FLAG: Lattice QCD tests of the SM and foretaste for beyond

A. Vladikas
INFN - TOR VERGATA

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The FLAG Collaboration

FLAG: Flavour Lattice Averaging Group

The FLAG Collaboration

- Lattice simulations performed by different groups involve different choices both at the level of formalism (lattice actions, number of sea flavours etc.) and at the level of resources (lattice volumes, quark masses etc.).

- Often this amounts to making different compromises which in turn introduce different systematic effects; thus not all lattice results of a given quantity are directly comparable.

- FLAG aim: answer, in a way which is readily accessible to non-experts, the question: What is currently the "best lattice value" for a particular quantity?

- 2011: end of phase 1 (FLAG-1 consisted of 12 European members): G. Colangelo et al., "Review of Lattice Results Concerning Low-Energy Particle Physics", Eur. Phys. J. C 71 (2011) 1695

- 2014: end of phase 2 (FLAG-2 consisted of 28 American/Asian/European members): S. Aoki et al., "Review of Lattice Results Concerning Low-Energy Particle Physics", Eur. Phys. J. C 74 (2014) 2890

- Lattice collaborations which participated in FLAG-2: Alpha/CLS, BMW, ETMC, FNAL, HPQCD, JLQCD, PACS-CS, RBC/UKQCD

- Here a selection of FLAG-2 results are presented (NB: Closing date for reviewing lattice papers: 30th November 2013)

- Currently working on FLAG-3; should be ready by spring 2016.

The FLAG-2 Composition

- FLAG structure:
  - **Advisory Board:** S. Aoki (Japan), C. Bernard (USA), C. Sachrajda (UK)
  - **Editorial Board:** G.Colangelo (Bern), H.Leutwyler (Bern), A.V. (Rome-2), U. Wenger (Bern)
  - **WORKING GROUPS:**
    - Light quark masses: L. Lellouch, T. Blum, V. Lubicz
    - f_K, f_K/f_π, $f_+^{K\pi}(0)$ ➡ V_{us}, V_{ud}: A. Jüttner, T. Kaneko, S. Simula
    - LEC: S. Dürr, H. Fukaya, S. Necco
    - B_K: H. Wittig, J. Laiho, S. Sharpe
    - α_S: R. Sommer, R. Horsley, T. Onogi
    - f_D, B_D, f_B, B_B: A. El Khadra, Y. Aoki, M. Della Morte, J. Shigemitsu
    - D --> K/π l ν, B --> K/π l ν: R. van de Water, E. Lunghi, C. Pena

Quality Criteria

- A number of criteria have been fixed, providing compact information on the quality of a computation.
- For most quantities these criteria refer to the quality/reliability of:
  - the continuum limit extrapolations;
  - the chiral limit extrapolations;
  - the finite volume effects;
  - the renormalization (perturbative, non-perturbative).
- For heavy flavours and α_{strong} the criteria are different.
- Criteria:

★ the relevant systematic error estimated in a satisfactory manner and under control.

○ a reasonable attempt at estimating systematic error; can be improved.

■ no attempt or unsatisfactory attempt at controlling a systematic error (result is dropped!).

Quality Criteria

- Many more issues; e.g. how to average, how to make an estimate if and average is not possible, how to combine/correlate errors, how (not) to take conference proceedings into account, ...

- Simulations are carried out either for $N_f = 2$, or $N_f = 2+1$, or $N_f = 2+1+1$ sea quarks (two light flavours are isospin symmetric).

- Quenched results ($N_f = 0$) are omitted, except for α_{strong}, where they are reported without averages

- NB: FLAG averages/estimates reported at fixed N_f and are **not averaged** for different N_f

- **FIGURES:** for each N_f value, we use different symbols as follows:

  ■ FLAG average or estimate;

  ■ results which from which the FLAG average/estimate is obtained;

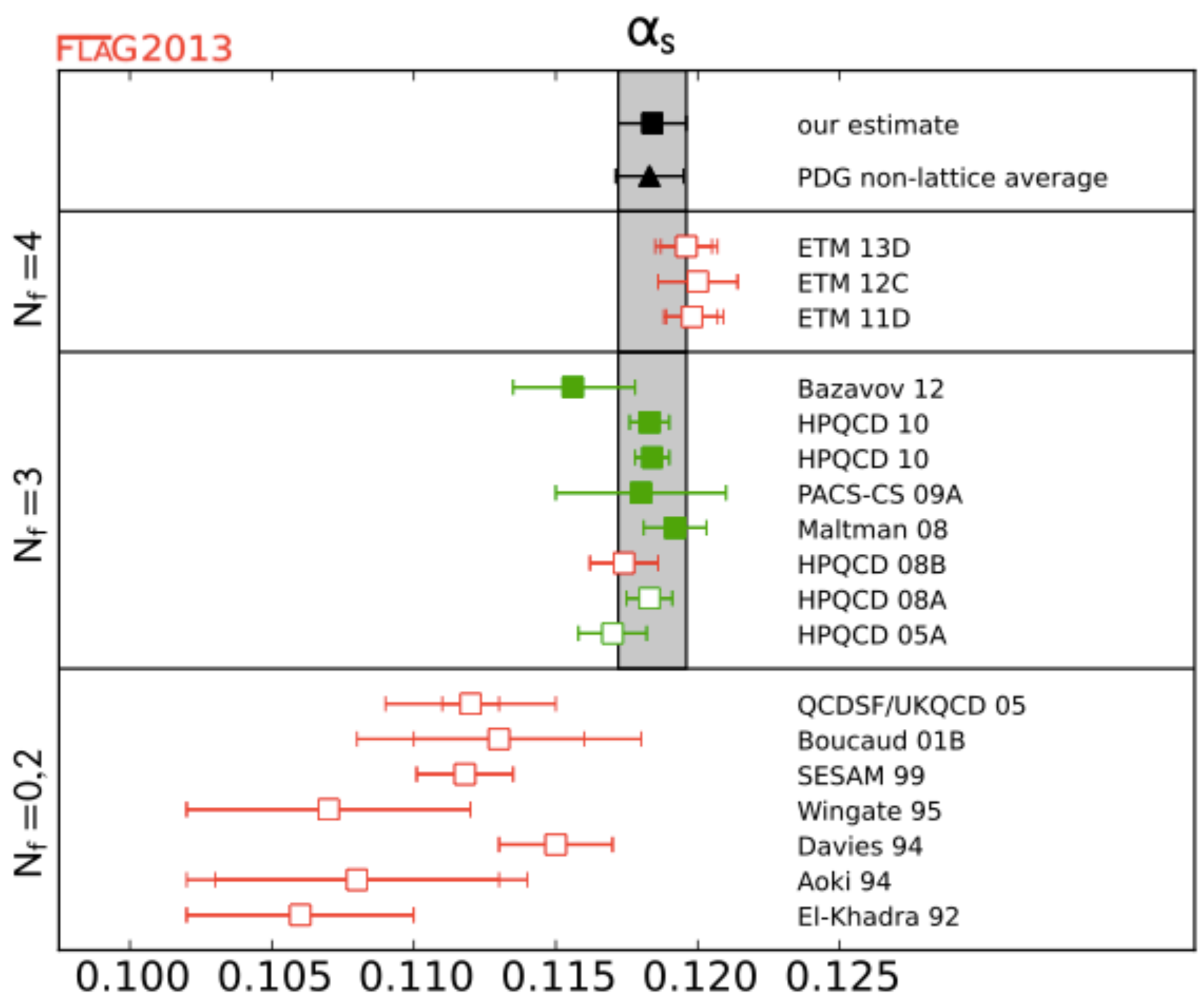
  □ results without red tags (i.e. good control of the systematics) but not included in the average for some reason; e.g. not published in peer reviewed journals, superseded by later results of the same collaboration, some other effect has not been controlled...

  □ results are not included in the average because they do not pass the criteria;

  ● non-lattice results.

Quality Criteria

- The importance of quality criteria is seen in our estimate of α_{strong}



- FLAG estimate has conservative error (not all FLAG agrees)
- PDG total average takes all lattice results at face value
- PDG without lattice agrees with FLAG

FLAG estimate: $$\alpha^{(5)}_{\overline{MS}}(M_Z) = 0.1184(12)$$

PDG average $$\alpha^{(5)}_{\overline{MS}}(M_Z) = 0.1185(5)$$

PDG average (non lattice) $$\alpha^{(5)}_{\overline{MS}}(M_Z) = 0.1183(12)$$

Strange Physics

f_π, f_K, $f_+(0)$, $|V_{ud}|$, $|V_{us}|$

CKM first row unitarity

Form factor, decay constants and unitarity

- Leptonic pion and Kaon decays associated with hadronic matrix elements, expressed in terms of decay constants $f^{\pm}_{\pi}$ and $f^{\pm}_{K}$:

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 u|\pi^\pm(\vec{p})\rangle = i p_\mu f_{\pi^\pm} \qquad \langle 0|\bar{s}\gamma_\mu\gamma_5 u|K^\pm(\vec{p})\rangle = i p_\mu f_{K^\pm}$$

- Semi-leptonic Kaon decays associated with form factor $f+(q^2)$ at momentum transfer to lepton pair q^2:

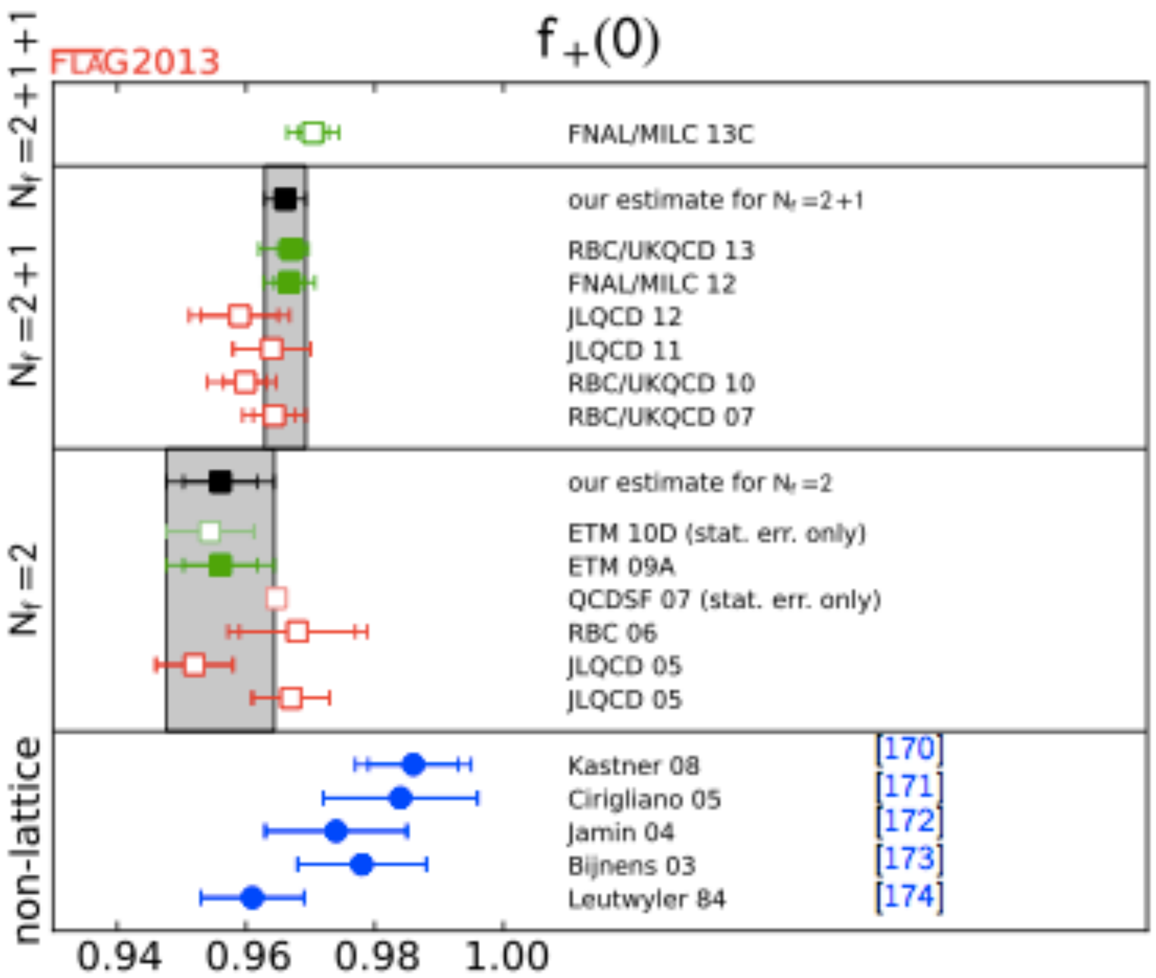
$$K^0 \to \pi^- \nu\, l^+$$

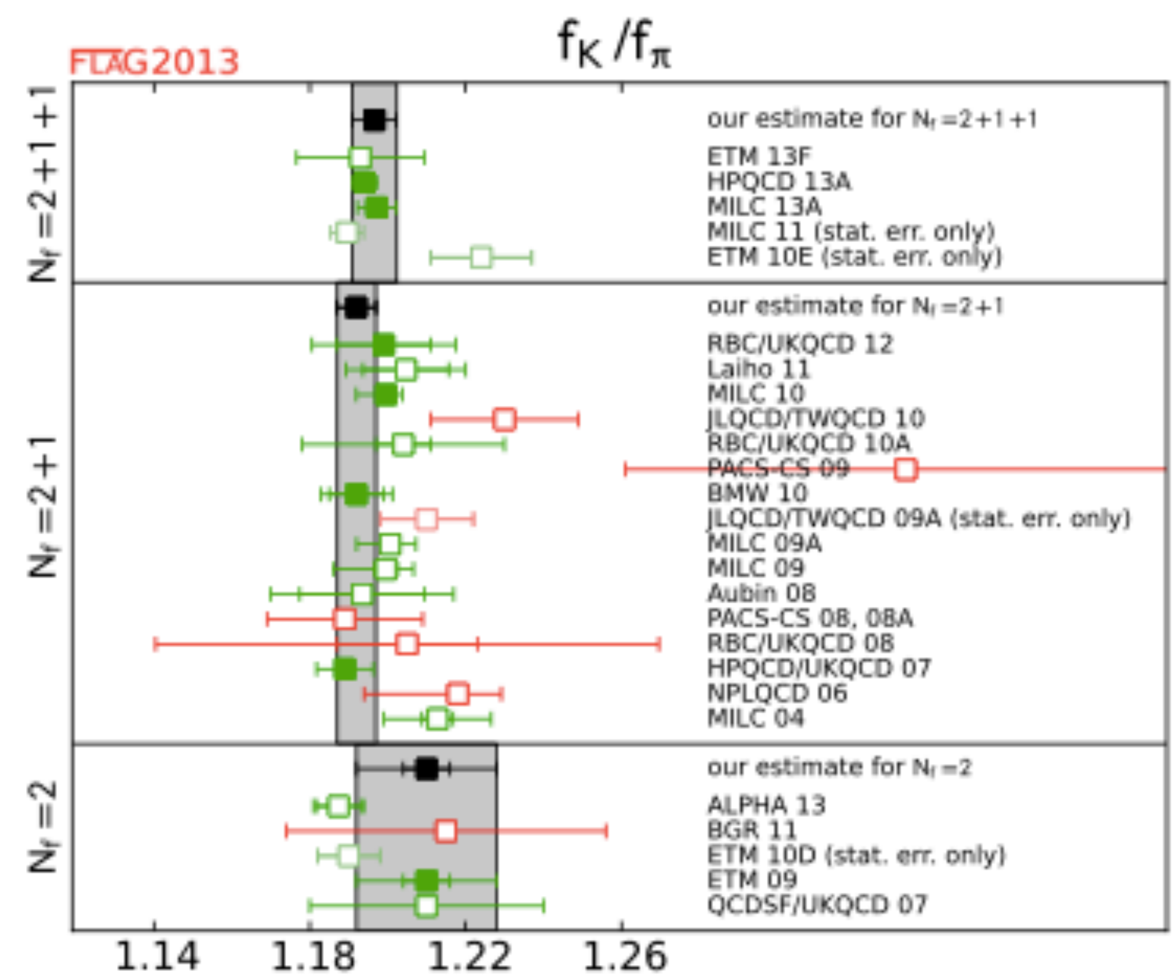
- M.Antonelli et al., Eur.Phys.J. C69(2010)399 results from high accuracy experimental data:

$$|V_{us}|\; f_+(0) = 0.2163(5) \qquad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5)$$

form factor @ zero momentum transfer

Form factor, decay constants and unitarity





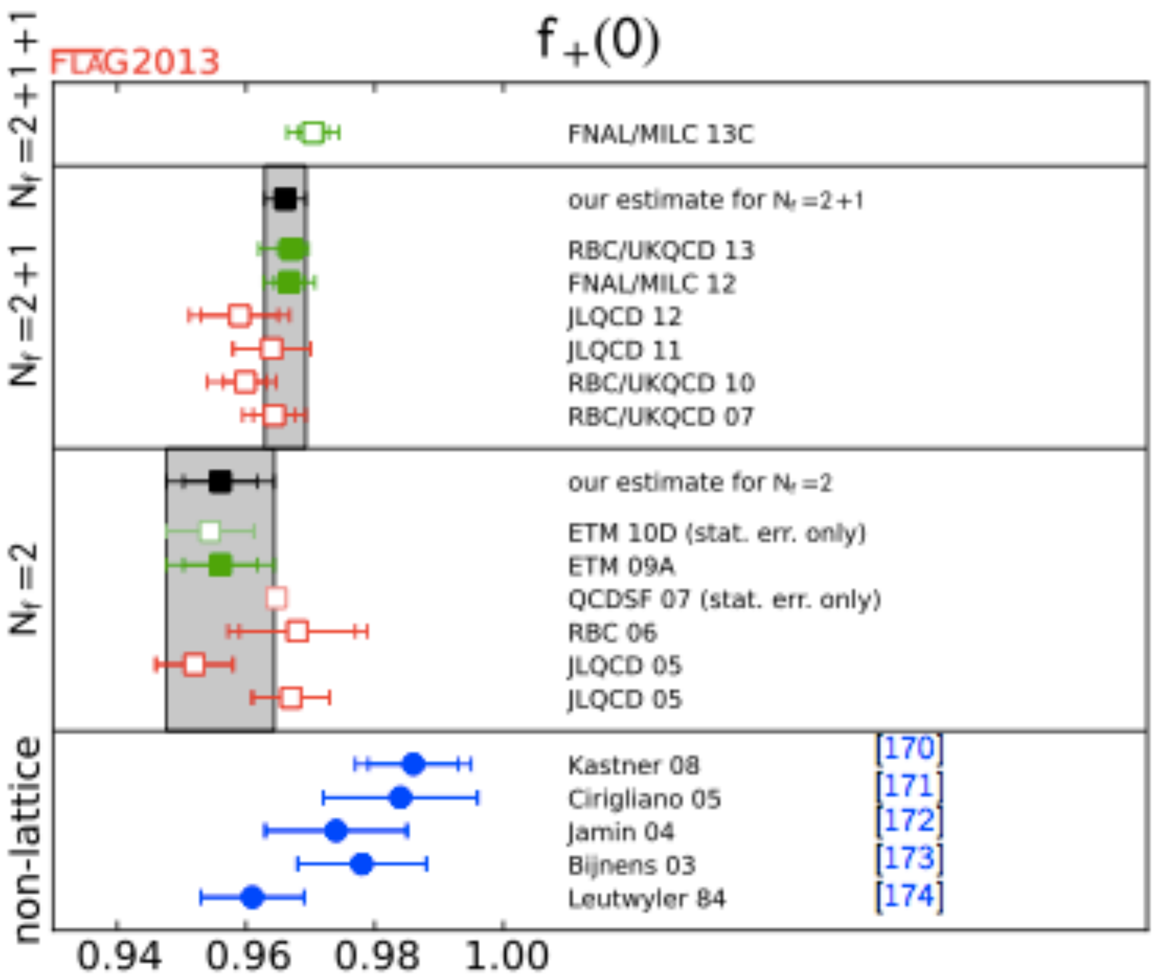
$$f_+(0) = 0.9661(32) \text{ MeV} \qquad N_f = 2+1$$

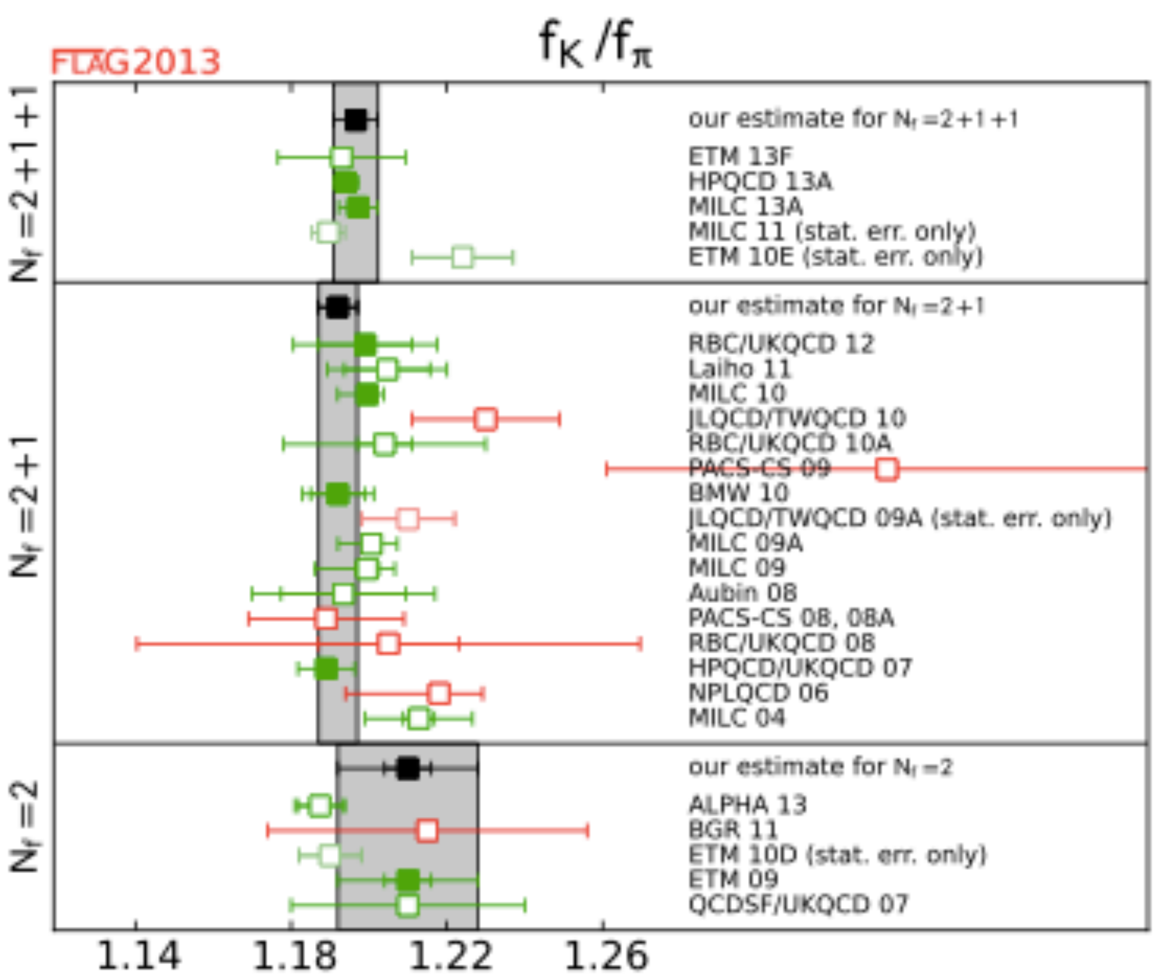
$$f_+(0) = 0.9560(57)(62) \text{ MeV} \qquad N_f = 2$$

ETM datum; no FLAG average

Systematic, mostly due to chiral extrapolations

Form factor, decay constants and unitarity





$$\frac{f_K^\pm}{f_\pi^\pm} = 1.194(5)\ \mathrm{MeV} \qquad N_f = 2+1+1$$

$$\frac{f_K^\pm}{f_\pi^\pm} = 1.192(5)\ \mathrm{MeV} \qquad N_f = 2+1$$

$$\frac{f_K^\pm}{f_\pi^\pm} = 1.205(6)(17)\ \mathrm{MeV} \qquad N_f = 2$$

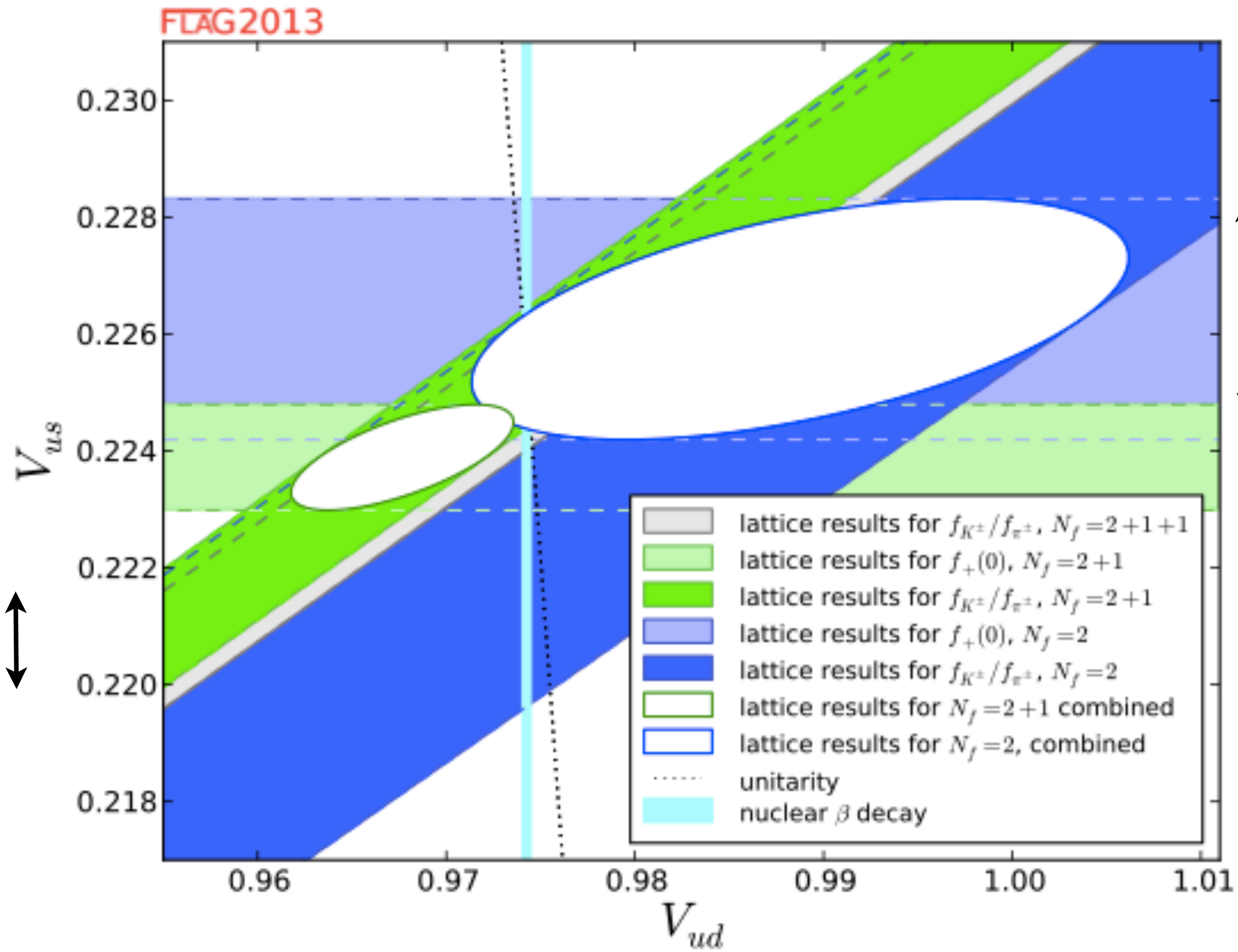
NLO χPT used to get $f_K^\pm / f_\pi^\pm$ from f_K / f_π

Form factor, decay constants and unitarity

N_f=2+1

$|V_{us}|$ from $f_+(0)$

$|V_{us}|/|V_{ud}|$ from $f_{K^\pm}/f_{\pi^\pm}$



$|V_{us}|/|V_{ud}|$ from $f_{K^\pm}/f_{\pi^\pm}$

$|V_{us}|$ from $f_+(0)$

N_f=2

$$|V_{us}|\ f_+(0) \ = \ 0.2163(5)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} \ = \ 0.2758(5)$$

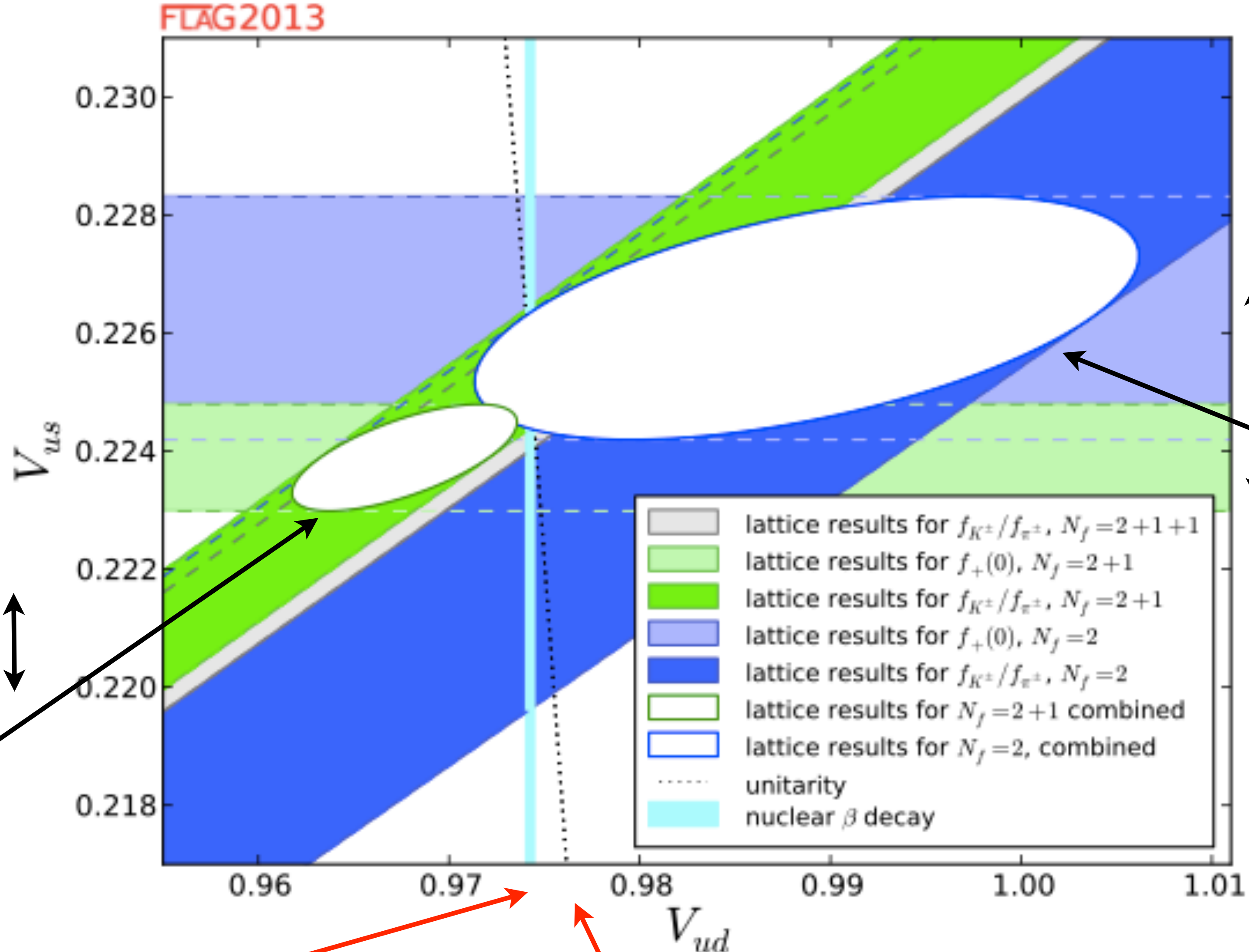
M.Antonelli et al., Eur.Phys.J. C69(2010)399

Form factor, decay constants and unitarity

N_f=2+1

|V_{us}| from $f_+(0)$

|V_{us}|/|V_{ud}| from $f_{K^\pm}/f_{\pi^\pm}$



|V_{us}|/|V_{ud}| from $f_{K^\pm}/f_{\pi^\pm}$

|V_{us}| from $f_+(0)$

68% likelihood

N_f=2

68% likelihood

|V_{ud}| from nuclear β-decay; agreement with lattice

|V_{us}| vs. |V_{ud}| from CKM unitarity --- small tension

NB: |V_{ub}| known!

Form factor, decay constants and unitarity

- 1st row unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- PDG experiment: $$|V_{ub}| = 4.15(49) \cdot 10^{-3}$$

- From lattice data for N_f=2+1 and kaon decay branching rations we see the slight tension of previous plot (small ellipse vs dotted curve)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.987(10)$$

- From lattice result for $f_+(0)$ and nuclear β-decay for $|V_{ud}|$ the test sharpens:

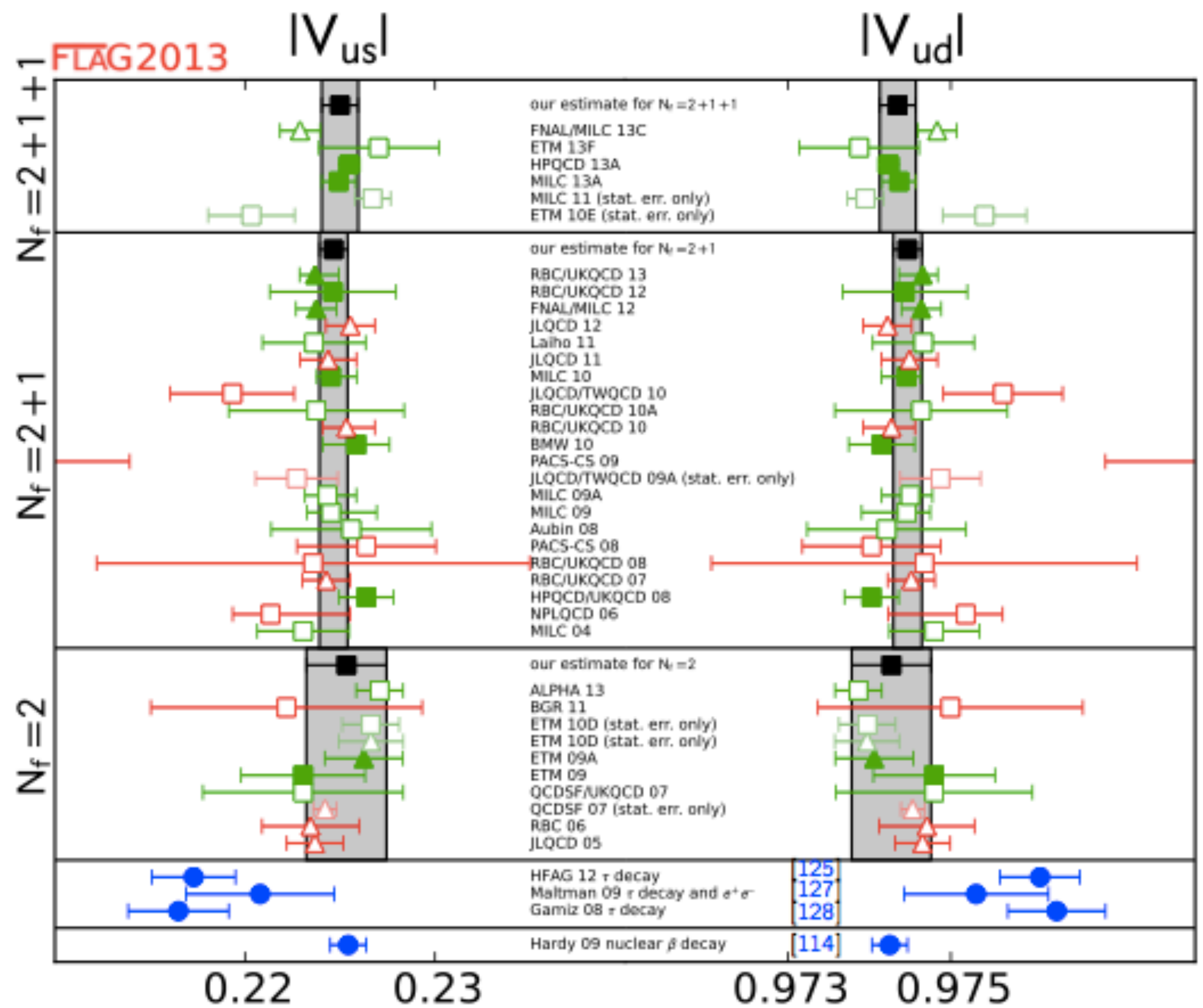
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9993(5)$$

- From lattice result for $f_{K^\pm}/f_{\pi^\pm}$ and nuclear β-decay for $|V_{ud}|$ the test sharpens:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0000(6)$$

- Full agreement with unitarity for N_f=2 (bigger errors) and N_f=2+1+1 (only from $f_{K^\pm}/f_{\pi^\pm}$)

Form factor, decay constants and unitarity



agreement for different N_f

some tension between lattice and τ-decay

agreement between lattice and β-decay

* Estimates obtained from an analysis of the lattice data within the Standard Model, see section 4.5.

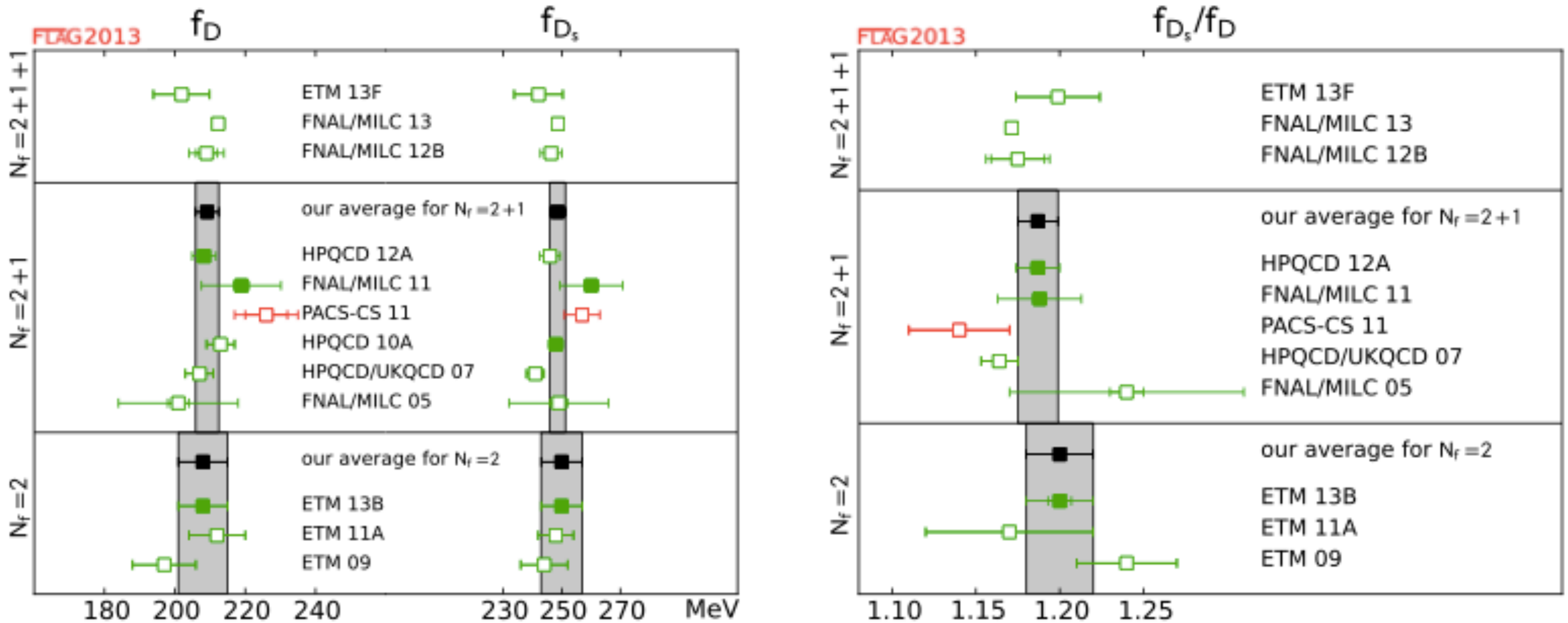
Figure 6: Results for $|V_{us}|$ and $|V_{ud}|$ that follow from the lattice data for $f_+(0)$ (triangles) and $f_{K^\pm}/f_{\pi^\pm}$ (squares), on the basis of the assumption that the CKM matrix is unitary. The black squares and the grey bands represent our estimates, obtained by combining these two different ways of measuring $|V_{us}|$ and $|V_{ud}|$ on a lattice. For comparison, the figure also indicates the results obtained if the data on nuclear β decay and τ decay are analysed within the Standard Model.

Charm Physics

f_D, f_{D_s}, $f_+(0)$, $|V_{cd}|$, $|V_{cs}|$

CKM second row unitarity

Leptonic decay constants f_D and f_{Ds}



$$f_D = 208 \pm 7 \text{ MeV} \qquad N_f = 2$$

$$f_D = 209.2 \pm 3.3 \text{ MeV} \qquad N_f = 2+1$$

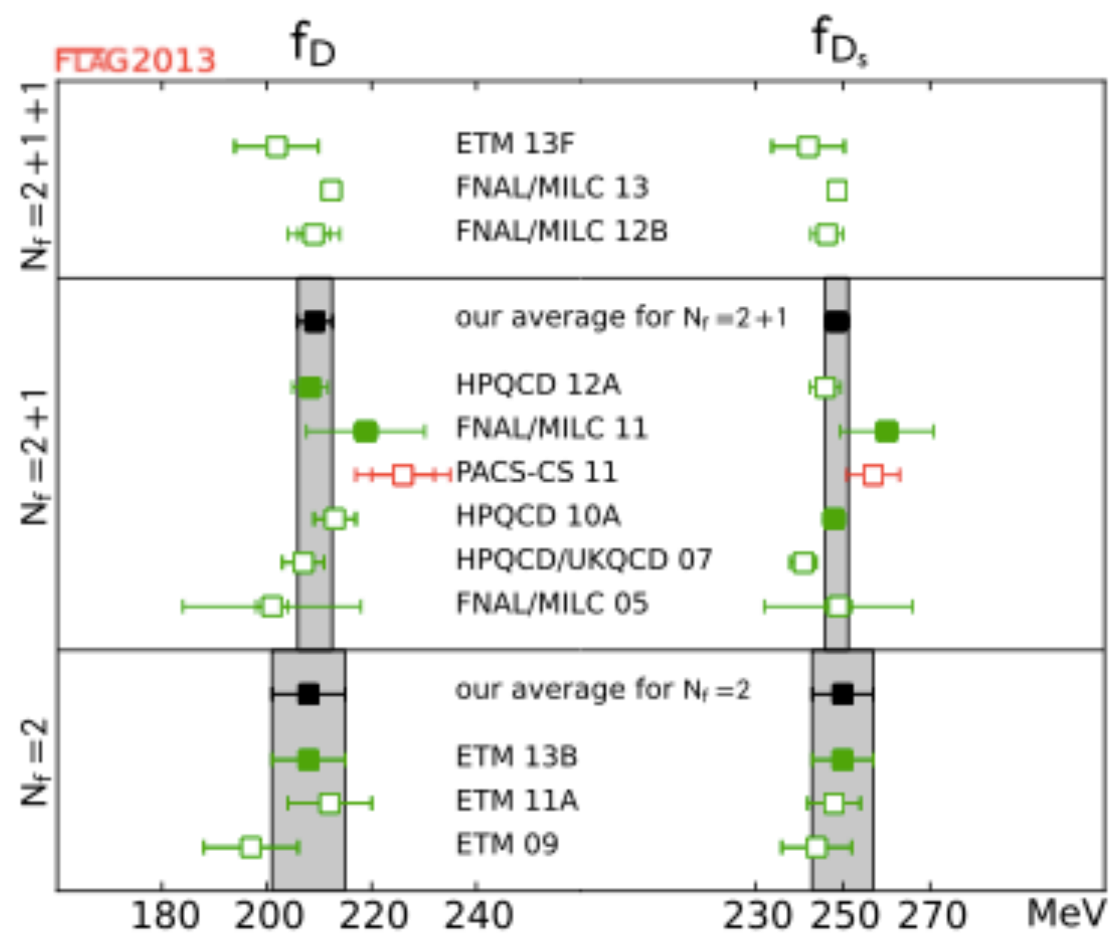
$$f_{D_s} = 250 \pm 7 \text{ MeV} \qquad N_f = 2$$

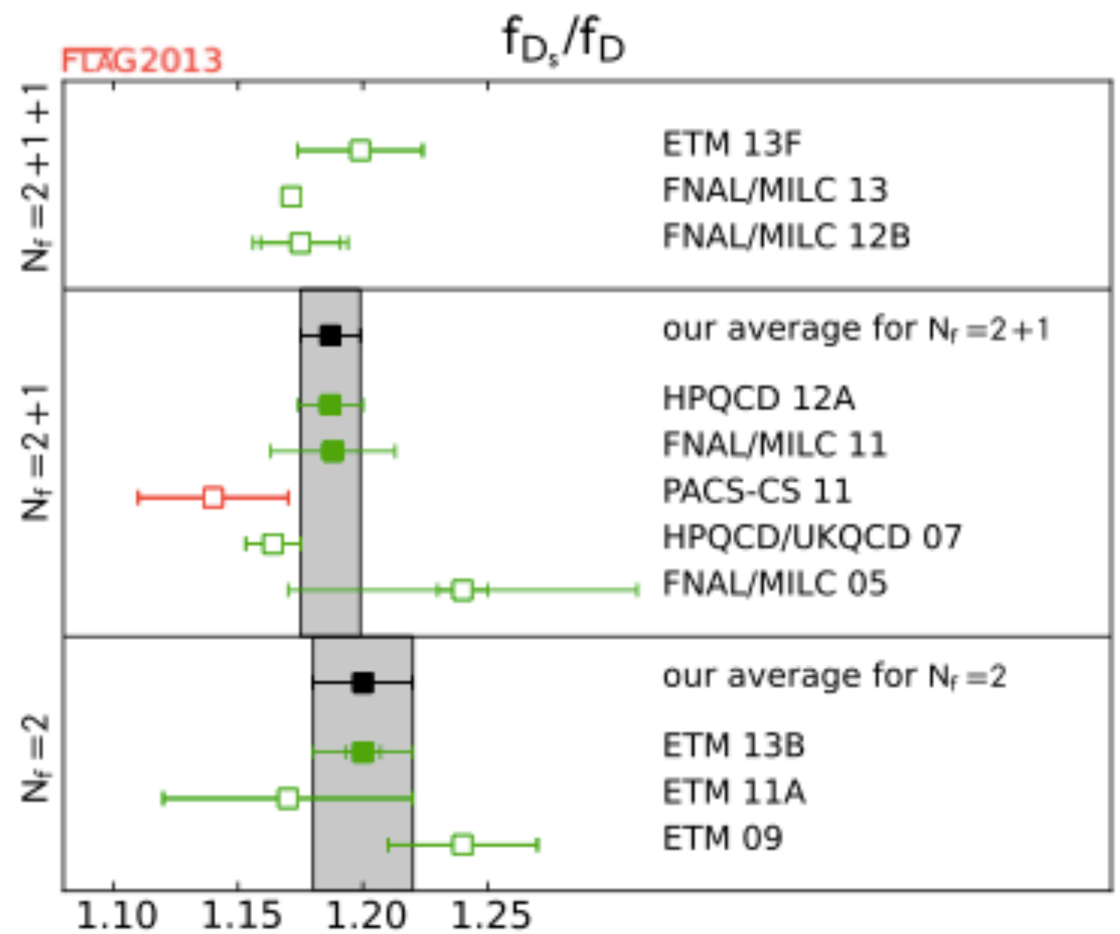
$$f_{D_s} = 248.6 \pm 2.7 \text{ MeV} \qquad N_f = 2+1$$

$$\frac{f_{D_s}}{f_D} = 1.20 \pm 0.02 \qquad N_f = 2$$

$$\frac{f_{D_s}}{f_D} = 1.187 \pm 0.012 \qquad N_f = 2+1$$

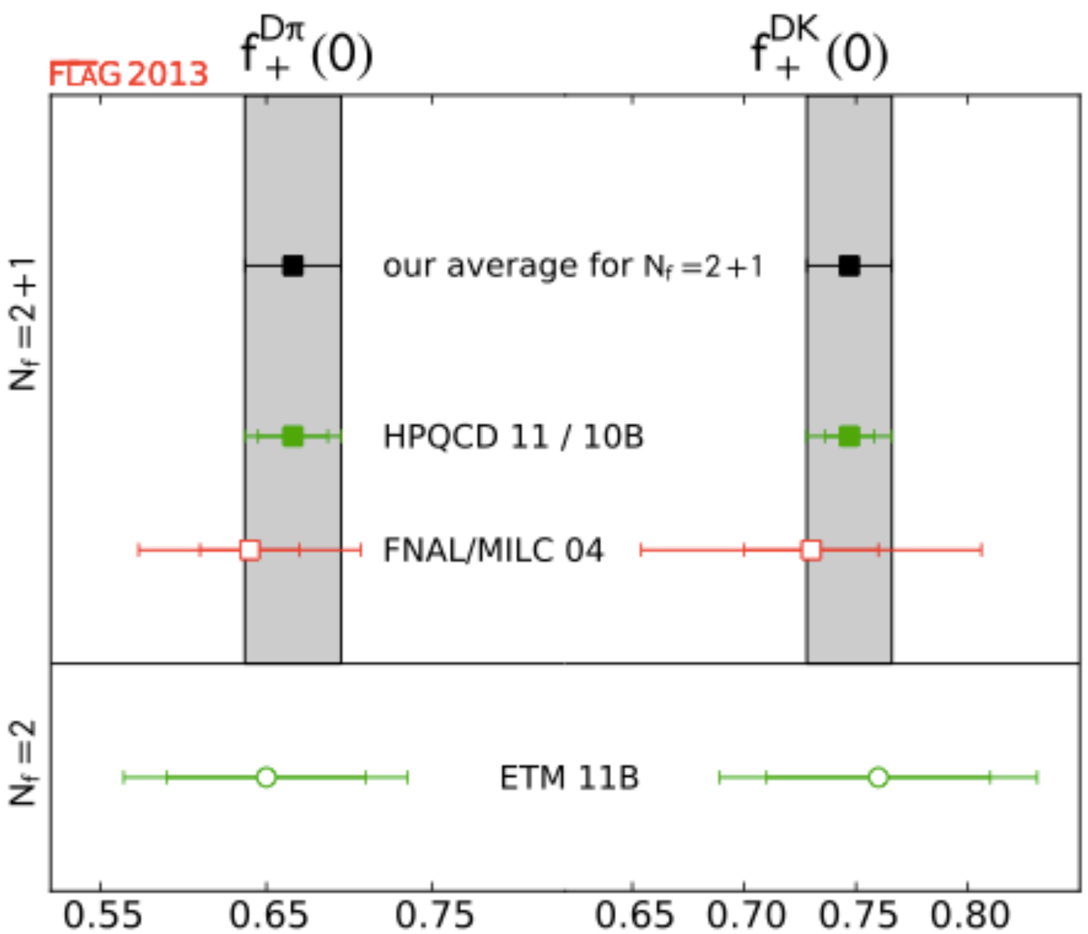
Leptonic decay constants f_D and f_{Ds}





- NB: as the quality of the simulations improves in the near future, we should distinguish between f_{D+} (FNAL/MILK) and the average between f_{D+} and f_{D0} (HPQCD, PACS-CS, ETM).

Semileptonic decay form factor $f_+(0)$



- N_f=2
  - ETM (proceedings)
- N_f=2+1
  - FNAL/MILK (single lattice spacing) predicted shape of $f^{DK}_+(q^2)$ by FOCUS & Belle
  - HPQCD (more accurate)
- N_f=2+1+1
  - in the works (ETM)

- only HPQCD datum; no FLAG average

$$f_+^{D\pi}(0) \;=\; 0.666 \pm 0.029 \text{ MeV} \qquad N_f = 2+1$$
$$f_+^{DK}(0) \;=\; 0.747 \pm 0.019 \text{ MeV} \qquad N_f = 2+1$$

CKM angles $|V_{cd}|$ and $|V_{cs}|$

- J.L.Rosner & S.Stone, arXiv:1201.2401

$$f_D|V_{cd}| = 46.40 \pm 1.98\text{MeV} \qquad f_{D_s}|V_{cs}| = 253.1 \pm 5.3\text{MeV}$$

- N_f=2 $\qquad |V_{cd}| = 0.2231(95)(75) \qquad |V_{cs}| = 1.012(21)(28)$
- N_f=2+1 $\qquad |V_{cd}| = 0.2218(35)(95) \qquad |V_{cs}| = 1.018(11)(21)$

lattice

non-lattice th. & exp.

- N_f=2+1: lattice errors (HPQCD dominated) much smaller than other errors

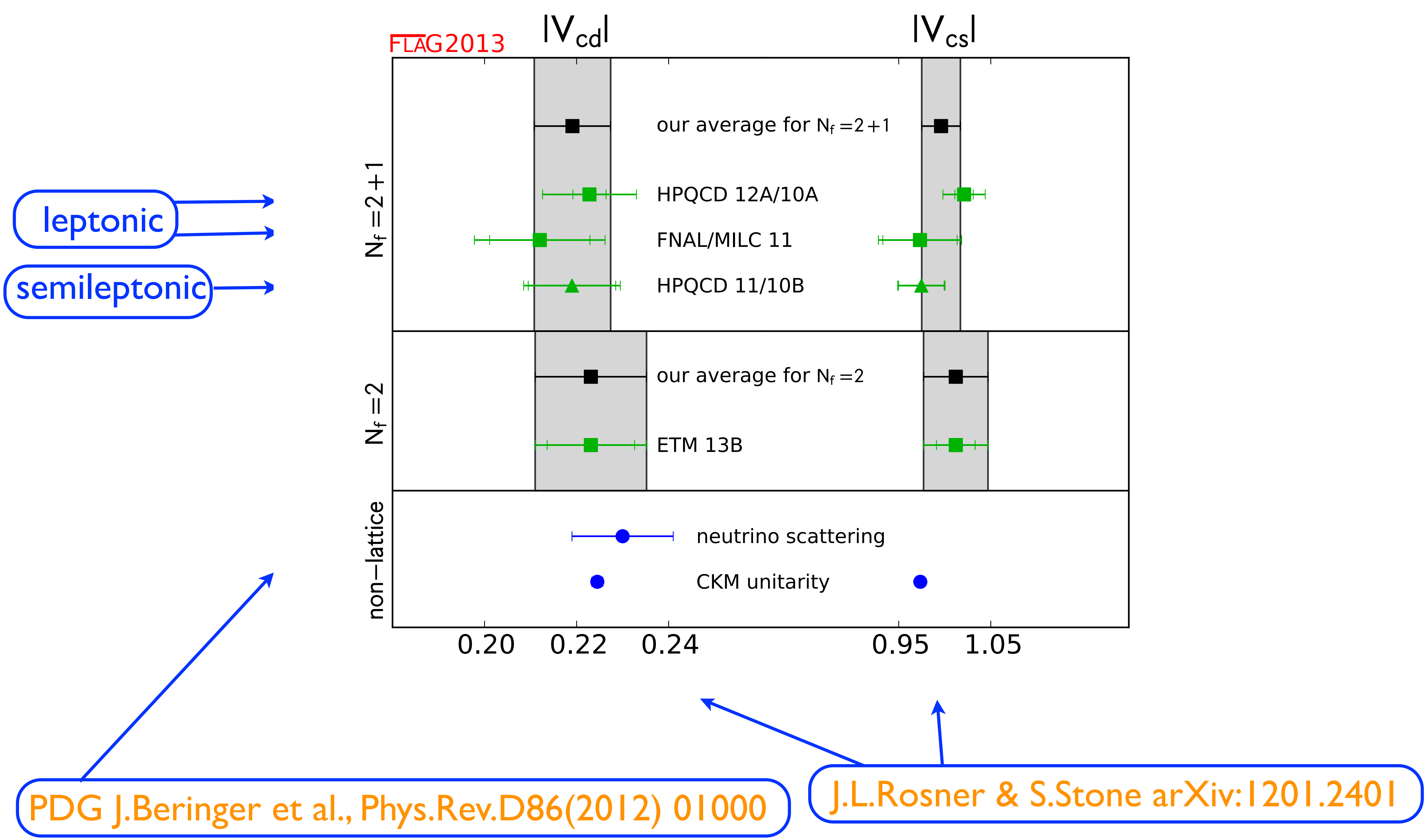
CKM angles $|V_{cd}|$ and $|V_{cs}|$

- HFAG: Y. Amhis et al., arXiv:1207.1158

$$f_+^{D\pi}(0)|V_{cd}| = 0.146 \pm 0.003 \qquad f_+^{DK}(0)|V_{cs}| = 0.728 \pm 0.005$$

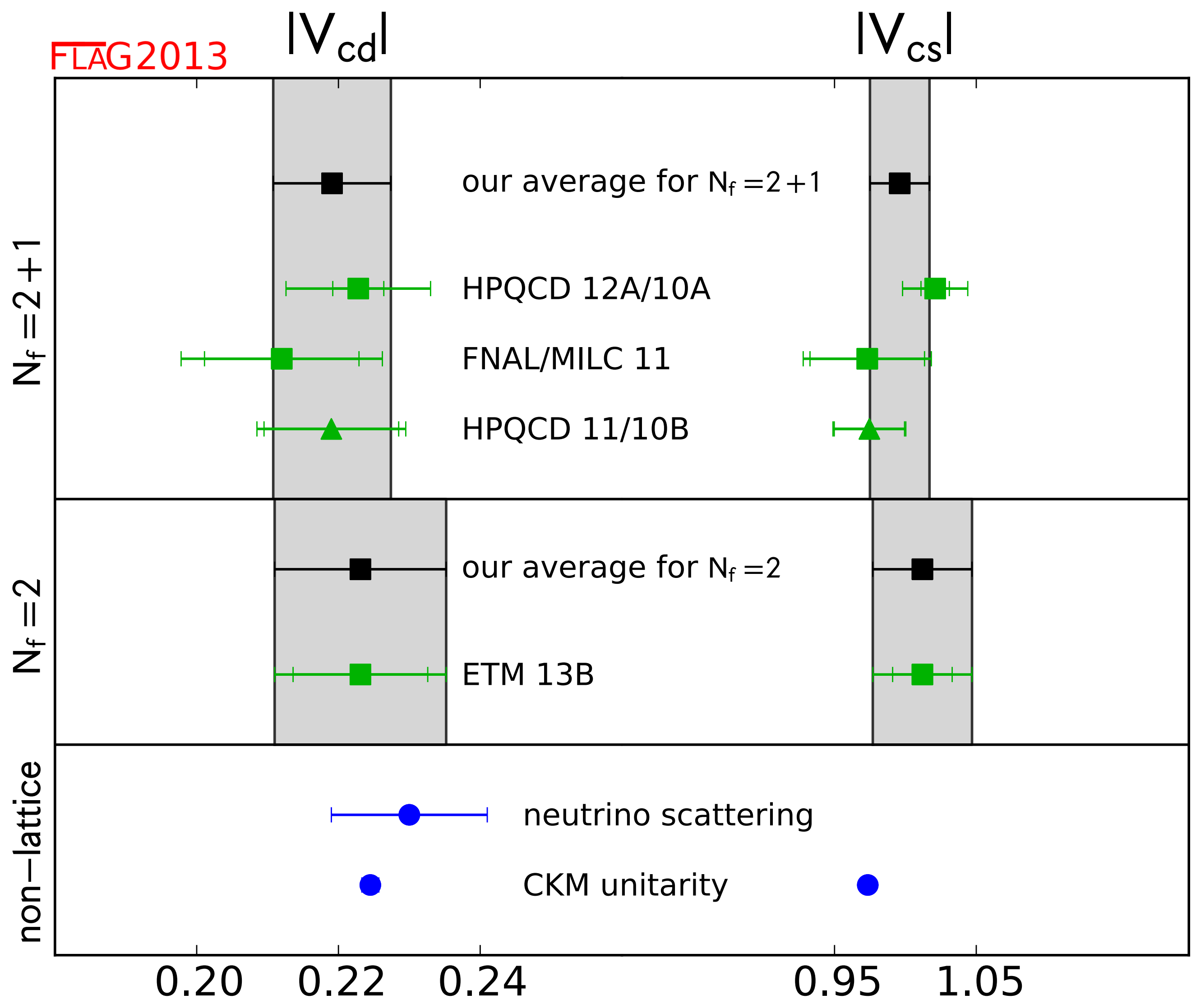
- N_f=2+1 $$|V_{cd}| = 0.2192(\underbrace{95}_{\text{lattice}})(\underbrace{45}_{\text{non-lattice th. \& exp.}}) \qquad |V_{cs}| = 0.9746(248)(67)$$

CKM angles $|V_{cd}|$ and $|V_{cs}|$



- *Vcd*: agreement
- *Vcs*: 1.2σ between leptonic/semileptonic; 1.9σ between leptonic and CKM-unit. (driven by HPQCD result; but note that the lattice estimate at N_f=2+1 supported by that at N_f=2)

CKM angles $|V_{cd}|$ and $|V_{cs}|$



$$|V_{cd}| \;=\; 0.2191(83) \qquad |V_{cs}| \;=\; 0.996(21) \qquad N_f = 2+1$$

- 2nd row unitarity agrees with SM (independently of $|V_{cb}| = O(10^{-2})$):

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 \;=\; 0.04(6) \qquad N_f = 2+1$$

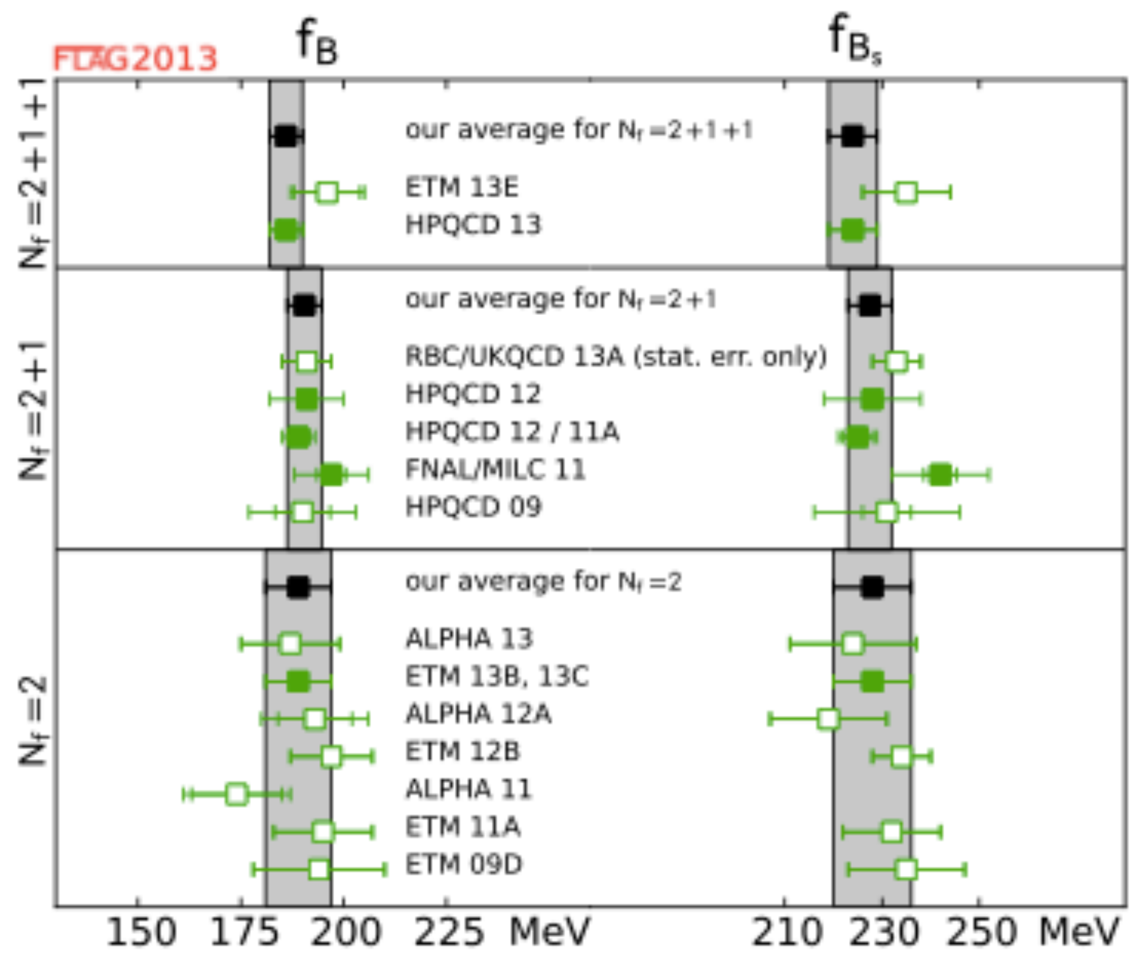
Bottom Physics

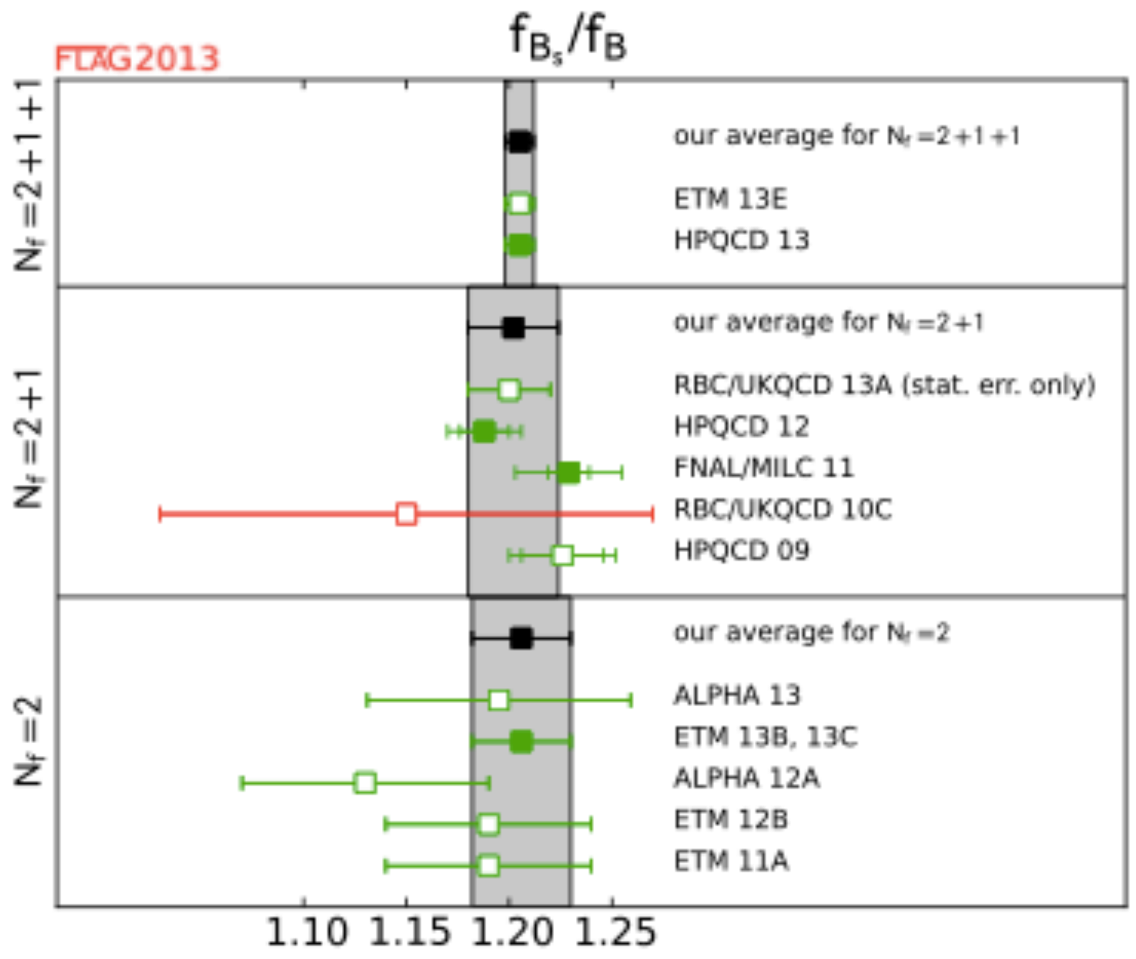
f_B, f_{Bs}, $f_+(0)$, $|V_{ub}|$

Generalities

- In present day computations $m_b \sim \Lambda_{UV} \sim 1/a$ so this mass cannot be simulated directly. Therefore:
  - introduce effective theories (HQET, NRQCD) and a systematic expansion in $1/m_b$ (non-relativistic treatment);
  - simulate with lighter than physical bottom masses of $O(1/m_c)$ and extrapolate to physical point m_b, or interpolate to HQET point.
- This results to new problems (matching of HQET to QCD, renormalization, control of discretization effects).
- There are less results than in light-quark Physics; situation is rapidly improving.

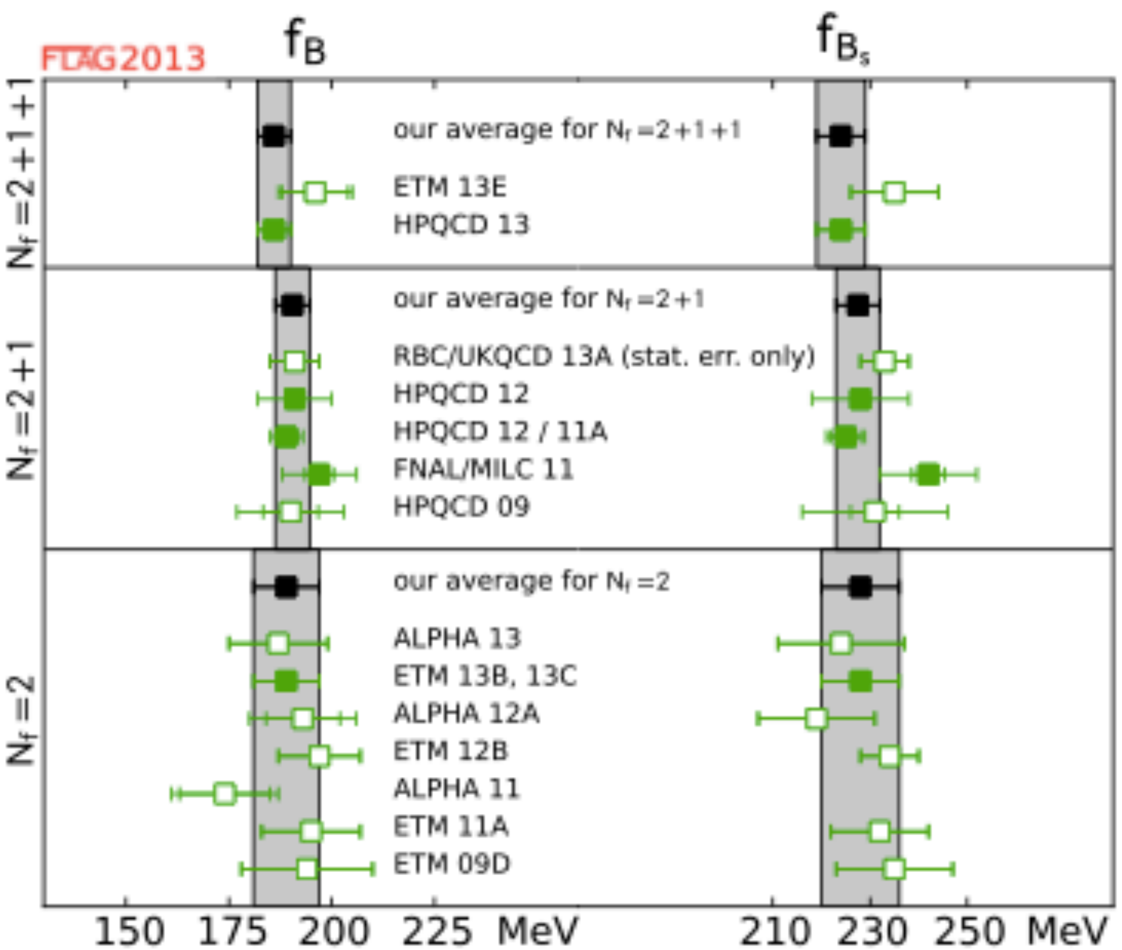
Leptonic decay constants f_B and f_{Bs}

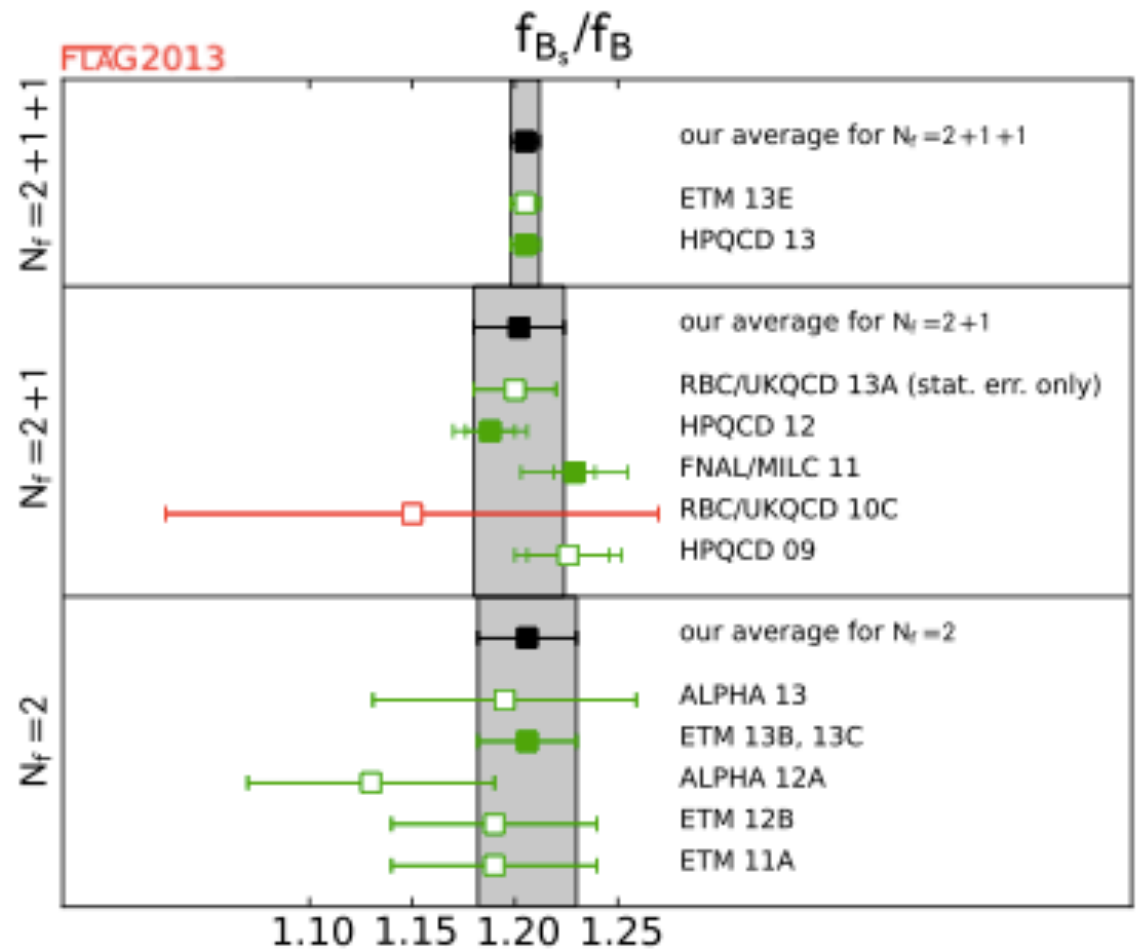




$$
\begin{aligned}
f_B &= 189 \pm 8 \text{ MeV} & N_f &= 2 \\
f_B &= 190.5 \pm 4.2 \text{ MeV} & N_f &= 2+1 \\
f_B &= 186 \pm 4 \text{ MeV} & N_f &= 2+1+1
\end{aligned}
$$

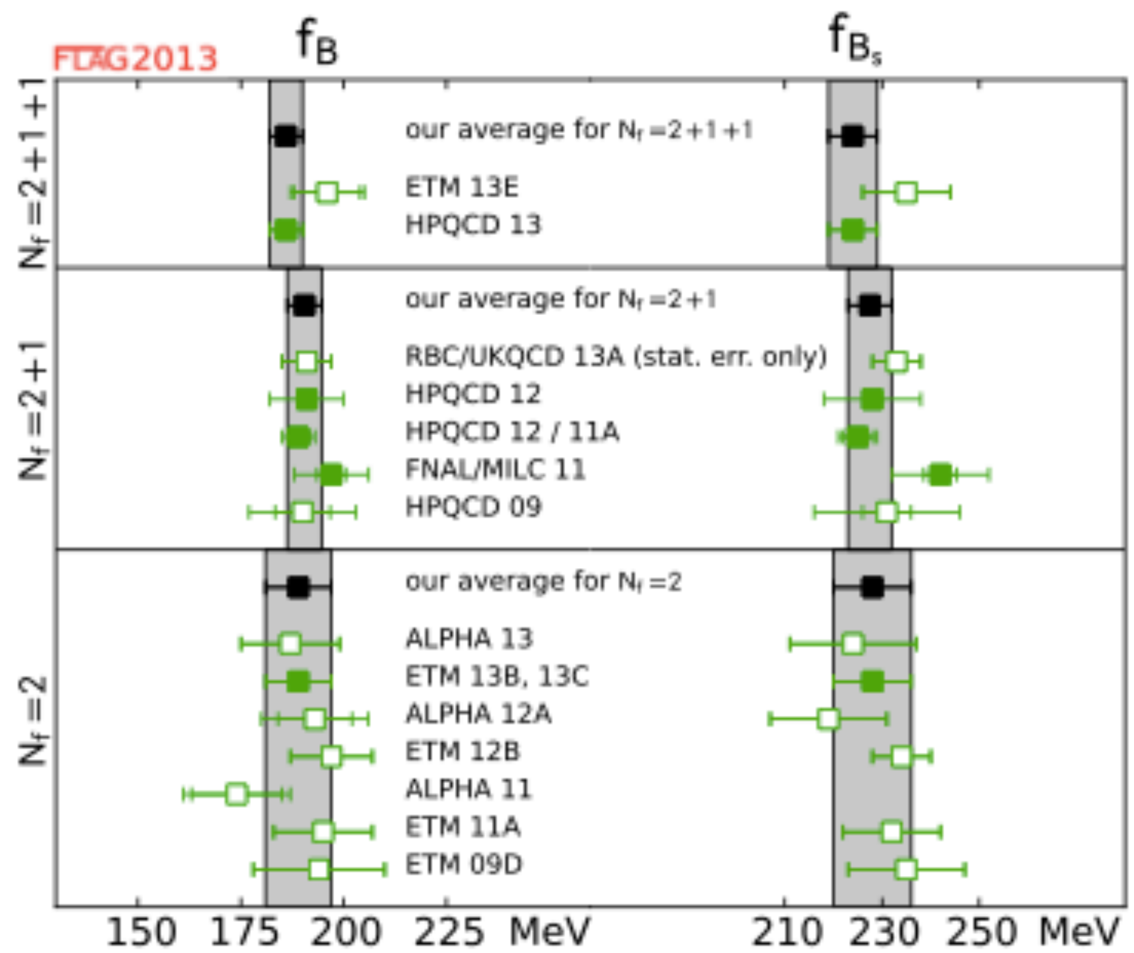
Leptonic decay constants f_B and f_{Bs}

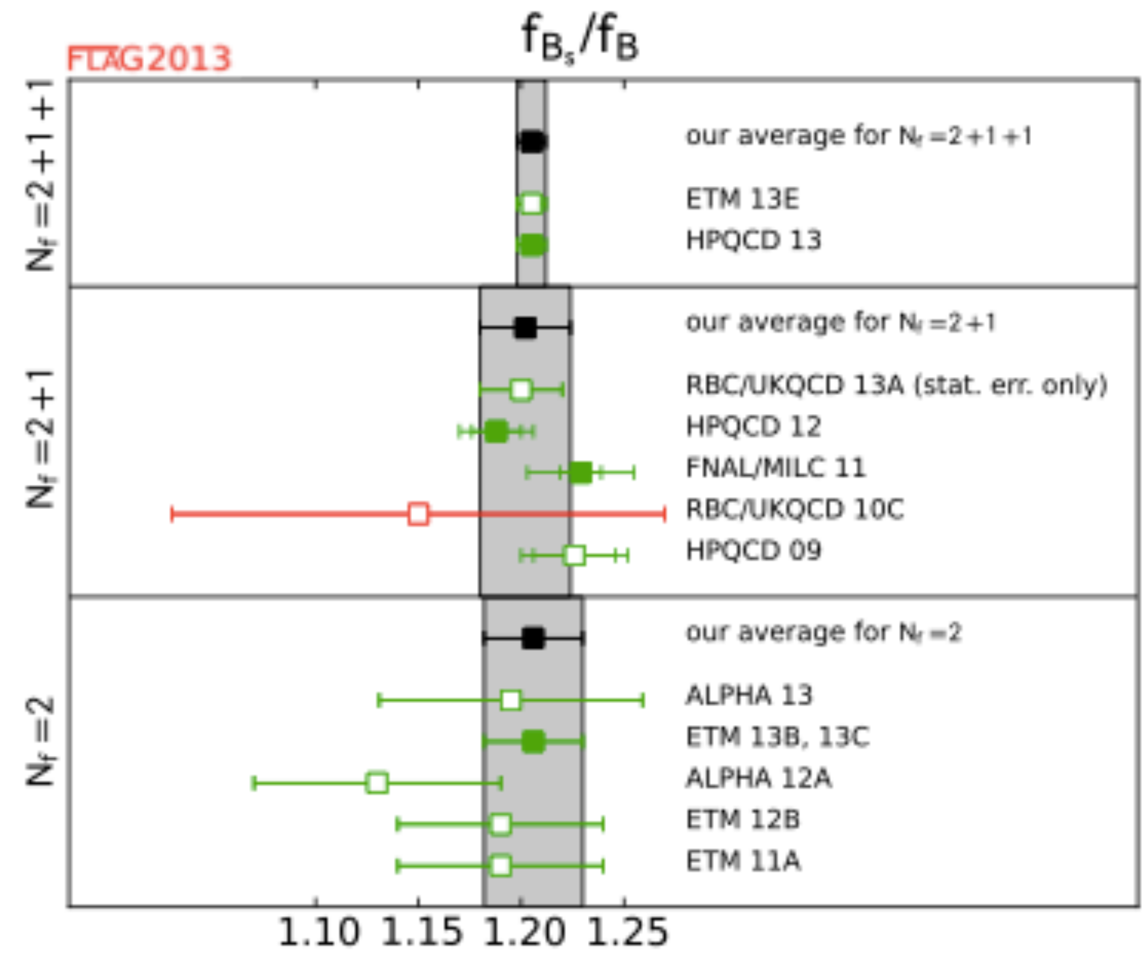




$$f_{B_s} = 228 \pm 8 \text{ MeV} \qquad N_f = 2$$
$$f_{B_s} = 227.7 \pm 4.5 \text{ MeV} \qquad N_f = 2+1$$
$$f_{B_s} = 224 \pm 5 \text{ MeV} \qquad N_f = 2+1+1$$

Leptonic decay constants f_B and f_{Bs}



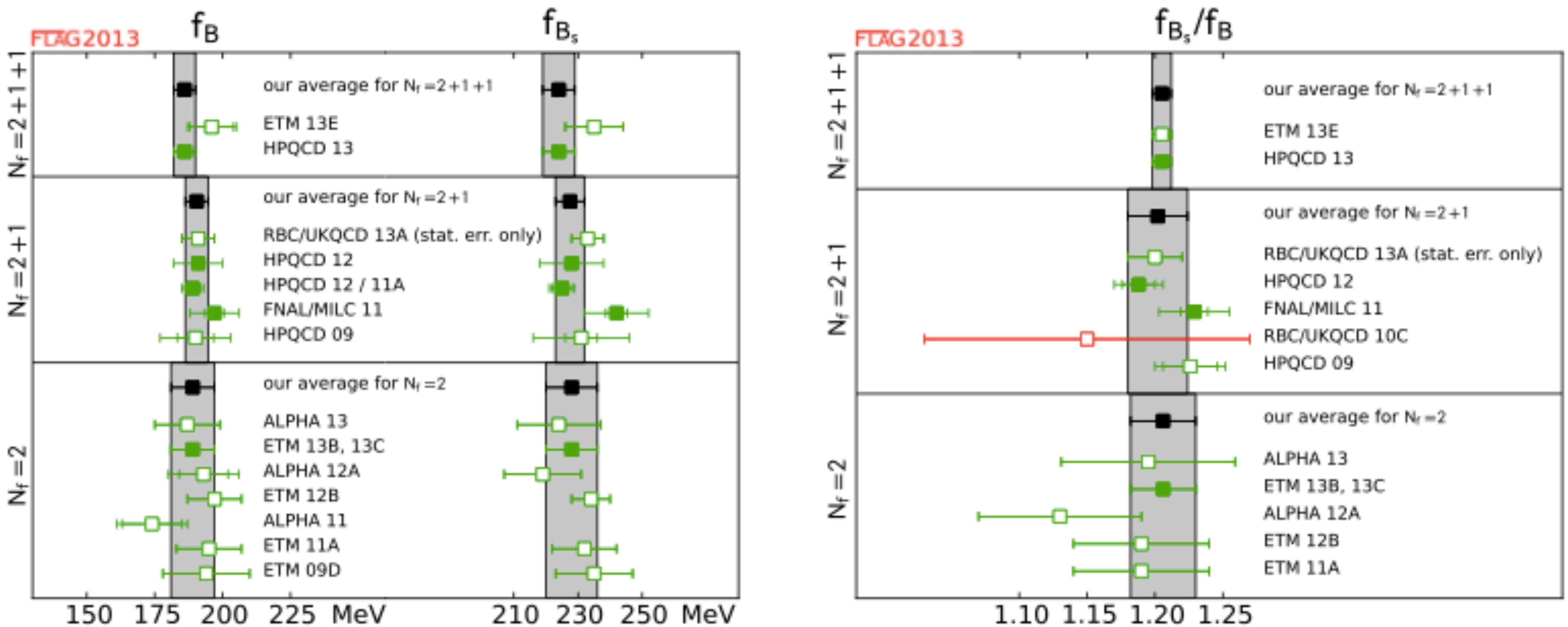


$$\frac{f_{B_s}}{f_B} = 1.206 \pm 0.024 \text{ MeV} \qquad N_f = 2$$

$$\frac{f_{B_s}}{f_B} = 1.202 \pm 0.022 \text{ MeV} \qquad N_f = 2+1$$

$$\frac{f_{B_s}}{f_B} = 1.205 \pm 0.007 \text{ MeV} \qquad N_f = 2+1+1$$

Leptonic decay constants f_B and f_{Bs}



- *NB:*
  - Most results, obtained with degenerate light quarks, refer to average decay constants for B^+ and B^0. Some collaborations (FNAL/MILC, HPQCD) have started giving distinct results (they differ by about 2%). As errors decrease with time, collaborations should start giving B^+ and B^0 results separately.

CKM angle $|V_{ub}|$

- Branching ratio for $B^+ \to \tau^+ \nu_\tau$ measured by Belle and BaBar

BaBar: B. Aubert et al., Phys.Rev.D81 (2010) 051101; J. Lees et al., Phys.Rev.D88 (2013) 031102

Belle: K.Hara et al., Phys.Rev.D82(2010) 071101; I. Adachi et al., Phys.Rev.Lett.110 (2013)131801

$$|V_{ub}| = 4.21(53)(18) \cdot 10^{-3} \qquad N_f = 2$$
$$|V_{ub}| = 4.18(52)(9) \cdot 10^{-3} \qquad N_f = 2+1$$
$$|V_{ub}| = 4.28(53)(9) \cdot 10^{-3} \qquad N_f = 2+1+1$$

- 1st error: experiment
- 2nd error: lattice

- Branching ratio for $B^0 \to \pi^- l^+ \nu$ ratio measured by Belle and BaBar

BaBar: J. Lees et al., Phys.Rev.D86 (2012) 092004; J. Lees et al., Phys.Rev.D88 (2013) 031102

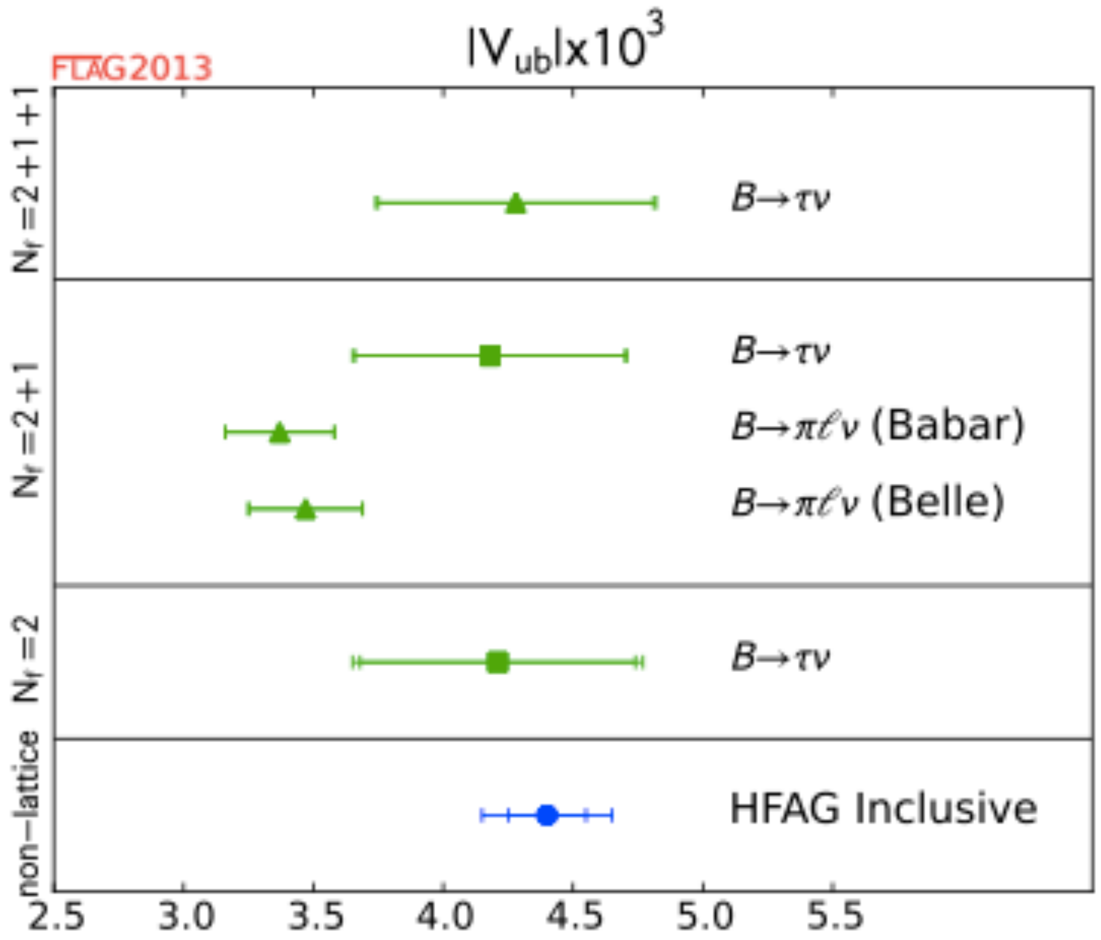
Belle: H.Ha et al., Phys.Rev.D83(2011) 071101; I. Adachi et al., Phys.Rev.Lett.110 (2013)131801

- Lattice form factor estimates are from FNAL/MILC (2008) and HPQCD (2006) for N_f=2+1

$$|V_{ub}| = 3.37(21) \cdot 10^{-3} \qquad N_f = 2+1 \qquad \text{BaBar}$$
$$|V_{ub}| = 3.47(22) \cdot 10^{-3} \qquad N_f = 2+1 \qquad \text{Belle}$$

Results reported separately, as experimental correlations cannot be properly taken into account

CKM angle $|V_{ub}|$



- Lattice central value from $B^+ \to \tau^+\nu_\tau$ lies between HFAG (inclusive) and lattice from $B^0 \to \pi^- l^+\nu$ (inclusive); due to big error it agrees within ~ 1.5σ with other determinations

- Tension ~ 3σ between HFAG (inclusive) and lattice from $B^0 \to \pi^- l^+\nu$ (inclusive)

- Situation still unclear; too much spread

- lattice improvements expected soon for the semi-leptonic $B^0 \to \pi^- l^+\nu$ determination of $|V_{ub}|$

- Belle II data (as from 2016) will improve leptonic $B^+ \to \tau^+\nu_\tau$ determination of $|V_{ub}|$

K^0-Meson Oscillations

B_K

in the SM (and beyond - preliminary)

B_K in the SM

- $\Delta S = 2$ transitions are governed by the transition amplitude of the effective Hamiltonian:

$$\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle \; = \; \frac{G_F^2 M_{\rm W}^2}{16\pi^2} \Big[\lambda_c^2 S_0(x_c)\eta_1 \; + \; \lambda_t^2 S_0(x_t)\eta_2 \; + \; 2\lambda_c\lambda_t S_0(x_c, x_t)\eta_3 \Big]$$

$$\times \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_{\rm R}^{\Delta S=2}(\mu) | K^0 \rangle \; + \; \text{h.c.}$$

B_K in the SM

- $\Delta S = 2$ transitions are governed by the transition amplitude of the effective Hamiltonian:

$$\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 M_{\rm W}^2}{16\pi^2} \left[\lambda_c^2 S_0(x_c)\eta_1 + \lambda_t^2 S_0(x_t)\eta_2 + 2\lambda_c\lambda_t S_0(x_c, x_t)\eta_3 \right]$$

$$\times \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_{\rm R}^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.}$$

- Four fermion $L \otimes L$ operator of dim=6 ($Q^{\Delta S=2}{}_{\rm R}$: renormalized; parity-even part contributes):

$$Q^{\Delta S=2} = [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma_\mu(1-\gamma_5)d] \equiv O_{\rm VV+AA} - O_{\rm VA+AV}$$

B_K in the SM

- $\Delta S = 2$ transitions are governed by the transition amplitude of the effective Hamiltonian:

$$\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 M_{\rm W}^2}{16\pi^2} \left[\lambda_c^2 S_0(x_c)\eta_1 + \lambda_t^2 S_0(x_t)\eta_2 + 2\lambda_c\lambda_t S_0(x_c, x_t)\eta_3 \right]$$

$$\times \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_{\rm R}^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.}$$

- Four fermion $L \otimes L$ operator of dim=6 ($Q^{\Delta S=2}{}_{\rm R}$: renormalized; parity-even part contributes):

$$Q^{\Delta S=2} = [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma_\mu(1-\gamma_5)d] \equiv O_{\rm VV+AA} - O_{\rm VA+AV}$$

- Computed on the lattice through its B_K-parameter:

$$B_{\rm K}(\mu) = \frac{\langle \bar{K}^0 \left| Q_{\rm R}^{\Delta S=2}(\mu) \right| K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$$

B_K in the SM

- $\Delta S = 2$ transitions are governed by the transition amplitude of the effective Hamiltonian:

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} \left[\lambda_c^2 S_0(x_c)\eta_1 + \lambda_t^2 S_0(x_t)\eta_2 + 2\lambda_c\lambda_t S_0(x_c, x_t)\eta_3 \right]$$

$$\times \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.}$$

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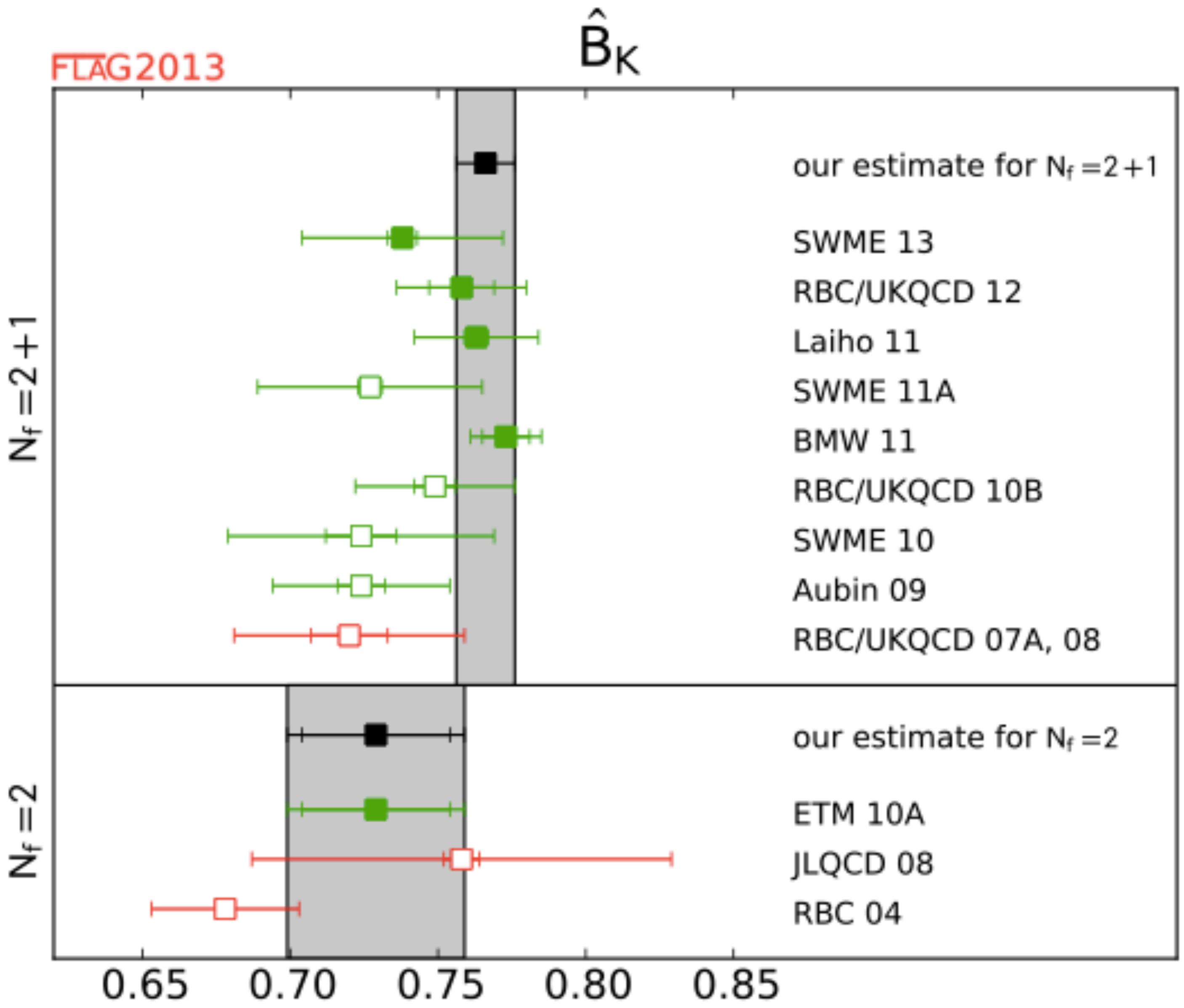
- Computed on the lattice through its B_K-parameter:

$$B_K(\mu) = \frac{\left\langle \bar{K}^0 \left| Q_R^{\Delta S=2}(\mu) \right| K^0 \right\rangle}{\frac{8}{3} f_K^2 m_K^2}$$

RGI (scale μ-independent at NLO)

$$\hat{B}_K$$

B_K in the SM



$$\hat{B}_K = 0.7661(99) \qquad N_f = 2+1$$
$$\hat{B}_K = 0.729(25)(17) \qquad N_f = 2$$

$$B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.5596(72) \qquad N_f = 2+1$$
$$B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.533(18)(12) \qquad N_f = 2$$

B_K-in the SM and beyond

NB: non-FLAG results

SWME: J.A. Bailey et al., arXiv:1503.06613

ETM: V.Bertone et al., JHEP03(2013)089

B_K in the SM

- Self consistency of ε_K, the role of B_K- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\Big(C_\epsilon \hat{B}_K X_{\rm SD} + \xi_0 + \xi_{\rm LD}\Big) \; + \; \cdots$$

known factor: $$C_\epsilon = \frac{G_F^2 F_K^2 m_{K^0} M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}$$ $$x_{c,t} \equiv m_{c,t}^2/M_W^2$$

short distance:

$$X_{\rm SD} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

Inami-Lim functions: $S_0(x_{c,t})$ $\quad S_0(x_c, x_t)$

Coefficients known to NLO, NNLO, NNLO: η_{tt} $\quad \eta_{ct}$ $\quad \eta_{cc}$

$$r = \{\eta_{cc}S_0(x_c) - 2\eta_{ct}S_0(x_c,x_t)\}/\{\eta_{tt}S_0(x_t)\}$$

B_K in the SM

- Self consistency of ε_K, the role of B_K- and $|V_{cb}|$ NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\left(C_\epsilon \hat{B}_K X_{\rm SD} + \xi_0 + \xi_{\rm LD}\right) \; + \; \cdots$$

short distance:

$$X_{\rm SD} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

long distance effect from absorptive part (-7% effect): $\xi_0 = {\rm Im}(A_0)/{\rm Re}(A_0)$

RBC/UKQCD T. Blum et al., Phys.Rev.Lett.108 (2012)141601

long distance effect from dispersive part (2% effect - neglected): $\xi_{\rm LD}$

RBC/UKQCD N. Christ et al., Phys.Rev.D88 (2013)014508

B_K in the SM

- Self consistency of ε_K, the role of B_K- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\left(C_\epsilon \hat{B}_K X_{\mathrm{SD}} + \xi_0 + \xi_{\mathrm{LD}}\right) \; + \; \cdots$$

short distance:

$$X_{\mathrm{SD}} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

Wolfenstein parameters NOT from UTfit / CKMfitter (they contain unwanted dependence on B_K, |Vcb| and ε_K)

Prefer Angle-Only-Fit (AOF) of A. Bevan, M. Bona et al., Nucl.Phys.Proc.Suppl.241-242 (2013) 89 for ρ and η

|Vus| ≈ λ from $K_{\mu 2}$ and K_{l3}

|Vcb| ≈ Aλ^2 NB: 4th POWER!

B_K in the SM

- Self consistency of ε_K, the role of B_K- and $|V_{cb}|$ NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\Big(C_\epsilon \hat{B}_K X_{\mathrm{SD}} + \xi_0 + \xi_{\mathrm{LD}}\Big) \; + \; \cdots$$

short distance:

$$X_{\mathrm{SD}} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

1.Use $N_f = 2+1$ FLAG result for B_K

2. Use inclusive channel ($B \to X_c\, l\, \nu$ and $B \to X_s\, \gamma$ decays) for $|V_{cb}|$ = 42.21(78)

A. Alberti, et al., Phys.Rev.Lett.114 (2015) 061802

Use exclusive channel ($B \to D^*\, l\, \nu$ decays) for $|V_{cb}|$ = 39.04(49)(53)(19)

FNAL/MILC: J.A. Bailey Phys.Rev.D89 (2014)014504

3. Calculate $|\epsilon_K^{\mathrm{SM}}|$ and compare it to $|\epsilon_K^{\mathrm{exp}}| = (2.228 \pm 0.011) \times 10^{-3}$

B_K in the SM

- Self consistency of ε_K, the role of B_K- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\left(C_\epsilon \hat{B}_K X_{\rm SD} + \xi_0 + \xi_{\rm LD}\right) + \cdots$$

short distance:

$$X_{\rm SD} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

- $|\epsilon_K^{\rm exp}| = (2.228 \pm 0.011) \times 10^{-3}$
- $|\epsilon_K^{\rm SM}| = (1.58 \pm 0.18) \times 10^{-3}$ exclusive |Vcb|
- $|\epsilon_K^{\rm SM}| = (2.13 \pm 0.23) \times 10^{-3}$ inclusive |Vcb|

$$\Delta\epsilon_{\rm K} \equiv |\epsilon_{\rm K}^{\rm SM}| - |\epsilon_{\rm K}^{\rm exp}|$$

$$\Delta\epsilon_{\rm K} = 3.6(2)\sigma \qquad \text{exclusive } |V_{cb}|$$

$$\Delta\epsilon_{\rm K} = 0.44(24)\sigma \qquad \text{inclusive } |V_{cb}|$$

neglected $\xi_{\rm LD}$ contribution (2%) cannot explain this 30% gap in $\Delta\varepsilon_K$

B_K in the SM

- Self consistency of ε_K, the role of B_K- and $|V_{cb}|$ NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\Big(C_\epsilon \hat{B}_K X_{\mathrm{SD}} + \xi_0 + \xi_{\mathrm{LD}}\Big) + \cdots$$

short distance:

$$X_{\mathrm{SD}} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

TABLE IX. Fractional error budget for $\varepsilon_K^{\mathrm{SM}}$ obtained using the AOF method, the exclusive V_{cb}, and the FLAG $\hat{B}_K$.

source	error (%)	memo
V_{cb}	40.7	FNAL/MILC
$\bar{\eta}$	21.0	AOF
η_{ct}	17.2	$c-t$ Box
η_{cc}	7.3	$c-c$ Box
$\bar{\rho}$	4.7	AOF
m_t	2.5	
ξ_0	2.2	RBC/UKQCD
$\hat{B}_K$	1.6	FLAG
m_c	1.0	
$\vdots$	$\vdots$	

Error budget tells us that B_K is not the dominant uncertainty

B_K beyond the SM

- Analyze New Physics (NP) effects in a model-independent way: assume a generalization of the effective $\Delta S = 2$ Hamiltonian which contains operators beyond the SM one; the amplitude is:

$$< \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 > = C_1 < \bar{K}^0 | O_1 | K^0 > + \sum_{i=2}^{5} C_i < \bar{K}^0 | O_i | K^0 >$$

$$\begin{aligned}
O_1 &= [\bar{s}^\alpha \gamma_\mu (1-\gamma_5) d^\alpha][\bar{s}^\beta \gamma_\mu (1-\gamma_5) d^\beta] \\
O_2 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1-\gamma_5) d^\beta] \\
O_3 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1-\gamma_5) d^\alpha] \\
O_4 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1+\gamma_5) d^\beta] \\
O_5 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1+\gamma_5) d^\alpha]
\end{aligned}$$

SM contributions

NP coupling

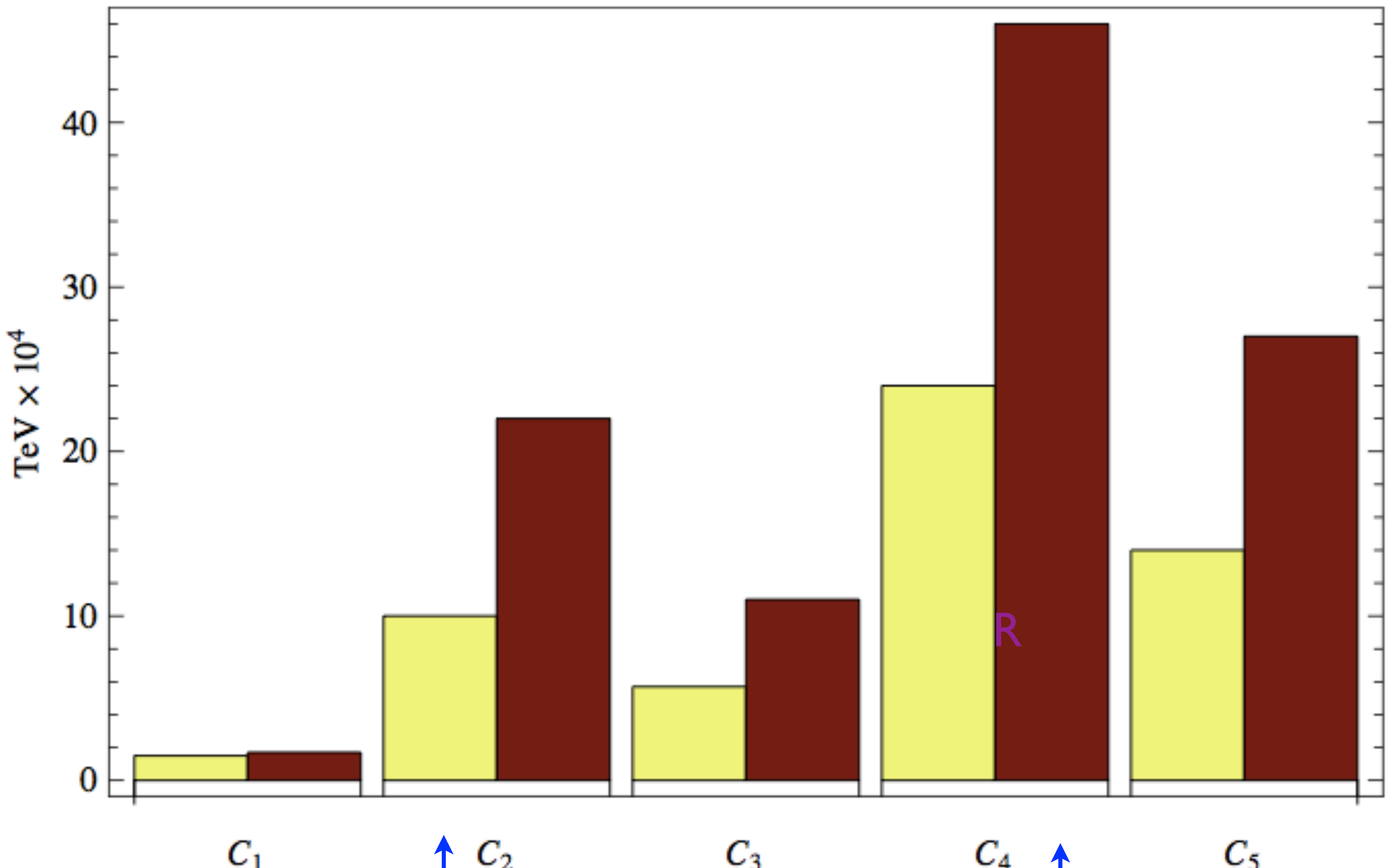
Loop factor coupling dependent

$$C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2} \qquad i = 2, \cdots, 5$$

NP scale

Assuming $F_i \sim L_i \sim 1$, generalized UT-fit analysis produces lower bounds for Λ; these depend very strongly (several orders of magnitude) on this assumption.

B_K beyond the SM



$$R_i = \frac{< \bar{K}^0|O_i|K^0 >}{< \bar{K}^0|O_1|K^0 >} \qquad i = 2, \cdots, 5$$

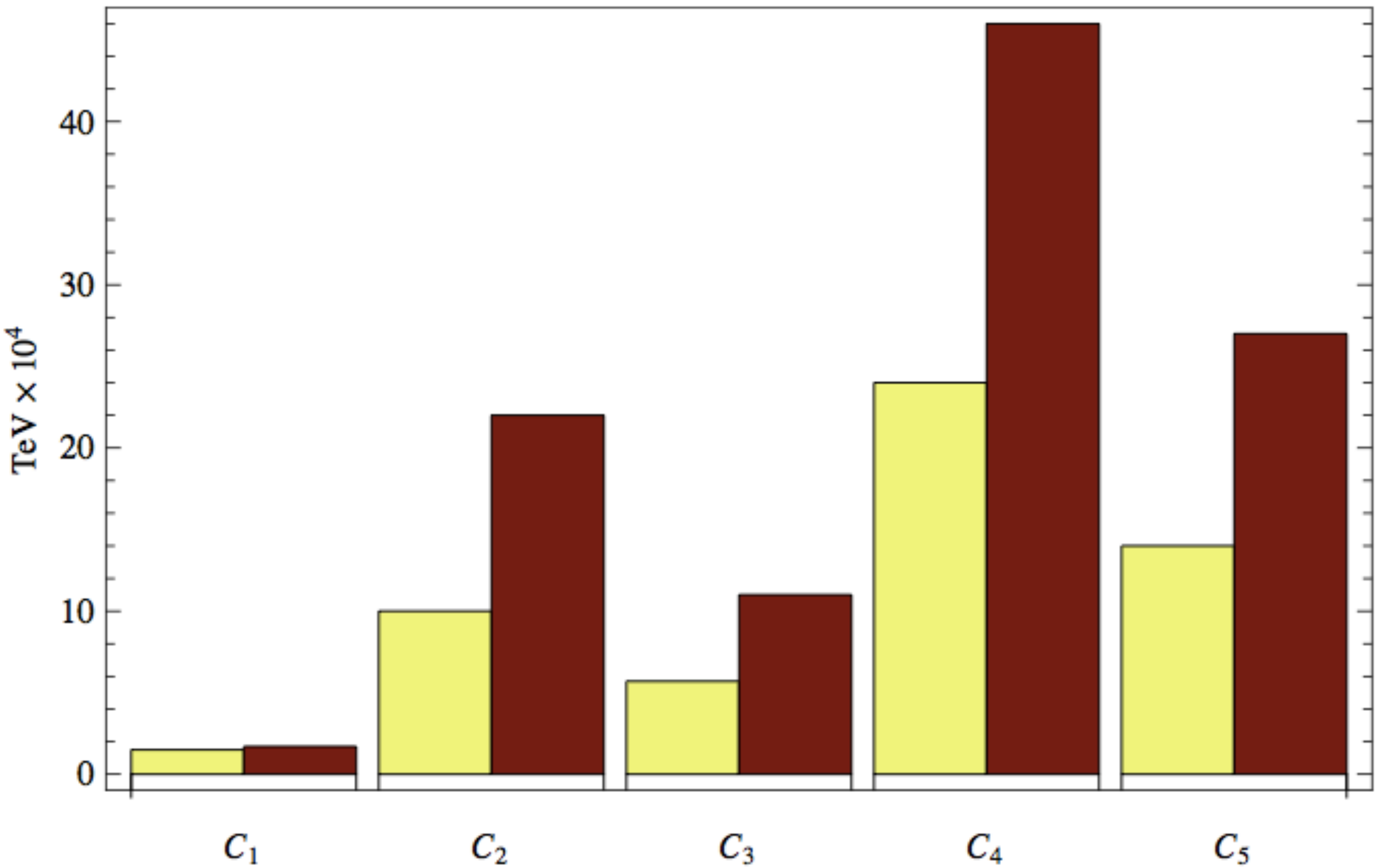
$N_f=0$ data; accuracy of ratios $R_i \sim$ 20%-23%

UTfit: M.Bona et al., JHEP03(2008)049

$N_f=2$ data; accuracy of ratios $R_i \sim$ 3%-6%

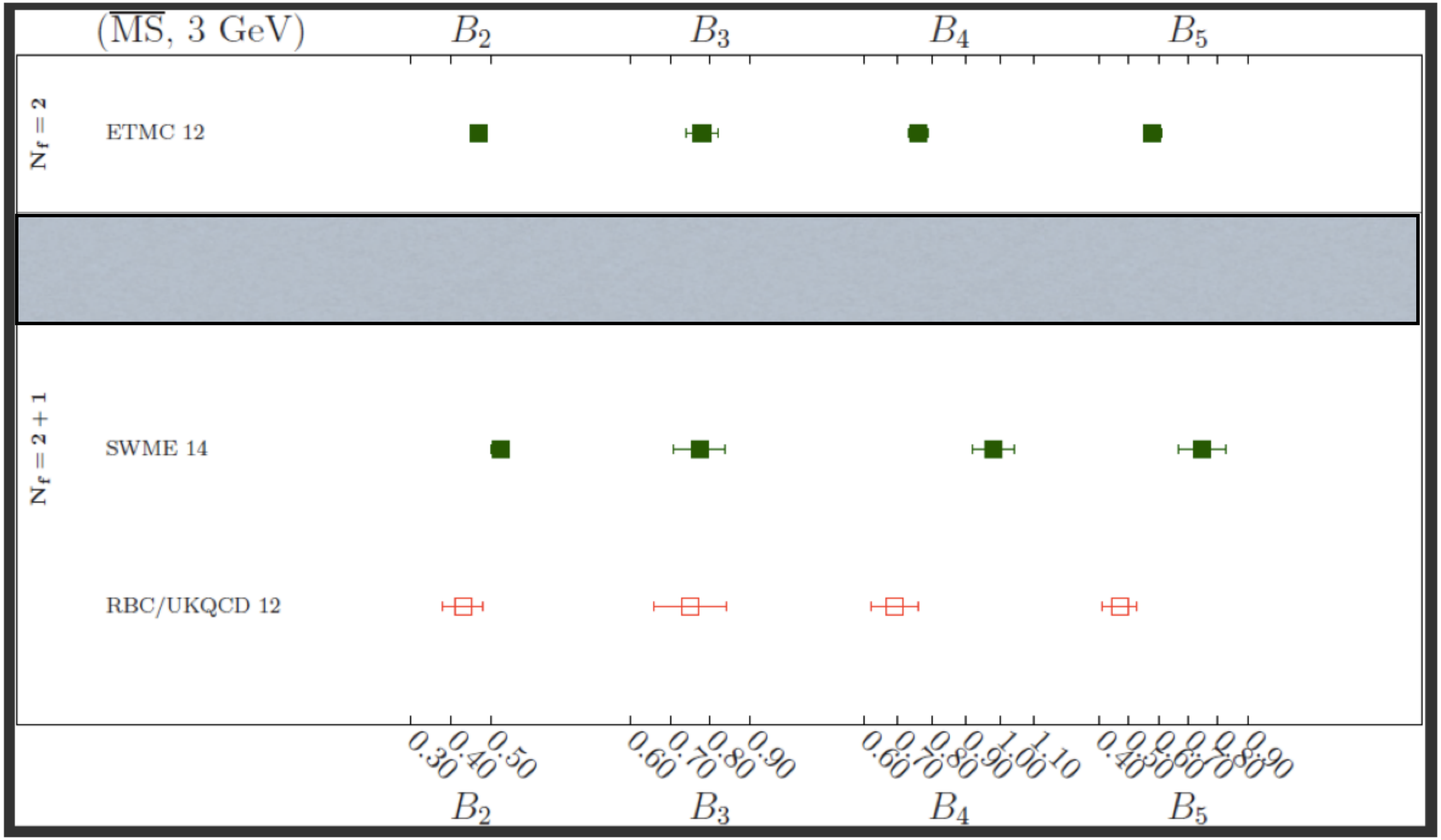
ETM: V.Bertone et al., JHEP03(2013)089

B_K beyond the SM



- NB: each contribution analyzed separately (avoids accidental cancellations).
- NB: SM bound is several orders of magnitude weaker thank those arising form BSM operators.

B_K beyond the SM



Courtesy of P. Dimopoulos

PRELIMINARY!!!!!!!

Conclusions

- Lattice is now credible and competes with the accuracy of experiments (in recent years we moved from 5% to 1%-2%).

- It is the responsibility of the lattice community to provide experimentalists and non-lattice theorists with a review of phenomenologically relevant lattice results with conservative error estimates.

- FLAG rates lattice output according to some quality criteria, performs averages or proposes estimates and is sometimes trying to push the analysis beyond that (e.g. CKM unitarity), stopping short of a UT analysis.

- FLAG has entered its third phase with a larger group and a slightly amplified Physics scope (charm and bottom quark masses, B_K beyond SM).

- The initiative is gaining momentum and the support of the lattice community as well as recognition in the wider high energy community.

FLAG-3 composition

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