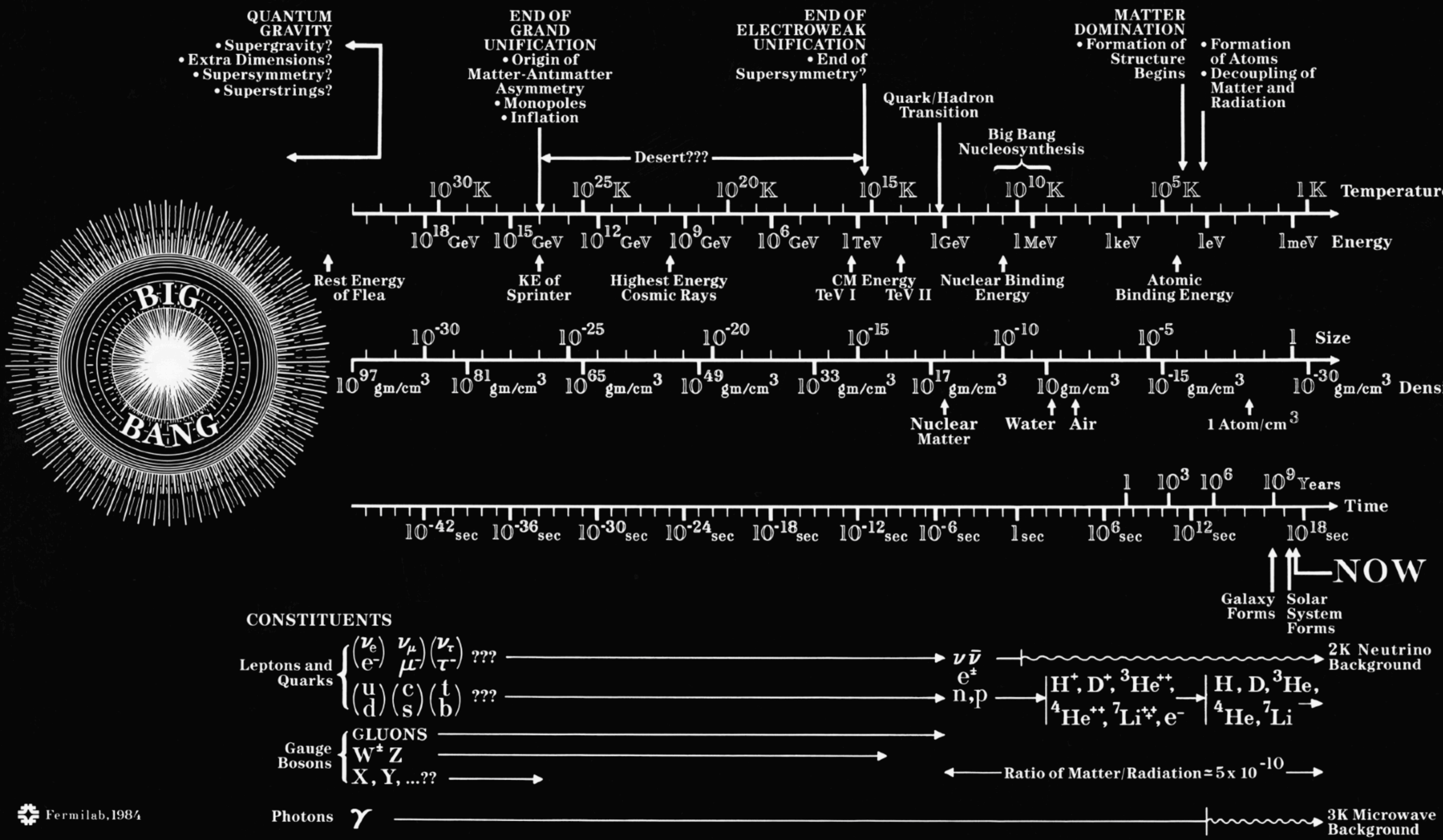
→Reduce the calculation cost!
→Clear way to tell When & Why EDM is induced!







Backup

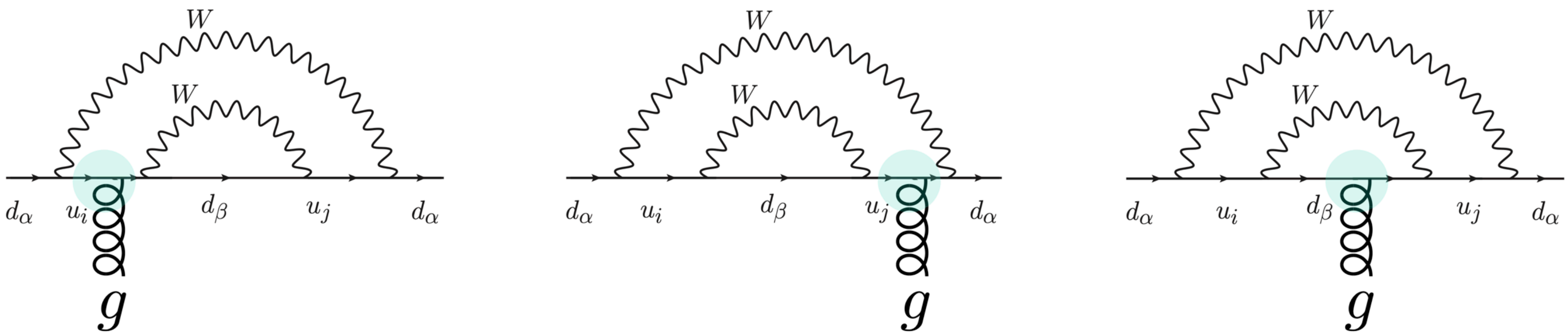

QUANTUM GRAVITY
• Supergravity?
• Extra Dimensions?
• Supersymmetry?
• Superstrings?
END OF GRAND UNIFICATION
• Origin of Matter-Antimatter Asymmetry
• Monopoles
• Inflation
END OF ELECTROWEAK UNIFICATION
• End of Supersymmetry?
MATTER DOMINATION
• Formation of Structure Begins
• Formation of Atoms
• Decoupling of Matter and Radiation
Quark/Hadron Transition
Big Bang Nucleosynthesis
Desert???
10^30 K
10^25 K
10^20 K
10^15 K
10^10 K
10^5 K
1 K
Temperature
10^18 GeV
10^15 GeV
10^12 GeV
10^9 GeV
10^6 GeV
1 TeV
1 GeV
1 MeV
1 keV
1 eV
1 meV
Energy
Rest Energy of Flea
KE of Sprinter
Highest Energy Cosmic Rays
CM Energy
TeV I
TeV II
Nuclear Binding Energy
Atomic Binding Energy
BIG BANG
10^-30
10^-25
10^-20
10^-15
10^-10
10^-5
1
Size
10^97 gm/cm^3
10^81 gm/cm^3
10^65 gm/cm^3
10^49 gm/cm^3
10^33 gm/cm^3
10^17 gm/cm^3
10 gm/cm^3
10^-15 gm/cm^3
10^-30 gm/cm^3
Density
Nuclear Matter
Water
Air
1 Atom/cm^3
1
10^3
10^6
10^9 Years
Time
10^-42 sec
10^-36 sec
10^-30 sec
10^-24 sec
10^-18 sec
10^-12 sec
10^-6 sec
1 sec
10^6 sec
10^12 sec
10^18 sec
NOW
Galaxy Forms
Solar System Forms
CONSTITUENTS
Leptons and Quarks
(νe e-) (νμ μ-) (ντ τ-) ???
(u d) (c s) (t b) ???
ν ν̄
e±
n,p
H+, D+, 3He++, 4He++, 7Li+++, e-
H, D, 3He, 4He, 7Li
2K Neutrino Background
Gauge Bosons
GLUONS
W± Z
X, Y, ...??
Ratio of Matter/Radiation ≃ 5 x 10^-10
Photons γ
3K Microwave Background
Fermilab, 1984

Frequently asked questions

Q. Generalization for Chromo EDM?

A. **YES**, since QED-like identity holds for the external vertices even in non-abelian theory (eg. The SM Chromo EDM is cancelled @2-loop level)

[L. F. Abbott (1981)]
[A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou (2014)]



Q. Neutron EDM in the SM is induced @3-loop level?

A. **YES,** because we also have the diagrams **other than the rainbow type**

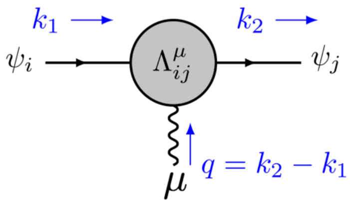
- No complete calculation @3-loop level
- LO contribution comes from the long-distance contribution

$$d_n \sim 10^{-31}\ ecm$$

[T. Mannel, N. Uraltsev (2012)]

Fermion off-shell vertices (Backup)

12 basis to expand Λ^μ [J. S. Ball, T. W. Chiu (1980)] [J. Bernstein (1968)]



$$
\begin{array}{llll}
\mathcal{V}_1^\mu = k^\mu, & \mathcal{V}_2^\mu = q^\mu, & \mathcal{V}_3^\mu = \gamma^\mu, & \mathcal{V}_4^\mu = i\sigma^{\mu\nu}k_\nu, \\
\mathcal{V}_5^\mu = i\sigma^{\mu\nu}q_\nu, & \mathcal{V}_6^\mu = \not{k}q^\mu, & \mathcal{V}_7^\mu = \not{q}k^\mu, & \mathcal{V}_8^\mu = \not{k}k^\mu, \\
\mathcal{V}_9^\mu = \not{q}q^\mu, & \mathcal{V}_{10}^\mu = i\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu k_\rho q_\sigma, & \mathcal{V}_{11}^\mu = \not{k}\not{q}k^\mu, & \mathcal{V}_{12}^\mu = \not{k}\not{q}q^\mu,
\end{array}
$$

Ingredients:

$$\{k^\mu, q^\mu, \gamma^\mu, \sigma^{\mu\nu}, \epsilon^{\mu\nu\rho\sigma}\}$$

- 4 of them can be eliminated to satisfy W-T id.
- $\mathscr{V}_5^\mu$, $\mathscr{V}_{10}^\mu$: Trans. only

Expressed in form factors for self-energy

(1) Longitudinal

$$
\begin{aligned}
\Lambda_{\mathrm{L}}^\mu(k_1,k_2) = & \left[\frac{A^L(k_1^2)-A^L(k_2^2)}{(k\cdot q)}k^\mu\not{k} - \frac{A^L(k_1^2)+A^L(k_2^2)}{2}\gamma^\mu + \frac{B(k_1^2)-B(k_2^2)}{(k\cdot q)}k^\mu\right]P_L \\
& + \left[\frac{A^R(k_1^2)-A^R(k_2^2)}{(k\cdot q)}k^\mu\not{k} - \frac{A^R(k_1^2)+A^R(k_2^2)}{2}\gamma^\mu + \frac{B^\dagger(k_1^2)-B^\dagger(k_2^2)}{(k\cdot q)}k^\mu\right]P_R,
\end{aligned}
$$

6 basis

(2) Transverse

$$\Lambda_{\mathrm{T}}^\mu(k_1,k_2) = \sum_{a=1}^{8}\left[C_a^L(k_1,k_2)V_a^\mu P_L + C_a^R(k_1,k_2)V_a^\mu P_R\right],$$

8 basis ×2 (for L & R)

$$
\left(
\begin{array}{ll}
V_1^\mu = (k\cdot q)\,q^\mu - q^2k^\mu, & V_2^\mu = \not{k}\left[(k\cdot q)\,q^\mu - q^2k^\mu\right], \\
V_3^\mu = \not{k}\not{q}\left[(k\cdot q)\,q^\mu - q^2k^\mu\right], & V_4^\mu = \not{q}q^\mu - q^2\gamma^\mu, \\
V_5^\mu = (k\cdot q)\,\gamma^\mu - \not{q}k^\mu, & V_6^\mu = \not{k}\left[(k\cdot q)\,\gamma^\mu - \not{q}k^\mu\right], \\
V_7^\mu = i\sigma^{\mu\nu}q_\nu, & V_8^\mu = i\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu k_\rho q_\sigma,
\end{array}
\right)
$$

← $\mathscr{O}(q)$ contribution

BU

EDM operators

Non-zero EDM condition

- We need internal chirality flip in the internal fermion operators to pick up CP phase
  → One more chirality flip by external fermion mass is needed in chiral model
  (eg. The SM)

$$\mathcal{L} \supset \frac{C_{in}}{\Lambda} \underbrace{\overline{L}}_{\text{R}} \gamma^5 \sigma^{\mu\nu} F_{\mu\nu} \underbrace{L_L}_{\text{L}} + h.c. \qquad \text{dim. 5}$$

Using EoM of external fermion

$$\mathcal{L} \supset \frac{C_{ex}}{\Lambda^2} \underbrace{\overline{L}}_{\text{L}} \gamma^5 \sigma^{\mu\nu} F_{\mu\nu} \not{D} \underbrace{L_L}_{\text{L}} + h.c. \qquad \text{dim. 6}$$

$$\downarrow$$

$$m_{\text{ext}} L_L$$

Fin.

Thanks!

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