KMI 2019, Feb. 18-20, 2019 @ Nagoya

Recent developments in AdS/CFT
-Emergent Space from Quantum Information-

Tadashi Takayanagi

Yukawa Institute for Theoretical Physics
Kyoto University

It from Qubit
Simons Collaboration

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

① Introduction

Microscopes are often crucial for experiments in science.

In particle physics, accelerators are used as microscopes.

What are microscopes in gravitational physics ?

⇒ **Holography provides microscopes !**
(AdS/CFT, gauge/gravity duality)

Holography is not a real experiment but provides a crucial device in thought experiments.

Indeed, holography magnifies a gravitational spacetime into tiny bits of quantum information as we will review.

Bekentein-Hawking Formula of BH Entropy

$$\boxed{S_{BH} = \frac{c^3}{\hbar} \times \frac{A}{4G_N}}$$

➡ BH thermodynamics !

$$\left.\begin{array}{l}\textbf{A}= \text{Surface Area of Black hole } \Rightarrow \textbf{Geometry} \\ \textbf{G}_\textbf{N}=\text{Newton constant } \Rightarrow \textbf{Gravity} \\ \hbar = \text{Planck constant } \Rightarrow \textbf{Quantum Mechanics}\end{array}\right\} \textbf{Quantum Gravity!}$$

BH Entropy is proportional to the area, not to the volume !

➡ Degrees of Freedom in Gravity $\propto$ Surface Area !

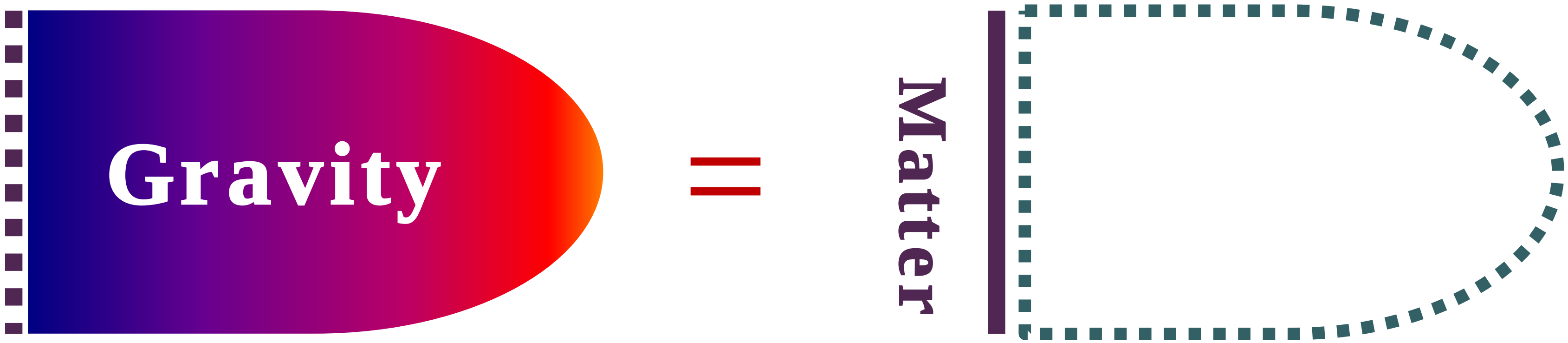
The idea of holographic principle !

['t Hooft 1993, Susskind 1994]

Bdy of M

Holography

Gravity on M = Quantum Matter on ∂M



BH entropy($\propto$Area)= Thermal Entropy of Matter ($\propto$Volume)

[Strominger-Vafa 1996]

However, we still do not know the origin of BH entropy from the viewpoint of gravity even now !

The best example of holography in string theory:

<u>AdS/CFT Correspondence</u> [Maldacena 1997]

AdS/CFT

Gravity (String theory) on D+1 dim. AdS (anti de-Sitter space) = **Conformal Field Theory (CFT) on D dim. Minkowski spacetime**

⬇ Classical limit

General relativity with $\Lambda < 0$

⬇ Large N + Strong coupling

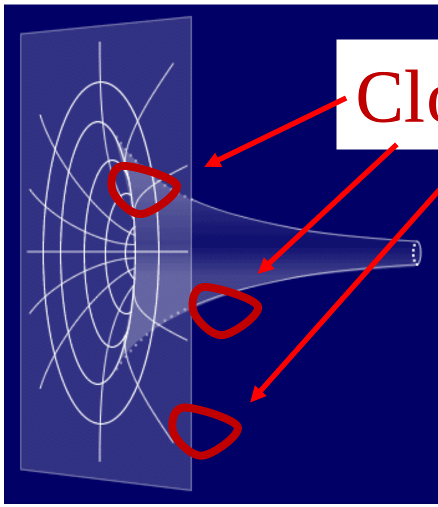
Strongly interacting Quantum Field Theories

Basic Principle
(Bulk-Boundary relation) :

$$\boxed{Z_{Gravity} = Z_{CFT}}$$

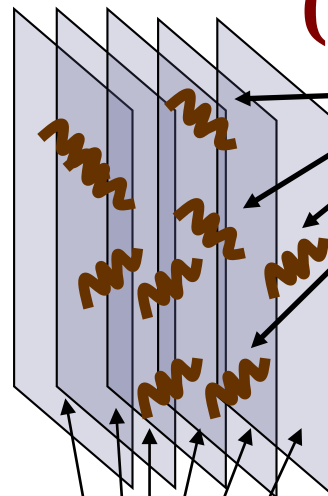
Gravity in Anti de-Sitter space

Conformal Field Theories (CFTs)



Closed Strings

Equivalent AdS/CFT



Open Strings

SU(N) gauge theories

N D-branes

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Equivalent

Thermodynamics of Black holes (branes)

Thermodynamics of various materials

Caution

Strongly Coupled SU(N) gauge theories in the large N limit ⟺ **Classical Gravity**

Closely related examples:

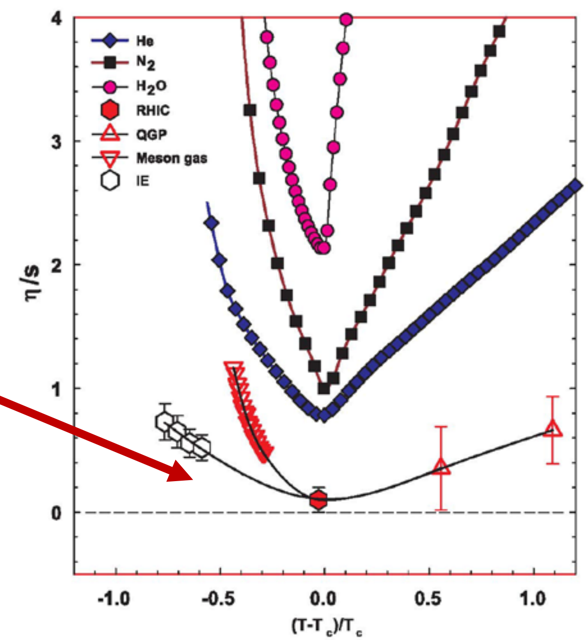
Quark-Gluon plasma

Cold atoms, High Tc SC, etc.

AdS/CFT result $\boxed{\dfrac{\eta}{s} = \dfrac{1}{4\pi}}$

[Kovtun-Son-Starinets 2004]

Viscosity/Entropy Ratio



[Lacey et.al. 2006]

This strongly coupled property also leads to holographic studies of hadron physics. [Sakai-Sugimoto 2004]

Recent Applications to strongly coupled systems

(1) Quantum Chaos (Lyapunov exponent: λ)

$$\langle [W(t), V(0)]^2 \rangle \approx \frac{1}{N^2} e^{\lambda t}$$

$$\lambda_{AdS} = \frac{2\pi k_B T}{\hbar}$$ In general, we have $\lambda \leq \frac{2\pi k_B T}{\hbar}$.

[Maldacena-Shenker-Stanford 2015]

(2) Specific Heat

2D strongly interacting metal (∃Fermi Surface)

$C_{AdS} \propto T^{\alpha}$ with $\alpha \leq \frac{2}{3}$ (Non Fermi Liquids)

[Ogawa-Ugajin-Takayanagi 2011]

cf. α=1 for Landau Fermi liquids

Contents

② Quantum Entanglement and Holography

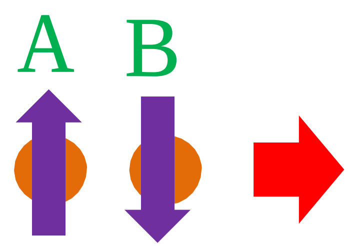
(2-1) Quantum Entanglement (QE)

QE = quantum correlations between two subsystems

Simple example: 2 Qubits system

(1) Direct Product State

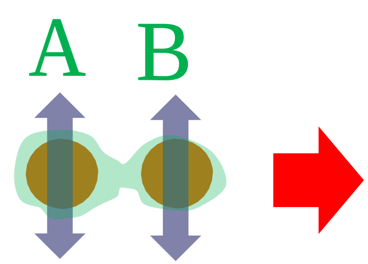
$$\left|\Psi_c\right\rangle = \left|\uparrow\right\rangle_A \otimes \left|\downarrow\right\rangle_B$$



Not Entangled

(2) EPR (Bell) States

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_A \otimes \left|\downarrow\right\rangle_B \pm \left|\downarrow\right\rangle_A \otimes \left|\uparrow\right\rangle_B\right)$$



Entangled

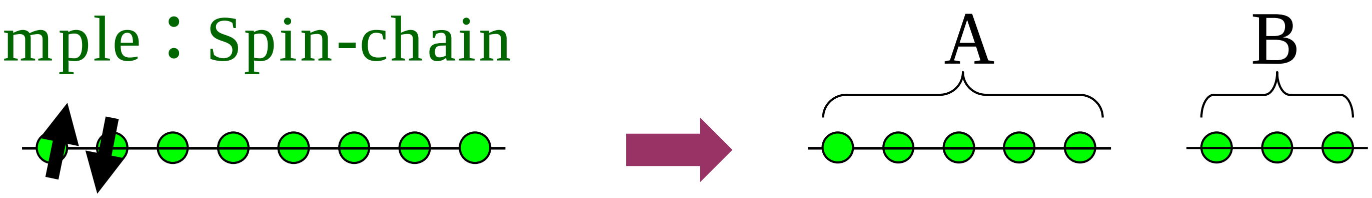
EPR Pair

Entanglement Entropy (EE)

$\boxed{\text{Amount of QE} = \#\ \text{of EPR Pairs}}$ ➡ **Entanglement Entropy (EE)**

First we decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$.

Example：Spin-chain



We introduce the reduced density matrix ρ_A

by tracing out B $\rho_A = \mathrm{Tr}_B\left[|\Psi_{tot}\rangle\langle\Psi_{tot}| \right].$

The entanglement entropy (EE) S_A is defined by

$$\boxed{S_A = -\mathrm{Tr}[\rho_A \log \rho_A]}$$

(2-2) Holographic Entanglement Entropy (HEE)

[Ryu-Takayanagi 2006, Hubeny-Rangamani-Takayanagi 2007]

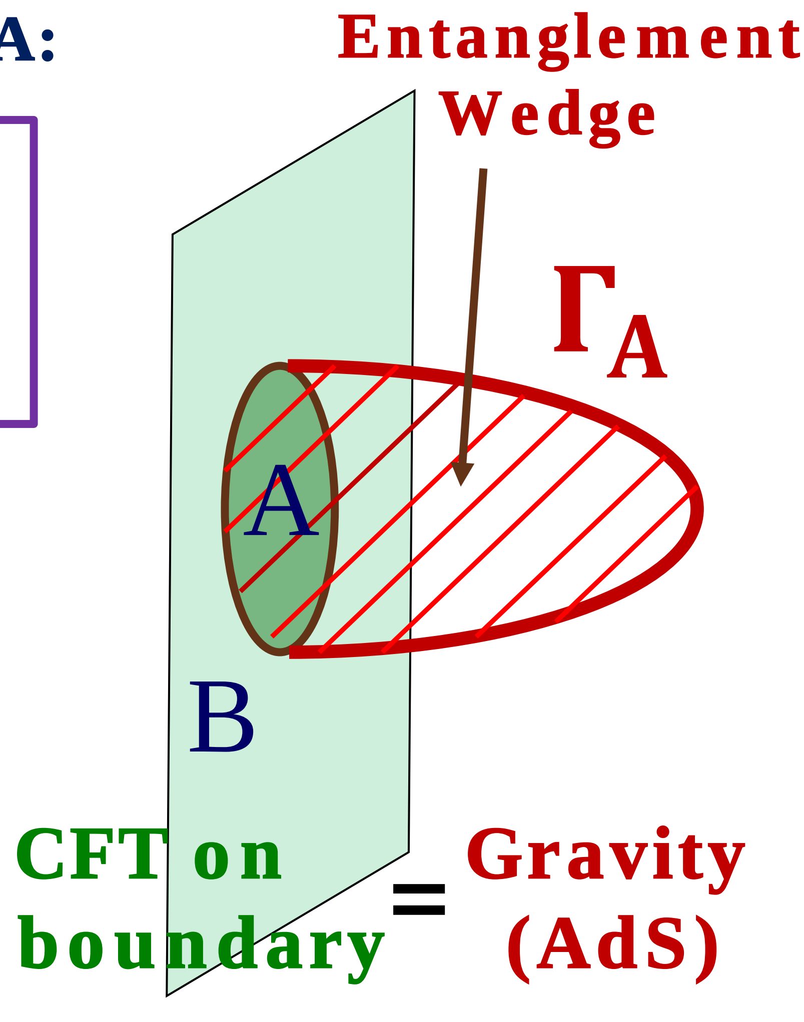
EE in CFT: SA can be computed from the minimal area surface ΓA:

$$S_A = \min_{\Gamma_A}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N}\right]$$

Note: The bdy of ΓA =The bdy of A.

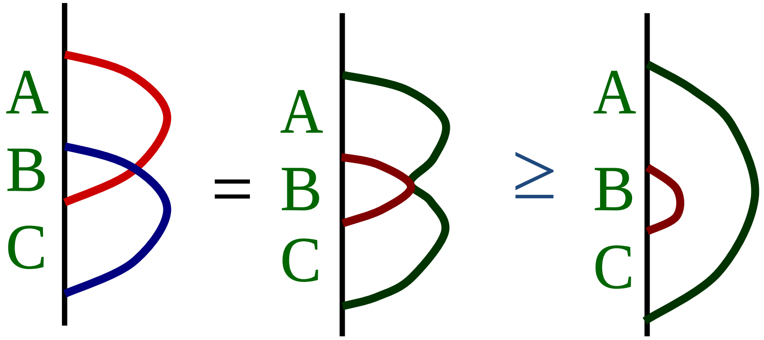
Many evidences for this conjecture have been found for these 10 years.

This formula was proved by Lewkowycz-Maldacena 2013 based on the bulk-bdy relation of AdS/CFT.

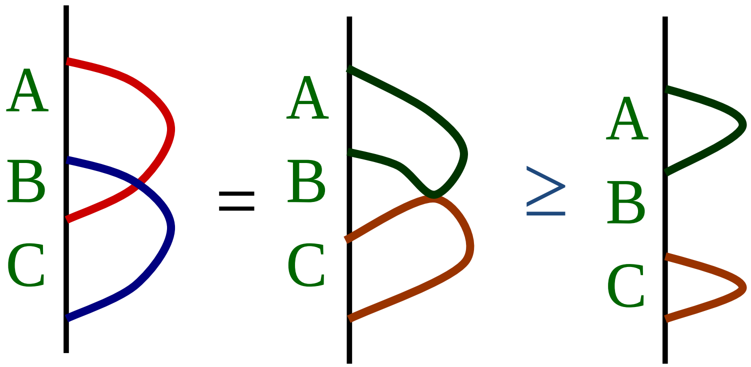


Holographic Proof of Strong Subadditivity

[Headrick-TT 07]



$$\Rightarrow \quad S_{AB} + S_{BC} \geq S_{ABC} + S_{B}$$



$$\Rightarrow \quad S_{AB} + S_{BC} \geq S_{A} + S_{C}$$

$$(\text{Note:} \quad AB \equiv A \cup B)$$

Algebraic relations in Quantum Info. Theory
$\Leftrightarrow$ Geometric properties in Gravity

(2-3) Einstein Equation from QE

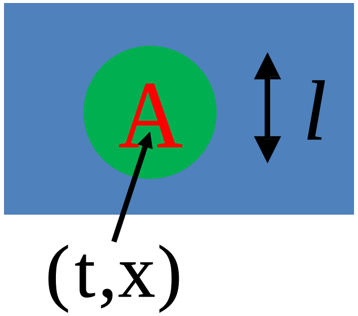
<u>First Law of EE</u>

[Casini-Huerta-Myers 2013,
Bhattachrrya-Nozaki-Ugajin-Takayanagi 2013]

$$\boxed{\Delta S_A \simeq \Delta H_A}$$

[H_A=-logρ_A: Modular Hamiltonian]

$$\boxed{\left(\partial_l^2 - \partial_l - \partial_x^2 - \frac{3}{l^2}\right)\Delta S_A(t,\vec{x},l) = \langle O\rangle\langle O\rangle}$$



$$\phi \leftrightarrow O$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

Kinetic term **C.C.** **Matter contributions**

➡ The 1st law of EE explains the perturbative Einstein eq.

[Raamsdonk et.al. 2013, Faulkner et.al 2013, 2017, Sarosi-Ugajin. 2017]

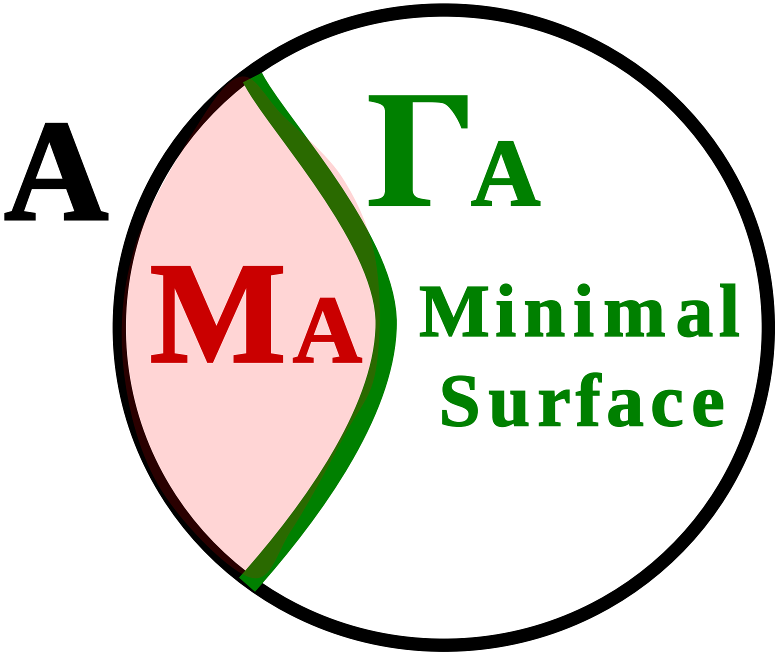
Next leading order analysis

(2-4) Entanglement Wedges and QEC

Which bulk region is dual to a given region A in CFT ?

⇒ Entanglement Wedge M_A

M_A = A region surrounded by A and Γ_A

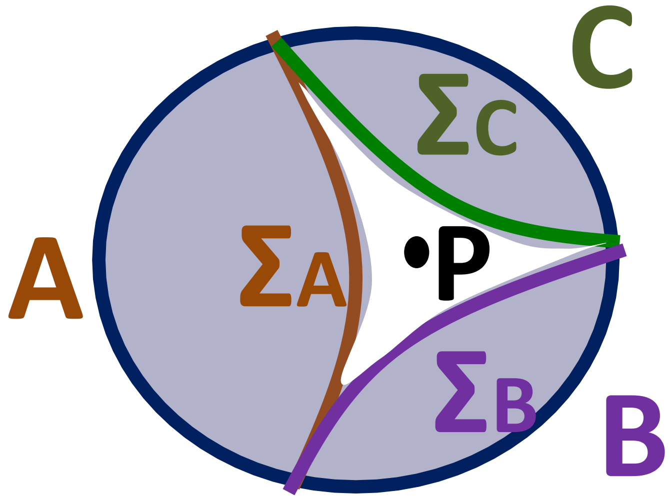


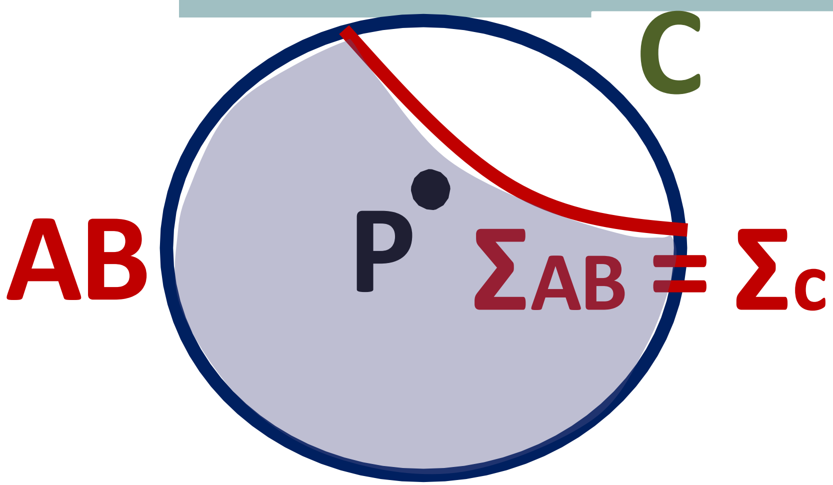
Time slice of AdS

ρ_A in CFT (Low energy info)

$\Leftrightarrow$ ρ_{MA} in AdS gravity

[Hamilton-Kabat-Lifschytz-Lowe 2006, Czech-Karczmarek-Nogueira-Raamsdonk 2012, Wall 2012, Headrick-Hubeny-Lawrence-Rangamani 2014, Jafferis-Lewkowycz-Maldacena-Suh 2015, Dong-Harlow-Wall 2016, . . .]





We can reconstruct the bulk information at P from ρ_{AB}.

But we cannot do so from ρ_A, ρ_B, ρ_C .

Property of Quantum Error Correcting Codes

[Almheiri-Dong-Harlow 2014]

Physical Space = all CFT states = quantum gravity

$\cup$

Code Subspace = BPS states in CFT = supergravity

Protected by QEC

(2-5) Entanglement of Purification

Entanglement of Purification (EoP)
$\Rightarrow$ A mixed state extension of <u>EE</u>

Good only for pure states

<u>Purification</u>

For a given density matrix: $\rho_C = \sum_i \lambda_i |i\rangle_C \langle i|$,

we can always describe this as a pure state $|\Psi\rangle_{CD} = \sum_i \sqrt{\lambda_i}\,|i\rangle_C |i\rangle_D$

by extending the Hilbert space: $H_C \rightarrow H_C \otimes H_D$

such that $\rho_C = \mathrm{Tr}_D\left[|\Psi\rangle\langle\Psi|\right]$.

Note: Purified states depend on the basis $|i\rangle_D$ we chose.

EoP

Consider all purifications $|\Psi\rangle_{A\tilde{A}B\tilde{B}}$ of ρ_{AB} in the extended Hilbert space: $H_A \otimes H_B \to H_A \otimes H_B \otimes H_{\tilde{A}} \otimes H_{\tilde{B}}$.

Entanglement of Purification (EoP) is defined by

$$\boxed{\mathrm{E_P}(\rho_{AB}) = \underset{\text{All purifications } |\Psi\rangle \text{of } \rho_{\mathrm{AB}}}{\mathrm{Min}} S_{A\tilde{A}}\left(|\Psi\rangle_{A\tilde{A}B\tilde{B}}\right)}$$

$$\rho_{AB} = \mathrm{Tr}_{\tilde{A}\tilde{B}}\left[|\Psi\rangle\langle\Psi|\right].$$

Entanglement Entrop

Note: $\mathrm{E_p}(\rho_{AB}) \geq 0$ and $\mathrm{E_p}(\rho_{AB}) = 0 \Leftrightarrow \rho_{AB} = \rho_A \otimes \rho_B$.

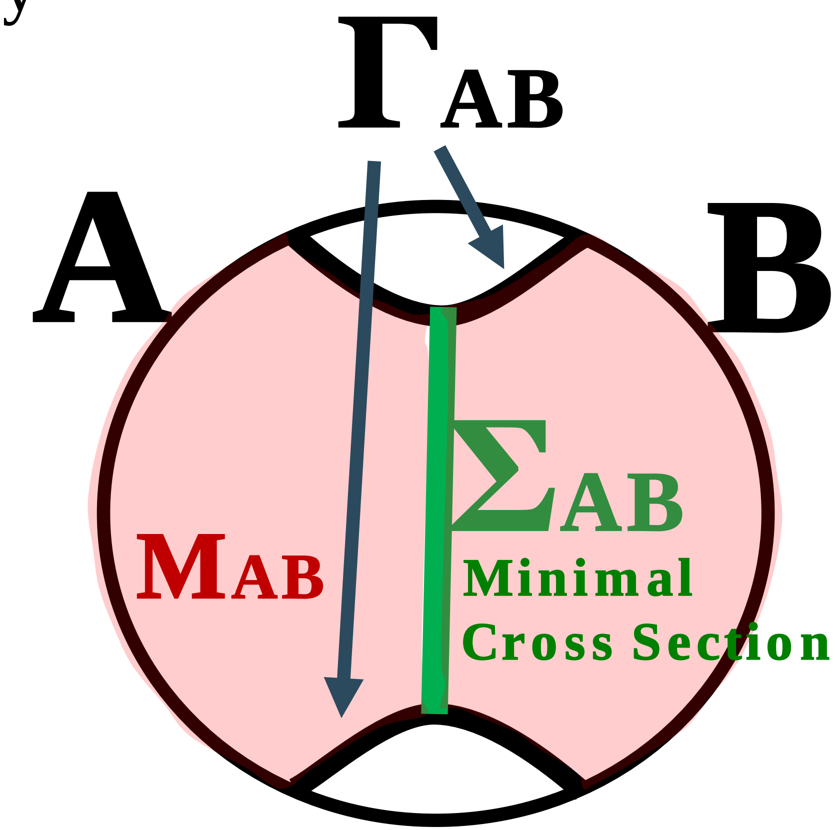
Holographic Entanglement of Purification

Define the **EW cross section** by

$$\boxed{E_W(\rho_{AB}) = \frac{\mathrm{Area}(\Sigma_{AB})}{4G_N}}$$

We propose that this quantity is a holographic dual of EoP:

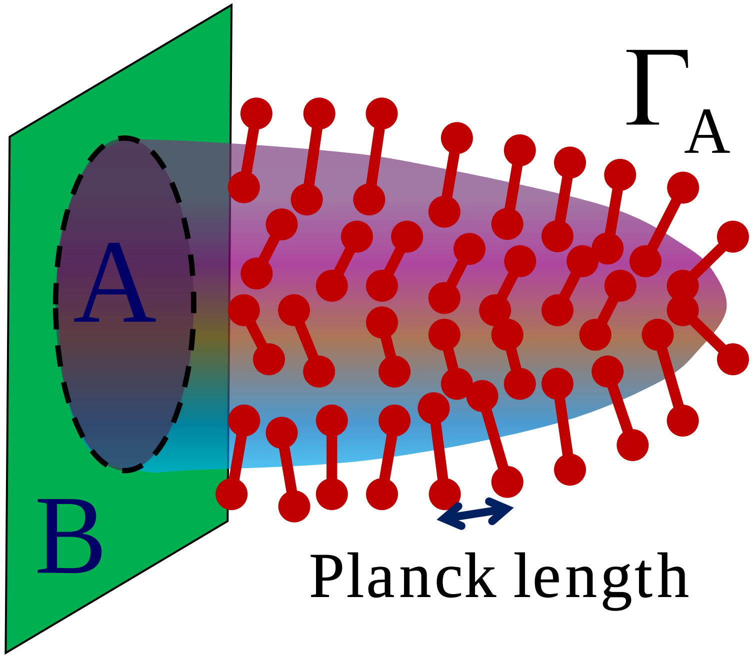
$$\boxed{E_W(\rho_{AB}) = E_P(\rho_{AB})}$$



[Umemoto-TT 2017, Nguyen-Devakul-Halbasch-Zaletel-Swingle 2017,
Explicit Checks from CFT calculations: Caputa-Miyaji-Umemoto-TT 2018]

③ Emergent Space from Quantum Information

The HEE suggests that there is one qubit of entanglement for each Planck length area !



$$S_A = \frac{\mathbf{Area}(\Gamma_A)}{4 l_{pl}^{D-1}}$$

~10^{65} qubits per $1\mathrm{cm}^2$!

We can change the position and size of A as we like.

➡ Gravitational Space may consist of EPR pairs !

(3-1) Tensor Network (TN) and AdS/CFT

Tensor network = Graphical description of quantum states

$\Rightarrow$ Network of quantum entanglement

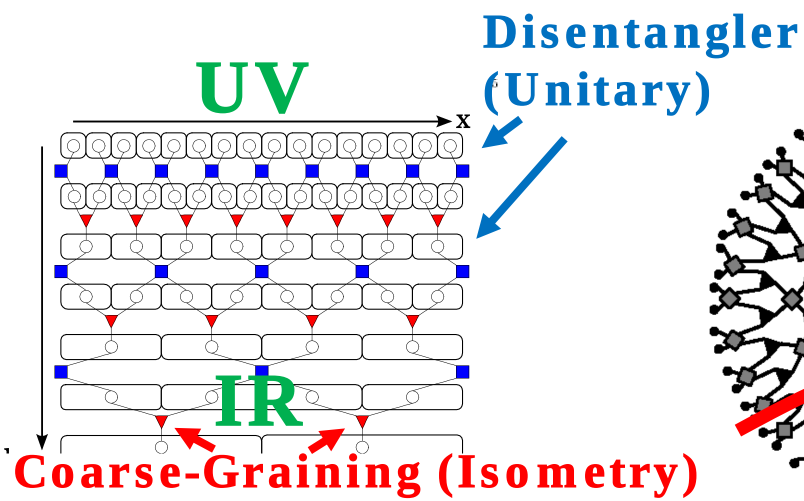
MERA (Multi-scale Entanglement Renormalization Ansatz) [Vidal 2005]

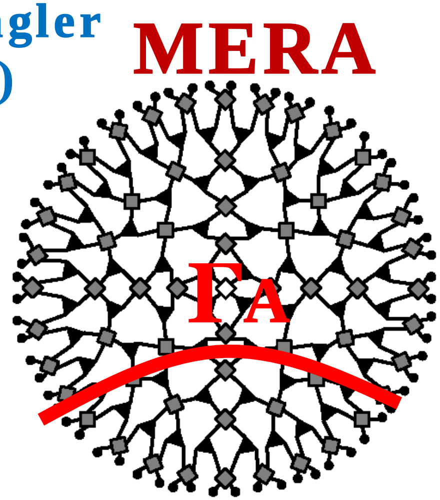
$\Rightarrow$ a TN suitable for CFTs.

Conjecture [Swingle 2009,...]: **a slice of AdS**

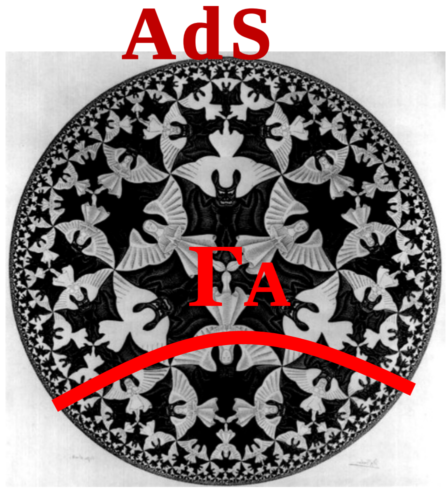
= a TN (MERA,..etc.)

MERA TN



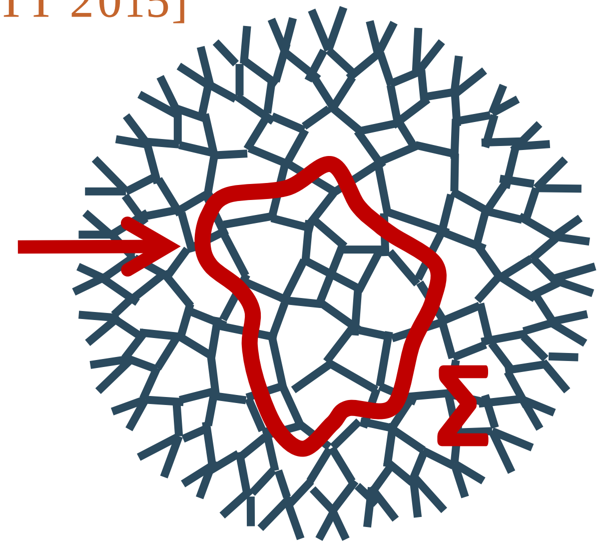


$\approx$



(3-2) Surface/State duality [Miyaji-TT 2015]

A basic relation in AdS/TN conjecture is the surface/state duality: $|\Psi(\Sigma)\rangle$



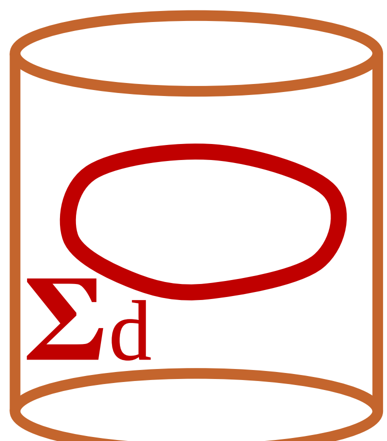
For Einstein gravity on a d+2 dim. AdS, the surface/state duality argues:

Σ : a d dim. convex space-like surface in AdSd+2. which is closed and homologically trivial

Dual

$$\boxed{|\Psi(\Sigma)\rangle \in H_{CFT}}$$

A pure state



Gravity

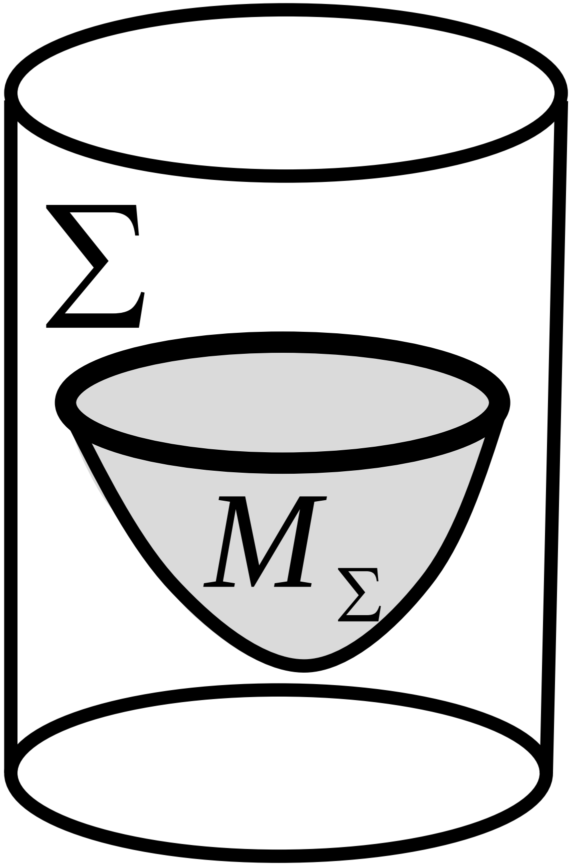
AdSd+2

(3-3) Slice = Quantum Circuit Conjecture [TT 2018]

A (d+1) dim. slice M_Σ in AdS_{d+2}

Dual

A quantum circuit which creates the state $|\Psi(\Sigma)\rangle$ by path-integrals



$$\Psi(\Sigma)[\varphi_0] = \int D\varphi\, e^{-S_{CFT}[\varphi]}\delta[\varphi|_\Sigma - \varphi_0]$$

M_Σ

A CFT action on the curved space M_Σ
with an appropriate coarse-graining s.t. z= lattice spacing

(3-4) Path-integral Optimization and Complexity

[Caputa-Kundu-Miyaji-Watanabe-TT 17]

Consider two dimensional CFTs. We write the metric:

$$ds^2 = e^{2\phi(x,z)}(dx^2 + dz^2) \quad \text{with} \quad e^{2\phi}\big|_{z=\varepsilon} = \varepsilon^{-2}.$$

space time

The rule of UV cut off: a lattice site for a unit area.

Path-integral Optimization for |ψ>

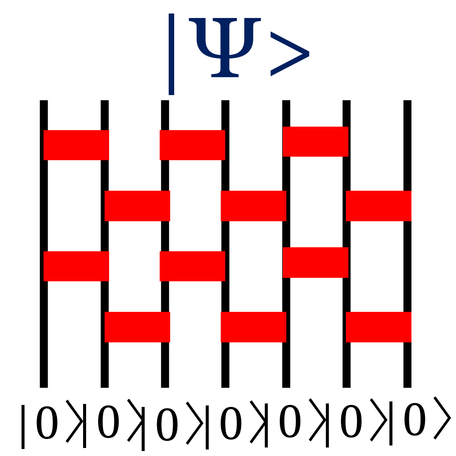
= a special Weyl transformation which

(i) preserves the quantum state |ψ> at the time z=ε,

(ii) minimizes the path-integral "**complexity**"

Comment on Complexity

Computational Complexity of a quantum state $|\Psi\rangle$
= Min [# of Quantum Gates]

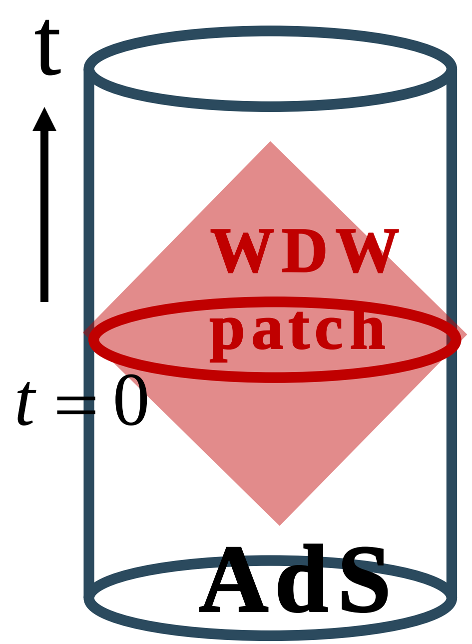


A Holographic Complexity Proposal

Holographic Complexity
= Gravity Action in Wheeler-DeWitt (WDW) patch of AdS

[Brown-Roberts-Susskind-Swingle-Zhao 15]

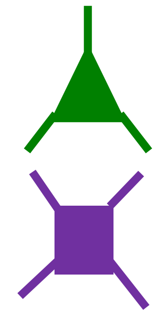
➡ **So far no clear derivation of this formula.**



Path-Integral Complexity I[φ]

$$I[\phi] = \mathrm{Log}\left[\frac{\Psi_{g=e^{2\phi}\delta_{ab}}}{\Psi_{g=\delta_{ab}}}\right] = S_L[\phi],$$

# of Isometries



[Czech 17]

Liouville Action SL[φ]

# of Unitaries

$$S_L[\phi] = \frac{c}{24\pi}\int dxdz\left[\overbrace{(\partial_x\phi)^2 + (\partial_z\phi)^2} + e^{2\phi}\right]$$

$$= \frac{c}{24\pi}\int dxdz\left[(\partial_x\phi)^2 + \left(\partial_z\phi + e^{\phi}\right)^2\right] + (\text{surface term})$$

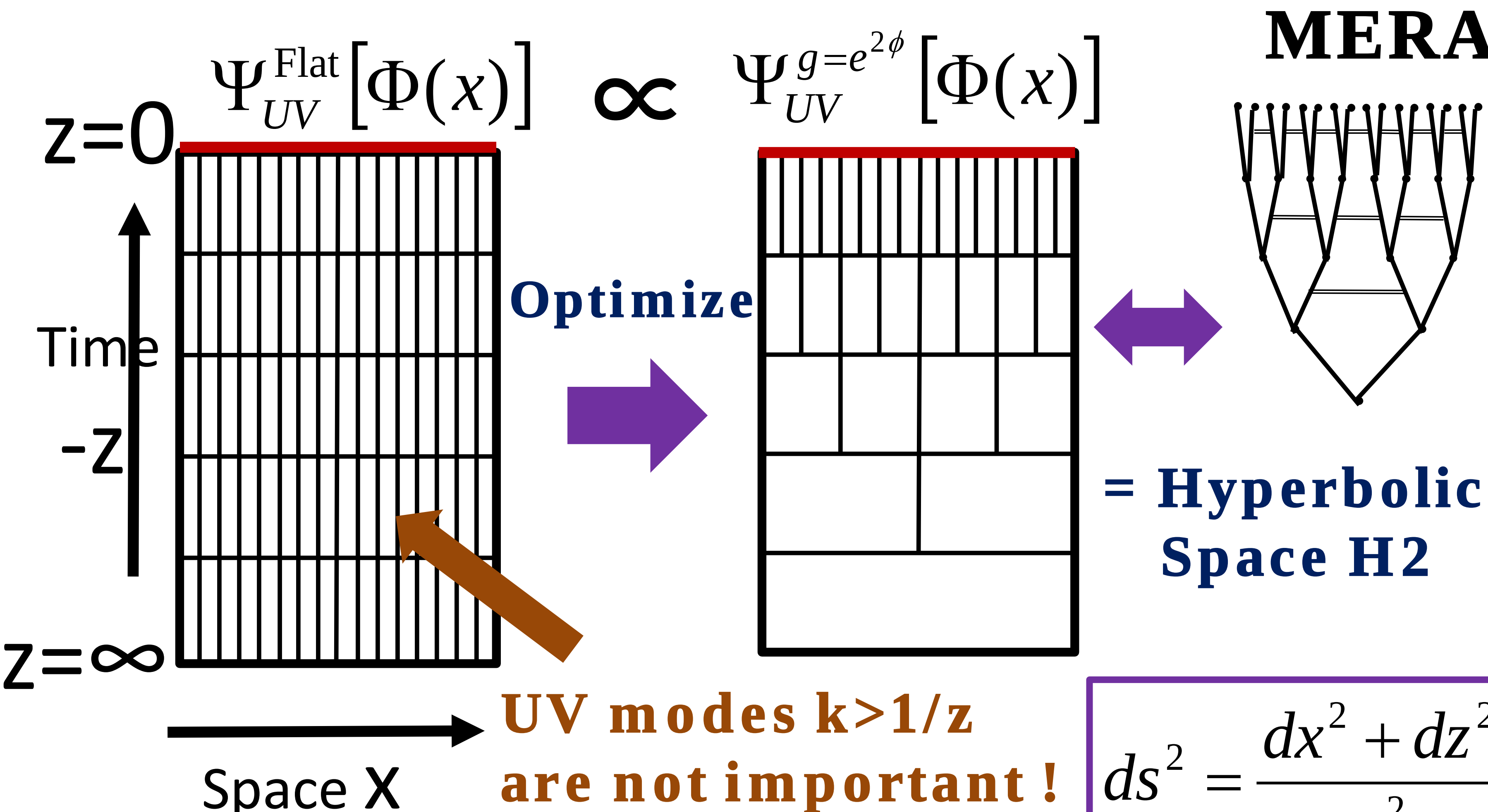
$$\Rightarrow \text{Minimum}: \; e^{2\varphi} = \frac{\epsilon^2}{z^2}$$

Hyperbolic plane (H2)
= Time slice of AdS3

[Caputa-Kundu-Miyaji-Watanabe-TT 17]

A Sketch: Optimization of Path-Integral

$$\Psi_{UV}^{\mathrm{Flat}}\left[\Phi(x)\right] \propto \Psi_{UV}^{g=e^{2\phi}}\left[\Phi(x)\right]$$



UV modes k>1/z are not important !

$$ds^2 = \frac{dx^2 + dz^2}{z^2}$$

Recently, this analysis gives a CFT confirmation of the holographic EoP conjecture: [Caputa-Miyaji-Umemoto-TT 2018].

④ Conclusions

- The holography (or AdS/CFT) provides a very powerful tool to study quantum gravity.
- The holographic counterpart of entanglement entropy is given by minimal surface areas in AdS.
- This inspires the new idea of *emergent spacetime from quantum entanglement*. This idea is explicitly realized in tensor networks and path-integral optimization.
- There have recently been interesting progresses on other quantum information theoretic quantities.

e.g. Complexity, Information metric,

entanglement of purification, relatively entropy, etc.

Future problems

- An explicit proof of AdS/CFT [$\Leftarrow$Quantum info. ?]
- Generalization of AdS/CFT to other spacetimes
  (e.g. cosmological spacetimes such as de Sitter spaces)
- Clear explanation of Black hole information paradox
  : show explicitly recovery of information from radiations.
- More quantum information quantities
  from AdS/CFT   (e.g. mixed state entanglement)

:

: