# Measurement of cross section of light hadron production in e<sup>+</sup>e<sup>-</sup> collisions in the Belle II experiment

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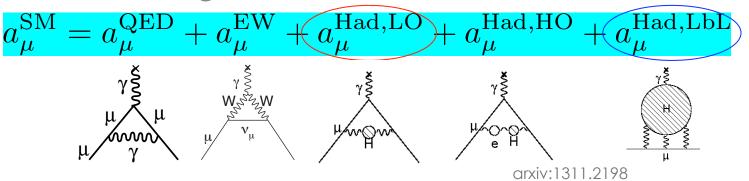
2018 WPI-next mini-workshop "Hints for New Physics in Heavy Flavors"

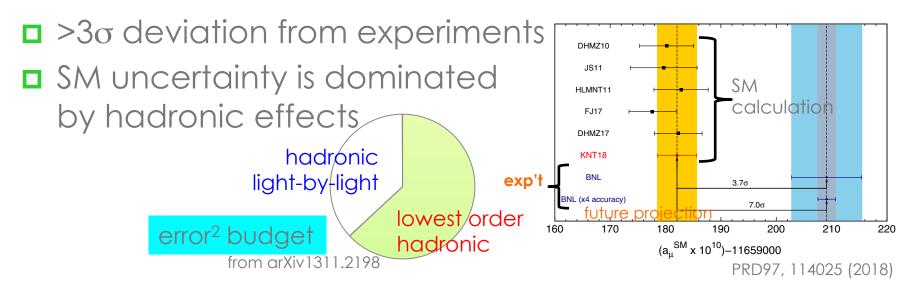


World Research Unit for **Heavy Flavor Particle Physics** 

## muon g-2 and the ee $\rightarrow \pi\pi$ process

□ muon g-2 SM value

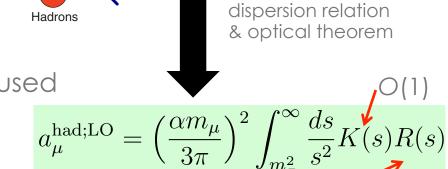




## muon g-2 and the ee $\rightarrow \pi\pi$ process

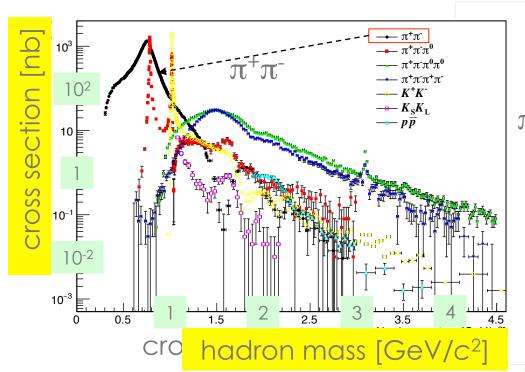
□ leading order hadronic effect  $Im\left(\cdots\right) = \cdots$ 

- hadronic loop
- involves low energy QCD → calculation is difficult
- □ but, ee→(hadrons) cross section data can be used
- Arr ee  $ightarrow \pi\pi$  gives the largest contribution



$$R_{\text{had}}(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$





 $\square$  ee $\rightarrow \pi\pi$  gives the largest contribution

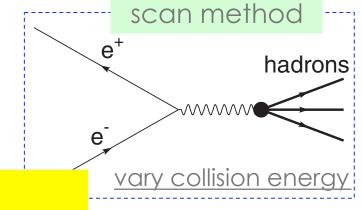
others  $\pi^+\pi^$ from arXiv1311.2198 contribution to a had;LO  $a_{\mu}^{\text{had;LO}} = \left(\frac{\alpha m_{\mu}}{3\pi}\right)^2 \int_{m_{\pi}^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$ 

$$R_{\rm had}(s) = \sigma(e^+e^- \to {\rm hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$



#### measurement methods

- direct scan:
  - change collision energy and measure # of events
  - e.g. CMD3 and SND in Novosibirsk
- ©fine scan is possible for sharp resonances
- ©different conditions among different energy
- Sdifficulty in handling low-momentum particles return method
  - □ radiative return method:
    - collision energy is fixed
    - □ require energetic γ
       (Initial State Radiation, ISR)
       →effectively low energy collision
    - measure mass spectrum of final state hadrons
    - e.g. BaBar, BES III, KLOE



 $e^{+}$ hadrons  $\sqrt{s} = M_{Y(4S)}$   $\rightarrow \sqrt{s'} = M_{had}$   $e^{-}$   $ISR \gamma$ 

fixed collision energy

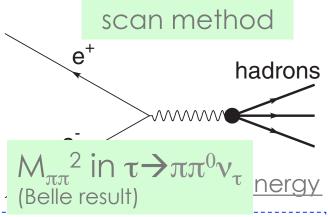


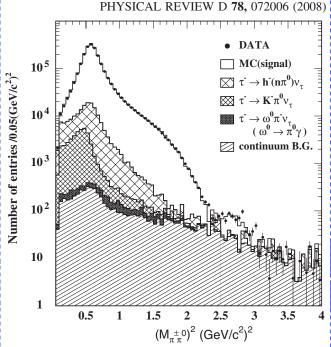
### measurement methods

- ©large statistics
- Suncertainty due to correction of iso-spin breaking effect

oirsk

- tau hadronic decay with CVC:
  - Conserved Vector Current hypothesis
  - □ ππ mass spectrum in  $\tau \rightarrow \pi \pi^0 \nu_{\tau}$
  - e.g. LEP exp'ts, CLEO, Belle
- □ radiative return method:
  - collision energy is fixed
  - □ require energetic γ
     (Initial State Radiation, ISR)
     →effectively low energy collision
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  - □ e.g. BaBar, BES III, KLOE

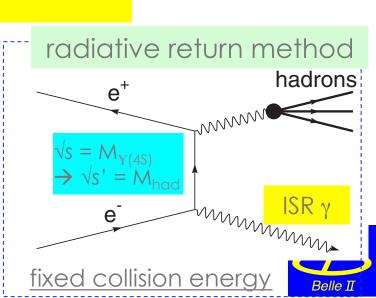




hadrons

#### measurement methods

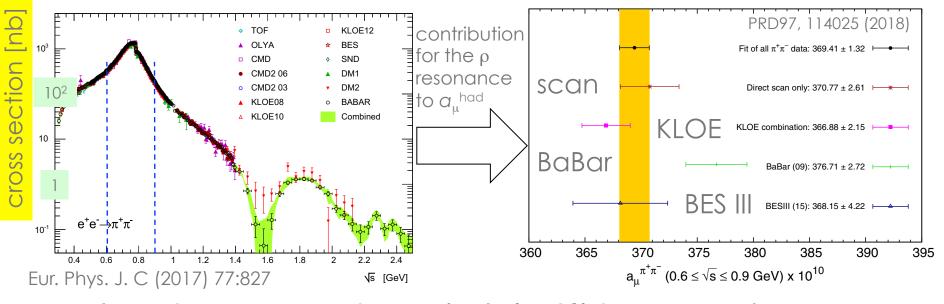
- direct scan:
  - change collision energy
- $\otimes$ low statistics due to ISR requirement ( $O(\alpha)$ )
- ©but is compensated high luminosity machines
- ©can scan cross section for wide energy range in the same experimental condition
  - □ e.g. LEP exp'ts, CLEO, Belle
  - radiative return method:
    - collision energy is fixed
    - □ require energetic γ
       (Initial State Radiation, ISR)
       →effectively low energy collision
    - measure mass spectrum of final state hadrons
    - e.g. BaBar, BES III, KLOE



scan method

vary collision energy

#### status of $\pi\pi$ cross section measurement



- □ Already measured precisely(≤1%) by several experiments
- □ small discrepancy (a few %) among measurements
- must be confirmed by Belle II
- □ target: 0.5% precision (similar or better than Babar)



## advantages in Belle II

- large statistics
  - signal events themselves
  - control samples for estimation of systematic uncertainty
- well-designed triggers
  - Neither Belle and BaBar had optimized trigger for this measurement
  - Belle suffered from large efficiency loss due to trigger
- larger detector coverage
- better generator
- lessons from the BaBar measurement
  - All are giving comparable uncertainty, but <u>PID-related</u> ones are relatively large

list of systematic errors in BaBar (PRD86 032013)

#### Sources

Trigger/filter
Tracking  $\pi$ -ID
Background

Acceptance

Kinematic fit  $(\chi^2)$ 

Correl.  $\mu\mu$  ID loss

 $\pi\pi/\mu\mu$  non-cancel.

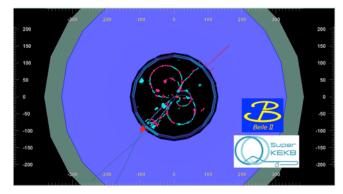
Unfolding

**ISR** luminosity

Sum (cross section)

#### first look at the Belle II data

- Belle II phase2 operation
  - commissioning of the accelerator with collisions
  - end of March middle of Jul
  - □ the first collision at 26th April
- □ full data of 472 pb<sup>-1</sup> was used
- goal of the analysis
  - to observe ρ meson peak in the mass spectrum
  - yield comparison with MC simulation
  - study of trigger efficiency



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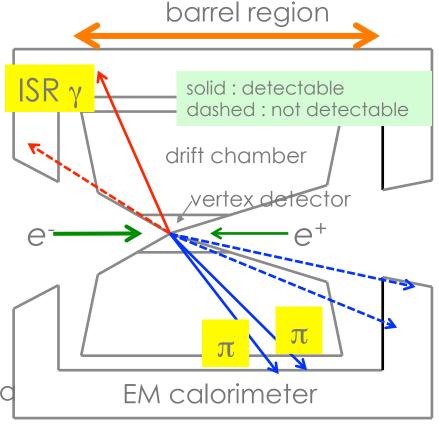


celebration of the first collision (26<sup>th</sup> Apr.)



## analysis procedure

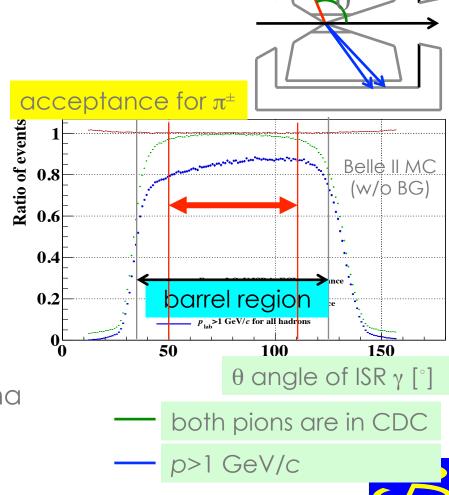
- select events with
  - □ one energetic photon (ECMS>3 GeV)
  - two charged tracks (p>1 GeV/c)
- selection criteria
  - □ photon points to central part of the barrel region  $(50^{\circ} < \theta_{ISR} < 110^{\circ})$
  - E/p<0.8</li>→remove Radiative Bhabho (ee→eeγ) contribution
  - $10 < M(\pi\pi\gamma) < 11 \text{ GeV/c}^2$  $\rightarrow$  no other extra particles





analysis procedure

- select events with
  - □ one energetic photon (E<sup>CMS</sup>>3 GeV)
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analysis procedure

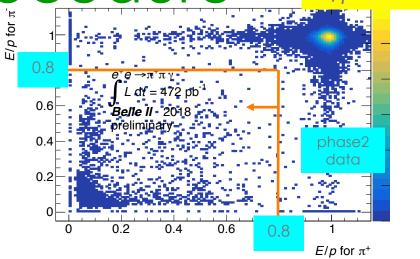
E/p ratio

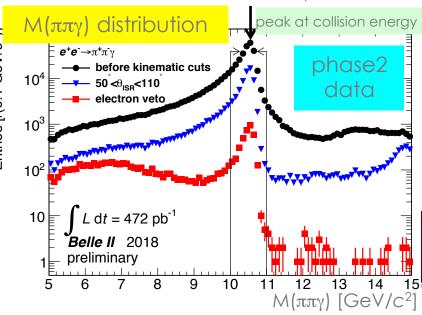
- select events with
  - one energetic photon (ECMS>3 GeV)
  - two charged tracks (p>1 GeV/c)
- selection criteria
  - photon points to central photon points to central part of the barrel region 50°<θ<sub>ISR</sub><110°

    E/p<0.8

    → remove Radiative Bhabha

    Leader Contribution
  - □ E/p<0.8 (ee → ee<sub>Y</sub>) contribution
  - □ 10<M(ππγ)<11 GeV/c<sup>2</sup> →no other extra particles



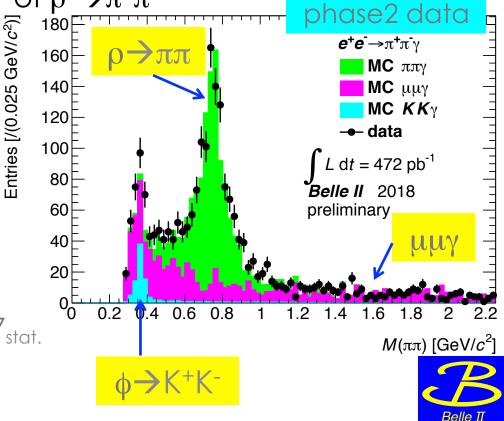


## ππ mass spectrum

□ ρ meson peak is clearly observed! Belle II first "rediscovery" of ρ<sup>0</sup>→ $\pi$ <sup>+</sup> $\pi$ <sup>-</sup>

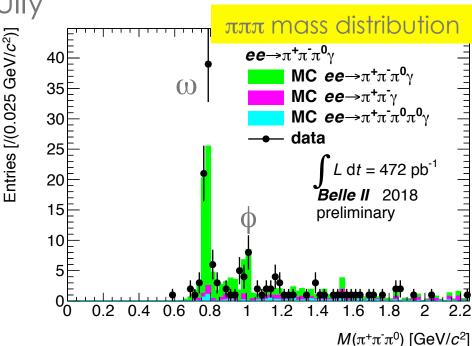
- no PID cuts except for the E/p cut
   →contribution from μμγ / ΚΚγ
  - □ peak at low mass due to φ→K<sup>+</sup>K<sup>-</sup>
  - high mass (>1 GeV/c²)
     is dominated by μμγ
- reasonable data/MC agreement
  - $\Box$  data/MC = 1.065±0.037<sub>stat.</sub> (0.5-1 GeV/c<sup>2</sup>)

MC trigger efficiency is assumed to be 100%



### results for other modes, K, K,

- the ee→πππγ process is also studied with phase2 data
  - 2nd biggest contribution to a<sub>u</sub>had;LO
- ω, φ peaks are successfully observed
  - "rediscovery"
- reasonable data/MC agreement



 $\pi^+\pi^-\pi^0$ 

contribution of each mode

to  $a_{\mu}^{\text{had;LO}}$  ( $\sqrt{s} < 1.8 \text{ GeV}$ )

 $\pi^+\pi^-$ 

## trigger efficiency for ππγ

high trigger efficiency is necessary

for precision measurement

■ Belle II trigger for ee → ππγ

total calorimeter energy1 GeV

■ Bhabha veto←loss of this vetomust be small



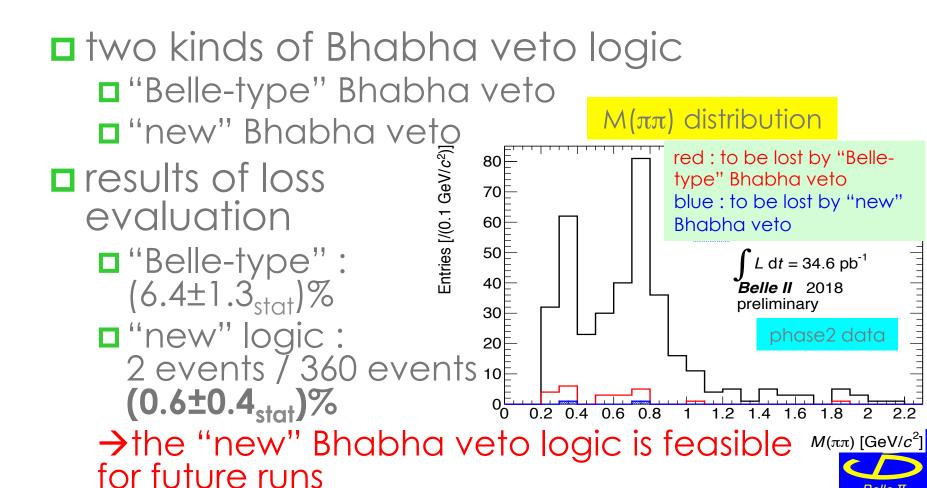
Belle trigger simulation

- □ large loss by Bhabha veto in Belle→precision measurement was difficult
- □ all Bhabha events were collected in phase2
  - Efficiency loss can be easily evaluated by counting the number of events with Bhabha trig.



Belle II

## efficiency loss by Bhabha veto



expected performance by MC sim.

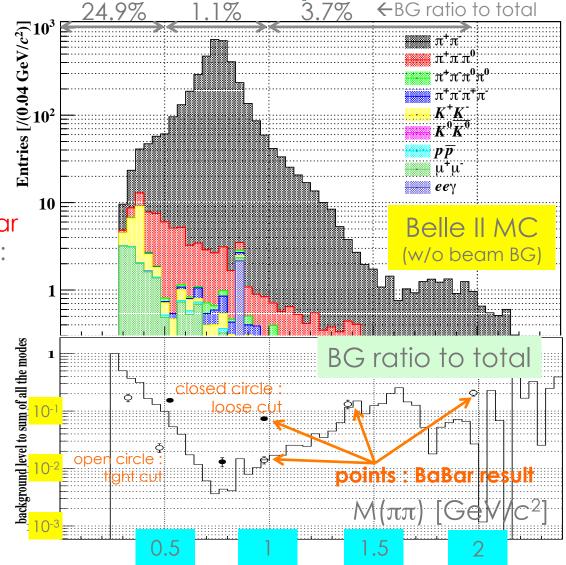


- BG contribution
  - other ISR modes  $(\pi^+\pi^-\pi^0, K^+K^-,...)$
  - O(%) level BG; same level with BaBar
  - **n** high BG at low mass:  $\pi\pi\pi^0$  with low-E  $\pi^0$

←can be reduced (kinematic fit...)

- efficiency
  - 49% for  $50 < \theta_{ISR} < 110^{\circ}$
  - expect > 1M events with 500 fb<sup>-1</sup>

→can have results with early Belle II data!!



#### summary

- □ ee→ππ cross section measurement in Belle II with ISR method is critical to reduce uncertainty of theoretical value for muon g-2
- In Phase2 data, ρ meson peak was clearly observed and good data-MC agreement was confirmed
- $\blacksquare$  Peaks for  $\omega$ ,  $\phi \rightarrow \pi^+\pi^-\pi^0$  are also observed.
- □ Although Belle suffered from large efficiency loss due to Bhabha veto in the trigger level, such loss is evaluated to be small (≤1%) with a new Bhabha veto logic in phase2 data.
- The first O(100) fb<sup>-1</sup> data will give enough signal events, which will be expected in a few years

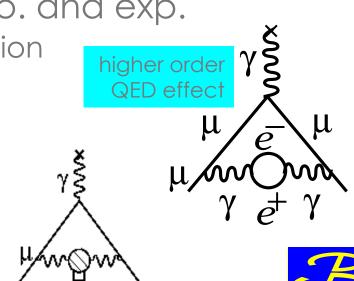




## muon g-2

$$\vec{\mu}_{\ell} = g_{\ell} \frac{Qe}{2m_{\ell}} \vec{s}$$

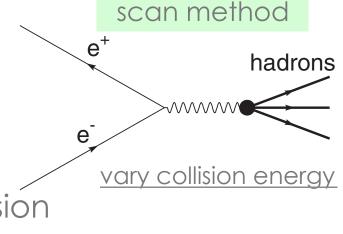
- "g-factor" of μ (also e) is slightly larger than 2 due to QED effect
  - $\Box a_{\mu} = (g-2)/2$
  - $\square$  ~3 $\sigma$  discrepancy btw theo. and exp.
    - □ both have ~0.5 ppm precision
- strong interaction and weak interaction also contribute
  - □ strong: ~60 ppm
  - weak : ~1.3 ppm

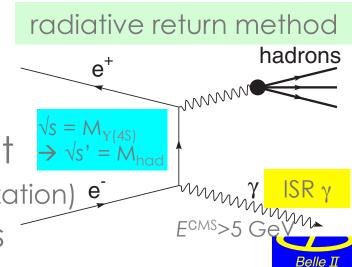


"Schwinger"

#### ee $\rightarrow \pi\pi$ measurement at Belle II

- radiative return method:
   detect ee → ππγ events
  - require energetic γ (Initial State Radiation, ISR)
    - →effectively low energy collision
  - □ hadron inv. mass distribution
     →corrections
     (BG, eff., unfolding...)
    - →cross section for each √s
- simultaneous measurement
  - of  $\pi\pi\gamma$  (signal) and  $\mu\mu\gamma$  (normalization)
    - cancellation of various errors





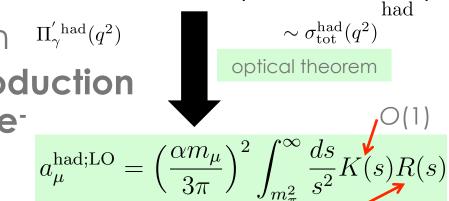
fixed collision energy

## hadronic contribution

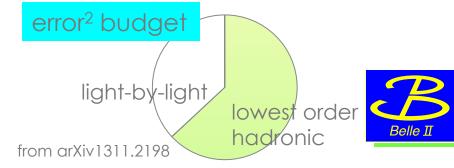
Physics Reports 477 (2009) 1–110



- $\square$  ~60 ppm contribution  $\Pi_{\gamma}^{' \text{had}}(q^2)$
- □ related to hadron production cross section from e<sup>+</sup>e<sup>-</sup>
- dominating theo. uncertainty
- higher order
  - □ smaller uncertainty
- □ light-by-light
  - □ (not discussed here)



$$R_{\rm had}(s) = \sigma(e^+e^- \to {\rm hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$



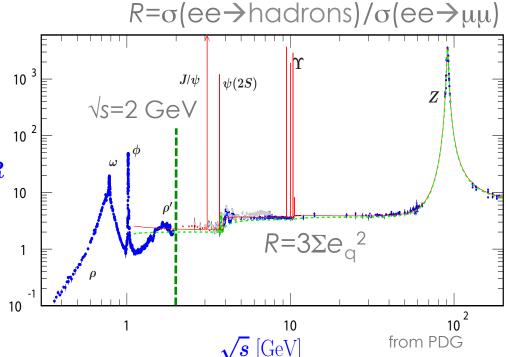
## Fermion pair production in e<sup>+</sup>e<sup>-</sup> collisions

cross section is well understood

can be neglected at  $M_u^2/s \ll 1$ 

$$\sigma(e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}) = \underbrace{\frac{4\pi\alpha^{2}}{3s}}\sqrt{1 - 4M_{\mu}^{2}/s} (1 + 2M_{\mu}^{2}/s)$$
86.85 nb / (s [GeV<sup>2</sup>/c<sup>4</sup>])

- quark production is also well described at large √s
  - charge/flavor/color
- for small √s (<2 GeV), <sup>R</sup> experimental data is necessary
  - low energy QCD



syst. error table

## detection eff. study

- reduction of systematic errors is crucial →need to understand each efficiency within 0.5%
- important to keep high efficiency
  - geometrical acceptance
  - trigger efficiency
  - reconstruction efficiency
  - cut efficiency
    - momentum threshold
    - □ PID cut
  - background / unfolding / normalization...

in the BaBar result Sources PRD86 032013

Trigger/filter **Tracking**  $\pi$ -ID

Background Acceptance

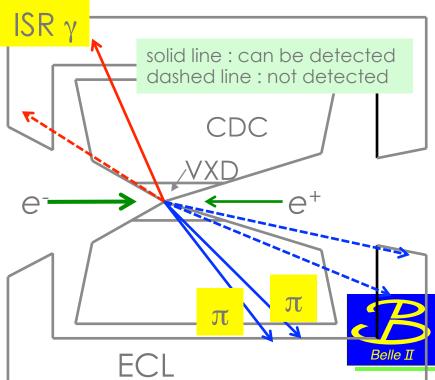
Kinematic fit  $(\chi^2)$ 

Correl.  $\mu\mu$  ID loss  $\pi\pi/\mu\mu$  non-cancel.

Unfolding

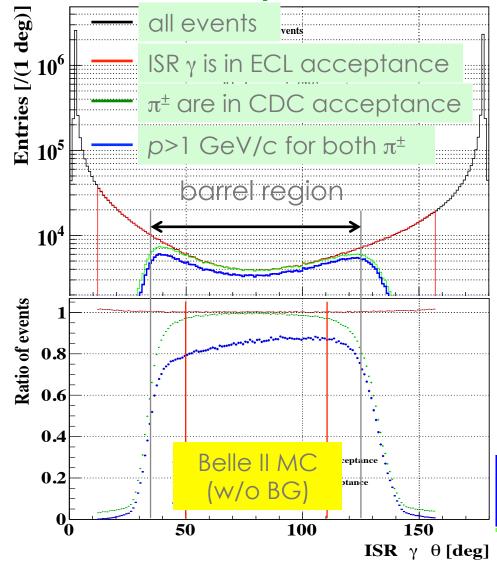
**ISR** luminosity

Sum (cross section)

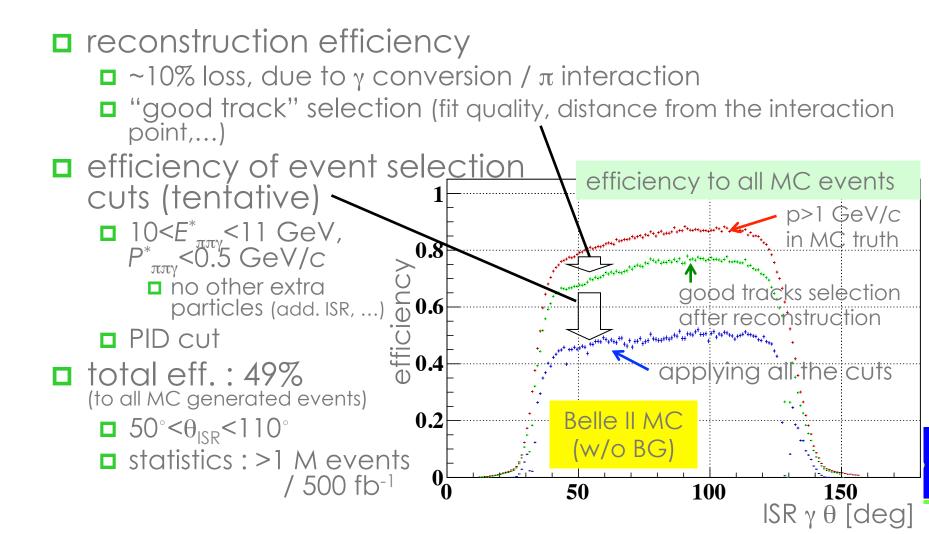


## acceptance study

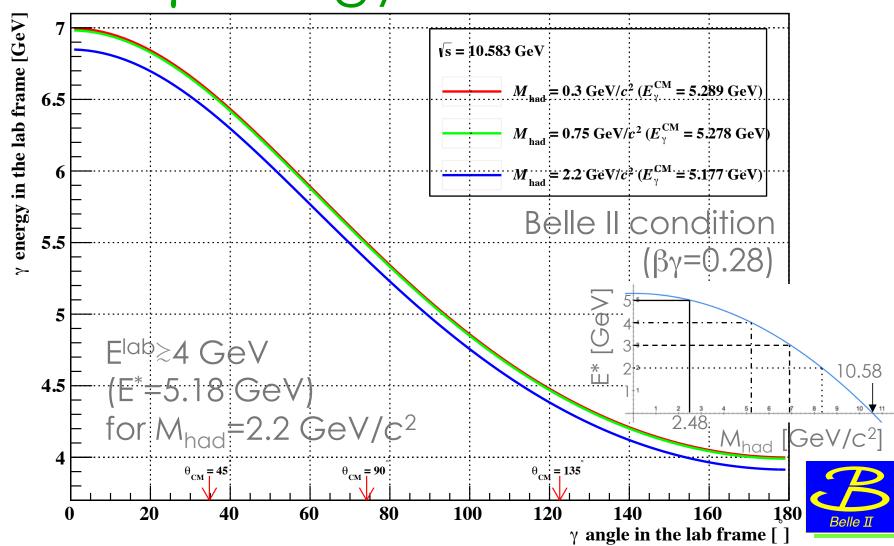
- efficiency is flat for large angle ISR γ
   by limiting ISR γ θ angle, acceptance can be kept high
  - lose some events, but can be easily compensated by Belle II high stat.
- □ 10-20% loss due to momentum cut (p>1 GeV/c)
  - for good muon-ID



## efficiency for each selection



## ISR y energy in lab frame

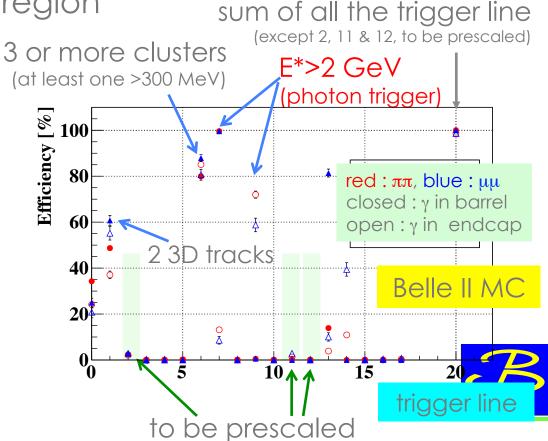


## trigger simulation

100% efficiency for good events with ISR γ pointing the barrel region

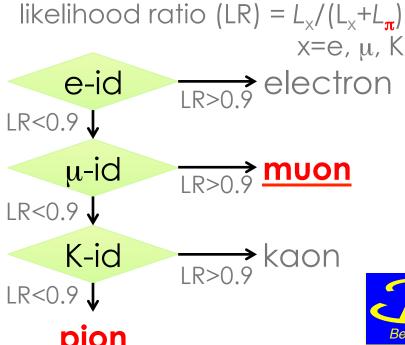
Bhabha veto is considered

- some loss (O(%))
   for endcap,
   as designed
   (but these events are not used as discussed later)
- photon trigger is working effectively as expected



## PID algorithm

- assign unique PID for each track
- require both tracks to be identified as the particle of interest
- study items
  - $\square$   $\mu\mu \leftarrow \rightarrow \pi\pi$  cross feed
  - correlated efficiency loss





KLM module

structure

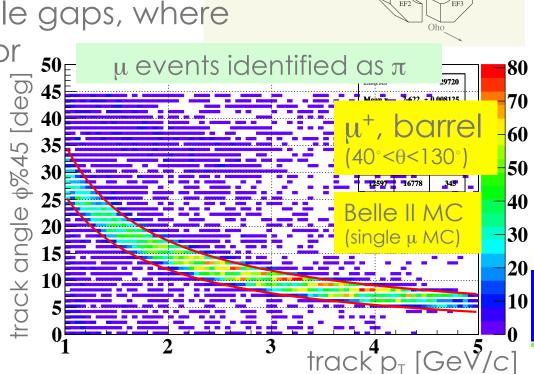
## muon/pion separation

mis-identified muons tend to be recognized as pions
 →μ-id ineff. = fake π

■ avoiding KLM module gaps, where

 $\mu$ -id efficiency is poor

- visible in p<sub>T</sub>-φ plane
- set veto regions (for barrel/endcap, positive/negative μ)
- require at least one track to be outside of the veto regions

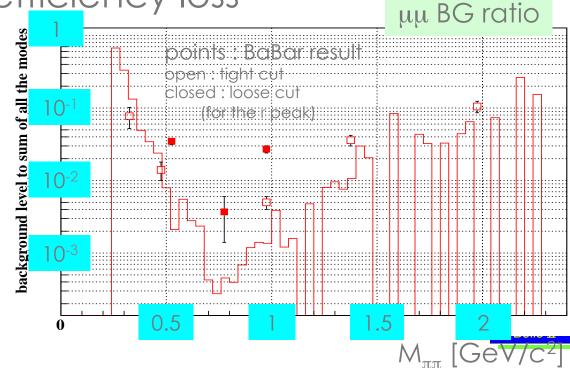


## μμ BG in ππ analysis

reduction by a factor of 5 by introduction of KLM module gap veto

■ 9% additional efficiency loss

■ the same level with BaBar



#### correlated loss of PID eff.

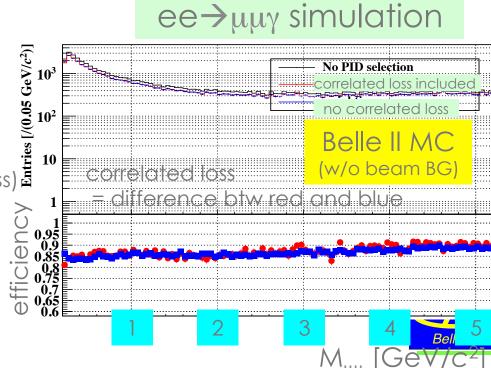
additional efficiency loss can exist due to two tracks close to each other

compare two efficiencies

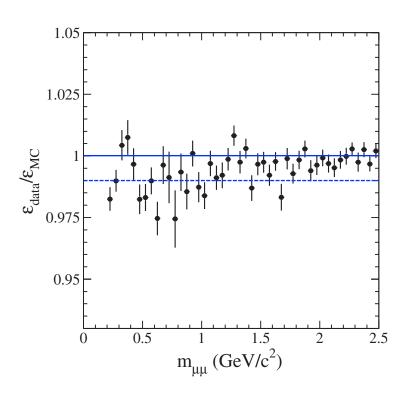
 $\square$   $\mu$ -id for both tracks

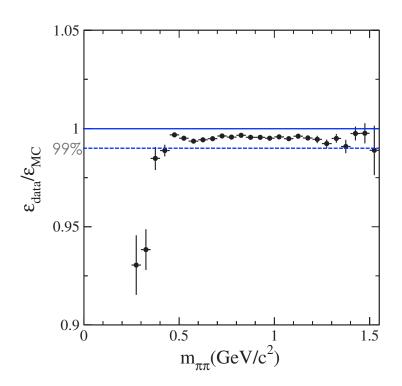
ificant correlated loss γ ancy loss γ product of μ-id

significant correlated efficiency loss was not seen



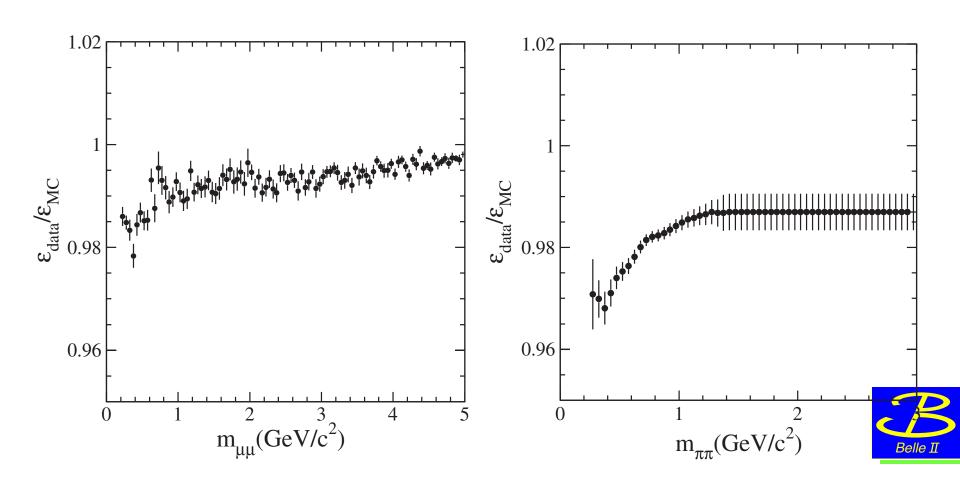
## BaBar trigger/filter eff. correction







## BaBar tracking eff. correction



## L1 trigger menu

Bit	Phase 2 description	Prescale Phase 2	Changes for 2020	Prescale 2020
0	3 or more 3D tracks			
1	2 3D tracks, ≥1 within 25 cm, not a trkBhabha		2 3D tracks, ≥1 within 10 cm, not a trkBhabha	
2	2 3D tracks, not a trkBhabha	20		20
3	2 3D tracks, trkBhabha			2
4	1 track, <25cm, clust same hemi, no 2 GeV clust		1 track, <10cm, clust same hemi, no 2 GeV clust	
5	1 track, <25cm, clust opp hemi, no 2 GeV clust		1 track, <10cm, clust opp hemi, no 2 GeV clust	
6	≥3 clusters inc. ≥1 300 MeV, not an eclBhabha		≥3 clusters inc. ≥2 300 MeV, not an eclBhabha	
7	2 GeV E* in [4,14], not a trkBhabha			
8	2 GeV E* in [4,14], trkBhabha			2
9	2 GeV E* in 2,3,15,16, not eclBhabha			
10	2 GeV E* in 2,3,15 or 16, eclBhabha			
11	2 GeV E* in 1 or 17, not eclBhabha	10		20
12	2 GeV E* in 1 or 17, eclBhabha	10		20
13	exactly 1 E*>1 GeV and 1 E>300 MeV, in [4,15]			
14	exactly 1 E*>1 GeV and 1 E>300 MeV, in 2,3 or 16			5
15	clusters back-to-back in phi, both >250 MeV, no 2 GeV			
16	clusters back-to-back in phi, 1 <250 MeV, no 2 GeV		clust back-to-back in phi, <250 MeV, no 2 GeV, no trk>25cm	3
17	clusters back-to-back in 3D, no 2 GeV			5

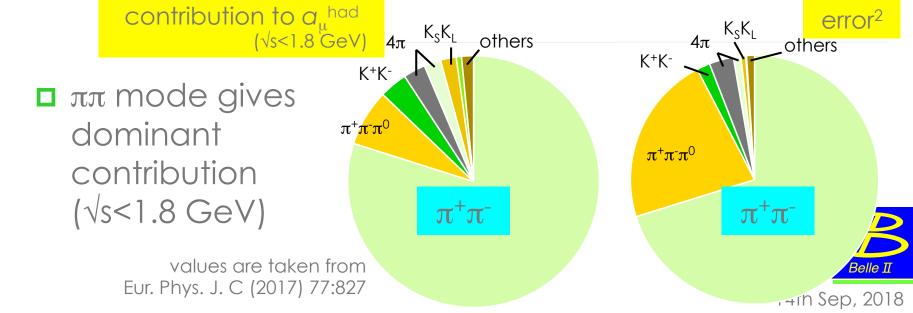


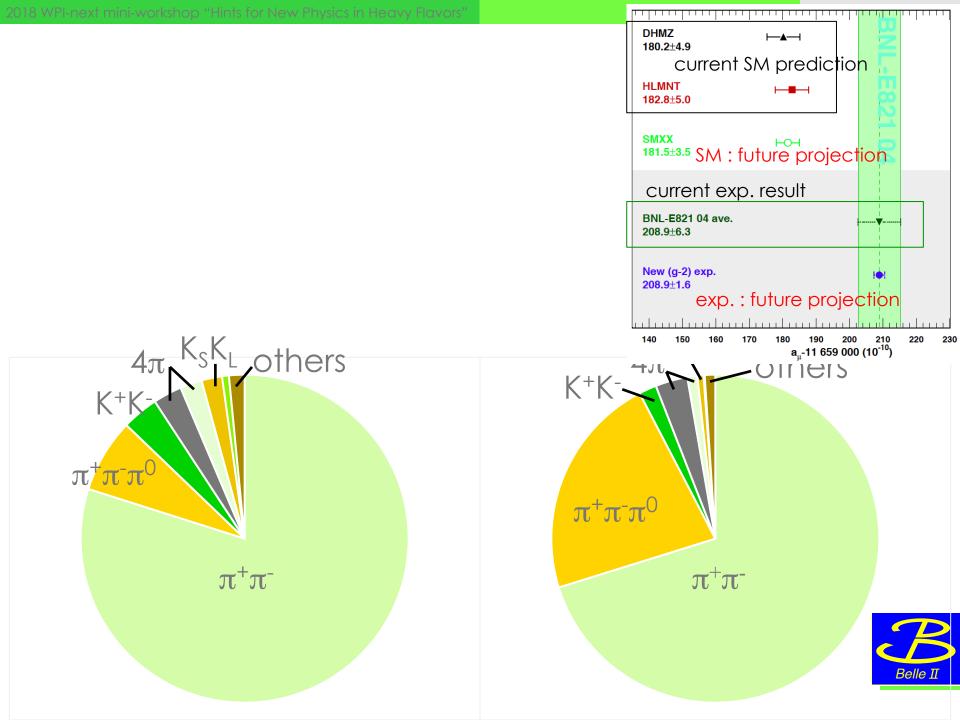
## light hadron production

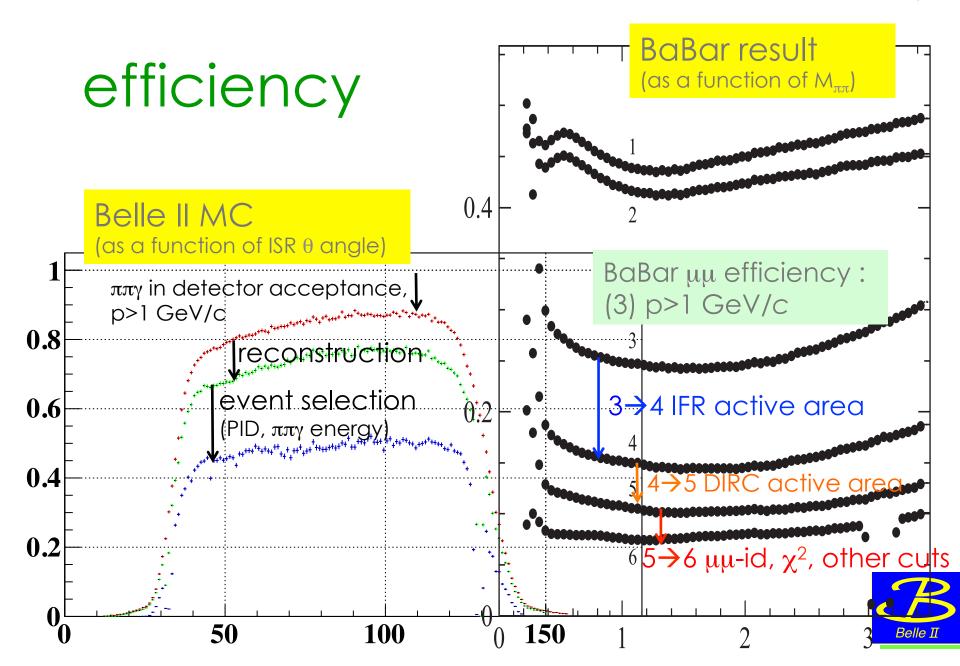
 $\blacksquare$  Hadron production cross section is an important input for hadronic contribution  $a_{\mu}^{\ \ had}$  of  $\mu$  g-2

$$a_{\mu}^{(4)}(\text{vap, had}) = \left(\frac{\alpha m_{\mu}}{3\pi}\right)^2 \left(\int_{m_{\pi^0}^2}^{E_{\text{cut}}^2} \mathrm{d}s \frac{R_{\text{had}}^{\text{data}}(s)}{s^2} \hat{K}(s) + \int_{E_{\text{cut}}^2}^{\infty} \mathrm{d}s \frac{R_{\text{had}}^{\text{pQCD}}(s) \hat{K}(s)}{s^2}\right)$$

$$K(s) : \text{Kernel function}$$





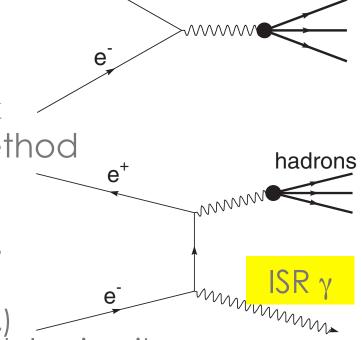


hadrons

#### R measurement

$$a_{\mu}^{\mathrm{had;LO}} = \left(\frac{\alpha m_{\mu}}{3\pi}\right)^2 \int_{m_{\pi}^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$$
 R(s) dependence

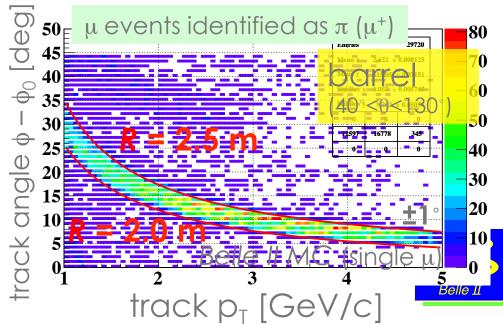
- scan method
  - □ ©large statistics
  - □ ⊗limited energy range
  - ⊗point-to-point errors
  - being performed in Novosibirsk
- Initial State Radiation (ISR) method (colliders with fixed energy)
  - □ tag ISR photon (E>3 GeV)
  - ©can scan wide energy range
  - ©same exp'tal condition
  - □ Slower statistics due to ISR  $O(\alpha)$  ←can be compensated by high luminosity
  - performed by BaBar / BES / KLOE



endca KLM gap effect ■ muon ID inefficiency  $\rightarrow$ fake  $\pi$ derived from module gaps of the  $K_1$ - $\mu$  detector (KLM) □ also very forward (□ 160 0 140 0  $\mu^+$  identified as  $\pi^+$ 60 50 region ( $\theta$ <25°), not  $\Xi_{120}$ 40 covered by KLM 100 Avoiding this region Avoidi 30 20 helps to reduce 10 μμ <del>)</del>ππ bkg 20 -200-100100 [deg]

### KLM-gap veto cut

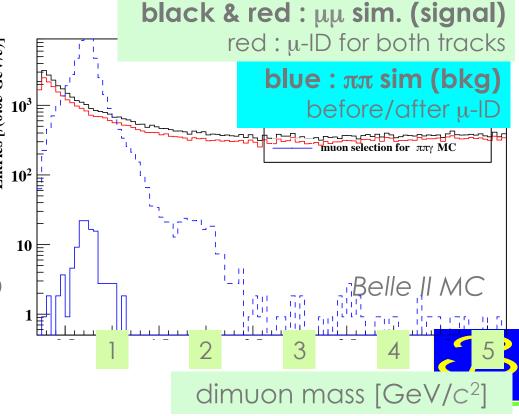
- $\Box$  veto regions in track  $p_T$ - $\phi$  plane  $(\phi$  is measured with respect to gap angle  $\phi_0)$ 
  - $\blacksquare$  defined for each of particle charge and  $\theta$  direction (endcap or barrel)
- prequire at least one track to be outside this veto region when track  $\phi=90^{\circ}$  when track  $\phi=90^{\circ}$  when  $\phi^*=\cos^{-1}\frac{cBR}{\sqrt{20}}$



### PID performance – μμ mode

μμ/ππ modes can be background for each other

- MC stat.:
  ~5 fb<sup>-1</sup> equiv.
- □ μμ-ID eff.
  - □~80%
  - loss by veto cut: 5%
- □ππ→μμ bkg. ratio
  - □~0.4% (M<sub>uu</sub><1 GeV/c²)

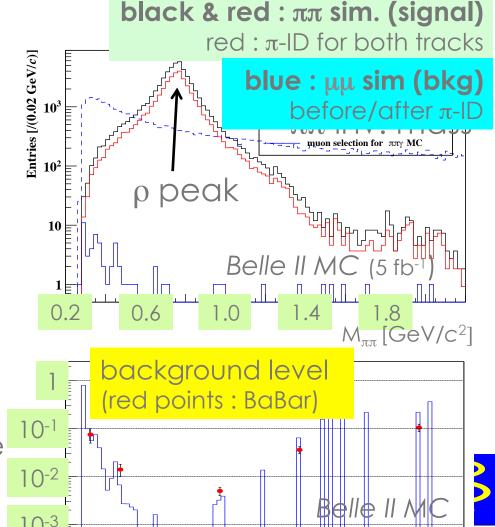


 $M_{\pi\pi} \frac{2}{[\text{GeV}/c]}$ 

#### PID performance – $\pi\pi$ mode

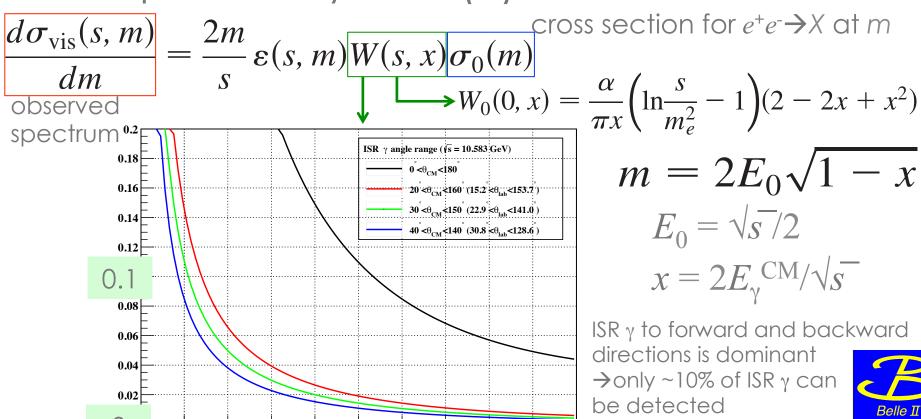
- □ ππ-ID cut efficiency
  - **□** 69%
  - □ loss by veto cut: 8.8%
- □ μμ→ππ background
  - □ 0.15% (<1 GeV/c<sup>2</sup>)
  - ←factor 5 reduction due to the veto cut
  - □ same level as BaBar
- □ required statistic
  - 5.3k evts / 5 fb⁻¹
     →>100 fb⁻¹
     possible in early stage of Belle II run

(BaBar: 232 fb<sup>-1</sup> PRD86 032013)



#### radiator function

 $\square$  probability to emit ISR  $\gamma$  to produce a particle system (X) with mass of m



 $x = 2E_{v}^{\text{CM}}/\sqrt{s}$ 

0.1

0.2

0.3

0.4

0.5

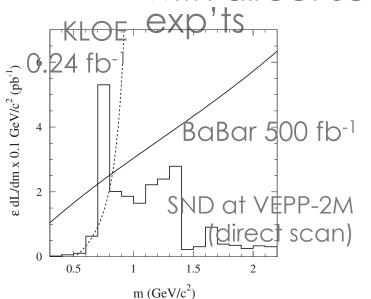
0.6

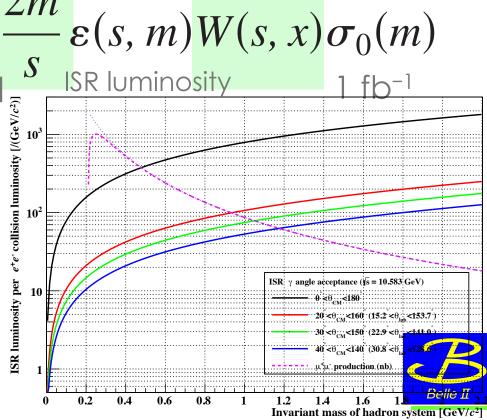
# ISR luminosity

 $m = 2E_0 \sqrt{1 - x}$ □ 2m/s: to change x to m

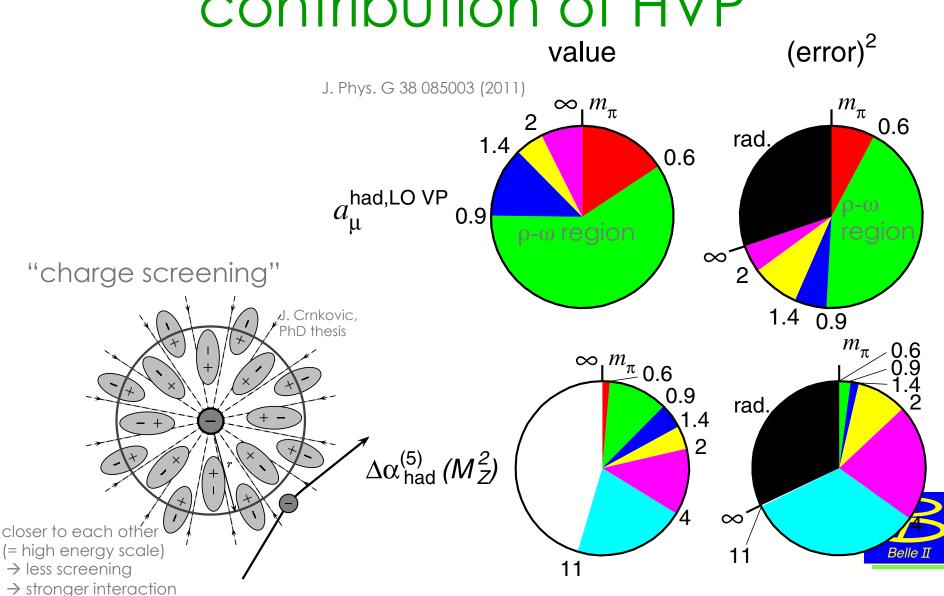
$$\frac{d\sigma_{\text{vis}}(s,m)}{s} = 2$$

 $\square$  can be compared with direct scan





#### contribution of HVP



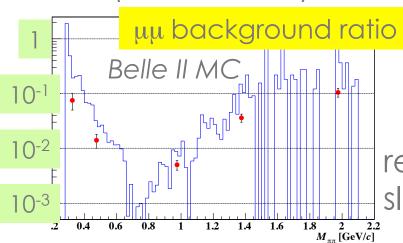
### without veto cuts $(\pi\pi)$

 $\square$   $\pi\pi$  efficiency ~ 75%

□ μμ→ππ bkg. ratios ~ 0.85%

□ comparison

□ comparison with BaBar ana. (PRD86 032013)



blue: μμ sim (bkg)

dashed/solid:

before/after π-ID

peak

Belle II MC

1.2

black & red :  $\pi\pi$  sim. (signal)

red:  $\pi$ -ID for both tracks

red points: BaBar analysis slightly worse in this analysis

0.8

0.6

0.4



 $M_{\pi\pi} = \frac{2}{\text{GeV}/c}$ 

1.8

1.6

#### cut optimization

	μμ efficiency	ππ→μμ BG	ππ efficiency	μμ→ππ BG
no veto cut	85.2%	0.39%	75.3%	0.83%
loose cut	80.9%	0.39%	68.7%	0.15%
tight cut	58.2%	0.40%	46.2%	0.10%

M<1 GeV/c<sup>2</sup>

tight cut (require both tracks to be outside the veto regions) loses efficiency, while background reduction is not so large

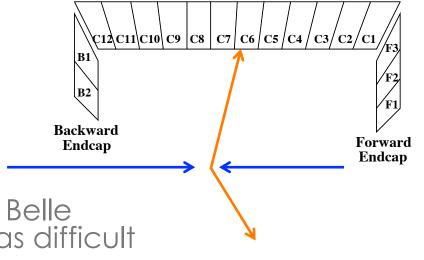


# trigger efficiency for ππγ

high trigger efficiency is necessary for precision measurement

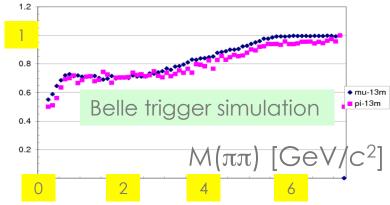
■ Belle II trigger for ee  $\rightarrow \pi \pi \gamma$ 

- total calorimeter energy1 GeV
- Bhabha veto←loss of this vetomust be small
- □ large loss by Bhabha veto in Belle
   → precision measurement was difficult
- Bhabha veto logic in Belle II
  - 2D Bhabha veto: rely only on θ information
  - 3D Bhabha veto: include  $\phi$  information



Barrel

Belle-type Bhabha veto



## trigger efficiency study

- All the Bhabha events were recorded in phase2 data due to low luminosity
  - no loss of events by Bhabha veto
  - can evaluate expected loss directly

loss = # of events triggered by Bhabha trigger

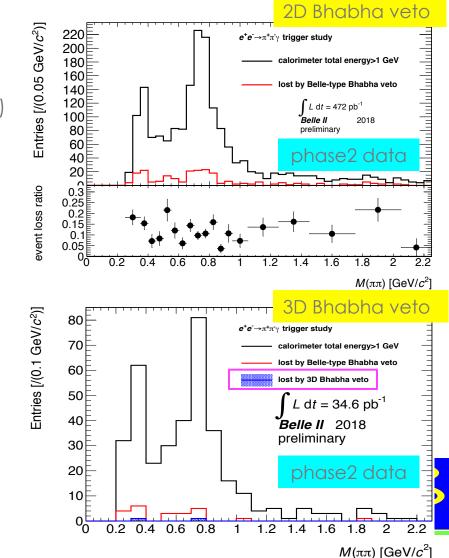
# of all events

standard calorimeter trigger (total E>1 GeV &&!2D-Bhabha)

2D-Bhabha trigger

#### event loss by Bhabha veto

- 2D Bhabha
  - $\square$  (12.3±0.8<sub>stat</sub>)% (M( $\pi\pi$ )<2 GeV/ $c^2$ )
  - significantly large
- □ 3D Bhabha
  - available only for the last short period
  - □ loosen γ angle cut to increase statistics [50°,110°] → [17°,128°]
  - 2 events / 360 events(0.6±0.4<sub>stat</sub>)%
  - much smaller loss
  - →can use the 3D Bhabha veto logic instead of the Belle -type Bhabha veto



#### current situation of e g-2

PRL100, 120801

```
\blacksquare measurement: a_e^{exp} = 1159652180.73(28) \times 10^{-12} \pm 0.24 \,\mathrm{ppb}
   (Harvard U) 8th and 10th order hadronic contribution
a_e(\text{theory}) = 1159652181.78(6)(4)(2)(77) \times 10^{-12} [0.67 ppb]
                                                           PRL109, 111807
     □ QED mass-dependent term:
                                   2.7478(2) \times 10^{-12}
     □ had a_e(\text{had.v.p.}) = 1.866(10)_{\text{exp}}(5)_{\text{rad}} \times 10^{-12}, 1.5 ppb
           a_e(\text{NLOhad.v.p.}) = -0.2234(12)_{\text{exp}}(7)_{\text{rad}} \times 10^{-12}
                  a_e(\text{had.} l\text{-}l) = 0.035(10) \times 10^{-12},
     ■ weak
             a_e(\text{weak}) = 0.0297 (5) \times 10^{-12}
```



### current situation of $\mu$ g-2

 $= 116592089(63) \times 10^{-11} \pm 0.54 \text{ ppm}$ 

■ measureme

(BNL E821)

theory

QED

8<sup>th</sup> and 10<sup>th</sup> order of QED calculation

lepton mass

order	with $\alpha^{-1}(Rb)$	with $\alpha^{-1}(a_e)$
2	116 140 973.318 (77)	116 140 973.213 (30)
4	413 217.6291 (90)	413 217.6284 (89)
6	30 141.902 48 (41)	30 141.902 39 (40)
8	381.008 (19)	381.008 (19)
10	5.0938 (70)	5.0938 (70)
$a_{\mu}(\text{QED}) \times 10^{11}$	116 584 718.951 (80)	116 584 718.846 (37)

PRL109, 111808

$$a_{\mu}^{\text{QED}} = 116\ 584\ 718.951\ (0.009)(0.019)(0.007)(.077) \times 10^{-11}$$

hadron

$$a_{\mu}^{\text{had;LO}} = (6.923 \pm 42) \times 10^{-11}$$
  
 $a_{\mu}^{\text{had;NLO}} = (-98.4 \pm 0.6_{\text{exp}} \pm 0.4_{\text{rad}}) \times 10^{-11}$   
 $a_{\mu}^{\text{HLbL}} = (105 \pm 26) \times 10^{-11}$ 

■weak

$$a_{\mu}^{\text{EW}} = (153.6 \pm 1.0) \times 10^{-11}$$

