CP Violation in the Standard Model (and Beyond)*

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Abstract

This is a written version of a series of lectures aimed at graduate students in particle physics. We explain the reasons for the interest in CP violation and in flavor physics. We describe flavor physics and the related CP violation within the Standard Model, and explain how the B-factories proved that the CKM (KM) mechanism dominates the flavor changing (CP violating) processes that have been observed in meson decays. We explain the implications of CP violation and of flavor physics for new physics, with emphasis on the "new physics flavor puzzle".

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I. INTRODUCTION

A. What is CP (violation)?

The CP transformation combines charge conjugation C with parity P. For example, a left-handed (LH) electron e_L^- transforms under CP into a right-handed (RH) positron e_R^+. CP is a good symmetry if there is a basis where all the parameters of the Lagrangian are real. We do not prove it here but provide a simple intuitive explanation of this statement.

Consider a theory with a single scalar, ϕ, and two sets of N fermions, ψ_L^i and ψ_R^i ($i = 1, 2, ..., N$). The Yukawa interactions are given by

$$-\mathcal{L}_{\rm Yuk} = Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + Y_{ij}^*\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \tag{1}$$

where we write the two hermitian conjugate terms explicitly. The CP transformation of the fields is defined as follows:

$$\phi \to \phi^\dagger, \qquad \psi_{Li} \to \overline{\psi_{Li}}, \qquad \psi_{Ri} \to \overline{\psi_{Ri}}. \tag{2}$$

Therefore, a CP transformation exchanges the operators

$$\overline{\psi_{Li}}\phi\psi_{Rj} \ \stackrel{\rm CP}{\longleftrightarrow}\ \overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \tag{3}$$

but leaves their coefficients, Y_{ij} and Y_{ij}^*, unchanged. This means that CP is a symmetry of $\mathcal{L}_{\rm Yuk}$ if $Y_{ij} = Y_{ij}^*$.

In practice, things are more subtle, since one can define the CP transformation in a more general way than Eq. (2):

$$\phi \to e^{i\theta}\phi^\dagger, \qquad \psi_L^i \to e^{i\theta_{Li}}\overline{\psi_L^i}, \qquad \psi_R^i \to e^{i\theta_{Ri}}\overline{\psi_R^i}, \tag{4}$$

with $\theta, \theta_{Li}, \theta_{Ri}$ convention-dependent phases. Then, there can be complex couplings, yet CP would be a good symmetry. The correct statement is that CP is violated if, using all freedom to redefine the phases of the fields, one cannot find any basis where all couplings are real. We note that a theory with only gauge interactions conserves CP as the coupling constants are real.

There are four main reasons for the interest in CP violation:

- CP asymmetries provide some of the theoretically cleanest probes of flavor physics. The reason for that is that CP is a good symmetry of the strong interactions. Consequently, for some hadronic decays, QCD-related uncertainties cancel out in the CP asymmetries.

- For some processes, the CP violating part is particularly suppressed within the SM, providing sensitivity to new physics at very high scales.

- Non-perturbative effects of the strong interactions are expected to generate an electric dipole moment (EDM) of the neutron that is some ten orders of magnitude above the current experimental upper bound. This situation is known as "the strong CP problem."

- There is a cosmological puzzle related to CP violation. The baryon asymmetry of the Universe is a CP violating observable, and it is many orders of magnitude larger than the SM prediction. Hence, there must exist new sources of CP violation beyond the single phase of the CKM matrix.

B. What is flavor (violation)?

The term "**flavors**" is used, in the jargon of particle physics, to describe mass eigenstates of the same gauge representation (but possibly different masses), namely several fields that are assigned the same quantum charges. Within the Standard Model, when thinking of its unbroken $SU(3)_{\rm C} \times U(1)_{\rm EM}$ gauge group, there are four different types of particles, each coming in three flavors:

- Up-type quarks in the $(3)_{+2/3}$ representation: u, c, t;

- Down-type quarks in the $(3)_{-1/3}$ representation: d, s, b;

- Charged leptons in the $(1)_{-1}$ representation: e, μ, τ;

- Neutrinos in the $(1)_0$ representation: ν_1, ν_2, ν_3.

The term "**flavor physics**" refers to interactions that distinguish between flavors. By definition, gauge interactions, namely interactions that are related to unbroken symmetries and mediated therefore by massless gauge bosons, do not distinguish among the flavors and do not constitute part of flavor physics. Within the Standard Model, flavor-physics refers to the weak and Yukawa interactions.

The term "**flavor parameters**" refers to parameters that carry flavor indices. Within the Standard Model, these are the nine masses of the charged fermions and the four "mixing parameters" (three angles and one phase) that describe the interactions of the charged weak-force carriers ($W^\pm$) with quark-antiquark pairs. If one augments the Standard Model with Majorana mass terms for the neutrinos, one should add to the list three neutrino masses and six mixing parameters (three angles and three phases) for the $W^\pm$ interactions with lepton-antilepton pairs.

The term "**flavor universal**" refers to interactions with couplings (or to parameters) that are proportional to the unit matrix in flavor space. Thus, the strong and electromagnetic interactions are flavor-universal. An alternative term for "flavor-universal" is "**flavor-blind**".

The term "**flavor diagonal**" refers to interactions with couplings (or to parameters) that are diagonal, but not necessarily universal, in the flavor space. Within the Standard Model, the Yukawa interactions of the Higgs boson are flavor diagonal.

The term "**flavor changing**" refers to processes where the initial and final flavor-numbers (that is, the number of particles of a certain flavor minus the number of anti-particles of the same flavor) are different. In "flavor changing charged current" (FCCC) processes, both up-type and down-type flavors, and/or both charged lepton and neutrino flavors are involved. Examples are (i) muon decay via $\mu \to e\bar{\nu}_e\nu_\mu$, (ii) $K^- \to \mu^-\bar{\nu}_\mu$ (which corresponds, at the quark level, to $s\bar{u} \to \mu^-\bar{\nu}_\mu$), and (iii) $B \to \psi K$ ($b \to c\bar{c}s$). Within the Standard Model, these processes are mediated by the W-bosons and occur at tree level. In "**flavor changing neutral current**" (FCNC) processes, either up-type or down-type flavors but not both, and/or either charged lepton or neutrino flavors but not both, are involved. Example are (i) muon decay via $\mu \to e\gamma$, (ii) $K_L \to \mu^+\mu^-$ (which corresponds, at the quark level, to $s\bar{d} \to \mu^+\mu^-$), and (iii) $B \to \phi K$ ($b \to s\bar{s}s$). Within the Standard Model, these processes do not occur at tree level, and are often highly suppressed.

Another useful term is "**flavor violation**". As we explain later in these lectures, if the Yukawa couplings had vanished, the SM would have gained a global $[SU(3)]^5$ symmetry. Interactions, or parameters, that break this symmetry are called flavor violating.

Flavor physics is interesting, on one hand, as a tool for discovery and, on the other hand, because of intrinsic puzzling features:

- Flavor physics can discover new physics or probe it before it is directly observed in experiments. More specifically, FCNC can be affected by new degrees of freedom that are much heavier than the energy scale of the experiment. Here are some examples from the past:
  - The smallness of $\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)}$ led to predicting a fourth (the charm) quark;
  - The size of Δm_K led to a successful prediction of the charm mass;
  - The size of Δm_B led to a successful prediction of the top mass;
  - The measurement of ε_K led to predicting the third generation.
  - The measurement of neutrino flavor transitions led to the discovery of neutrino masses.
- CP violation is closely related to flavor physics. Within the Standard Model, there is a single CP violating parameter, the Kobayashi-Maskawa phase $\delta_{\rm KM}$ [2]. Baryogenesis tells us, however, that there must exist new sources of CP violation. Measurements of CP violation in flavor changing processes might provide evidence for such sources.
- The fine-tuning problem of the Higgs mass, and the puzzle of the dark matter imply that there exists new physics at, or below, the TeV scale. If such new physics had a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. The question of why this does not happen constitutes the *new physics flavor puzzle.*
- Most of the charged fermion flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is the *Standard Model flavor puzzle.*
- The SM flavor puzzle became even deeper after neutrino masses and mixings were measured because, so far, neither smallness nor hierarchy in these parameters have been established. This is the *neutrino flavor puzzle.*

II. THE STANDARD MODEL

A model of elementary particles and their interactions is defined by the following ingredients: (i) The symmetries of the Lagrangian and the pattern of spontaneous symmetry breaking (SSB); (ii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows:

- The symmetry is a local

$$G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{5}$$

- It is spontaneously broken by the VEV of a single Higgs scalar,

$$\phi(1,2)_{+1/2}, \quad (\langle \phi^0 \rangle = v/\sqrt{2}), \tag{6}$$

$$G_{\rm SM} \to SU(3)_C \times U(1)_{\rm EM} \quad (Q_{\rm EM} = T_3 + Y). \tag{7}$$

- There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q_{Li}(3,2)_{+1/6}, \quad U_{Ri}(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3}, \quad L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1}. \tag{8}$$

A. The Lagrangian

The most general renormalizable Lagrangian with scalar and fermion fields can be decomposed into

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_\psi + \mathcal{L}_{\rm Yuk} + \mathcal{L}_\phi. \tag{9}$$

Here $\mathcal{L}_{\rm kin}$ describes free propagation in spacetime, as well as gauge interactions, $\mathcal{L}_\psi$ gives fermion mass terms, $\mathcal{L}_{\rm Yuk}$ describes the Yukawa interactions, and $\mathcal{L}_\phi$ gives the scalar potential. We now find the specific form of the Lagrangian made of the fermion fields Q_{Li}, U_{Ri}, D_{Ri}, L_{Li} and E_{Ri} (8), and the scalar field (6), subject to the gauge symmetry (5) and leading to the SSB of Eq. (7).

1. $\mathcal{L}_{\rm kin}$

The local symmetry requires the following gauge boson degrees of freedom:

$$G_a^\mu(8,1)_0, \quad W_a^\mu(1,3)_0, \quad B^\mu(1,1)_0. \tag{10}$$

The corresponding field strengths are given by

$$\begin{aligned}
G_a^{\mu\nu} &= \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu, \\
W_a^{\mu\nu} &= \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu, \\
B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu.
\end{aligned} \tag{11}$$

The covariant derivative is

$$D^\mu = \partial^\mu + i g_s G_a^\mu L_a + i g W_b^\mu T_b + i g' B^\mu Y, \tag{12}$$

where the L_a's are $SU(3)_C$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_L$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_Y$ charges. Explicitly, the covariant derivatives acting on the various scalar and fermion fields are given by

$$\begin{aligned}
D^\mu \phi &= \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{2} g' B^\mu\right) \phi, \\
D^\mu Q_{Li} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{6} g' B^\mu\right) Q_{Li}, \\
D^\mu U_{Ri} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{2i}{3} g' B^\mu\right) U_{Ri}, \\
D^\mu D_{Ri} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a - \frac{i}{3} g' B^\mu\right) D_{Ri}, \\
D^\mu L_{Li} &= \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b - \frac{i}{2} g' B^\mu\right) L_{Li}, \\
D^\mu E_{Ri} &= \left(\partial^\mu - i g' B^\mu\right) E_{Ri}.
\end{aligned} \tag{13}$$

$\mathcal{L}_{\rm kin}$ is given by

$$\begin{aligned}
\mathcal{L}_{\rm kin}^{\rm SM} = &-\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} - \frac{1}{4} W_b^{\mu\nu} W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\
&- i\overline{Q_{Li}} \not{D} Q_{Li} - i\overline{U_{Ri}} \not{D} U_{Ri} - i\overline{D_{Ri}} \not{D} D_{Ri} - i\overline{L_{Li}} \not{D} L_{Li} - i\overline{E_{Ri}} \not{D} E_{Ri} \\
&- (D^\mu \phi)^\dagger (D_\mu \phi).
\end{aligned} \tag{14}$$

This part of the interaction Lagrangian is flavor-universal. In addition, it conserves CP.

2. $\mathcal{L}_\psi$

There are no mass terms for the fermions in the SM. We cannot write Dirac mass terms for the fermions because they are assigned to chiral representations of the gauge symmetry.

We cannot write Majorana mass terms for the fermions because they all have $Y \neq 0$. Thus,

$$\mathcal{L}_{\psi}^{\rm SM} = 0. \tag{15}$$

3. $\mathcal{L}_{\rm Yuk}$

The Yukawa part of the Lagrangian is given by

$$\mathcal{L}_{\rm Y}^{\rm SM} = Y_{ij}^{d}\overline{Q_{Li}}\phi D_{Rj} + Y_{ij}^{u}\overline{Q_{Li}}\tilde{\phi} U_{Rj} + Y_{ij}^{e}\overline{L_{Li}}\phi E_{Rj} + \text{h.c.}, \tag{16}$$

where $\tilde{\phi} = i\tau_2\phi^\dagger$, and the Y^f are general 3×3 matrices of dimensionless couplings. This part of the Lagrangian is, in general, flavor-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

Without loss of generality, we can use a bi-unitary transformation,

$$Y^e \to \hat{Y}_e = U_{eL} Y^e U_{eR}^\dagger, \tag{17}$$

to change the basis to one where Y^e is diagonal and real:

$$\hat{Y}^e = \text{diag}(y_e, y_\mu, y_\tau). \tag{18}$$

In the basis defined in Eq. (18), we denote the components of the lepton $SU(2)$-doublets, and the three lepton $SU(2)$-singlets, as follows:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}; \quad e_R, \quad \mu_R, \quad \tau_R, \tag{19}$$

where e, μ, τ are ordered by the size of $y_{e,\mu,\tau}$ (from smallest to largest).

Similarly, without loss of generality, we can use a bi-unitary transformation,

$$Y^u \to \hat{Y}_u = V_{uL} Y^u V_{uR}^\dagger, \tag{20}$$

to change the basis to one where $\hat{Y}^u$ is diagonal and real:

$$\hat{Y}^u = \text{diag}(y_u, y_c, y_t). \tag{21}$$

In the basis defined in Eq. (21), we denote the components of the quark $SU(2)$-doublets, and the quark up $SU(2)$-singlets, as follows:

$$\begin{pmatrix} u_L \\ d_{uL} \end{pmatrix}, \quad \begin{pmatrix} c_L \\ d_{cL} \end{pmatrix}, \quad \begin{pmatrix} t_L \\ d_{tL} \end{pmatrix}; \quad u_R, \quad c_R, \quad t_R, \tag{22}$$

where u, c, t are ordered by the size of $y_{u,c,t}$ (from smallest to largest).

We can use yet another bi-unitary transformation,

$$Y^d \to \hat{Y}_d = V_{dL} Y^d V_{dR}^\dagger, \tag{23}$$

to change the basis to one where $\hat{Y}^d$ is diagonal and real:

$$\hat{Y}^d = \mathrm{diag}(y_d, y_s, y_b). \tag{24}$$

In the basis defined in Eq. (24), we denote the components of the quark $SU(2)$-doublets, and the quark down $SU(2)$-singlets, as follows:

$$\begin{pmatrix} u_{dL} \\ d_L \end{pmatrix}, \qquad \begin{pmatrix} u_{sL} \\ s_L \end{pmatrix}, \qquad \begin{pmatrix} u_{bL} \\ b_L \end{pmatrix}; \qquad d_R, \qquad s_R, \qquad b_R, \tag{25}$$

where d, s, b are ordered by the size of $y_{d,s,b}$ (from smallest to largest).

Note that if $V_{uL} \neq V_{dL}$, as is the general case, then the interaction basis defined by (21) is different from the interaction basis defined by (24). In the former, Y^d can be written as a unitary matrix times a diagonal one,

$$Y^u = \hat{Y}^u, \quad Y^d = V\hat{Y}^d. \tag{26}$$

In the latter, Y^u can be written as a unitary matrix times a diagonal one,

$$Y^d = \hat{Y}^d, \quad Y^u = V^\dagger \hat{Y}^u. \tag{27}$$

In either case, the matrix V is given by

$$V = V_{uL} V_{dL}^\dagger, \tag{28}$$

where V_{uL} and V_{dL} are defined in Eqs. (20) and (23), respectively. Note that V_{uL}, V_{uR}, V_{dL} and V_{dR} depend on the basis from which we start the diagonalization. The combination $V = V_{uL}V_{dL}^\dagger$, however, does not. This is a hint that V is physical. Indeed, below we see that it plays a crucial role in the charged current interactions.

4. $\mathcal{L}_\phi$

The scalar potential is given by

$$\mathcal{L}_\phi^{\mathrm{SM}} = -\mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger\phi)^2. \tag{29}$$

Choosing $\mu^2 < 0$ and $\lambda > 0$ leads to the required spontaneous symmetry breaking. This part of the Lagrangian is also CP conserving.

TABLE I: The SM particles

particle	spin	color	$Q_{\rm EM}$	mass $[v]$
$W^\pm$	1	(1)	±1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e,μ,τ	1/2	(1)	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e,ν_μ,ν_τ	1/2	(1)	0	0
u,c,t	1/2	(3)	$+2/3$	$y_{u,c,t}/\sqrt{2}$
d,s,b	1/2	(3)	$-1/3$	$y_{d,s,b}/\sqrt{2}$

5. *Summary*

The renormalizable part of the Standard Model Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_{\rm SM} = & -\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu}-\frac{1}{4}W_b^{\mu\nu}W_{b\mu\nu}-\frac{1}{4}B^{\mu\nu}B_{\mu\nu}-(D^\mu\phi)^\dagger(D_\mu\phi)\\
& - i\overline{Q_{Li}}\not{D}Q_{Li}-i\overline{U_{Ri}}\not{D}U_{Ri}-i\overline{D_{Ri}}\not{D}D_{Ri}-i\overline{L_{Li}}\not{D}L_{Li}-i\overline{E_{Ri}}\not{D}E_{Ri}\\
& + \left(Y_{ij}^u\overline{Q_{Li}}U_{Rj}\,\tilde{\phi}+Y_{ij}^d\overline{Q_{Li}}D_{Rj}\,\phi+Y_{ij}^e\overline{L_{Li}}E_{Rj}\,\phi+\text{h.c.}\right)\\
& - \lambda\left(\phi^\dagger\phi-v^2/2\right)^2\,,
\end{aligned} \tag{30}$$

where $i,j=1,2,3$.

The only complex couplings – and therefore the only potential sources of CP violation – are the Yukawa matrices, Y^u, Y^d and Y^e. In the basis defined by Eq. (18) and by either Eq. (26) or by Eq. (27), the only complex parameters – and therefore the only potential sources of CP violation – are in the unitary matrix V.

B. The spectrum

The spectrum of the SM is presented in Table I.

All masses are proportional to the VEV of the scalar field, v. For the three massive gauge bosons, and for the fermions, this is expected: In the absence of spontaneous symmetry breaking, the former would be protected by the gauge symmetry and the latter by their chiral nature. For the Higgs boson, the situation is different, as a mass-squared term does not violate any symmetry.

For the charged fermions, the spontaneous symmetry breaking allows their masses because they are in vector-like representations of the $SU(3)_C \times U(1)_{\rm EM}$ group: The LH and RH charged lepton fields, e, μ and τ, are in the $(1)_{-1}$ representation; The LH and RH up-type quark fields, u, c and t, are in the $(3)_{+2/3}$ representation; The LH and RH down-type quark fields, d, s and b, are in the $(3)_{-1/3}$ representation. On the other hand, the neutrinos remain massless in spite of the fact that they are in the $(1)_0$ representation of $SU(3)_C \times U(1)_{\rm EM}$, which allows for Majorana masses. Such masses require a VEV carried by a scalar field in the $(1,3)_{+1}$ representation of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry, but there is no such field in the SM.

The experimental values of the charged fermion masses are [1] [1]

$$\begin{aligned}
&m_e = 0.510998946(3)\ \text{MeV}, \quad m_\mu = 105.6583745(24)\ \text{MeV}, \quad m_\tau = 1776.86(12)\ \text{MeV}, \\
&m_u = 2.2^{+0.5}_{-0.4}\ \text{MeV}, \quad m_c = 1.275^{+0.025}_{-0.035}\ \text{GeV}, \quad m_t = 173.1 \pm 0.9\ \text{GeV}, \\
&m_d = 4.7^{+0.5}_{-0.3}\ \text{MeV}, \quad m_s = 95^{+9}_{-3}\ \text{MeV}, \quad m_b = 4.18^{+0.04}_{-0.03}\ \text{GeV}.
\end{aligned} \tag{31}$$

C. The interactions

Within the SM, the fermions have five types of interactions. These interactions are summarized in Table II. In the next few subsections, we explain the entries of this Table.

1. *EM and strong interactions*

By construction, a local $SU(3)_C \times U(1)_{\rm EM}$ symmetry survives the SSB. The SM has thus the photon and gluon massless gauge fields. All charged fermions interact with the photon:

$$\mathcal{L}_{{\rm QED},\psi} = -\frac{2e}{3}\overline{u_i}\not{A}u_i + \frac{e}{3}\overline{d_i}\not{A}d_i + e\overline{\ell_i}\not{A}\ell_i, \tag{32}$$

where $u_{1,2,3} = u, c, t$, $d_{1,2,3} = d, s, b$ and $\ell_{1,2,3} = e, \mu, \tau$. We emphasize the following points:

[1] See [1] for detailed explanations of the quoted quark masses. For $q = u, d, s, c, b$, m_q are the running quark masses in the $\overline{\rm MS}$ scheme, with $m_{u,d,s} = m_{u,d,s}(\mu = 2\ \text{GeV})$ and $m_{c,b} = m_{c,b}(\mu = m_{c,b})$.

TABLE II: The SM fermion interactions. CPV (CPC) stands for CP violating (conserving).

interaction	fermions	force carrier	coupling	flavor	CP
Electromagnetic	u, d, ℓ	A^0	eQ	universal	CPC
Strong	u, d	g	g_s	universal	CPC
NC weak	all	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	universal	CPC
CC weak	$\bar{u}d/\bar{\ell}\nu$	$W^\pm$	gV/g	non-universal/universal	CPV/CPC
Yukawa	u, d, ℓ	h	y_q	diagonal	CPC

1. The photon couplings are *vector-like* and *parity conserving.*

2. The photon couplings are *CP conserving.*

3. *Diagonality:* The photon couples to e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$, but not to $e^\pm\mu^\mp$, $e^\pm\tau^\mp$ or $\mu^\pm\tau^\mp$ pairs, and similarly in the up and down sectors.

4. *Universality:* The couplings of the photon to different generations are universal.

All colored fermions (namely, quarks) interact with the gluon:

$$\mathcal{L}_{\mathrm{QCD},\psi} = -\frac{g_s}{2}\bar{q}\lambda_a \not{G}_a q, \tag{33}$$

where $q = u, c, t, d, s, b$. We emphasize the following points:

1. The gluon couplings are *vector-like* and *parity conserving.*

2. The gluon couplings are *CP conserving.*

3. *Diagonality:* The gluon couples to $\bar{t}t$, $\bar{c}c$, *etc.*, but not to $\bar{t}c$ or any other flavor changing pair.

4. *Universality:* The couplings of the gluon to different quark generations are universal.

The universality of the photon and gluon couplings are a result of the $SU(3)_C \times U(1)_{\mathrm{EM}}$ gauge invariance, and thus hold in any model, and not just within the SM.

2. Z-mediated weak interactions

All SM fermions couple to the Z-boson:

$$\mathcal{L}_{Z,\psi} = \frac{e}{s_W c_W}\Big[-\left(\frac{1}{2}-s_W^2\right)\,\overline{e_{Li}}\not{Z}e_{Li}+s_W^2\,\overline{e_{Ri}}\not{Z}e_{Ri}+\frac{1}{2}\,\overline{\nu_{L\alpha}}\not{Z}\nu_{L\alpha} \tag{34}$$
$$+\left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\,\overline{u_{Li}}\not{Z}u_{Li}-\frac{2}{3}s_W^2\,\overline{u_{Ri}}\not{Z}u_{Ri}-\left(\frac{1}{2}-\frac{1}{3}s_W^2\right)\,\overline{d_{Li}}\not{Z}d_{Li}+\frac{1}{3}s_W^2\,\overline{d_{Ri}}\not{Z}d_{Ri}\Big].$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$. We emphasize the following points:

1. The Z-boson couplings are *chiral* and *parity violating.*

2. The Z-boson couplings are *CP conserving.*

3. *Diagonality:* The Z-boson couples diagonally and, as a result of this, there are no Z-mediated flavor changing neutral current (FCNC) processes.

4. *Universality:* The couplings of the Z-boson to different fermion generations are universal.

The universality is a result of a special feature of the SM: all fermions of given chirality and given charge come from the same $SU(2)_L \times U(1)_Y$ representation.

As an example to experimental tests of diagonality and universality, we can take the leptonic sector. The branching ratios of the Z-boson into charged lepton pairs [1],

$$\begin{aligned} \mathrm{BR}(Z\to e^+e^-) &= (3.363\pm 0.004)\%\,, \\ \mathrm{BR}(Z\to \mu^+\mu^-) &= (3.366\pm 0.007)\%\,, \\ \mathrm{BR}(Z\to \tau^+\tau^-) &= (3.367\pm 0.008)\%\,. \end{aligned} \tag{35}$$

beautifully confirms universality:

$$\begin{aligned} \Gamma(\mu^+\mu^-)/\Gamma(e^+e^-) &= 1.0009\pm 0.0028, \\ \Gamma(\tau^+\tau^-)/\Gamma(e^+e^-) &= 1.0019\pm 0.0032. \end{aligned}$$

Diagonality is also tested by the following experimental searches:

$$\begin{aligned} \mathrm{BR}(Z\to e^+\mu^-) &< 7.5\times 10^{-7}, \\ \mathrm{BR}(Z\to e^+\tau^-) &< 9.8\times 10^{-6}, \\ \mathrm{BR}(Z\to \mu^+\tau^-) &< 1.2\times 10^{-5}. \end{aligned} \tag{36}$$

3. W-mediated weak interactions

We now study the couplings of the charged vector bosons, $W^\pm$, to fermion pairs. For the lepton mass eigenstates, things are simple, because there exists an interaction basis that is also a mass basis. Thus,

$$\mathcal{L}_{W,\ell} = -\frac{g}{\sqrt{2}} \left(\overline{\nu_{eL}}\, \not{W}^+ e^-_L + \overline{\nu_{\mu L}}\, \not{W}^+ \mu^-_L + \overline{\nu_{\tau L}}\, \not{W}^+ \tau^-_L + \text{h.c.} \right). \tag{37}$$

Eq. (37) reveals some important features of the model:

1. Only left-handed particles take part in charged-current interactions. Consequently, parity is violated.

2. The W-boson couplings to leptons are CP conserving.

3. *Diagonality*: the charged current interactions couple each charged lepton to a single neutrino, and each neutrino to a single charged lepton. Note that a global $SU(2)$ symmetry would allow off-diagonal couplings; It is the local symmetry that leads to diagonality.

4. *Universality*: the couplings of the W-boson to $\tau\bar{\nu}_\tau$, to $\mu\bar{\nu}_\mu$ and to $e\bar{\nu}_e$ are equal. Again, a global symmetry would have allowed an independent coupling to each lepton pair.

All of these predictions have been experimentally tested. As an example of how well universality works, consider the decay rates of the W-bosons to the three lepton pairs [1]:

$$\begin{aligned} \text{BR}(W^+ \to e^+\nu_e) &= (10.71 \pm 0.16) \times 10^{-2}, \\ \text{BR}(W^+ \to \mu^+\nu_\mu) &= (10.63 \pm 0.15) \times 10^{-2}, \\ \text{BR}(W^+ \to \tau^+\nu_\tau) &= (11.38 \pm 0.21) \times 10^{-2}. \end{aligned} \tag{38}$$

As concerns quarks, things are more complicated, since there is no interaction basis that is also a mass basis. In the interaction basis where the down quarks are mass eigenstates (25), the W interactions have the following form:

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}} \left(\overline{u_{dL}}\, \not{W}^+ d_L + \overline{u_{sL}}\, \not{W}^+ s_L + \overline{u_{bL}}\, \not{W}^+ b_L + \text{h.c.} \right). \tag{39}$$

The Yukawa matrices in this basis have the form (27), and in particular, for the up sector, we have

$$\mathcal{L}^u_{\rm Yuk} = (\overline{u_{dL}}\ \overline{u_{sL}}\ \overline{u_{bL}})V^\dagger \hat{Y}^u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}, \tag{40}$$

which tells us straightforwardly how to transform to the mass basis:

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = V \begin{pmatrix} u_{dL} \\ u_{sL} \\ u_{bL} \end{pmatrix}. \tag{41}$$

Using Eq. (41), we obtain the form of the W interactions (39) in the mass basis:

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\ (\overline{u_L}\ \overline{c_L}\ \overline{t_L})\ \ V\ W\!\!\!/^{+} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}. \tag{42}$$

You can easily convince yourself that we would have obtained the same form starting from any arbitrary interaction basis. We remind you that

$$V = V_{uL} V_{dL}^\dagger \tag{43}$$

is basis independent. The matrix V is called the CKM matrix [2, 3].

Eq. (42) reveals some important features of the model:

1. Similarly to the leptons, only left-handed quarks take part in charged-current interactions and, consequently, parity is violated by these interactions.

2. The matrix V is, in general, complex. (We will analyze this point in more detail below.) Thus, the W-boson couplings to quarks are CP violating.

3. The W couplings to the quark mass eigenstates are neither universal nor diagonal. The universality of gauge interactions is hidden in the unitarity of the matrix V.

Omitting common factors (particularly, a factor of $g^2/4$) and phase space factors, we obtain the following predictions for the W decays:

$$\begin{aligned} \Gamma(W^+ \to \ell^+ \nu_\ell) &\propto 1, \\ \Gamma(W^+ \to u_i \overline{d_j}) &\propto 3|V_{ij}|^2 \quad (i=1,2;\ j=1,2,3). \end{aligned} \tag{44}$$

The top quark is not included because it is heavier than the W boson. Taking this fact into account, and the CKM unitarity relations

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1, \tag{45}$$

we obtain

$$\Gamma(W \to \text{hadrons})/\Gamma(W \to \text{leptons}) \approx 2. \tag{46}$$

Experimentally (we use [1] $\text{BR}(W \to \text{leptons}) = 0.326 \pm 0.003$ and $\text{BR}(W \to \text{hadrons}) = 0.674 \pm 0.003$)

$$\Gamma(W \to \text{hadrons})/\Gamma(W \to \text{leptons}) = 2.07 \pm 0.02, \tag{47}$$

which, taking into account radiative corrections, is in good agreement with the SM prediction. The (hidden) universality within the quark sector is tested by the prediction

$$\Gamma(W \to uX) = \Gamma(W \to cX) = \frac{1}{2}\Gamma(W \to \text{hadrons}). \tag{48}$$

Experimentally (we use [1] $\text{BR}(W \to cX) = 0.333 \pm 0.026$ and $\text{BR}(W \to \text{hadrons}) = 0.674 \pm 0.003$),

$$\Gamma(W \to cX)/\Gamma(W \to \text{hadrons}) = 0.49 \pm 0.04. \tag{49}$$

4. Yukawa interactions

The Yukawa interactions are given by

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} = &- \frac{h}{v}\left(m_e\, \overline{e_L}\, e_R + m_\mu\, \overline{\mu_L}\, \mu_R + m_\tau\, \overline{\tau_L}\, \tau_R\right.\\ &\left.+m_u\, \overline{u_L}\, u_R + m_c\, \overline{c_L}\, c_R + m_t\, \overline{t_L}\, t_R + m_d\, \overline{d_L}\, d_R + m_s\, \overline{s_L}\, s_R + m_b\, \overline{b_L}\, b_R + \text{h.c.}\right).\end{aligned}$$

To see that the Higgs boson couples diagonally to the quark mass eigenstates, let us start from an arbitrary interaction basis:

$$\begin{aligned} h\overline{D_L}Y^d D_R &= h\overline{D_L}(V_{dL}^\dagger V_{dL})Y^d(V_{dR}^\dagger V_{dR})D_R \\ &= h(\overline{D_L}V_{dL}^\dagger)(V_{dL}Y^dV_{dR}^\dagger)(V_{dR}D_R) \\ &= h(\overline{d_L}\ \overline{s_L}\ \overline{b_L})\hat{Y}^d(d_R\ s_R\ b_R)^T. \end{aligned} \tag{50}$$

We conclude that the Higgs couplings to the fermion mass eigenstates have the following features:

1. *Diagonality.*

2. *Non-universality.*

3. *Proportionality* to the fermion masses: the heavier the fermion, the stronger the coupling. The factor of proportionality is m_ψ/v.

4. *CP conserving.*

Thus, the Higgs boson decay is dominated by the heaviest particle which can be pair-produced in the decay. For $m_h \sim 125$ GeV, this is the bottom quark. Indeed, the SM predicts the following branching ratios quoted in Table III for the leading decay modes. The following comments are in order with regard to the predicted branching ratios:

1. From the seven branching ratios, three (b, τ, c) stand for two-body tree-level decays. Thus, at tree level, the respective branching ratios obey $\mathrm{BR}_{\bar{b}b} : \mathrm{BR}_{\tau^+\tau^-} : \mathrm{BR}_{c\bar{c}} = 3m_b^2 : m_\tau^2 : 3m_c^2$. QCD radiative corrections somewhat suppress the two modes with the quark final states (b, c) compared to one with the lepton final state (τ).

2. The WW^* and ZZ^* modes stand for the three-body tree-level decays, where one of the vector bosons is on-shell and the other off-shell.

3. The Higgs boson does not have a tree-level coupling to gluons since it carries no color (and the gluons have no mass). The decay into final gluons proceeds via loop diagrams. The dominant contribution comes from the top-quark loop.

4. Similarly, the Higgs decays into final two photons via loop diagrams with small ($\mathrm{BR}_{\gamma\gamma} \sim 0.002$), but observable, rate. The dominant contributions come from the W-boson and the top-quark loops which interfere destructively.

Experimentally, the decays into final ZZ^*, WW^*, $\gamma\gamma$, $b\bar{b}$ and $\tau^+\tau^-$ have been established.

D. Global symmetries

The SM has an accidental global symmetry:

$$G_{\mathrm{global}}^{\mathrm{SM}} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \tag{51}$$

This symmetry leads to various testable predictions. Here are a few examples:

TABLE III: Higgs decays: The SM predictions for the branching ratios, and the experimental μ values.

Mode	$\mathrm{BR}_{\mathrm{SM}}$	$\mu_{\mathrm{experiment}}$	Comments
$b\bar{b}$	0.58	0.98 ± 0.20	
WW^*	0.21	0.99 ± 0.15	3-body
gg	0.09		loop
$\tau^+\tau^-$	0.06	1.09 ± 0.23	
ZZ^*	0.03	1.17 ± 0.23	3-body
$c\bar{c}$	0.03		
$\gamma\gamma$	0.002	1.14 ± 0.14	loop

- The proton must not decay, *e.g.* $p \to e^+\pi^0$ is forbidden.
- FCNC decays of charged leptons must not occur, *e.g.* $\mu \to e\gamma$ is forbidden.
- Neutrinos are massless, $m_\nu = 0$.

The last prediction is, however, violated in Nature. Neutrino flavor transitions are observed, implying that at least two of the neutrino masses are different from zero.

Accidental symmetries are broken by higher-dimensional (non-renormalizable) terms. Two examples are the following:

- At dimension five, $\frac{z^\nu_{ij}}{\Lambda} L_i L_j \phi\phi$ terms break $U(1)_e \times U(1)_\mu \times U(1)_\tau$.
- At dimension six, $\frac{y_{ijkl}}{\Lambda^2} Q_i Q_j Q_k L_l$ terms break $U(1)_B$.

Thus, given that $m_\nu \neq 0$, we learn that the SM is, at best, a good low energy effective field theory.

In the absence of the Yukawa matrices, $\mathcal{L}_{\mathrm{Yuk}} = 0$, the SM would gain $[U(3)]^5$ global symmetry:

$$G^{\mathrm{SM}}_{\mathrm{global}}(Y^{u,d,e} = 0) = SU(3)^3_q \times SU(3)^2_\ell \times U(1)^5, \tag{52}$$

where

$$SU(3)^3_q = SU(3)_Q \times SU(3)_U \times SU(3)_D,$$

$$
\begin{aligned}
SU(3)_\ell^2 &= SU(3)_L \times SU(3)_E, \\
U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\rm PQ} \times U(1)_E. \qquad (53)
\end{aligned}
$$

Out of the five $U(1)$ charges, three can be identified with baryon number (B), lepton number (L) and hypercharge (Y), which are respected by the Yukawa interactions. The two remaining $U(1)$ groups can be identified with the PQ symmetry whereby the Higgs and D_R, E_R fields have opposite charges, and with a global rotation of E_R only.

The point that is important for our purposes is that $\mathcal{L}_{\rm kin}$ respects the non-Abelian flavor symmetry $SU(3)_q^3 \times SU(3)_\ell^2$, under which

$$
Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R, \qquad (54)
$$

where the V_i are unitary matrices. The Yukawa interactions (16) break the global symmetry into the subgroup of Eq. (51). (Of course, the gauged $U(1)_Y$ also remains a good symmetry.) Thus, the transformations of Eq. (54) are not a symmetry of $\mathcal{L}_{\rm SM}$. Instead, they correspond to a change of the interaction basis. These observations also offer an alternative way of defining flavor physics: it refers to interactions that break the $[SU(3)]^5$ symmetry (54). Thus, the term "**flavor violation**" is often used to describe processes or parameters that break the symmetry.

One can think of the quark Yukawa couplings as spurions that break the global $SU(3)_q^3$ symmetry (but are neutral under $U(1)_B$),

$$
Y^u \sim (3, \bar{3}, 1)_{SU(3)_q^3}, \quad Y^d \sim (3, 1, \bar{3})_{SU(3)_q^3}, \qquad (55)
$$

and of the lepton Yukawa couplings as spurions that break the global $SU(3)_\ell^2$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$
Y^e \sim (3, \bar{3})_{SU(3)_\ell^2}. \qquad (56)
$$

The spurion formalism is convenient for several purposes: parameter counting (see below), identification of flavor suppression factors (see Section VII), and the idea of minimal flavor violation (see Section VII B).

E. Counting parameters

How many independent parameters are there in $\mathcal{L}_{\rm Yuk}^q$? The two Yukawa matrices, Y^u and Y^d, are 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. The pattern of $G_{\rm global}$ breaking means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$). For example, we can use the unitary transformations $Q_L \to V_Q Q_L$, $U_R \to V_U U_R$ and $D_R \to V_D D_R$, to lead to the following interaction basis:

$$Y^d = \hat{Y}_d, \quad Y^u = V^\dagger \hat{Y}_u, \tag{57}$$

where $\hat{Y}_{d,u}$ are diagonal and real,

$$\hat{Y}_d = {\rm diag}(y_d, y_s, y_b), \quad \hat{Y}_u = {\rm diag}(y_u, y_c, y_t), \tag{58}$$

while V is a unitary matrix that depends on three real angles and one complex phase. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we identify the nine real parameters as six quark masses and three mixing angles, while the single phase is $\delta_{\rm KM}$.

How many independent parameters are there in $\mathcal{L}^\ell_{\rm Yuk}$? The Yukawa matrix Y^e is 3×3 and complex. Consequently, there are 9 real and 9 imaginary parameters in this matrix. There is, however, freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two 3×3 unitary matrices minus the phases related to $[U(1)]^3$). For example, we can use the unitary transformations $L_L \to V_L L_L$ and $E_R \to V_E E_R$, to lead to the following interaction basis:

$$Y^e = \hat{Y}_e = {\rm diag}(y_e, y_\mu, y_\tau), \tag{59}$$

where $y_{e,\mu,\tau}$ are real. We conclude that there are 3 real lepton flavor parameters. In the mass basis, we identify these parameters as the three charged lepton masses. We must, however, modify the model when we take into account the evidence for neutrino masses.

F. The strong CP problem

The counting of parameters described above is done at the classical level. Usually, when quantizing a system, the number of parameters is not changed. Yet, there are exceptions that are related to non-Abelian gauge groups. In the SM it turns out that there is one more renormalizable parameter that is unphysical at the classical level but is physical at the quantum level. This parameter is called $\theta_{\rm QCD}$:

$$\mathcal{L}_{\theta_{\rm QCD}} = \frac{\theta_{\rm QCD}}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}G_a^{\mu\nu}G_a^{\rho\sigma}. \tag{60}$$

This term violates P and CP. In particular, it leads to an electric dipole moment (EDM) of the neutron d_n. The experimental upper bound on the EDM of the neutron,

$$d_n < 2.9\times 10^{-26}\ e\ {\rm cm}, \tag{61}$$

implies that, $\theta_{\rm QCD} \lesssim 10^{-9}$, that is it is very small. The problem of why $\theta_{\rm QCD}$ is so small is known as the strong CP problem. We do not discuss it any further here.

III. THE CKM MATRIX

Among the SM interactions, the W-mediated interactions are the only ones that are not diagonal. Consequently, all flavor changing processes depend on the CKM parameters. The fact that there are only four independent CKM parameters, while the number of measured flavor changing processes is much larger, allows for stringent tests of the CKM mechanism for flavor changing processes and of the KM mechanism of CP violation.

A. Parametrization of the CKM matrix

The CKM matrix V is a 3×3 unitary matrix. Its form, however, is not unique:

(*i*) There is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1,u_2,u_3)\to(u,c,t)$ and $(d_1,d_2,d_3)\to(d,s,b)$. The elements of V are written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{62}$$

(*ii*) There is further freedom in the phase structure of V. This means that the number of physical parameters in V is smaller than the number of parameters in a general unitary 3×3 matrix which is nine (three real angles and six phases). Let us define P_q $(q=u,d)$ to be diagonal unitary (phase) matrices. Then, if instead of using V_{qL} and V_{qR} for the rotations (20) and (23) to the mass basis we use $\tilde V_{qL}$ and $\tilde V_{qR}$, defined by $\tilde V_{qL} = P_q V_{qL}$ and

$\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since $M_q^{\rm diag}$ remains unchanged by such transformations. However, V does change:

$$V \to P_u V P_d^*. \tag{63}$$

This freedom is fixed by demanding that V has the minimal number of phases. In the three generation case V has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of CP violation in the quark sector of the Standard Model [2].

The fact that V is unitary and depends on only four independent physical parameters can be made manifest by choosing a specific parametrization. The standard choice is [4]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{64}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The θ_{ij}'s are the three real mixing parameters while δ is the Kobayashi-Maskawa phase. The experimental central values of the four parameters are given by

$$s_{12} = 0.225, \quad s_{23} = 0.042, \quad s_{13} = 0.0037, \quad \delta = 74^o. \tag{65}$$

As we will later see, experiments imply a hierarchy in the size of the CKM entries:

$$|V_{ub}|, |V_{td}| \ll |V_{cb}|, |V_{ts}| \ll |V_{us}|, |V_{cd}| \ll |V_{ud}|, |V_{cs}|, |V_{tb}|. \tag{66}$$

Consequently, it is convenient to choose an approximate expression where this hierarchy is manifest. This is the Wolfenstein parametrization, where the four mixing parameters are (λ, A, ρ, η) with $\lambda = |V_{us}| \approx 0.23$ playing the role of an expansion parameter and η representing the CP violating phase [5, 6]:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \tag{67}$$

The experimental ranges for the four parameters are given by

$$\begin{aligned} \lambda &= 0.2251 \pm 0.0005, \\ A &= 0.81 \pm 0.03, \\ \rho &= +0.160 \pm 0.007, \\ \eta &= +0.350 \pm 0.006. \end{aligned} \tag{68}$$

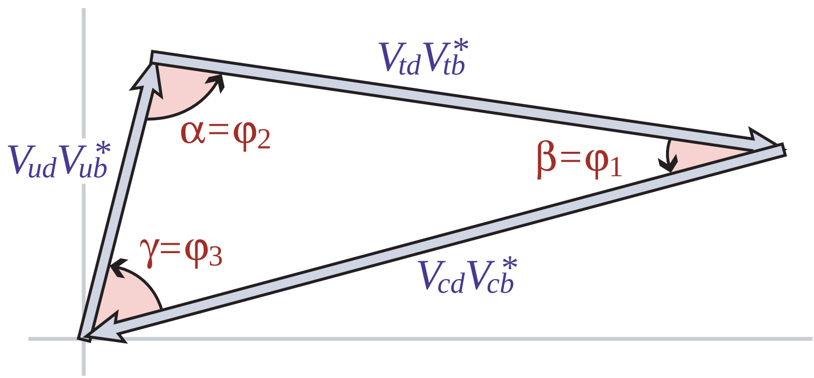

FIG. 1: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

B. Unitarity triangles

A very useful concept is that of the *unitarity triangles*. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{69}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{70}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{71}$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (71) only. The unitarity triangle related to Eq. (71) is depicted in Fig. 1.

The rescaled unitarity triangle is derived from (71) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η). The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the (ρ, η) plane, the lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2 + \eta^2}, \quad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2 + \eta^2}. \tag{72}$$

TABLE IV: FCCC processes and CKM entries. The values of the parameters are taken from Ref. [9].

Process	CKM
$u \to d\ell^+\nu$	$\lvert V_{ud}\rvert = 0.97420 \pm 0.00021$
$s \to u\ell^-\bar{\nu}$	$\lvert V_{us}\rvert = 0.2243 \pm 0.0005$
$c \to d\ell^+\nu$ or $\nu_\mu + d \to c + \mu^-$	$\lvert V_{cd}\rvert = 0.218 \pm 0.004$
$c \to s\ell^+\nu$ or $c\bar{s} \to \ell^+\nu$	$\lvert V_{cs}\rvert = 0.997 \pm 0.017$
$b \to c\ell^-\bar{\nu}$	$\lvert V_{cb}\rvert = 0.0422 \pm 0.0008$
$b \to u\ell^-\bar{\nu}$	$\lvert V_{ub}\rvert = 0.0039 \pm 0.0004$
$pp \to tX$	$\lvert V_{tb}\rvert = 1.02 \pm 0.03$
$b \to sc\bar{u}$ and $b \to su\bar{c}$	$\gamma = 73 \pm 5^o$

The three angles of the unitarity triangle are defined as follows [7, 8]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{73}$$

They are physical quantities and can be independently measured by CP asymmetries in B decays. It is also useful to define the two small angles of the unitarity triangles (70,69):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{74}$$

In terms of the Wolfenstein parameter λ, the angles $\alpha, \beta, \gamma = \mathcal{O}(1)$, $\beta_s = \mathcal{O}(\lambda^2)$ and $\beta_K = \mathcal{O}(\lambda^4)$.

C. The CKM matrix from tree level processes

The absolute values of seven entries and, in addition, one phase of the CKM matrix are extracted from tree level processes, see Table IV.

These eight measurements already over-constrain the four Wolfenstein parameters, but the CKM mechanism passes this test successfully. The ranges that are consistent with all tree level measurements are the following:

$$\lambda = 0.2245 \pm 0.0005, \quad A = 0.84 \pm 0.02, \quad \rho = +0.14 \pm 0.04, \quad \eta = +0.37 \pm 0.03. \tag{75}$$

The λ and A parameters are very well determined. The main effort in CKM measurements is thus aimed at further improving our knowledge of ρ and η. The present status of our knowledge is best seen in a plot of the various constraints and the final allowed region in the $\rho - \eta$ plane. This is shown in Fig. 2. The present status of our knowledge of the absolute values of the various entries in the CKM matrix can be summarized as follows:

$$|V| = \begin{pmatrix} 0.97434 \pm 0.00012 & 0.22506 \pm 0.00050 & (3.57 \pm 0.15) \times 10^{-3} \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ (8.75 \pm 0.33) \times 10^{-3} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}. \qquad (76)$$

D. CP violation

In this section we prove that CP violation in a two generation SM is impossible. In contrast, in a three generation SM, CP is violated in general. This CP violation requires, however, a long list of necessary conditions on the SM flavor parameters.

1. SM2: CP conserving

Consider a two generation Standard Model, SM2. This model is similar to the one defined in Section II, which in this section will be referred to as SM3, except that there are two, rather than three fermion generations. Many features of SM2 are similar to SM3, but there is one important difference: CP is a good symmetry of SM2, but not of SM3. To see how this difference comes about, let us examine the accidental symmetries of SM2. We follow here the line of analysis of SM3 in Section II E.

If we set the Yukawa couplings to zero, $\mathcal{L}_{\rm Yuk}^{\rm SM2} = 0$, SM2 gains an accidental global symmetry:

$$G_{\rm SM2}^{\rm global}(Y^{u,d,e} = 0) = U(2)_Q \times U(2)_U \times U(2)_D \times U(2)_L \times U(2)_E, \qquad (77)$$

where the two generations of each gauge representation are a doublet of the corresponding $U(2)$. The Yukawa couplings break this symmetry into the subgroup

$$G_{\rm SM2}^{\rm global} = U(1)_B \times U(1)_e \times U(1)_\mu. \qquad (78)$$

A-priori, the Yukawa terms depend on three 2×2 complex matrices, namely $12_R + 12_I$ parameters. The global symmetry breaking, $[U(2)]^5 \to [U(1)]^3$, implies that we can remove $5 \times (1_R + 3_I) - 3_I = 5_R + 12_I$ parameters. Thus the number of physical flavor parameters is 7 real parameters and no imaginary parameter. The real parameters can be identified as two charged lepton masses, four quark masses, and the single real mixing angle, $\sin\theta_c = |V_{us}|$.

The important conclusion for our purposes is that all imaginary couplings can be removed from SM2, and CP is an accidental symmetry of the model.

2. *SM3: Not necessarily CP violating*

A-priori, CP is not necessarily violated in SM3. If two quarks of the same charge had equal masses, one mixing angle and the phase could be removed from V. This can be written as a condition on the quark mass differences. CP violation requires

$$(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \neq 0. \tag{79}$$

Likewise, if the value of any of the three mixing angles were 0 or $\pi/2$, then the phase can be removed. Finally, CP would not be violated if the value of the single phase were 0 or π. These last eight conditions are elegantly incorporated into one, parametrization-independent condition. To find this condition, note that the unitarity of the CKM matrix, $VV^\dagger = 1$, requires that for any choice of $i,j,k,l = 1,2,3$,

$$\mathcal{I}m[V_{ij}V_{kl}V_{il}^*V_{kj}^*] = J\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln}. \tag{80}$$

In terms of the explicit parameterizations given in Eqs. (64) and (67), we have

$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta \approx \lambda^6 A^2 \eta. \tag{81}$$

The conditions on the mixing parameters are summarized by

$$J \neq 0. \tag{82}$$

The quantity J is called the Jarlskog invariant [10] and is of much interest in the study of CP violation from the CKM matrix. The maximum value that J could assume in principle is $1/(6\sqrt{3}) \approx 0.1$, but it is found to be [9]

$$J = (3.18 \pm 0.15) \times 10^{-5}. \tag{83}$$

It is interesting to note that the areas of all six unitarity triangles are the same, and equal $J/2$.

TABLE V: Measurements related to neutral meson mixing

Sector	CP-conserving	CP-violating
sd	$\Delta m_K/m_K = 7.0 \times 10^{-15}$	$\epsilon_K = 2.3 \times 10^{-3}$
cu	$\Delta m_D/m_D = 8.7 \times 10^{-15}$	$A_\Gamma/y_{\rm CP} \lesssim 0.2$
bd	$\Delta m_B/m_B = 6.3 \times 10^{-14}$	$S_{\psi K} = +0.70 \pm 0.02$
bs	$\Delta m_{B_s}/m_{B_s} = 2.1 \times 10^{-12}$	$S_{\psi\phi} = -0.04 \pm 0.06$

The fourteen conditions incorporated in Eqs. (79) and (82) can all be written as a single requirement on the quark mass matrices in the interaction basis:

$$X_{CP} \equiv \mathcal{I}m\left\{\det\left[M_d M_d^\dagger, M_u M_u^\dagger\right]\right\} \neq 0 \ \Leftrightarrow \ \text{CP violation.} \tag{84}$$

This is a convention independent condition.

IV. FLAVOR CHANGING NEUTRAL CURRENT (FCNC) PROCESSES

A very useful class of FCNC is that of neutral meson mixing. Nature provides us with four pairs of neutral mesons: $K^0 - \overline{K}^0$, $B^0 - \overline{B}^0$, $B_s^0 - \overline{B}_s^0$, and $D^0 - \overline{D}^0$. Mixing in this context refers to a transition such as $K^0 \to \overline{K}^0$ ($\bar{s}d \to \bar{d}s$).[2] The experimental results for CP conserving and CP violating observables related to neutral meson mixing (mass splittings and CP asymmetries in tree level decays, respectively) are given in Table V.

Our aim in the following subsections is to explain the suppression factors that affect FCNC and the special role of CP violation within the SM.

A. Loop suppression

The W-boson cannot mediate FCNC processes at tree level, since it couples to up-down pairs, or to neutrino-charged lepton pairs. Obviously, only neutral bosons can mediate FCNC at tree level. The SM has four neutral bosons: the gluon, the photon, the Z-boson

[2] These transitions involve four-quark operators. When calculating the matrix elements of these operators between meson-antimeson states, approximate symmetries of QCD are of no help. Instead, one uses lattice calculations to relate, for example, the $B^0 \to \overline{B}^0$ transition to the corresponding quark process, $\bar{b}d \to \bar{d}b$.

and the Higgs-boson. As concerns the massless gauge bosons, the gluon and the photon, their couplings are flavor-universal and, in particular, flavor-diagonal. This is guaranteed by gauge invariance. The universality of the kinetic terms in the canonical basis requires universality of the gauge couplings related to the unbroken symmetries. Hence neither the gluon nor the photon can mediate flavor changing processes at tree level. The situation concerning the Z-boson and the Higgs-boson is more complicated. In fact, the diagonality of their tree-level couplings is a consequence of special features of the SM, and can be violated with new physics.

The Z-boson, similarly to the W-boson, does not correspond to an unbroken gauge symmetry (as manifest in the fact that it is massive). Hence, there is no fundamental symmetry principle that forbids flavor changing couplings. Yet, as mentioned in Section II C 2, in the SM this does not happen. The key point is the following. For each sector of mass eigenstates, characterized by spin, $SU(3)_C$ representation and $U(1)_{\rm EM}$ charge, there are two possibilities:

1. All mass eigenstates in this sector originate from interaction eigenstates in the same $SU(2)_L \times U(1)_Y$ representation.

2. The mass eigenstates in this sector mix interaction eigenstates of different $SU(2)_L \times U(1)_Y$ representations (but, of course, with the same $T_3 + Y$).

Let us examine the Z couplings in the interaction basis in the subspace of all states that mix within a given sector of mass eigenstates:

1. In the first class, the Z couplings in this subspace are universal, namely they are proportional to the unit matrix (times $T_3 - Q\sin^2\theta_W$ of the relevant interaction eigenstates). The rotation to the mass basis maintains the universality: $V_{fM} \times \mathbf{1} \times V_{fM}^\dagger = \mathbf{1}$ ($f = u, d, e$; $M = L, R$).

2. In the second class, the Z couplings are only "block-universal". In each sub-block i of m_i interaction eigenstates that have the same $(T_3)_i$, they are proportional to the $m_i \times m_i$ unit matrix, but the overall factor of $(T_3)_i - Q\sin^2\theta_W$ is different between the sub-blocks. In this case, the rotation to the mass basis, $V_{fM} \times {\rm diag}\{[(T_3)_1 - Qs_W^2]\mathbf{1}_{m_1}, [(T_3)_2 - Qs_W^2]\mathbf{1}_{m_2}, \ldots\} \times V_{fM}^\dagger$, does not maintain the universality, nor even the diagonality.

The special feature of the SM fermions is that they belong to the first class: All fermion mass eigenstates in a given $SU(3)_C \times U(1)_{\rm EM}$ representation come from the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ representation.[3] For example, all the left-handed up quark mass eigenstates, which are in the $(3)_{+2/3}$ representation, come from interaction eigenstates in the $(3,2)_{+1/6}$ representation. This is the reason that the SM predicts universal Z couplings to fermions. If, for example, Nature had left-handed quarks also in the $(3,1)_{+2/3}$ representation, then the Z couplings in the left-handed up sector would be non-universal and the Z could mediate FCNC.

The Yukawa couplings of the Higgs boson are not universal. In fact, in the interaction basis, they are given by completely general 3×3 matrices. Yet, as explained in Section II C 4, in the fermion mass basis they are diagonal. The reason is that the fermion mass matrix is proportional to the corresponding Yukawa matrix. Consequently, the mass matrix and the Yukawa matrix are simultaneously diagonalized. The special features of the SM in this regard are the following:

1. All the SM fermions are chiral, and therefore there are no bare mass terms.

2. The scalar sector has a single Higgs doublet.

In contrast, either of the following possible extensions would lead to flavor changing Higgs couplings:

1. There are quarks or leptons in vector-like representations, and thus there are bare mass terms.

2. There is more than one $SU(2)_L$-doublet scalar.

We conclude that within the SM, all FCNC processes are loop suppressed. However, in extensions of the SM, FCNC can appear at the tree level, mediated by the Z boson or by the Higgs boson or by new massive bosons.

[3] This is not true for the SM bosons. The vector boson mass eigenstates in the $(1)_0$ representation come from interaction eigenstates in the $(1,3)_0$ and $(1,1)_0$ representations (W_3 and B, respectively).

B. CKM suppression

Obviously, all flavor changing processes are proportional to off-diagonal entries in the CKM matrix. A quick look at the absolute values of the off-diagonal entries of the CKM matrix (76) reveals that they are small. A rough estimate of the CKM suppression can be acquired by counting powers of λ in the Wolfenstein parametrization (67): $|V_{us}|$ and $|V_{cd}|$ are suppressed by λ, $|V_{cb}|$ and $|V_{ts}|$ by λ^2, $|V_{ub}|$ and $|V_{td}|$ by λ^3.

For example, the amplitude for $b \to s\gamma$ decay comes from penguin diagrams, dominated by the intermediate top quark, and suppressed by $|V_{tb}V_{ts}| \sim \lambda^2$. As another example, the $B^0 - \overline{B}^0$ mixing amplitude comes from box diagrams, dominated by intermediate top quarks, and suppressed by $|V_{tb}V_{td}|^2 \sim \lambda^6$.

C. GIM suppression

If all quarks in a given sector were degenerate, then there would be no flavor changing W-couplings. A consequence of this fact is that FCNC in the down (up) sector are proportional to mass-squared differences between the quarks of the up (down) sector. For FCNC processes that involve only quarks of the first two generations, this leads to a strong suppression factor related to the light quark masses, and known as Glashow-Iliopoulos-Maiani (GIM) suppression [11].

Let us take as an example Δm_K, the mass splitting between the two neutral K-mesons. We have $\Delta m_K = 2|M_{K\bar{K}}|$, where $M_{K\bar{K}}$ corresponds to the $\overline{K}^0 \to K^0$ transition and comes from box diagrams. The top contribution is CKM-suppressed compared to the contributions from intermediate up and charm, so we consider only the latter:

$$M_{K\bar{K}} \simeq \sum_{i,j=u,c} \frac{G_F^2 m_W^2}{16\pi^2} \langle K^0 | (\bar{d}_L \gamma^\mu s_L)^2 | \overline{K}^0 \rangle (V_{is}V_{id}^* V_{js} V_{jd}^*) \times F(x_i, x_j), \tag{85}$$

where $x_i = m_i^2/m_W^2$. If we had $m_u = m_c$, the amplitude would be proportional to $(V_{us}V_{ud}^* + V_{cs}V_{cd}^*)^2$, which vanishes in the two generation limit. We conclude that $\Delta m_K \propto (m_c^2 - m_u^2)/m_W^2$, which is the GIM suppression factor.

For the $B^0 - \overline{B}^0$ and $B_s - \overline{B}_s$ mixing amplitudes, the top-mediated contribution is not CKM suppressed compared to the lighter generations. The mass ratio m_t^2/m_W^2 enhances, rather than suppresses, the top contribution. Consequently, the $M_{B\bar{B}}$ amplitude is dominated by the top contribution:

$$M_{B\bar{B}} \simeq \frac{G_F^2 m_W^2}{16\pi^2} \langle B^0 | (\bar{d}_L \gamma^\mu b_L)^2 | \overline{B}^0 \rangle (V_{tb} V_{td}^*)^2 \times F(x_t, x_t) \,. \tag{86}$$

D. CPV suppression

In some cases, CP violating (CPV) observables related to neutral meson mixing are CKM suppressed beyond their CP conserving (CPC) counterparts. The two most relevant examples are the suppression of ϵ_K, related to $K^0 - \overline{K}^0$ mixing, and the suppression of $S_{\psi\phi}$, related to $B_s - \overline{B_s}$ mixing.

Numerically, the effect of the extra suppression is most significant in the case of CPV in kaon mixing. The ϵ_K observable depends on the relative phase between the CKM combination that contributes to $K^0 - \overline{K}^0$ mixing, $(V_{cd} V_{cs}^*)^2$, and the CKM combination that is related to K^0 and $\overline{K}^0$ decays into $\pi\pi$, $(V_{ud} V_{us}^*)^2$. This phase is of order $|(V_{td} V_{ts}^*)/(V_{ud} V_{us}^*)| \sim \lambda^4$, which explains why $\epsilon_K = \mathcal{O}(10^{-3})$. It can also be understood by examining the $s - d$ unitarity triangle: it is of the order of the ratio between its area (representing CPV) and its long side squared (representing CPC).

The effect of the extra suppression is also significant in the case of CPV in the B_s system. The $S_{\psi\phi}$ observable depends on the relative phase between the CKM combination that contributes to $B_s - \overline{B_s}$ mixing, $(V_{tb} V_{ts}^*)^2$, and the CKM combination that is related to B_s and $\overline{B_s}$ decays into $\psi\phi$, $(V_{cb} V_{cs}^*)^2$. This phase is of order $|(V_{ub} V_{us}^*)/(V_{cb} V_{cs}^*)| \sim \lambda^2$, which explains why $S_{\psi\phi} = \mathcal{O}(10^{-2})$. It can also be understood by examining the $b - s$ unitarity triangle: it is of the order of the ratio between its area (representing CPV) and its long side squared (representing CPC).

V. SM CALCULATIONS OF CPV

In this section we give several examples of CPV observables that are used to test the CKM mechanism of flavor violation and to search for or constrain new flavor physics.

A. CPV in decay: $B \to DK$

In order to have CP violation in decay, two amplitudes with different weak and different strong phases have to contribute. In most cases, the calculation involves large hadronic uncertainties, as the size of an amplitude and its strong phase are inherently non-perturbative. Yet, there are a few cases where we can use approximate symmetries of QCD or related measurements to determine the hadronic parameters. In these cases, CP violation in decay serves as a clean probe of CKM parameters. Here we discuss one of these cases: the extraction of the phase γ from $B \to DK$ decays.

Consider the following three $B \to DK$ decay modes:

$$B^+ \to D^0 K^+, \qquad B^+ \to \overline{D}^0 K^+, \qquad B^+ \to D_{CP} K^+, \tag{87}$$

where D_{CP} is a state that decays into a CP eigenstate. For the sake of concreteness, we take $D_{CP} = (K^+K^-)_D$, namely the state that is tagged by a final K^+K^- state with $(p_{K^+} + p_{K^-})^2 = m_D^2$. In what follows, we neglect $D^0 - \overline{D}^0$ mixing and CP violation in D decays. In this case, the CP-even D_{CP} can be written as

$$D_{CP} = \frac{1}{\sqrt{2}}(D^0 + \overline{D}^0). \tag{88}$$

The $B^+ \to D_{CP} K^+$ can proceed in two ways:

1. $B^+ \to \overline{D}^0 K^+$ followed by $\overline{D}^0 \to K^+K^-$. The corresponding quark transitions are $\bar{b} \to \bar{c} u \bar{s}$ followed by $\bar{c} \to \bar{s} s \bar{u}$. The CKM factors are

$$(V_{cb}^* V_{us})(V_{cs} V_{us}^*). \tag{89}$$

2. $B^+ \to D^0 K^+$ followed by $D^0 \to K^+K^-$. The corresponding quark transitions are $\bar{b} \to \bar{u} c \bar{s}$ followed by $c \to s \bar{s} u$. The CKM factors are

$$(V_{ub}^* V_{cs})(V_{cs}^* V_{us}). \tag{90}$$

CP violation comes from the interference between these decay amplitudes. It is proportional to the relative weak phase, namely

$$\arg\left(\frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}}\right) = -\gamma. \tag{91}$$

Let us define the following pairs of CP conjugate amplitudes:

$$\begin{aligned}
A_{D^0K^+} &\equiv A(B^+ \to D^0K^+), & \overline{A}_{\overline{D}^0K^-} &\equiv A(B^- \to \overline{D}^0K^-),\\
A_{\overline{D}^0K^+} &\equiv A(B^+ \to \overline{D}^0K^+), & \overline{A}_{D^0K^-} &\equiv A(B^- \to D^0K^-),\\
A_{D_{CP}K^+} &\equiv A(B^+ \to D_{CP}K^+), & \overline{A}_{D_{CP}K^-} &\equiv A(B^- \to D_{CP}K^-)\,. \qquad (92)
\end{aligned}$$

Given Eq. (88) we have

$$A_{D_{CP}K^+} = \frac{A_{D^0K^+} + A_{\overline{D}^0K^+}}{\sqrt{2}}, \qquad \overline{A}_{D_{CP}K^-} = \frac{\overline{A}_{\overline{D}^0K^-} + \overline{A}_{D^0K^-}}{\sqrt{2}}\,. \qquad (93)$$

The decay amplitudes $A_{D^0K^+}$ and $A_{\overline{D}^0K^+}$ differ in magnitude, weak phase and strong phase. The weak phase difference is γ, see Eq. (91). We define the ratio of sizes r and the strong phase difference δ via

$$\frac{A_{\overline{D}^0K^+}}{A_{D^0K^+}} = re^{i(\delta+\gamma)}, \qquad \frac{\overline{A}_{D^0K^-}}{\overline{A}_{\overline{D}^0K^-}} = re^{i(\delta-\gamma)}. \qquad (94)$$

We obtain the following decay rates for the flavor-specific final states:

$$\begin{aligned}
\Gamma(B^+ \to D^0K^+) &= \Gamma(B^- \to \overline{D}^0K^-) = |A_{D^0K^+}|^2,\\
\Gamma(B^+ \to \overline{D}^0K^+) &= \Gamma(B^- \to D^0K^-) = |A_{D^0K^+}|^2 r^2, \qquad (95)
\end{aligned}$$

and for the CP eigenstate final states:

$$\begin{aligned}
\Gamma(B^+ \to D_{CP}K^+) &= |A_{D_{CP}K^+}|^2 = \frac{|A_{D^0K^+}|^2}{2}\left[1 + r^2 + 2r\cos(\delta+\gamma)\right],\\
\Gamma(B^- \to D_{CP}K^-) &= |\overline{A}_{D_{CP}K^-}|^2 = \frac{|A_{D^0K^+}|^2}{2}\left[1 + r^2 + 2r\cos(\delta-\gamma)\right]. \qquad (96)
\end{aligned}$$

There are two points to be made here. First, since the two rates in Eq. (96) are CP conjugates of each other we can define the CP asymmetry in decay as in Eq. (B5) with $f^\pm = D_{CP}K^\pm$ and we obtain, for $r \ll 1$,

$$\mathcal{A}_{D_{CP}K} = 2r\sin\gamma\sin\delta. \qquad (97)$$

We thus demonstrated that CP violation in decay arises when we have two interfering amplitudes with different weak and different strong phases.

The second point is that all the parameters that are necessary to extract γ, that is $|A_{D^0K^+}|, r$ and δ, can be determined experimentally. The flavor eigenstates D^0 and $\overline{D}^0$ can be tagged with semileptonic decays so that the two rates of Eq. (95) enable us to measure

$|A_{D^0K^+}|$ and r. Then, the two rates of Eq. (96) allow us to further determine δ and γ (up to a discrete ambiguity in the phases). The crucial point is that we do not need to calculate the size of the amplitudes and the strong phase. Instead, they are extracted from the measurements. Thus, $B \to DK$ decays provide a theoretically clean determination of the CP violating phase γ.

B. CPV in interference of decays with and without mixing: $B \to \psi K_S$

We give here an example of the SM contribution to CP violation in the interference of decays with and without mixing in the $B \to \psi K_S$ mode [12, 13]. This is often called "the golden mode" with regard to CP violation, as its theoretical calculation is uniquely clean of hadronic uncertainties. In fact, the CP asymmetry can be translated into a value of $\sin 2\beta$ [β is defined in Eq. (73)], with a theoretical uncertainty smaller than one percent.

For the neutral B meson system, $|\Gamma_{B\bar{B}}/M_{B\bar{B}}| \ll 1$ holds. Rewriting Eq. (98) as

$$M_{B\bar{B}} \simeq \frac{G_F^2 m_W^2}{12\pi^2} m_B m_W^2 (B_B f_B^2) S(x_t) (V_{tb}V_{td}^*)^2, \tag{98}$$

we obtain

$$\frac{M_{B\bar{B}}^*}{|M_{B\bar{B}}|} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}. \tag{99}$$

The K_S meson is the CP-even kaon state. For the sake of concreteness, we take $K_S = (\pi^+\pi^-)_K$, namely the state that is tagged by a final $\pi^+\pi^-$ state with $(p_{\pi^+} + p_{\pi^-})^2 = m_K^2$. Thus,

- $B^0 \to \psi K_S$ proceeds via $B^0 \to \psi K^0$ followed by $K^0 \to \pi^+\pi^-$. The corresponding quark transitions are $\bar{b} \to \bar{c}c\bar{s}$, followed by $\bar{s} \to \bar{u}u\bar{d}$. The CKM factors are

$$(V_{cb}^* V_{cs})(V_{us}^* V_{ud}). \tag{100}$$

- $\overline{B}^0 \to \psi K_S$ proceeds via $\overline{B}^0 \to \psi \overline{K}^0$ followed by $\overline{K}^0 \to \pi^+\pi^-$. The corresponding quark transitions are $b \to c\bar{c}s$, followed by $s \to u\bar{u}d$. The CKM factors are

$$(V_{cb} V_{cs}^*)(V_{us} V_{ud}^*). \tag{101}$$

We further use the fact that the $s - d$ unitarity triangle is very squashed and thus

$$V_{us}^* V_{ud} \approx -V_{cs}^* V_{cd}. \tag{102}$$

We obtain:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}. \tag{103}$$

Combining Eq. (99) and Eq. (103), we obtain

$$\lambda_{B\to\psi K_S} = \frac{M_{B\bar{B}}^*}{|M_{B\bar{B}}|}\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -e^{-2i\beta} \implies \mathcal{I}m(\lambda_{B\to\psi K_S}) = \sin 2\beta. \tag{104}$$

This result demonstrates the power of CP asymmetries in measuring CKM parameters. The experimental measurement of $\mathcal{I}m(\lambda_{\psi K_S})$ translates directly into the value of a CKM parameter, β, with very little hadronic parameters. A crucial role is played by the CP symmetry of the strong interactions. The size and the strong phase of the amplitude $A_{\psi K}$ cannot be calculated, but they are the same in the CP conjugate amplitudes $\overline{A}_{\psi K_S}$ and $A_{\psi K_S}$ and therefore cancel out when their ratio is taken.

A related example is that of the $B_s \to \psi\phi$. The analysis goes along similar lines to that of $B \to \psi K$. We obtain

$$\lambda_{B_s\to\psi\phi} = \frac{M_{B_s\bar{B}_s}^*}{|M_{B_s\bar{B}_s}|}\frac{\overline{A}_{\psi\phi}}{A_{\psi\phi}} = -e^{-2i\beta_s} \implies \mathcal{I}m(\lambda_{B_s\to\psi\phi}) = \sin 2\beta_s. \tag{105}$$

with β_s the equivalent of β in the $b-s$ unitarity triangle, see Eq. (74).

C. CPV in mixing: $K \to \pi\pi$ and $K \to \pi\ell\nu$

CPV was discovered in neutral kaon decays. We consider the following two CP asymmetries: The δ_L asymmetry in the $K \to \pi\ell\nu$ decays, defined in Eq. (B10), and the $A_{\pi\pi}^{\rm mass}$ asymmetry in the $K \to \pi\pi$ decays, defined in Eq. (B14). (To leading order, the result is the same for $f = \pi^+\pi^-$ and $f = \pi^0\pi^0$ final state, so we do not distinguish between them.)

CP violation in the $K \to \pi\pi$ and $K \to \pi\ell\nu$ decays has two features that allow to circumvent most of the hadronic uncertainties in its analysis:

- CP violation in mixing and in the interference of mixing and decays is small, of $\mathcal{O}(10^{-3})$.
- CP violation in decays is negligibly small, of $\mathcal{O}(10^{-6})$ for $K \to \pi\pi$ and even smaller for $K \to \pi\ell\nu$.

Neglecting CP violation in decay, we can write

$$\lambda_{\pi\pi} = -|q/p|e^{i\phi}. \tag{106}$$

Note that $\sin\phi \neq 0$ signifies CPV in the interference of decays with and without mixing, while $|q/p| \neq 1$ signifies CPV in mixing. One defines a CP violating ϵ_K via

$$\epsilon_K \equiv \frac{1+\lambda_{\pi\pi}}{1-\lambda_{\pi\pi}}. \tag{107}$$

Given the smallness of CPV, we can approximate

$$\mathcal{R}e(\epsilon_K) \approx \frac{1}{2}(1-|q/p|), \qquad \mathcal{I}m(\epsilon_K) \approx -\frac{1}{2}\tan\phi. \tag{108}$$

Thus, $\mathcal{I}m(\epsilon_K)$ signifies CPV in the interference of decays with and without mixing, while $\mathcal{R}e(\epsilon_K)$ signifies CPV in mixing.

There are two important measurements that probe ϵ_K:

1. CPV in the $K \to \pi\ell\nu$ decays, defined in Eq. (B10), provides a measurement of $\mathcal{R}e(\epsilon_K)$:

$$\delta_L = 2\mathcal{R}e(\epsilon_K) \approx (1-|q/p|). \tag{109}$$

2. CPV in $K_L \to \pi\pi$, defined Eq. (B14) with $f = \pi\pi$, provides a measurement of $|\epsilon_K|$:

$$\mathcal{A}_{\pi\pi}^{\rm mass} = |\epsilon_K|^2 \approx \frac{\tan^2\phi + (1-|q/p|)^2}{4}. \tag{110}$$

Combining δ_L and $\mathcal{A}_{\pi\pi}^{\rm mass}$ enables one to determine the magnitude and phase of ϵ_K. The data gives

$$|\epsilon_K| = 2.23 \times 10^{-3}, \qquad \arg(\epsilon_K) = 43.5^o. \tag{111}$$

In order to calculate ϵ_K we then need to calculate $|q/p|$ or ϕ. Both of them involve hadronic uncertainties, and here we explain how the SM prediction for $\mathcal{R}e(\epsilon_K) = \frac{1}{2}(1-|q/p|)$ is obtained. What we need to calculate is $\Gamma_{K\bar{K}}$. To overcome the large hadronic uncertainties in such a calculation, we use the experimental result that $\Delta\Gamma_K/\Delta m_K \approx -2$. Furthermore, given that the relevant CP violating effects are experimentally determined to be small, we have $\Delta\Gamma_K/\Delta m_K \simeq |\Gamma_{K\bar{K}}/M_{K\bar{K}}|$. With these two ingredients and in the phase convention where $\Gamma_{K\bar{K}}$ is real, we obtain

$$\mathcal{R}e(\epsilon) \approx \frac{\mathcal{I}m(M_{K\bar{K}})}{4\Delta m_K}. \tag{112}$$

Thus we need to calculate $\mathcal{I}m(M_{K\bar{K}})$.

The SM contribution to $M_{K\bar{K}}$ comes from box diagrams with intermediate up-type quarks. In the phase convention where $\Gamma_{K\bar{K}}$ is real, we have

$$\mathcal{I}m(M_{K\bar{K}}) \simeq \frac{G_F^2 m_W^2}{12\pi^2} m_K m_W^2 (B_K f_K^2) \sum_{i,j=c,t} S(x_i,x_j)\mathcal{I}m[(V_{is}V_{id}^* V_{js}V_{jd}^*)]. \tag{113}$$

While, as discussed in Section IV C, $|M_{K\bar{K}}|$ is dominated by the charm quark, this is not the case for the imaginary part. The reason is that, of the three relevant CKM combinations, the top-related one is highly suppressed: $|V_{td}^* V_{ts}| \sim \lambda^5$ compared to $|V_{cd}^* V_{cs}| \simeq \lambda$. Thus, Δm_K is dominated by the charm quark. As concerns $\mathcal{I}m(M_{K\bar{K}})$, however, the two relevant CKM combinations are equal in size: $\mathcal{I}m\frac{V_{cd}^* V_{cs}}{V_{ud}^* V_{us}} = -\mathcal{I}m\frac{V_{td}^* V_{ts}}{V_{ud}^* V_{us}}$, and thus the intermediate top and charm quarks give comparable contributions to ϵ_K.

VI. TESTING CKM

Measurements of rates, mixing, and CP asymmetries in B decays in the two B factories, BaBar and Belle, and in the two Tevatron detectors, CDF and D0, signified a new era in our understanding of flavor physics and CP violation. The progress has been both qualitative and quantitative. Various basic questions concerning CP and flavor violation have received, for the first time, answers based on experimental information. These questions include, for example,

- Is the Kobayashi-Maskawa mechanism at work (namely, is $\eta \neq 0$)?
- Does the KM phase dominate the observed CP violation?
- Does the CKM mechanism dominate FCNC?

As a first step, one may assume the SM and test the overall consistency of the various measurements. However, the richness of data from the B factories allow us to go a step further and answer these questions model independently, namely allowing new physics to contribute to the relevant processes. We here explain the way in which this analysis proceeds.

A. Is the CKM assumption Self-consistent?

The three generation standard model has room for CP violation, through the KM phase in the quark mixing matrix. Yet, one would like to make sure that indeed CP is violated by the SM interactions, namely that $\sin\delta_{\rm KM} \neq 0$. If we establish that this is the case, we would further like to know whether the SM contributions to CP violating observables are dominant. More quantitatively, we would like to put an upper bound on the ratio between the new physics and the SM contributions.

As a first step, one can assume that flavor changing processes are fully described by the SM, and check the consistency of the various measurements with this assumption. There are four relevant mixing parameters, which can be taken to be the Wolfenstein parameters λ, A, ρ and η defined in Eq. (67). The values of λ and A are known rather accurately [9] from, respectively, $K \to \pi\ell\nu$ and $b \to c\ell\nu$ decays:

$$\lambda = 0.2245 \pm 0.0005, \quad A = 0.836 \pm 0.015. \tag{114}$$

Then, one can express all the relevant observables as a function of the two remaining parameters, ρ and η, and check whether there is a range in the $\rho - \eta$ plane that is consistent with all measurements. The list of observables includes the following:

- The rates of inclusive and exclusive charmless semileptonic B decays depend on $|V_{ub}|^2 \propto \rho^2 + \eta^2$;
- The CP asymmetry in $B \to \psi K_S$, $S_{\psi K_S} = \sin 2\beta = \frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2}$;
- The rates of various $B \to DK$ decays depend on the phase γ, where $e^{i\gamma} = \frac{\rho+i\eta}{\sqrt{\rho^2+\eta^2}}$;
- The rates of various $B \to \pi\pi, \rho\pi, \rho\rho$ decays depend on the phase $\alpha = \pi - \beta - \gamma$;
- The ratio between the mass splittings in the neutral B and B_s systems is sensitive to $|V_{td}/V_{ts}|^2 = \lambda^2[(1-\rho)^2 + \eta^2]$;
- The CP violation in $K \to \pi\pi$ decays, ϵ_K, depends in a complicated way on ρ and η.

The resulting constraints are shown in Fig. 2.

The consistency of the various constraints is impressive. In particular, the following ranges for ρ and η can account for all the measurements [9]:

$$\rho = +0.122 \pm 0.018, \quad \eta = +0.355 \pm 0.012. \tag{115}$$

One can make then the following statements [15]:

Very likely, flavor changing processes are dominated by the Cabibbo-Kobayashi-Maskawa mechanism and, in particular, CP violation in flavor changing processes is dominated by the Kobayashi-Maskawa phase.

In the following subsections, we explain how we can remove the phrase "very likely" from this statement, and how we can quantify the ft(C)KM-dominance.

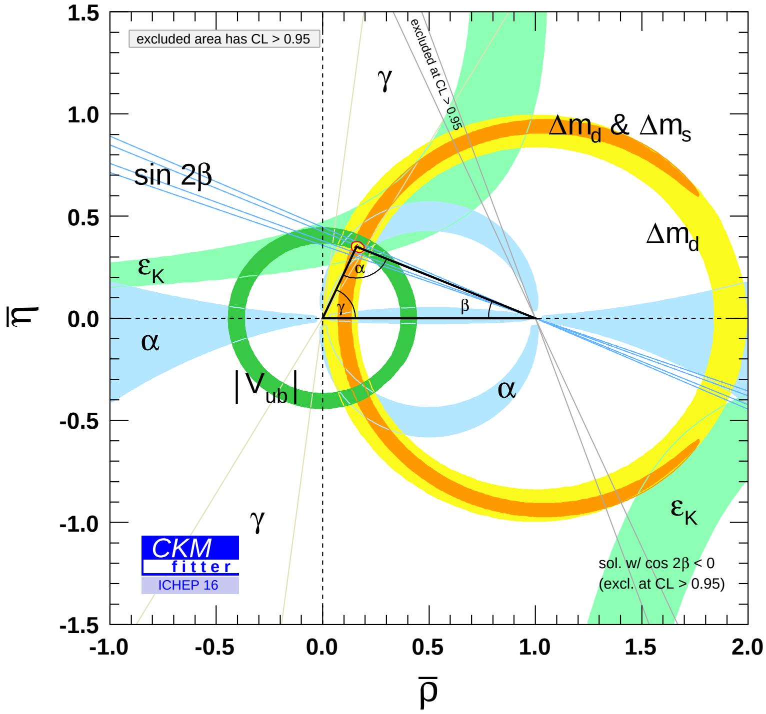

FIG. 2: Allowed region in the ρ, η plane. Superimposed are the individual constraints from charmless semileptonic B decays ($|V_{ub}|$), mass differences in the B^0 (Δm_d) and B_s (Δm_s) neutral meson systems, and CP violation in $K \to \pi\pi$ (ε_K), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ (α), and $B \to DK$ (γ). Taken from [14].

B. $S_{\psi K_S}$

As an example of how to use CPV and FCNC in probing new physics, we take $S_{\psi K_S}$. When we consider extensions of the SM, we still do not expect any significant new contribution to the tree level decay, $b \to c\bar{c}s$, beyond the SM W-mediated diagram. Thus, the expression $\bar{A}_{\psi K_S}/A_{\psi K_S} = (V_{cb}V_{cd}^*)/(V_{cb}^*V_{cd})$ remains valid, though the approximation of neglecting subdominant phases can be somewhat less accurate. On the other hand,since $B^0 - \overline{B}^0$ mixing is an FCNC process, $M_{B\bar{B}}$ can in principle get large and even dominant contributions from new physics. We can parameterize the modification to the SM in terms of a complex parameter Δ_d::

$$M_{B\bar{B}} = \Delta_d \; M_{B\bar{B}}^{\rm SM}(\rho, \eta). \tag{116}$$

Thus $|\Delta_d| \neq 1$ represents a new source of flavor violation:

$$\Delta m_B = |\Delta_d| \times 2|M_{B\bar{B}}^{\rm SM}(\rho,\eta)|, \tag{117}$$

while $\mathcal{I}m(\Delta_d)$ provides a new source of CP violation, leading to the following generalization of Eq. (104):

$$S_{\psi K_S} = \sin\left[2\arctan\left(\eta/(1-\rho)\right) + \arg(\Delta_d)\right], \quad C_{\psi K_S} = 0\ . \tag{118}$$

The experimental measurements give the following ranges [16]:

$$S_{\psi K_S} = +0.70 \pm 0.02, \quad C_{\psi K_S} = -0.005 \pm 0.015\ . \tag{119}$$

C. Is the KM mechanism at work?

In proving that the KM mechanism is at work, we assume that charged-current tree-level processes are dominated by the W-mediated SM diagrams (see, for example, [17]). This is a very plausible assumption. It is difficult to construct a model where new physics competes with the SM in FCCC processes, and does not violate the constraints from FCNC processes. Thus we can use all tree level processes and fit them to ρ and η, as we did before. The list of such processes includes the following:

1. Charmless semileptonic B-decays, $b \to u\ell\nu$, measure R_u [see Eq. (72)].
2. $B \to DK$ decays, which go through the quark transitions $b \to c\bar{u}s$ and $b \to u\bar{c}s$, measure the angle γ [see Eq. (73)].
3. $B \to \rho\rho$ decays (and, similarly, $B \to \pi\pi$ and $B \to \rho\pi$ decays) go through the quark transition $b \to u\bar{u}d$. With an isospin analysis, one can determine the relative phase between the tree decay amplitude and the mixing amplitude. By incorporating the measurement of $S_{\psi K_S}$, one can subtract the phase from the mixing amplitude, finally providing a measurement of the angle γ [see Eq. (73)].

In addition, we can use loop processes, but then we must allow for new physics contributions, in addition to the (ρ,η)-dependent SM contributions. Of course, if each such measurement adds a separate mode-dependent parameter, then we do not gain anything by using this information. However, there is a number of observables where the only relevant

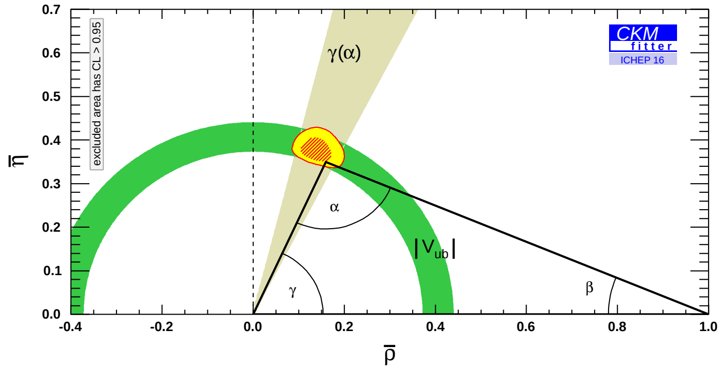

FIG. 3: The allowed region in the $\rho - \eta$ plane, assuming that tree diagrams are dominated by the Standard Model [14].

loop process is $B^0 - \overline{B}^0$ mixing. The list includes $S_{\psi K_S}$, Δm_B and the CP asymmetry in semileptonic B decays:

$$\begin{aligned} S_{\psi K_S} &= \sin\left[2\arctan\left(\eta/(1-\rho)\right) + \arg(\Delta_d)\right], \\ \Delta m_B &= 2|M_{B\bar{B}}^{\rm SM}(\rho,\eta)| \times |\Delta_d|, \\ \mathcal{A}_{\rm SL} &= -\mathcal{R}e\left(\frac{\Gamma_{B\bar{B}}}{M_{B\bar{B}}}\right)^{\rm SM} \frac{\sin[\arg(\Delta_d)]}{|\Delta_d|} + \mathcal{I}m\left(\frac{\Gamma_{B\bar{B}}}{M_{B\bar{B}}}\right)^{\rm SM} \frac{\cos[\arg(\Delta_d)]}{|\Delta_d|}. \end{aligned} \tag{120}$$

As explained above, such processes involve two new parameters [see Eq. (116)]. Since there are three relevant observables, we can further tighten the constraints in the (ρ,η)-plane. Similarly, one can use measurements related to $B_s - \overline{B}_s$ mixing. One gains three new observables at the cost of two new parameters (see, for example, [18]).

The results of such fit, projected on the $\rho - \eta$ plane, can be seen in Fig. 3. It gives [14]

$$\eta = 0.38 \pm 0.02. \tag{121}$$

It is clear that $\eta \neq 0$ is well established:

The Kobayashi-Maskawa mechanism of CP violation is at work.

The consistency of the experimental results (119) with the SM predictions (104) means that the KM mechanism of CP violation dominates the observed CP violation. In the next subsection, we make this statement more quantitative.

D. How much can new physics contribute to $B^0 - \overline{B}^0$ mixing?

All that we need to do in order to establish whether the SM dominates the observed CP violation, and to put an upper bound on the new physics contribution to $B^0 - \overline{B}^0$ mixing, is to project the results of the fit performed in the previous subsection on the $\mathcal{R}e(\Delta_d)-\mathcal{I}m(\Delta_d)$ plane. If we find that $|\mathcal{I}m(\Delta_d)| \ll 1$, then the SM dominance in the observed CP violation will be established. The constraints are shown in Fig. 4.

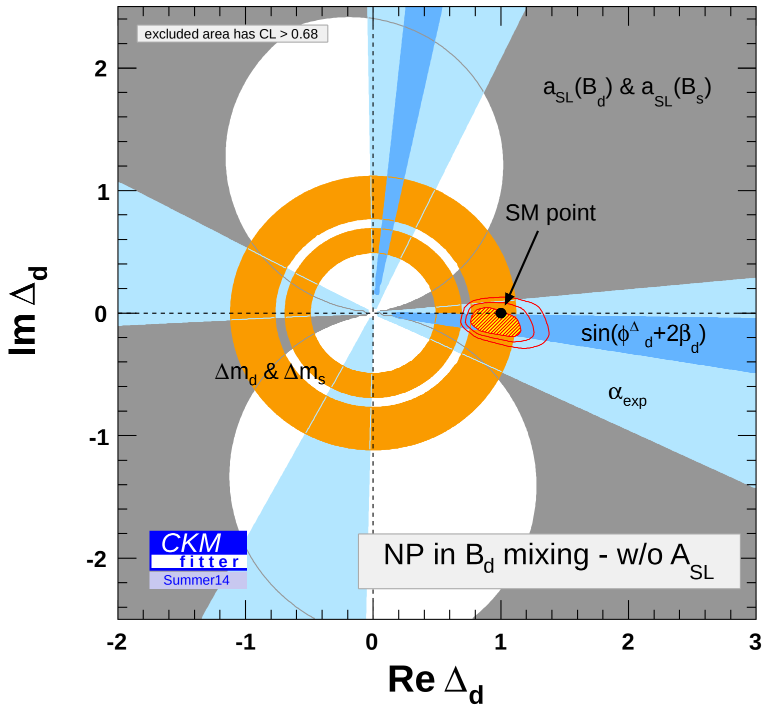

FIG. 4: Constraints in the $\mathcal{R}e(\Delta_d) - \mathcal{I}m(\Delta_d)$ plane, assuming that NP contributions to tree level processes are negligible [14].

We obtain:

$$\begin{aligned} \mathcal{R}e(\Delta_d) &= +0.94^{+0.18}_{-0.15}, \\ \mathcal{I}m(\Delta_d) &= -0.11^{+0.11}_{-0.05}. \end{aligned} \tag{122}$$

This can be translated into the following approximate (one sigma) upper bounds:

$$\begin{aligned} |M^{\rm NP}_{B\bar{B}}/M^{\rm SM}_{B\bar{B}}| &\lesssim 0.2, \\ \mathcal{I}m(M^{\rm NP}_{B\bar{B}}/M^{\rm SM}_{B\bar{B}}) &\lesssim 0.1. \end{aligned} \tag{123}$$

We can make the following two statements:

1. **A new physics contribution to $B^0 - \overline{B}^0$ mixing amplitude that carries a phase that is significantly different from the KM phase is constrained to lie below the 10% level.**

2. **A new physics contribution to the $B^0 - \overline{B}^0$ mixing amplitude which is aligned with the KM phase is constrained to lie below the 20% level.**

Analogous upper bounds can be obtained for new physics contributions to the $K^0 - \overline{K}^0$, $B_s^0 - \overline{B}_s^0$, and $D^0 - \overline{D}^0$ mixing amplitudes.

VII. THE NEW PHYSICS FLAVOR PUZZLE

A. A model independent discussion

It is clear that the Standard Model is not a complete theory of Nature:

1. It does not include gravity, and therefore it cannot be valid at energy scales above $m_{\rm Planck} \sim 10^{19}$ GeV:

2. It does not allow for neutrino masses, and therefore it cannot be valid at energy scales above $m_{\rm seesaw} \sim 10^{15}$ GeV;

3. The fine-tuning problem of the Higgs mass and the puzzle of the dark matter suggest that the scale where the SM is replaced with a more fundamental theory is actually much lower, $m_{\rm top-partners}, m_{\rm wimp} \lesssim$ a few TeV.

Given that the SM is only an effective low energy theory, non-renormalizable terms must be added to $\mathcal{L}_{\rm SM}$. These are terms of dimension higher than four in the fields which, therefore, have couplings that are inversely proportional to the scale of new physics $\Lambda_{\rm NP}$.

The lowest dimension non-renormalizable terms are dimension-five:

$$-\mathcal{L}^{\rm dim-5}_{\rm Seesaw} = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} L_{Li} L_{Lj} \phi\phi + {\rm h.c.}. \tag{124}$$

These are the seesaw terms, leading to neutrino masses.

Exercise 1: *How does the global symmetry breaking pattern (51) change when (124) is taken into account?*

Exercise 2: *What is the number of physical lepton flavor parameters in this case? Identify these parameters in the mass basis.*

As concerns quark flavor physics, consider, for example, the following dimension-six set of operators:

$$\mathcal{L}^{\rm dim-6}_{\Delta F=2} = \sum_{i\neq j} \frac{z_{ij}}{\Lambda^2} (\overline{Q_{Li}}\gamma_\mu Q_{Lj})^2, \tag{125}$$

where the z_{ij} are dimensionless couplings. These terms contribute to the mass splittings between the corresponding two neutral mesons. For example, the term $\mathcal{L}^{\rm dim-6}_{\Delta B=2} \propto (\overline{d_L}\gamma_\mu b_L)^2$ contributes to Δm_B, the mass difference between the two neutral B-mesons. We use

$$M^{\rm NP}_{B\bar{B}} = \frac{1}{6}\frac{z_{db}}{\Lambda^2} m_B f_B^2 B_B. \tag{126}$$

Analogous expressions hold for the other neutral mesons. Taking into account the bounds of Eq. (123), we obtain

$$\frac{|z_{db}|}{\Lambda^2} < \frac{2.3\times10^{-6}}{\rm TeV^2}, \quad \frac{\mathcal{I}m(z_{db})}{\Lambda^2} < \frac{1.1\times10^{-6}}{\rm TeV^2}. \tag{127}$$

A more detailed list of the bounds derived from the $\Delta F = 2$ observables in Table V is given in Table VI. The bounds refer to two representative sets of dimension-six operators: (i) left-left operators, that are also present in the SM, and (ii) operators with different chirality, where the bounds are strongest because of larger hadronic matrix elements.

We note that, as explained above, for the K and B_s meson mixing, the bounds from CPV observables are significantly stronger than those from CPC observables. In fact, the bound from ϵ_K is the strongest bound of all.

The first lesson that we draw from these bounds on Λ is that new physics can contribute to FCNC at a level comparable to the SM contributions even if it takes place at a scale that is six orders of magnitude above the electroweak scale. A second lesson is that if the new physics has a generic flavor structure, that is $z_{ij} = \mathcal{O}(1)$, then its scale must be above $10^4 - 10^5$ TeV (or, if the leading contributions involve electroweak loops, above $10^3 - 10^4$ TeV). *If indeed $\Lambda \gg TeV$, it means that we have misinterpreted the hints from the fine-tuning problem and the dark matter puzzle.*

A different lesson can be drawn from the bounds on z_{ij}. *It could be that the scale of new physics is of order TeV, but its flavor structure is far from generic.* Specifically, if new particles at the TeV scale couple to the SM fermions, then there are two ways in which their contributions to FCNC processes, such as neutral meson mixing, can be suppressed:

TABLE VI: Lower bounds on the scale of new physics Λ, in units of TeV, for $|z_{ij}| = 1$, and upper bounds on z_{ij}, assuming $\Lambda = 1$ TeV. Taken from [19].

Operator	Λ [TeV] CPC	Λ [TeV] CPV	$\lvert z_{ij}\rvert$	$\mathcal{I}m(z_{ij})$	Observables
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	$\Delta m_K;\ \epsilon_K$
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	$\Delta m_K;\ \epsilon_K$
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	$\Delta m_D;\ A_\Gamma$
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	$\Delta m_D;\ A_\Gamma$
$(\bar{b}_L\gamma^\mu d_L)^2$	6.6×10^2	9.3×10^2	2.3×10^{-6}	1.1×10^{-6}	$\Delta m_B;\ S_{\psi K}$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	2.5×10^3	3.6×10^3	3.9×10^{-7}	1.9×10^{-7}	$\Delta m_B;\ S_{\psi K}$
$(\bar{b}_L\gamma^\mu s_L)^2$	1.4×10^2	2.5×10^2	5.0×10^{-5}	1.7×10^{-5}	$\Delta m_{B_s};\ S_{\psi\phi}$
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	4.8×10^2	8.3×10^2	8.8×10^{-6}	2.9×10^{-6}	$\Delta m_{B_s};\ S_{\psi\phi}$

degeneracy and alignment. Either of these principles, or a combination of both, signifies non-generic structure.

One can use the language of effective operators also for the SM, integrating out all particles significantly heavier than the neutral mesons (that is, the top, the Higgs and the weak gauge bosons). Thus, the scale is $\Lambda_{\rm SM} \sim m_W$. Since the leading contributions to neutral meson mixings come from box diagrams, the z_{ij} coefficients are suppressed by α_2^2. To identify the relevant flavor suppression factor, one can employ the spurion formalism. For example, the flavor transition that is relevant to $B^0 - \overline{B}^0$ mixing involves $\overline{d_L} b_L$ which transforms as $(8,1,1)_{SU(3)_q^3}$. The leading contribution must then be proportional to $(Y^u Y^{u\dagger})_{13} \propto y_t^2 V_{tb} V_{td}^*$. Indeed, an explicit calculation (using VIA for the matrix element and neglecting QCD corrections) gives[4]

$$\frac{2M_{B\bar{B}}}{m_B} \approx -\frac{\alpha_2^2}{12}\frac{f_B^2}{m_W^2} S_0(x_t)(V_{tb}V_{td}^*)^2, \tag{128}$$

where $x_i = m_i^2/m_W^2$ and

$$S_0(x) = \frac{x}{(1-x)^2}\left[1 - \frac{11x}{4} + \frac{x^2}{4} - \frac{3x^2 \ln x}{2(1-x)}\right]. \tag{129}$$

[4] A detailed derivation can be found in Appendix B of [20].

Similar spurion analyses, or explicit calculations, allow us to extract the weak and flavor suppression factors that apply in the SM:

$$\begin{aligned}
\mathcal{I}m(z_{sd}^{\rm SM}) &\sim \alpha_2^2 y_t^2 |V_{td}V_{ts}|^2 \sim 1\times 10^{-10},\\
z_{sd}^{\rm SM} &\sim \alpha_2^2 y_c^2 |V_{cd}V_{cs}|^2 \sim 5\times 10^{-9},\\
\mathcal{I}m(z_{cu}^{\rm SM}) &\sim \alpha_2^2 y_b^2 |V_{ub}V_{cb}|^2 \sim 2\times 10^{-14},\\
z_{bd}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{td}V_{tb}|^2 \sim 7\times 10^{-8},\\
z_{bs}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{ts}V_{tb}|^2 \sim 2\times 10^{-6}.
\end{aligned} \tag{130}$$

(We did not include $z_{cu}^{\rm SM}$ in the list because it requires a more detailed consideration.)

It is clear then that contributions from new physics at $\Lambda_{\rm NP}\sim 1$ TeV should be suppressed by factors that are comparable or smaller than the SM ones. Why does that happen? This is the new physics flavor puzzle.

The fact that the flavor structure of new physics at the TeV scale must be non-generic means that flavor measurements are a good probe of the new physics. Perhaps the best-studied example is that of supersymmetry. Here, the spectrum of the superpartners and the structure of their couplings to the SM fermions will allow us to probe the mechanism of dynamical supersymmetry breaking.

B. Minimal flavor violation (MFV)

Models of gauge mediated supersymmetry breaking (GMSB) provide a concrete example of a large class of models that obey a simple principle called *minimal flavor violation* (MFV) [21]. This principle guarantees that low energy flavor changing processes deviate only very little from the SM predictions. The basic idea can be described as follows. The gauge interactions of the SM are universal in flavor space. The only breaking of this flavor universality comes from the three Yukawa matrices, Y^u, Y^d and Y^e. If this remains true in the presence of the new physics, namely Y^u, Y^d and Y^e are the only flavor non-universal parameters, then the model belongs to the MFV class.

Let us now formulate this principle in a more formal way, using the language of spurions that we presented in section II D. The Standard Model with vanishing Yukawa couplings has a large global symmetry (52,53). In this section we concentrate only on the quarks. The non-Abelian part of the flavor symmetry for the quarks is $SU(3)_q^3$ of Eq. (53) with the three generations of quark fields transforming as follows:

$$Q_L(3,1,1), \quad U_R(1,3,1), \quad D_R(1,1,3). \tag{131}$$

The Yukawa interactions,

$$\mathcal{L}^q_{\rm Yuk} = \overline{Q_L} Y^d D_R \phi + \overline{Q_L} Y^u U_R \tilde{\phi}, \tag{132}$$

break this symmetry. The Yukawa couplings can thus be thought of as spurions with the following transformation properties under $SU(3)^3_q$ [see Eq. (55)]:

$$Y^u \sim (3,\bar{3},1), \qquad Y^d \sim (3,1,\bar{3}). \tag{133}$$

When we say "spurions", we mean that we pretend that the Yukawa matrices are fields which transform under the flavor symmetry, and then require that all the Lagrangian terms, constructed from the SM fields, Y^d and Y^u, must be (formally) invariant under the flavor group $SU(3)^3_q$. Of course, in reality, $\mathcal{L}^q_{\rm Yuk}$ breaks $SU(3)^3_q$ precisely because $Y^{d,u}$ are *not* fields and do not transform under the symmetry.

The idea of minimal flavor violation is relevant to extensions of the SM, and can be applied in two ways:

1. If we consider the SM as a low energy effective theory, then all higher-dimension operators, constructed from SM-fields and Y-spurions, are formally invariant under $G_{\rm global}$.

2. If we consider a full high-energy theory that extends the SM, then all operators, constructed from SM and the new fields, and from Y-spurions, are formally invariant under $G_{\rm global}$.

That MFV allows new physics at the TeV scale is demonstrated in Table VII. Note that for the LL operators, MFV does not allow for new CPV phases [22].

Exercise 3: *Use the spurion formalism to argue that, in MFV models, the $K_L \to \pi^0 \nu\bar{\nu}$ decay amplitude is proportional to $y_t^2 V_{td} V_{ts}^*$.*

Examples of MFV models include models of supersymmetry with gauge-mediation or with anomaly-mediation of its breaking.

TABLE VII: The MFV values and the experimental bounds on the coefficients of $\Delta F = 2$ operators

Operator	$z_{ij} \propto$	CKM+GIM	$\lvert z_{ij}\rvert < (\frac{\Lambda}{\mathrm{TeV}})^2 \times$
$(\bar{s}_L \gamma^\mu d_L)^2$	$y_t^4 (V_{ts} V_{td}^*)^2$	10^{-7}	9.0×10^{-7}
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	$y_t^4 y_s y_d (V_{ts} V_{td}^*)^2$	10^{-14}	6.9×10^{-9}
$(\bar{c}_L \gamma^\mu u_L)^2$	$y_b^4 (V_{cb} V_{ub}^*)^2$	10^{-14}	5.6×10^{-7}
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	$y_b^4 y_c y_u (V_{cb} V_{ub}^*)^2$	10^{-20}	5.7×10^{-8}
$(\bar{b}_L \gamma^\mu d_L)^2$	$y_t^4 (V_{tb} V_{td}^*)^2$	10^{-4}	2.3×10^{-6}
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	$y_t^4 y_b y_d (V_{tb} V_{td}^*)^2$	10^{-9}	3.9×10^{-7}
$(\bar{b}_L \gamma^\mu s_L)^2$	$y_t^4 (V_{tb} V_{ts}^*)^2$	10^{-3}	5.0×10^{-5}
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	$y_t^4 y_b y_s (V_{tb} V_{ts}^*)^2$	10^{-6}	8.8×10^{-6}

VIII. CONCLUSIONS

- Measurements of CP violating B-meson decays have established that the Kobayashi-Maskawa mechanism is the dominant source of the observed CP violation.
- Measurements of flavor changing B-meson decays have established the the Cabibbo-Kobayashi-Maskawa mechanism is a major player in flavor violation.
- The consistency of all these measurements with the CKM predictions sharpens the new physics flavor puzzle: If there is new physics at, or below, the TeV scale, then its flavor structure must be highly non-generic.
- Extensions of the SM where new particles couple to quark- and/or lepton-pairs are constrained by flavor. If FCNC are still mediated at the loop level, and the new couplings are $\mathcal{O}(1)$, then the scale of new physics must be $\gtrsim 10^3$ TeV. If, furthermore, the new couplings carry phases of $\mathcal{O}(1)$, then the scale of new physics must be $\gtrsim 10^4$ TeV.
- There are two puzzles related to CP violation:
  - The strong CP problem: Why is $\theta_{\mathrm{QCD}} \ll 1$?
  - The baryon asymmetry: Why is $\eta \gg 10^{-20}$?

Precision measurements of CP asymmetries in meson decays, searches for EDMs, and the search for CP violation in the Higgs interaction, may lead to discovery of new physics, and perhaps also to progress on solving the CP-related puzzles.

Acknowledgments

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APPENDIX A: FORMALISM OF FLAVOR OSCILLATIONS

1. Formalism

Consider the case where initially, at $t = 0$, the neutral meson state is some specific combination of P^0 and $\overline{P}^0$:

$$|\psi_P(0)\rangle = a(0)|P^0\rangle + b(0)|\overline{P}^0\rangle \ . \tag{A1}$$

It evolves in time, and acquires components that correspond to all possible decay final states $\{f_1, f_2, \ldots\}$:

$$|\psi_P(t)\rangle = a(t)|P^0\rangle + b(t)|\overline{P}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \ . \tag{A2}$$

Our interest lies in obtaining only $a(t)$ and $b(t)$. For this aim, one can use a simplified formalism, where the full Hamiltonian, H, is replaced with a 2×2 effective Hamiltonian $\mathcal{H}$ that is not Hermitian. The non-Hermiticity is related to the possibility of decays, which makes the $\{P^0, \overline{P}^0\}$ system an open one.

Before we proceed, let us clarify a semantic issue. The effective Hamiltonian $\mathcal{H}$ and, similarly, its Hermitian part M, are (combinations of) operators. What we need for our purposes is its matrix element between specific meson states. We denote the operator by M_{ij} with $i, j = 1, 2$, and its matrix element by $M_{\alpha\beta}$ with $\alpha, \beta = P, \bar{P}$.

The complex matrix $\mathcal{H}$ can be written in terms of Hermitian matrices M and Γ as

$$\mathcal{H} = M - \frac{i}{2}\Gamma\,. \tag{A3}$$

The matrices M and Γ are associated with transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of M and Γ are associated with the flavor-conserving transitions $P^0 \to P^0$ and $\overline{P}^0 \to \overline{P}^0$. The CPT symmetry implies that $M_{PP} = M_{\bar{P}\bar{P}}$ and $\Gamma_{PP} = \Gamma_{\bar{P}\bar{P}}$. The off-diagonal elements are associated with the flavor changing transitions $P^0 \leftrightarrow \overline{P}^0$ and they are of significant interest for us. The phase

$$\theta_P = \arg(M_{P\bar{P}}\Gamma^*_{P\bar{P}}) \tag{A4}$$

is related to CP: $\mathcal{H}$ is CP symmetric if $\theta_P = 0$.

Since $\mathcal{H}$ is not a diagonal matrix, the states that have well defined masses and decay widths are not P^0 and $\overline{P}^0$, but rather the eigenvectors of $\mathcal{H}$. We denote the light and heavy eigenstates by P_L and P_H with masses $m_H > m_L$. Another possible choice, which is standard for K mesons, is to define the mass eigenstates according to their lifetimes. We denote the short-lived and long-lived eigenstates by K_S and K_L with decay widths $\Gamma_S > \Gamma_L$. (The K_L meson is experimentally found to be the heavier state.)

The eigenstates of $\mathcal{H}$ are given by

$$|P_{L,H}\rangle = p|P^0\rangle \pm q|\overline{P}^0\rangle, \tag{A5}$$

where

$$\left(\frac{q}{p}\right)^2 = \frac{M^*_{P\bar{P}} - (i/2)\Gamma^*_{P\bar{P}}}{M_{P\bar{P}} - (i/2)\Gamma_{P\bar{P}}}\,, \qquad |p|^2 + |q|^2 = 1. \tag{A6}$$

(Note that the phase of q/p is convention dependent, and not a physical observable.) Since $\mathcal{H}$ is not Hermitian, the eigenstates need not be orthogonal to each other, that is $\langle P_H|P_L\rangle = |p|^2 - |q|^2$ can be different from zero.

The eigenvalues of $\mathcal{H}$ can be written as

$$\mu_{H,L} = m_{H,L} + \frac{i}{2}\Gamma_{H,L}, \tag{A7}$$

such that the masses and decay-widths of the eigenstate are given by the real and imaginary parts of the eigenvalues, respectively. The average mass and the average width are given by

$$m \equiv \frac{m_H + m_L}{2}, \qquad \Gamma \equiv \frac{\Gamma_H + \Gamma_L}{2}. \tag{A8}$$

The mass difference Δm and the width difference $\Delta\Gamma$ are defined as follows:

$$\Delta m \equiv m_H - m_L, \qquad \Delta\Gamma \equiv \Gamma_H - \Gamma_L. \tag{A9}$$

Here Δm is positive by definition, while the sign of $\Delta\Gamma$ is to be determined experimentally. (Alternatively, one can use the states defined by their lifetimes to have $\Delta\Gamma \equiv \Gamma_S - \Gamma_L$ positive by definition, in which case the sign of Δm has to be determined experimentally.) Solving the eigenvalue equation gives

$$(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{P\bar{P}}|^2 - |\Gamma_{P\bar{P}}|^2, \qquad \Delta m\Delta\Gamma = 4\mathcal{R}e(M_{P\bar{P}}\Gamma^*_{P\bar{P}}). \tag{A10}$$

The above expressions simplify for $\mathcal{H}$ that is CP symmetric, in which case $\theta_P = 0$. As concerns the eigenvectors, Eq. (A6) gives

$$|q/p| = 1\ . \tag{A11}$$

It follows that the mass eigenstates are also CP eigenstates, and are orthogonal to each other, $\langle P_H|P_L\rangle = 0$. As concerns the eigenvalues, Eq. (A10) gives

$$\Delta m = 2|M_{P\bar{P}}|, \qquad |\Delta\Gamma| = 2|\Gamma_{P\bar{P}}|. \tag{A12}$$

Another limit of interest is when $|\Gamma_{P\bar{P}}| \ll |M_{P\bar{P}}|$. In that case,

$$\Delta m = 2|M_{P\bar{P}}|, \qquad |\Delta\Gamma| = 2|\Gamma_{P\bar{P}}|\cos\theta_P. \tag{A13}$$

It is interesting to note that for the four mesons in Nature to a very good approximation $\Delta m = 2|M_{P\bar{P}}|$. For B and B_s this is because $|\Gamma_{P\bar{P}}| \ll |M_{P\bar{P}}|$ while for K and D it is because CP is conserved to a very good approximation, $|\theta_P| \ll 1$.

To study the time evolution of the neutral mesons, it is convenient to define the dimensionless ratios,

$$x \equiv \frac{\Delta m}{\Gamma}, \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}, \tag{A14}$$

the decay amplitudes of P^0 and its CP conjugate $\overline{P}^0$ into a final state f,

$$A_f = \langle f|H|P^0\rangle, \qquad \overline{A}_f = \langle f|H|\overline{P}^0\rangle, \tag{A15}$$

and the parameter λ_f:

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f}. \tag{A16}$$

Our normalization is such that

$$\Gamma(P^0 \to f) = |A_f|^2, \qquad \Gamma(\overline{P}^0 \to f) = |\overline{A}_f|^2. \tag{A17}$$

2. Time evolution

Let us denote the time-evolved state of an initial state $|P^0\rangle$ by $|P^0(t)\rangle$, and of an initial state $|\overline{P}^0\rangle$ by $|\overline{P}^0(t)\rangle$. For mass eigenstates, the time evolution is simple,

$$|P_{L,H}(t)\rangle = e^{-im_{L,H}t-\frac{1}{2}\Gamma_{L,H}t}|P_{L,H}\rangle. \tag{A18}$$

But the time evolution of $|P^0(t)\rangle$ and $|\overline{P}^0(t)\rangle$ is more complicated:

$$|P^0(t)\rangle = g_+(t)|P^0\rangle - (q/p)g_-(t)|\overline{P}^0\rangle\,, \qquad |\overline{P}^0(t)\rangle = g_+(t)|\overline{P}^0\rangle - (p/q)g_-(t)|P^0\rangle\,, \tag{A19}$$

where

$$g_\pm(t) = \frac{1}{2}\left(e^{-im_Ht-\frac{1}{2}\Gamma_Ht} \pm e^{-im_Lt-\frac{1}{2}\Gamma_Lt}\right). \tag{A20}$$

The time dependent decay rates of $P^0 \to f$ and $\overline{P}^0 \to f$ are given by

$$\begin{aligned}
\frac{d\Gamma[P^0(t)\to f]/dt}{e^{-\Gamma t}|A_f|^2} &= (1+|\lambda_f|^2)\cosh(y\Gamma t) + (1-|\lambda_f|^2)\cos(x\Gamma t) \\
&\quad +2\mathcal{R}e(\lambda_f)\sinh(y\Gamma t) - 2\mathcal{I}m(\lambda_f)\sin(x\Gamma t), \\
\frac{d\Gamma[\overline{P}^0(t)\to f]/dt}{e^{-\Gamma t}|\overline{A}_f|^2} &= (1+|\lambda_f|^{-2})\cosh(y\Gamma t) + (1-|\lambda_f|^{-2})\cos(x\Gamma t) \\
&\quad +2\mathcal{R}e(\lambda_f^{-1})\sinh(y\Gamma t) - 2\mathcal{I}m(\lambda_f^{-1})\sin(x\Gamma t).
\end{aligned} \tag{A21}$$

We need to introduce the notion of "flavor tagging." The flavor eigenstates P^0 and $\overline{P}^0$ have a well defined flavor content. For example, B^0 ($\overline{B}^0$) is a $\bar{b}d$ ($b\bar{d}$) bound state. The term flavor tagging refers to the experimental determination of whether a neutral P meson is in a P^0 or $\overline{P}^0$ state. Flavor tagging is provided to us by Nature, when the meson decays into a flavor-specific final state, namely a state that can come from either P^0 or $\overline{P}^0$ state, but not from both. In other words, flavor specific decays refer to cases where either $\overline{A}_f = 0$ or $A_f = 0$. (Final states that are common to the decays of both P and $\overline{P}$ are also very useful in flavor physics and, in particular, to the study of CP violation. They are discussed in Section B.)

Semi-leptonic decays provide very good flavor tags. Take, for example, semileptonic b (anti)quark decays:

$$b \to c\mu^-\bar{\nu}, \qquad \bar{b} \to \bar{c}\mu^+\nu. \tag{A22}$$

Thus, the charge of the lepton tells us the flavor: μ^+ comes from a B^0 (or B^+) decay, $\overline{A}_{\mu^+\nu X} = 0$, while μ^- comes from a $\overline{B}^0$ (or B^-) decay, $A_{\mu^-\bar{\nu}X} = 0$. Of course, before the meson decays it could be in a superposition of B^0 and a $\overline{B}^0$. The decay acts as a quantum measurement. In the case of semileptonic decay, it acts as a measurement of flavor *vs.* anti-flavor.

The oscillation formalism is simplified in the case of flavor tagged decay. Take the case of $\overline{A}_f = 0$ and, therefore, $\lambda_f = 0$. We also simplify the discussion by assuming $|q/p| = 1$ and $y = 0$. We get

$$\begin{aligned}
\frac{d\Gamma[P^0(t) \to f]}{dt} &= e^{-\Gamma t}|A_f|^2 \left[1 + \cos(x\Gamma t)\right] \\
\frac{d\Gamma[\overline{P}^0(t) \to f]}{dt} &= e^{-\Gamma t}|A_f|^2 \left[1 - \cos(x\Gamma t)\right].
\end{aligned} \tag{A23}$$

We see that the flavor oscillates with frequency of Δm in the rest frame. The general case of Eq. (A21) involves deviations from pure exponential decay which depend on both Δm and $\Delta\Gamma$.

APPENDIX B: CP VIOLATION

CP asymmetries arise when two processes related by CP conjugation differ in their rates. To date, CP violation has been observed (at a level higher than 5σ) in about thirty different hadron decay modes, involving b or s decays. It has not been established in t and in c decays, or in the leptonic sector, or in flavor diagonal processes, such as electric dipole moments. Here we present the formalism relevant to measuring CP asymmetries in meson decays.

The experimental observation of CP violation is challenging for several reasons:

1. CP violation is related to interference. In order to have a sizable effect, we need contributions from two amplitudes of similar size but significantly different CP violating phases.

2. In order that there will be a CP asymmetry in a decay process, the presence of strong phases (defined below) is needed. These phases might be small (or vanish) and suppress the CP asymmetry (or make it vanish).

3. CPT implies that the total width of a particle and its anti-particle are the same. Thus, any CP violation in one channel must be compensated by CP violation with an opposite sign in other channels. Consequently, CP violation is suppressed in inclusive measurements.

4. Within the SM, CP violation arises only when all three generations are involved. With the smallness of the CKM mixing angles, this means that either the CP asymmetries are small, or they appear in modes with small branching ratios.

1. Notations and formalism

Our starting point is Eqs. (A21), which give the time-dependent decay rates $\Gamma(B^0(t) \to f)$ and $\Gamma(\overline{B}^0(t) \to f)$. Before we proceed, we present some physics ingredients concerning the decay amplitudes, and some further notations. We do so for the specific case of B-meson decays, but our discussion applies to all meson decays.

Consider A_f, the $B \to f$ decay amplitude, and $\overline{A}_{\overline{f}}$, the amplitude of the CP conjugate process, $\overline{B} \to \overline{f}$. There are two types of phases that may appear in these decay amplitudes:

- CP-odd phases, also known as weak phases. They are complex parameters in any Lagrangian term that contributes to A_f, and appear in a complex conjugate form in $\overline{A}_{\overline{f}}$. In other words, CP violating phases change sign between A_f and $\overline{A}_{\overline{f}}$. In the SM, these phases appear only in the couplings of the $W^\pm$-bosons, hence the CP violating phases are called "weak phases."

- CP-even phases, also known as strong phases. Phases can appear in decay amplitudes even when the Lagrangian parameters are all real. They arise from contributions of intermediate on-shell states, and can be identified with the e^{iHt} term in the time evolution Schrödinger equation. These CP conserving phases appear with the same sign in A_f and $\overline{A}_{\overline{f}}$. In meson decays, such rescattering is usually driven by strong interactions, hence the CP conserving phases are called "strong phases."

It is useful to factorize an amplitude into three parts: the magnitude $|a_i|$, the weak phase ϕ_i, and the strong phase δ_i. If there are two such contributions, $A_f = a_1 + a_2$, we write

$$A_f = |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}, \qquad \overline{A}_{\overline{f}} = |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}. \tag{B1}$$

It is further useful to define

$$\phi_f \equiv \phi_2 - \phi_1, \qquad \delta_f \equiv \delta_2 - \delta_1, \qquad r_f \equiv |a_2/a_1|. \tag{B2}$$

For neutral meson mixing, it is useful to write

$$M_{B\bar{B}} = |M_{B\bar{B}}| e^{i\phi_M}, \qquad \Gamma_{B\bar{B}} = |\Gamma_{B\bar{B}}| e^{i\phi_\Gamma} \ . \tag{B3}$$

Each of the phases appearing in Eqs. (B1) and (B3) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$, and others, are physical.

In neutral meson decays the phenomenology of CP violation is particularly rich thanks to the fact that meson mixing can contribute to the CP violating interference effects. One distinguishes three types of CP violation in meson decays, depending on which amplitudes interfere:

1. In decay: The interference is between two decay amplitudes.

2. In mixing: The interference is between the absorptive and dispersive mixing amplitudes.

3. In interference of decays with and without mixing: The interference is between the direct decay amplitude and a first-mix-then-decay amplitude.

We discuss these three types below.

2. CP violation in decay

CP violation in decay corresponds to

$$|\overline{A}_{\overline{f}}/A_f| \neq 1. \tag{B4}$$

In charged particle decays, this is the only possible contribution to the CP asymmetry:

$$\mathcal{A}_f \equiv \frac{\Gamma(B^- \to f^-) - \Gamma(B^+ \to f^+)}{\Gamma(B^- \to f^-) + \Gamma(B^+ \to f^+)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2 - 1}{|\overline{A}_{f^-}/A_{f^+}|^2 + 1}. \tag{B5}$$

Using Eq. (B1), we obtain, for $r_f \ll 1$,

$$\mathcal{A}_f = 2 r_f \sin\phi_f \sin\delta_f. \tag{B6}$$

This result shows explicitly that we need two decay amplitudes, that is, $r_f \neq 0$, with different weak phases, $\phi_f \neq 0, \pi$ and different strong phases $\delta_f \neq 0, \pi$.

A few comments are in order:

1. In order to have a large CP asymmetry, we need each of the three factors in (B6) not to be $\ll 1$.

2. A similar expression holds for the contribution of CP violation in decay in neutral meson decays. In this case there are, however, additional contributions.

3. Another complication with regard to neutral meson decays is that it is not always possible to tell the flavor of the decaying meson, that is, if it is B^0 or $\overline{B}^0$. This can be a problem or a virtue.

4. In general the strong phase is not calculable since it is related to QCD. This is not a problem if the aim is just to demonstrate CP violation, but it is if we want to extract the weak parameter ϕ_f. In some cases, however, the phase can be independently measured, eliminating this particular source of theoretical uncertainty.

3. CP violation in mixing

CP violation in mixing corresponds to

$$|q/p| \neq 1 \ . \tag{B7}$$

In decays of neutral mesons into flavor specific final states ($\overline{A}_f = 0$ and, consequently, $\lambda_f = 0$), and, in particular, semileptonic neutral meson decays, this is the only source of CP violation:

$$\mathcal{A}_{\rm SL}(t) \equiv \frac{\Gamma[\overline{B}^0(t) \to \ell^+ X] - \Gamma[B^0(t) \to \ell^- X]}{\Gamma[\overline{B}^0(t) \to \ell^+ X] + \Gamma[B^0(t) \to \ell^- X]} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \tag{B8}$$

Using Eq. (A6), we obtain for $|\Gamma_{B\bar{B}}/M_{B\bar{B}}| \ll 1$,

$$\mathcal{A}_{\rm SL} = -\,|\Gamma_{B\bar{B}}/M_{B\bar{B}}| \sin(\phi_M - \phi_\Gamma). \tag{B9}$$

A few comments are in order:

1. Eq. (B8) implies that this asymmetry of time-dependent decay rates is actually time independent.

2. The calculation of $|\Gamma_{P\bar{P}}/M_{P\bar{P}}|$ is difficult, since it depends on low-energy QCD effects. Hence, the extraction of the value of the CP violating phase $\phi_M - \phi_\Gamma$ from a measurement of $\mathcal{A}_{\rm SL}$ involves, in general, large uncertainties.

CP violation in $K^0-\overline{K}^0$ mixing is measured via a semileptonic asymmetry which is defined as follows:

$$\delta_L \equiv \frac{\Gamma(K_L\to\ell^+\nu_\ell\pi^-)-\Gamma(K_L\to\ell^-\nu_\ell\pi^+)}{\Gamma(K_L\to\ell^+\nu_\ell\pi^-)+\Gamma(K_L\to\ell^-\nu_\ell\pi^+)} = \frac{1-|q/p|^2}{1+|q/p|^2}. \tag{B10}$$

This asymmetry is somewhat different from the one defined in Eq. (B8), in that the decaying meson is the neutral mass eigenstate, rather than the flavor eigenstate. Hence also the different dependence on $|q/p|$.

4. CP violation in interference of decays with and without mixing

CP violation in interference of decays with and without mixing corresponds to

$$\frac{\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2} \neq 0. \tag{B11}$$

A particular simple case is the CP asymmetry in decays into final CP eigenstates. Moreover, a situation that is relevant in many cases is when the effects of CP violation in decay are negligible, $|\overline{A}_{f_{CP}}/A_{f_{CP}}| \simeq 1$, and the effects of CP violation in mixing are small, $|q/p| \simeq 1$. In this case, $\lambda_{f_{CP}}$ is a pure phase, $|\lambda_{f_{CP}}| = 1$. Further consider the case where $y = 0$. We obtain the very simple result:

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{\Gamma[\overline{B}^0(t)\to f_{CP}]-\Gamma[B^0(t)\to f_{CP}]}{\Gamma[\overline{B}^0(t)\to f_{CP}]+\Gamma[B^0(t)\to f_{CP}]} = \mathcal{I}m(\lambda_{f_{CP}})\sin(\Delta m_B t). \tag{B12}$$

Using Eq. (A16), we obtain, for $|\Gamma_{B\bar{B}}/M_{B\bar{B}}| \ll 1$,

$$\mathcal{I}m(\lambda_{f_{CP}}) = \mathcal{I}m\left(\frac{M^*_{B\bar{B}}}{|M_{B\bar{B}}|}\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}}\right) = -\sin(\phi_M+2\phi_1). \tag{B13}$$

The phase ϕ_M is defined in Eq. (B3), while the phase ϕ_1 is defined in Eq. (B1), and we assume that a_2 can be neglected.

For the case where we measure decays of mass eigenstate into final CP eigenstates, as can be done for kaons, one can average over the oscillation terms and and obtain

$$\mathcal{A}_f^{\rm mass} \equiv \frac{\Gamma(K_L\to f)}{\Gamma(K_S\to f)} = \left|\frac{1+\lambda_f}{1-\lambda_f}\right|^2. \tag{B14}$$

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[1] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no. 3, 030001 (2018). doi:10.1103/PhysRevD.98.030001

[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[3] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

[4] L. Chau and W. Keung, Phys. Rev. Lett. **53**, 1802 (1984).

[5] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[6] A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D **50**, 3433 (1994) [arXiv:hep-ph/9403384].

[7] C. Dib, I. Dunietz, F. J. Gilman and Y. Nir, Phys. Rev. D **41**, 1522 (1990).

[8] J. L. Rosner, A. I. Sanda and M. P. Schmidt, EFI-88-12-CHICAGO [Presented at Workshop on High Sensitivity Beauty Physics, Batavia, IL, Nov 11-14, 1987].

[9] A. Ceccucci, Z. Ligeti and Y. Sakai, in [1].

[10] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985). doi:10.1103/PhysRevLett.55.1039

[11] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D **2**, 1285 (1970). doi:10.1103/PhysRevD.2.1285

[12] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981).

[13] I. I. Y. Bigi and A. I. Sanda, Nucl. Phys. B **193**, 85 (1981).

[14] CKMfitter Group (J. Charles et al.), Eur. Phys. J. **C41**, 1 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

[15] Y. Nir, Nucl. Phys. Proc. Suppl. **117**, 111 (2003) [arXiv:hep-ph/0208080].

[16] Y. Amhis *et al.* [Heavy Flavor Averaging Group Collaboration], arXiv:1207.1158 [hep-ex] and online update at http://www.slac.stanford.edu/xorg/hfag.

[17] Y. Grossman, Y. Nir and M. P. Worah, Phys. Lett. B **407**, 307 (1997) [hep-ph/9704287].

[18] Y. Grossman, Y. Nir and G. Raz, Phys. Rev. Lett. **97**, 151801 (2006) [arXiv:hep-ph/0605028].

[19] G. Isidori, Y. Nir and G. Perez, Ann. Rev. Nucl. Part. Sci. **60**, 355 (2010) [arXiv:1002.0900 [hep-ph]].

[20] G. C. Branco, L. Lavoura and J. P. Silva, *CP violation,* Clarendon Press, Oxford (1999).

[21] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) [arXiv:hep-ph/0207036].

[22] A. L. Kagan, G. Perez, T. Volansky and J. Zupan, Phys. Rev. D **80**, 076002 (2009) [arXiv:0903.1794 [hep-ph]].