Muon g-2: a new data-based analysis

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talk at 2018 WPI-next Mini-workshop
"Hints for New Physics in Heavy Flavors"
@ Nagoya U.

November 16, 2018

Ref: A. Keshavarzi, DN and T. Teubner **(KNT)**
Phys. Rev. D97 (2018) 114025
[arXiv:1802.02995]

Muon g-2: introduction

Lepton magnetic moment $\vec{\mu}$:

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad (\vec{s} = \frac{1}{2}\vec{\sigma} \text{ (spin)}, \qquad g = 2 + 2F_2(0))$$

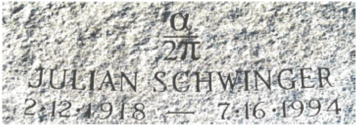
where

$$\overline{u}(p+q)\Gamma^\mu u(p) = \overline{u}(p+q)\left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)



Today, still important, since...

★ One of the **most precisely measured** quantities:

$$\boxed{a_\mu^{\text{exp}} = 11\ 659\ 208.9(6.3) \times 10^{-10}} \quad [0.5\text{ppm}] \qquad \text{(Bennett } et\ al\text{)}$$

★ **Extremely useful** in **probing/constraining physics beyond the SM**

Muon g-2: previous exp. (after 1960)

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]	Sensitivity
CERN I	1961	μ^+	11 450 000(220 000)	4300	2-loop QED contrib. (3600 ppm)
CERN II	1962-1968	μ^+	11 661 600(3100)	270	3-loop QED contrib. (260 ppm)
CERN III	1974-1976	μ^+	11 659 100(110)	10	hadronic vacuum polarization contrib. (60 ppm)
CERN III	1975-1976	μ^-	11 659 360(120)	10	
BNL	1997	μ^+	11 659 251(150)	13	
BNL	1998	μ^+	11 659 191(59)	5	4-loop QED contrib. (3.3 ppm)
BNL	1999	μ^+	11 659 202(15)	1.3	electroweak contrib. (1.3 ppm)
BNL	2000	μ^+	11 659 204(9)	0.73	hadronic light-by-light contrib. (0.86 ppm)
BNL	2001	μ^-	11 659 214(9)	0.72	hadronic NLO vacuum pol. contrib. (-0.85 ppm)
Average			11 659 208.0(6.3)	0.54	

Table from BNL-E821 final report, Phys. Rev. D 73 (2006) 072003

Testing the SM for more than 50 years!
2 more exp. to come: those from Fermilab and J-PARC

Breakdown of SM prediction for muon g-2

	2011		2018	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	**3.7σ**	this work

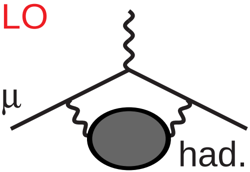
(HVP: Hadronic Vacuum Polarization)

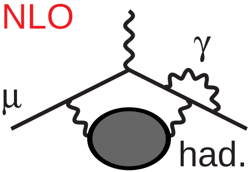
(HLbL: Hadronic Light-by-Light)

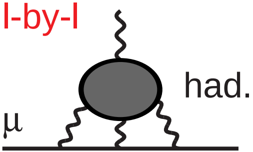
Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

Hadronic Contributions

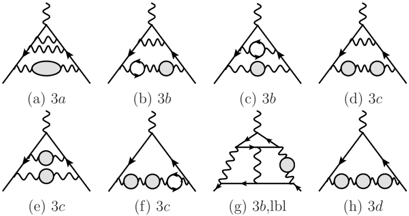
There are several hadronic contributions:



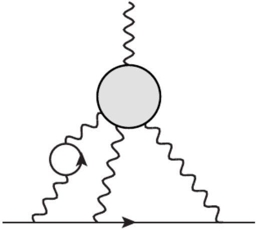




LO: Leading Order (or Vacuum Polarization) Hadronic Contribution
NLO: Next-to-Leading Order Hadronic Contribution
l-by-l: Hadronic light-by-light Contribution

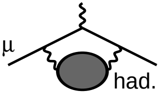


NNLO Hadronic Contributions

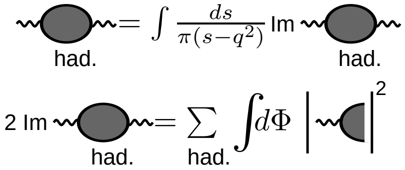


Hadronic l-by-l NLO Contrib.

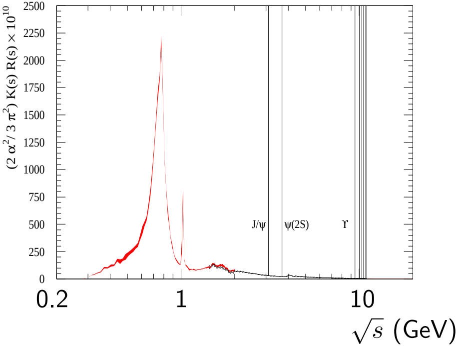
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s} \hat{K}(s) \sigma_{\rm had}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

Main improvements between HLMNT11 and KNT18

- Lots of new input $\sigma(e^+e^- \to \text{hadrons})$ data
- Improvements in the estimates of uncertainties due to radiative corrections (Vacuum Polarization Radiative Corrections & Final State Radiations)
- Improvements in data-combination method

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Channel	Energy range [GeV]	$a_\mu^{\rm had, LO\, VP} \times 10^{10}$	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) \times 10^4$	New data
		Chiral perturbation theory (ChPT) threshold contributions		
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	⋯
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	⋯
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	⋯
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	⋯
		Data based channels ($\sqrt{s} \le 1.937$ GeV)		
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\rm no\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	⋯
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\rm no\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	⋯
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\rm no\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	⋯
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	⋯
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	⋯
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to {\rm npp})K\bar{K}_{{\rm no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
		Estimated contributions ($\sqrt{s} \le 1.937$ GeV)		
$(\pi^+\pi^-3\pi^0)_{\rm no\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	⋯
$(\pi^+\pi^-4\pi^0)_{\rm no\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	⋯
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	⋯
$\omega(\to {\rm npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	⋯
$\omega(\to {\rm npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	⋯
$\omega(\to {\rm npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	⋯
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	⋯
		Other contributions ($\sqrt{s} > 1.937$ GeV)		
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	⋯	6.26 ± 0.19	7.07 ± 0.22	⋯
ψ'	⋯	1.58 ± 0.04	2.51 ± 0.06	⋯
$\Upsilon(1S-4S)$	⋯	0.09 ± 0.00	1.06 ± 0.02	⋯
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	⋯
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	⋯

Table from KNT18, Phys. Rev. D97 (2018) 114025

Breakdown of contributions to a_μ(had, LO VP) from various hadronic final states

We have included new data sets from $\sim$ 30 papers, in addition to those included in the HLMNT11 analysis

We have included $\sim$ 30 hadronic final states

At $2 \lesssim \sqrt{s} \lesssim 11$ GeV, we use inclusively measured data

At higher energies $\gtrsim 11$ GeV, we use pQCD

- Lots of new input $\sigma(e^+e^- \to \text{hadrons})$ data
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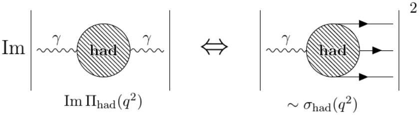
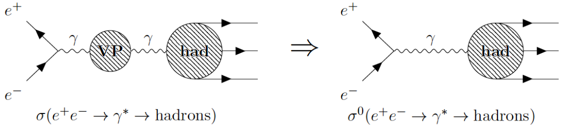
$\sigma^0_{\mathrm{had},\gamma}$: vacuum polarisation corrections

⇒ Reconsider the optical theorem:



⇒ Photon VP corresponds to higher order contributions to $a_\mu^{\mathrm{had,\,VP}}$

→ Must subtract VP:



⇒ Fully updated, self-consistent VP routine: [vp_knt_v3_0], available for distribution

→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\mathrm{had}}(s) = \sigma_{\mathrm{had}}(s)|1-\Pi(s)|^2$

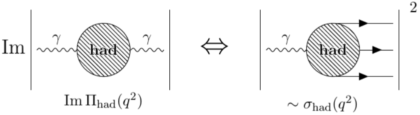
⇒ If correcting data, apply corresponding radiative correction uncertainty

→ Take $\frac{1}{3}$ of total correction per channel as conservative extra uncertainty

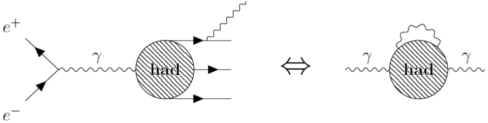
Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

$\sigma^0_{\text{had},\gamma}$: final state radiation corrections

⇒ Reconsider the optical theorem:



⇒ Photon FSR formally higher order corrections to $a_\mu^{\text{had, VP}}$



⇒ Cannot be unambiguously separated, not accounted for in HO contributions

→ Must be included as part of 1PI hadronic blobs

⇒ Experiment may cut/miss photon FSR → Must be added back

⇒ For $\pi^+\pi^-$, sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

⇒ For higher multiplicity states, difficult to estimate correction

∴ Apply conservative uncertainty

Need new, more developed tools to increase precision here

(e.g. - CARLOMAT 3.1 [Eur.Phys.J. C77 (2017) no.4, 254]?)

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

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Data Combination

To evaluate the vacuum polarization contribution, we have to combine lots of experimental data.

To do so, we usually construct a χ^2 function and find the value of $R(s)$ at each bin which minimizes χ^2.

Naively, the χ^2 function defined as

$$\chi^2(\{\overline{R}_i\}) \equiv \sum_{n=1}^{N_{\text{exp}}} \sum_{i=1}^{N_{\text{bin}}} \sum_{j=1}^{N_{\text{bin}}} (R_i^{(n)} - \overline{R}_i)(V_n^{-1})_{ij}(R_j^{(n)} - \overline{R}_j) \ ,$$

where V_n is the cov. matrix of the n-th exp.,

$$V_{n,ij} = \begin{cases} (\delta R_{i,\text{stat}}^{(n)})^2 + (\delta R_{i,\text{sys}}^{(n)})^2 & (\text{for } i = j) \\ (\delta R_{i,\text{sys}}^{(n)})(\delta R_{j,\text{sys}}^{(n)}) & (\text{for } i \neq j) \end{cases}$$

may seem OK, but when there are non-negligible normalization uncertainties in the data, we have to be more careful.

χ^2 vs normalization error: d'Agostini bias

G. D'Agostini, Nucl. Instrum. Meth. A346 (1994) 306

We first consider an observable x whose true value is 1. Suppose that there is an experiment which measures x and whose normalization uncertainty is 10%.
Now, assume that this experiment measured x twice:

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ ,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ .$$

Taking the systematic errors 0.09 and 0.11, respectively, the covariance matrix and the χ^2 function are

$$(\text{cov.}) = \begin{pmatrix} 0.1^2 + 0.09^2 & 0.09 \cdot 0.11 \\ 0.09 \cdot 0.11 & 0.1^2 + 0.11^2 \end{pmatrix} \ ,$$
$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} (\text{cov.})^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} \ .$$

χ^2 takes its minimum at $x = 0.98$: Biased downwards!

d'Agostini bias (2): improvement by iterations

What was wrong? In the previous page,

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \,,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \,.$$

we took the syst. errors 0.09 and 0.11, respectively, which made the downward bias. Instead, we should take 10% of some estimator $\bar{x}$ as the syst. errors. Then,

$$\text{(cov.)} = \begin{pmatrix} 0.1^2 + (0.1\bar{x})^2 & (0.1\bar{x})^2 \\ (0.1\bar{x})^2 & 0.1^2 + (0.1\bar{x})^2 \end{pmatrix} \,,$$

$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} \text{(cov.)}^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} \,.$$

χ^2 takes its minimum at $x = 1.00$: Unbiased!

In more general cases, we use iterations: we find an estimator for the next round of iteration by χ^2-minimization.

R.D.Ball et al, JHEP 1005 (2010) 075.

$\sigma(e^+e^- \to \pi^+\pi^-)$ data

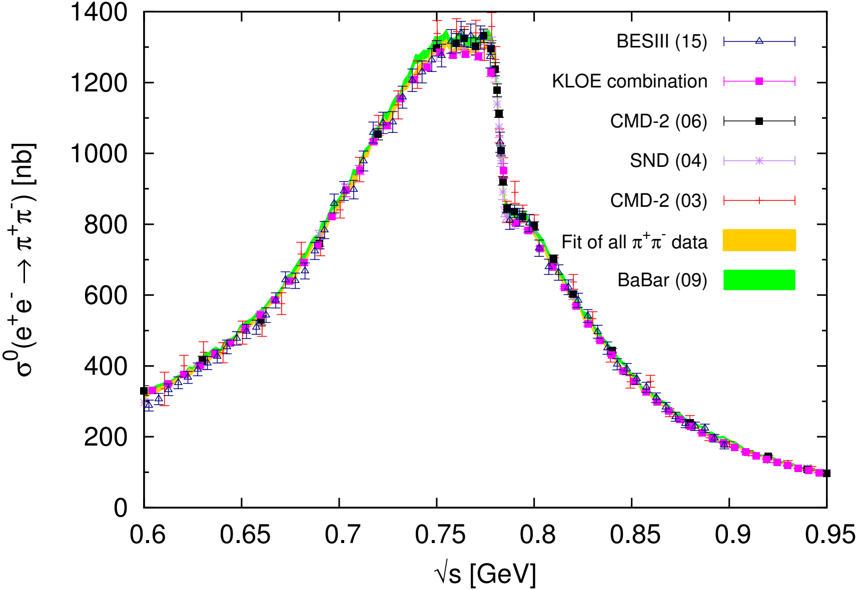


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

$\sigma(e^+e^- \to \pi^+\pi^-)$: ρ-ω interference region

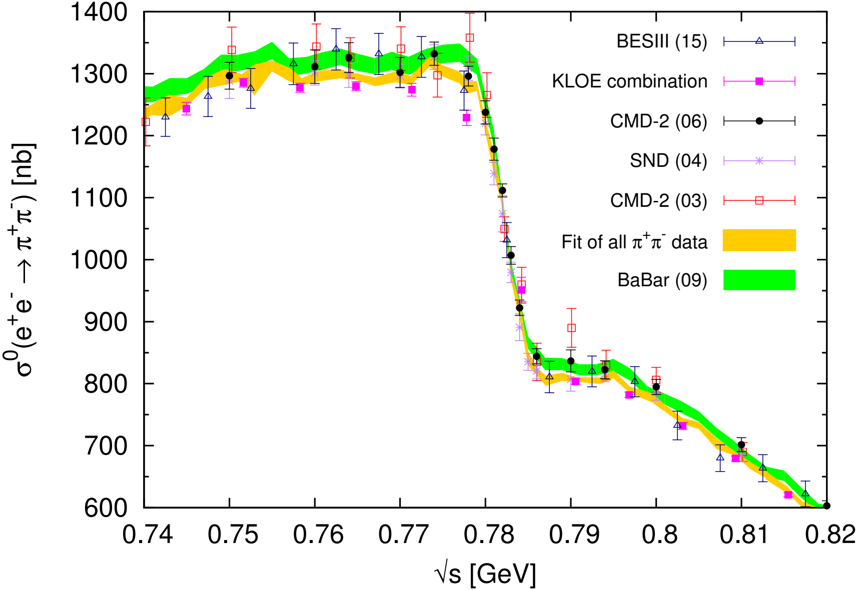


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

$\sigma(e^+e^- \to \pi^+\pi^-)$: relative differences

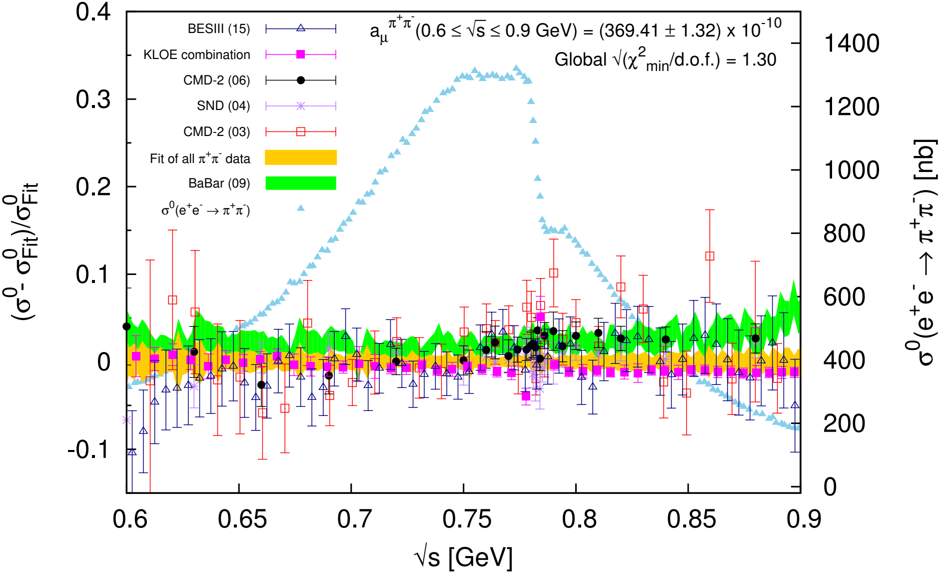


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

Contribution to $(g-2)_\mu$ from $\pi^+\pi^-$ channel

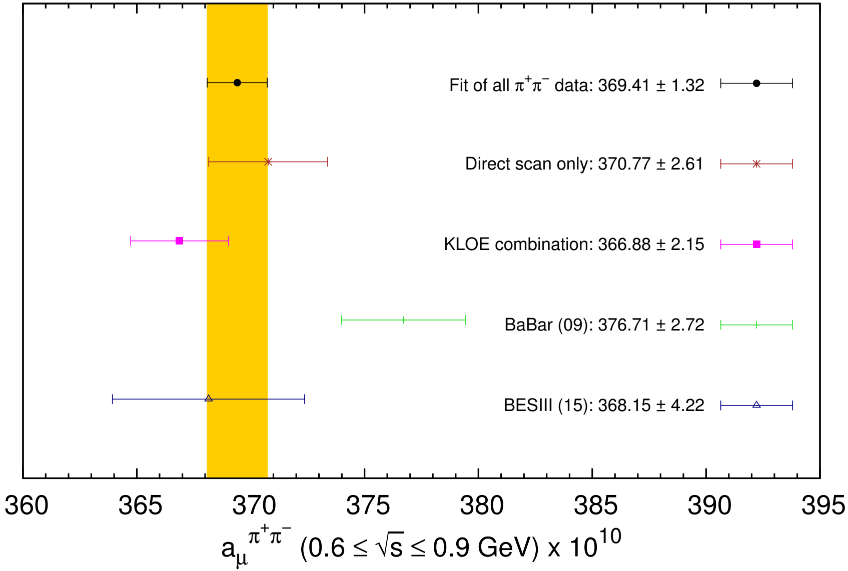
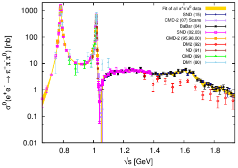


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

Other notable exclusive channels [KNT18: arXiv:1802.02995, PRD (in press)]

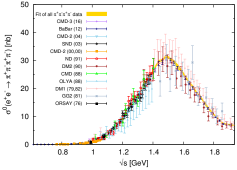
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT18: 47.92 ± 0.89

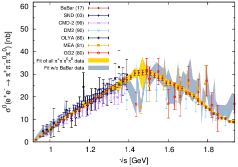
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT18: 14.87 ± 0.20

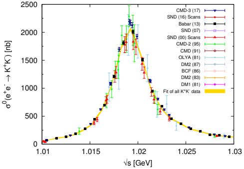
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT18: 19.39 ± 0.78

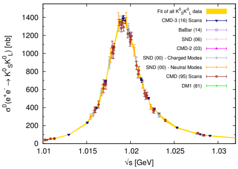
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT18: 23.03 ± 0.22

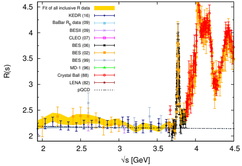
$K^0_S K^0_L$



HLMNT11: 13.33 ± 0.16

KNT18: 13.04 ± 0.19

Inclusive ($\sqrt{s} > 2$ GeV)



HLMNT11: 41.40 ± 0.87

KNT18: 41.27 ± 0.62

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

KNT18 $a_\mu^{\text{had, VP}}$ update

HLMNT(11): 694.91 ± 4.27

$\downarrow$

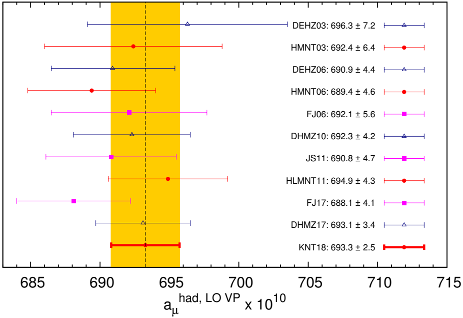
This work: $a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}}$

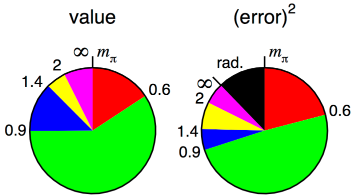
$= 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$

$= 693.27 \pm 2.46_{\text{tot}}$

$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$

$\Rightarrow$ Accuracy better then 0.4%
(uncertainties include all available correlations)





$\Rightarrow$ 2π dominance

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Breakdown of SM prediction for muon g-2

	2011		2018	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	**3.7σ**	this work

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)
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Exp. value of muon g-2 vs SM prediction

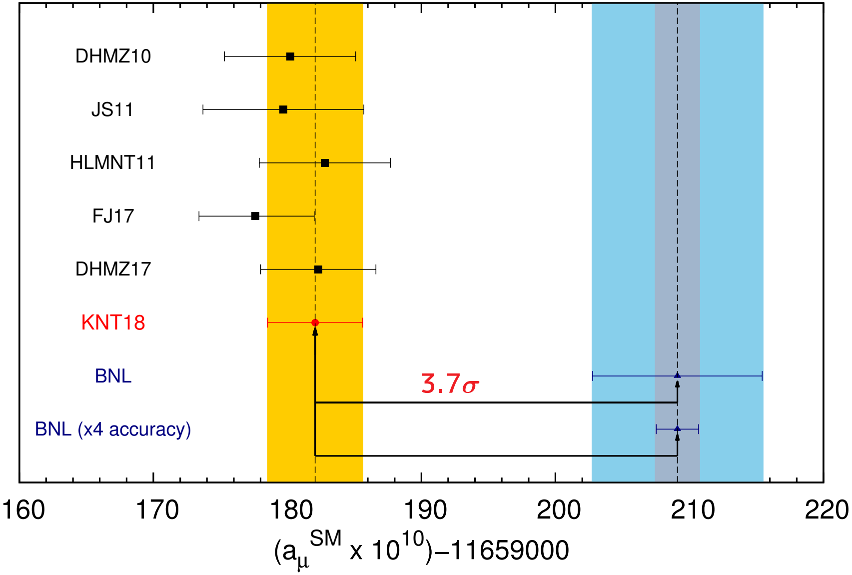


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

Comparison with HLMNT11

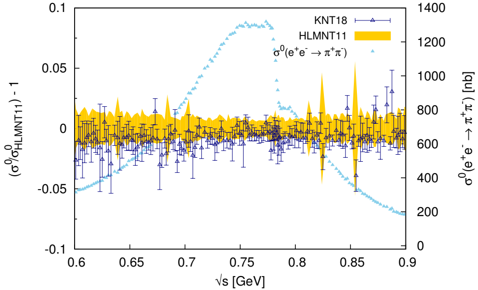
Channel	This work (KNT18)	HLMNT11	Difference
$\pi^+\pi^-$	502.99 ± 1.97	505.77 ± 3.09	-2.78 ± 3.66
$\pi^+\pi^-\pi^0$	47.82 ± 0.89	47.51 ± 0.99	0.31 ± 1.33
$\pi^+\pi^-\pi^+\pi^-$	15.17 ± 0.21	14.65 ± 0.47	0.52 ± 0.51
$\pi^+\pi^-\pi^0\pi^0$	19.80 ± 0.79	20.37 ± 1.26	-0.57 ± 1.49
K^+K^-	23.05 ± 0.22	22.15 ± 0.46	0.90 ± 0.51
$K^0_S K^0_L$	13.05 ± 0.19	13.33 ± 0.16	-0.28 ± 0.25
Inclusive channel	41.27 ± 0.62	41.40 ± 0.87	-0.13 ± 1.07
Total	693.27 ± 2.46	694.91 ± 4.27	-1.64 ± 4.93

⇒ Biggest difference in 2π channel
→ large reduction in mean and uncertainty

⇒ Tensions with HLMNT11 analysis for both two-kaon channels

⇒ Overall agreement with HLMNT11

⇒ Notable improvement of about one third in uncertainty



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Comparison with other similar works

Channel	This work (KNT18)	DHMZ17	Difference
$\pi^+\pi^-$	503.74 ± 1.96	507.14 ± 2.58	-3.40 ± 3.24
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50 ± 1.70
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31 ± 0.36
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12 ± 0.92
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19 ± 0.47
$K_S^0K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22 ± 0.31
$1.8 \leq \sqrt{s} \leq 3.7$ GeV	34.54 ± 0.56 (data)	33.45 ± 0.65 (pQCD)	1.09 ± 0.86
Total	693.3 ± 2.5	693.1 ± 3.4	0.2 ± 4.2

⇒ Total estimates from two analyses in very good agreement

⇒ Masks much larger differences in the estimates from individual channels

⇒ Unexpected tension for 2π considering the data input likely to be similar

→ Points to marked differences in way data are combined

→ From 2π discussion: $a_\mu^{\pi^+\pi^-}$ (Weighted average) $= 509.1 \pm 2.9$

⇒ Compensated by lower estimates in other channels

→ For example, the choice to use pQCD instead of data above 1.8 GeV

⇒ FJ17: $a_{\mu,\,\mathrm{FJ17}}^{\mathrm{had,\,LO\,VP}} = 688.07 \pm 41.4$

→ Much lower mean value, but in agreement within errors

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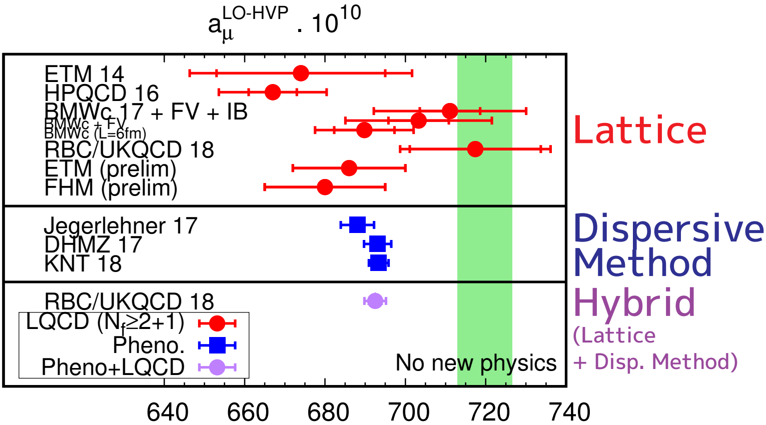
Comparison with Lattice Results



- Lattice errors $\sim$ 2% vs phenomenology errors $\sim$ 0.4%
- Some lattice results suggest new physics others not but all compatible with phenomenology

Slide by L. Lellouch (Marseille) at 'Muon g-2 Workshop' at Mainz, June 18-22, 2018

Summary

- Standard Model prediction for $(g-2)_\mu$: $\gtrsim 3.5\sigma$ deviation from measured value $\Longrightarrow$ New Physics?
- Recent data-driven evaluations of hadronic vacuum polarization contributions seem convergent (Similar mean values from KNT18 and Davier et al with slightly smaller uncertainty from KNT18.)
- To better establish the $g-2$ anomaly, better data for $e^+e^- \to \pi^+\pi^-$ welcome (from Belle II, CMD-3, SND, . . .)
- Lattice calculations still suffer from large uncertainties (but a hybrid approach gives a slight improvement)
- New exp. at Fermilab and J-PARC expected to reduce the uncertainty of $(g-2)_\mu$ by a factor of 4