Is quark-hadron duality valid in charm decays and mixing?

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Based on JHEP 09 (2021) 066 + 遂行中の研究

Exclusive

concrete hadronic final states

Examples:

$$B \to K^*\gamma, \bar{B} \to Dl\nu$$

$$D^0 \to K^-\pi^+ \to \bar{D}^0$$

theory: difficult

✔ Inclusive $\overset{?}{=}$ sum of exclusive

(quark-hadron duality)

◉ Heavy quark expansion (HQE).

$1/m_Q$: expansion parameter

$$\Gamma \propto \Big(\overset{\text{LO}}{1} + \underbrace{\frac{c_2}{m_Q^2} + \frac{c_3}{m_Q^3}}_{\text{higher order corrections}} + \cdots \Big), \quad Q = b, c$$

theory: easy(?)

(1) Is the above expansion convergent, expecially for charm quark?

(2) Is quark-hadron duality valid?

Inclusive processes

(1) Semi-leptonic decays of B meson

$$\Gamma[\bar{B} \to X_c l\nu] \stackrel{?}{=} \Gamma[\bar{B} \to D l\nu] + \Gamma[\bar{B} \to D^* l\nu] + \cdots$$

- Decay rate with kinematical cut is considered in practice.
- This is applied for the determination of $|V_{cb}|$. also for $c \to u$

✔ (2) Total width of B and D mesons

✔ (3) Width differences in $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ systems

- $1/m_Q$ expansion & duality can be testable.

Outline

(A) Inclusive analysis

— Decays: B and D mesons

— Mixings: $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$

(B) Quark hadron duality in the ’t Hooft model

— Mixing: $D^0 - \bar{D}^0$

— Decays: D mesons (preliminary)

(C) Summary

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(C) Summary

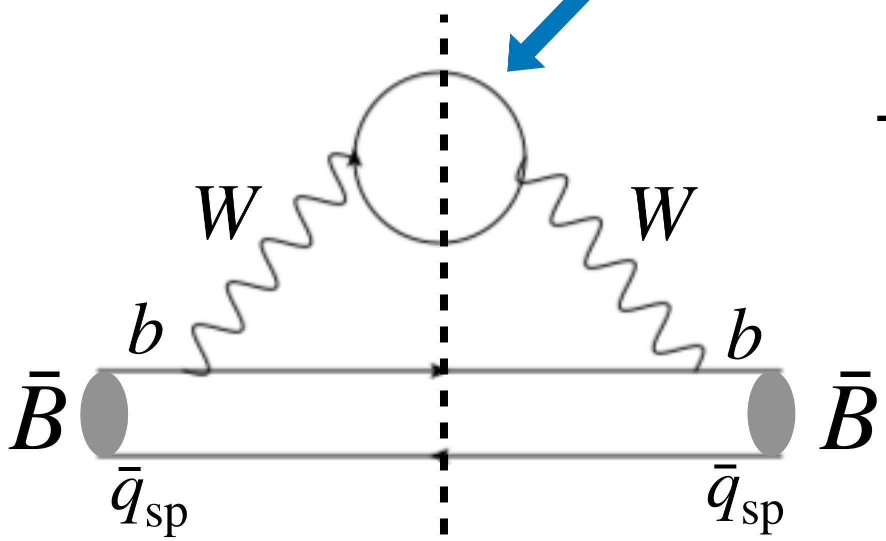
total width of B and D

$1/m_Q$ expansion $\qquad (Q = b, c)$

$$\Gamma = \frac{G_F^2 m_Q^5}{192\pi^3} |V_{\rm CKM}|^2 \Bigg[\overset{\rm LO}{c_3} - c_3 \frac{\mu_\pi^2}{2m_Q^2} + c_G \frac{\mu_G^2}{2m_Q^2} + \frac{c_6}{m_Q^3} \frac{< H_Q | (\bar{Q}q)(\bar{q}Q) | H_Q >}{M_{H_Q}} + \cdots \Bigg]$$

$1/m_Q^2$ corrections $\qquad \mathcal{O}(1/m_Q^3)$ correction

vanish for $m_Q \to \infty$



The spectator does not join interaction.

Lifetimes are common for

$B^+(\bar{b}u), B_d(\bar{b}d), B_s(\bar{b}s)$ and $D^0(c\bar{u}), D^+(c\bar{d}), D_s^+(c\bar{s})$

if $\mathcal{O}(1/m_Q^2)$ corrections are negligible.

lifetime ratio:
$$\frac{\tau(H_1)}{\tau(H_2)} = \overset{\rm LO}{1} + \frac{G_F^2 m_b^3}{384\pi^3} V_{cb}^2 \tau_1 \left[c_3 \left(\mu_\pi^2(H_1) - \mu_\pi^2(H_2) \right) + c_G \left(\mu_G^2(H_2) - \mu_G^2(H_1) \right) \right]$$
$$+ \frac{G_F^2 m_b^2}{192\pi^3} V_{cb}^2 \tau_1 \left[\frac{c_6(H_2)\langle H_2|Q|H_2\rangle - c_6(H_1)\langle H_1|Q|H_1\rangle}{M_B} + \mathcal{O}\left(\frac{\Lambda}{m_b}\right) \right]$$

If the lifetime ratio ≈ 1, the $1/m_Q$ expansion is convergent.
(provided that no cancellation occurs.)

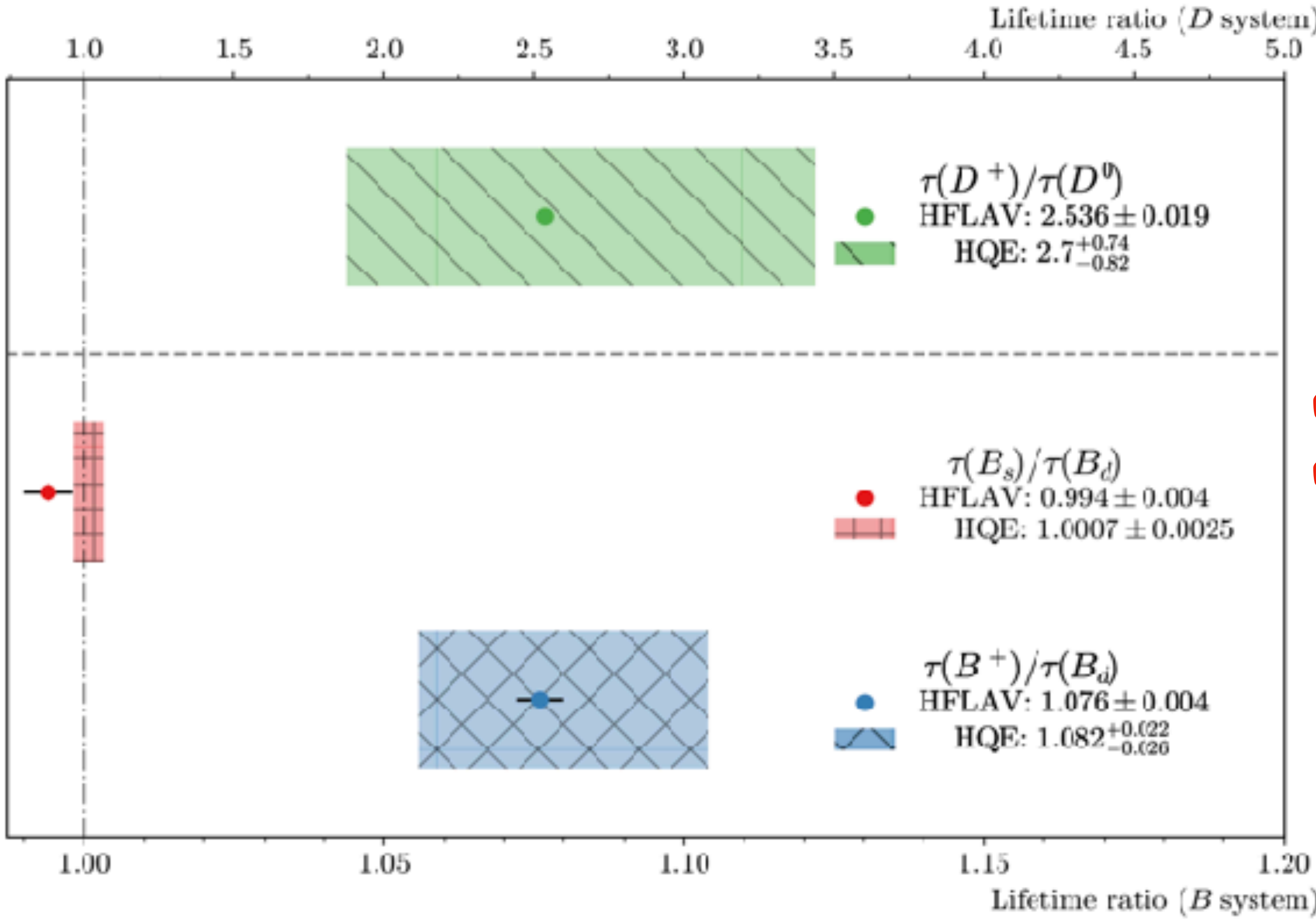
If not, the convergence is questioned.

What happens for realistic b, c?

$\longrightarrow$ Next slide

Lifetime ratios for B and D mesons

Kirk, Lenz and Rauh [1711.02100v4]



King *et al.* [2109.13219]

including the Darwin operator

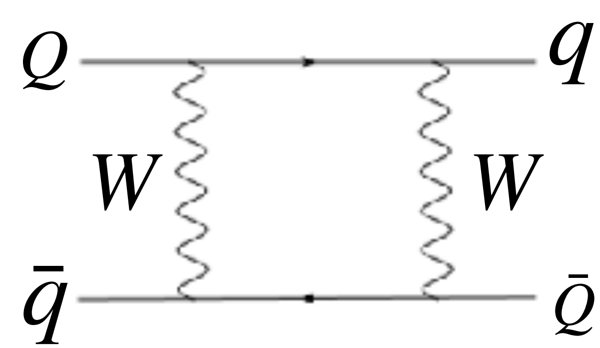
Observable	HQE prediction	Exp. value
$\Gamma(D^0)[\text{ps}^{-1}]$	$1.59 \pm 0.36^{+0.45}_{-0.36}{}^{+0.01}_{-0.01}$	2.44 ± 0.01
$\Gamma(D^+)[\text{ps}^{-1}]$	$-0.15 \pm 0.76^{+0.58}_{-0.27}{}^{+0.25}_{-0.10}$	0.96 ± 0.01
$\bar{\Gamma}(D_s^+)[\text{ps}^{-1}]$	$1.57 \pm 0.43^{+0.51}_{-0.40}{}^{+0.02}_{-0.01}$	1.88 ± 0.02
$\tau(D^+)/\tau(D^0)$	$2.80 \pm 0.85^{+0.01}_{-0.14}{}^{+0.11}_{-0.26}$	2.54 ± 0.02
$\bar{\tau}(D_s^+)/\tau(D^0)$	$1.01 \pm 0.15^{+0.02}_{-0.03}{}^{+0.01}_{-0.01}$	1.30 ± 0.01
$B_{sl}^{D^0}$ [%]	$5.91 \pm 1.57^{-0.33}_{-0.28}$	6.49 ± 0.11
$B_{sl}^{D^+}$ [%]	$15.0 \pm 4.04^{+0.83}_{-0.72}$	16.07 ± 0.30
$B_{sl}^{D_s^+}$ [%]	$7.76 \pm 2.62^{-0.43}_{-0.38}$	6.30 ± 0.16
$\Gamma_{sl}^{D^+}/\Gamma_{sl}^{D^0}$	$1.001 \pm 0.008 \pm 0.001$	0.985 ± 0.028
$\Gamma_{sl}^{D_s^+}/\Gamma_{sl}^{D^0}$	$1.06 \pm 0.23 \pm 0.01$	0.790 ± 0.026

- $\tau(B_s)/\tau(B_d)$ and $\tau(B^+)/\tau(B_d)$ are close to 1. ➡ $1/m_b$ expansion works.
- $\tau(D^+)/\tau(D^0)$ is much larger than 1. ➡ $1/m_c$ expansion may be non-convergent series?

⟶ Nevertheless, a good agreement is obtained within theoretical errors.

Mixings: $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$

Box diagram



$$M_{12} - \frac{i}{2}\Gamma_{12} = \frac{< H_Q | \mathscr{H}_{\text{eff}} | \bar{H}_Q >}{2M_{H_Q}}$$

Γ_{12} : on − shell contribution

$1/m_Q$ expansion

$$\Gamma_{12} = -\frac{G_F^2 m_Q^2}{24\pi M_{H_Q}} (V_{\text{CKM}})^2 [C(z) < H_Q | (\bar{q}Q)_{V-A} (\bar{q}Q)_{V-A} | \bar{H}_Q > \\ + C_S(z) < H_Q | (\bar{q}Q)_{S-P} (\bar{q}Q)_{S-P} | \bar{H}_Q > \overbrace{+ \cdots}^{1/m_Q \text{ corrections}}]$$

◉ $D^0 - \bar{D}^0$ mixing is strongly suppressed by GIM cancellation.

⟶ A tiny error may be enlarged.

Comparison between theory and experiment

D meson	B_s meson	B_d meson
Box diagram: Hagelin 1981, Cheng 1982; Buras, Slominski and Steger 1984	Lenz and Tetlalmatzi-Xolocotzi 2019	Lenz and Tetlalmatzi-Xolocotzi 2019
NLO QCD: ✔Golowich and Petrov 2005; Bobrowski *et al.* 2010		
SM $y \simeq 6 \cdot 10^{-7}$	SM $\Delta\Gamma_s = (0.091 \pm 0.013)\ \mathrm{ps}^{-1}$	SM $\Delta\Gamma_d = (2.6 \pm 0.4) \times 10^{-3}\ \mathrm{ps}^{-1}$
suppressed by GIM mechanism		
$y = \Gamma_{12}/\Gamma_D$		
HFLAV 2019	HFLAV 2019	HFLAV 2019
Exp. $y = (6.51^{+0.63}_{-0.69}) \times 10^{-3}$	Exp. $\Delta\Gamma_s = (0.090 \pm 0.005)\ \mathrm{ps}^{-1}$	Exp. $\Delta\Gamma_d = (0.002 \pm 0.020)\ \mathrm{ps}^{-1}$

- For B_s meson, the HQE gives good agreement with data.
- For B_d meson, the HQE is consistent with data within the error in data.
- For D meson, the order of magnitude is not reproduced within the leading-HQE.

Comparison between theory and experiment

D meson

<u>Box diagram</u> Hagelin 1981, Cheng 1982
Buras, Slominski and Steger 1984

<u>NLO QCD</u> ✔Golowich and Petrov 2005
Bobrowski *et al.* 2010

$\boxed{\text{SM}}$ $y \simeq 6 \cdot 10^{-7}$

suppressed by GIM mechanism

$$y = \Gamma_{12}/\Gamma_D$$

HFLAV 2019

$\boxed{\text{Exp.}}$ $y = (6.51^{+0.63}_{-0.69}) \times 10^{-3}$

Le

$\boxed{\text{SM}}$

$\boxed{\text{Exp.}}$

Possibilities discussed in the literature

✔ Violation of quark-hadron duality?

— 20% violation explains the data. (based on a simple model)

Jubb, Kirk, Lenz and Tetlalmatzi-Xolocotzi, 2017

Contribution of higher dim. operators?

suggested by Georgi, 1992, prior to the experimental measurement

— it may give a source of SU(3) breaking linear in m_s,
avoiding severe GIM cancellation?

$x, y \sim \mathcal{O}(10^{-3})$ Bigi and Uraltsev, 2001

— With some assumption about hadronic matrix elements,
$x \sim y \lesssim 10^{-3}$ Falk, Grossman, Ligeti and Petrov, 2001

Beyond the standard model?

e.g., Golowich, Pakvasa and Petrov, 2007
Golowich, Hewett, Pakvasa and Petrov, 2009

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— Decays: D mesons (preliminary result)

(C) Summary

Method to investigate duality violation

$$\Gamma_{\rm inc} \overset{?}{\neq} \sum \Gamma_{\rm exc}$$

<u>Comparison between $d = 4$ and $d = 2$</u>

	inclusive (HQE)	exclusive (Exp.)	exclusive (Theo.)
$d = 4$	✔	✔	✘
$d = 2$	✔	✘	✔

QCD is solvable in the large-N_c limit

't Hooft, 1974

The 't Hooft model (QCD_2 in the large-N_c limit)

◎ Asymptotic free theory.

◎ Gluon is not dynamical.

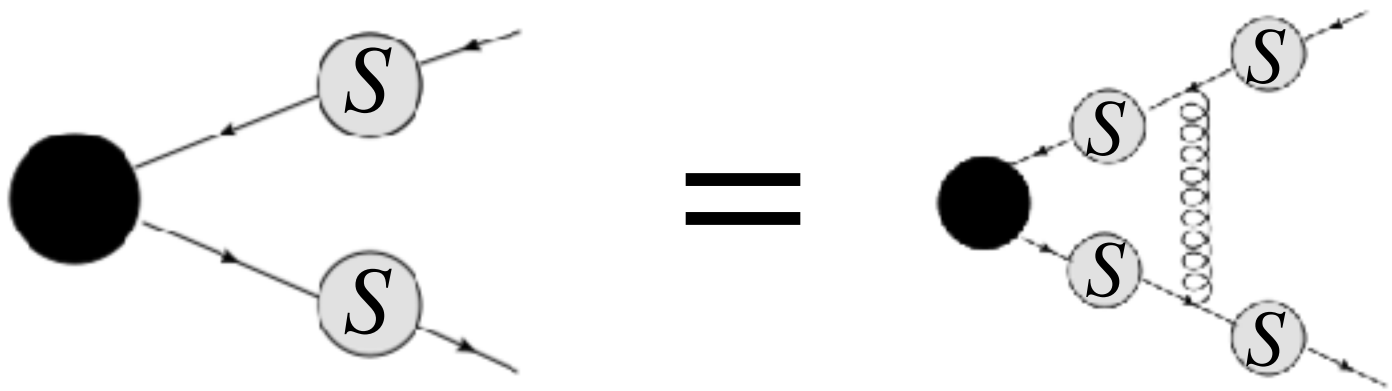
◎ Confinement is built-in.

◎ Phase space offen has singularity in $d = 2$.

➡ Duality violation is qualitatively testable.

Bound state equation in the 't Hooft model

Bethe-Salpeter equation (in the light-cone gauge):



the 't Hooft equation: $M_k^2 \phi_k(x) = \left(\frac{m_1^2 - \beta^2}{x} + \frac{m_2^2 - \beta^2}{1-x} \right) \phi_k(x) - \beta^2 \mathrm{Pr} \int_0^1 \frac{\phi_k(y) \mathrm{d}y}{(x-y)^2}$

notation of QCD coupling: $\beta^2 = \frac{g^2}{2\pi} \left(N_c - \frac{1}{N_c} \right), \quad \lim_{N_c \to \infty} \beta = \mathrm{const.}$

Masses and wavefunctions for mesons can be determined within the formalism.

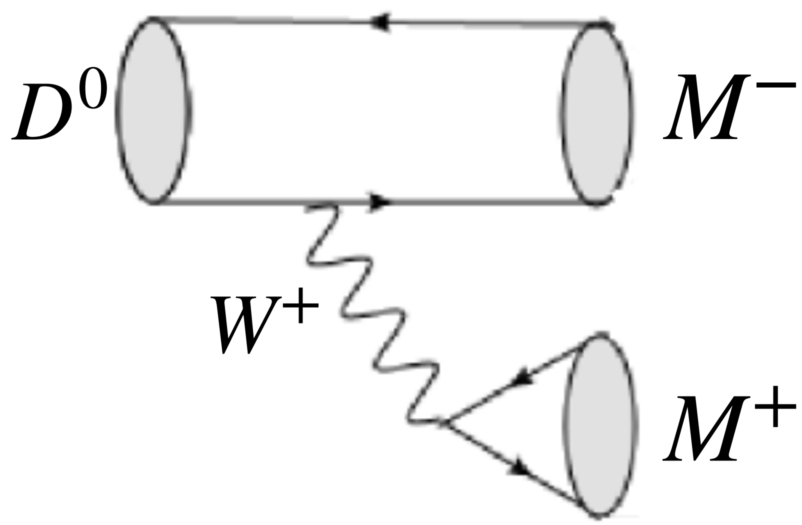
Exclusive processes for $D^0 - \bar{D}^0$ mixing

Width difference in CP conserving limit:

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 \,|\, p_{km} \,|}$$

phase space in 2D

$T^{(k,m)}$: color-allowed tree diagram



$M = \pi, K, +$ (excited states)

calculated by Grinstein and Lebed, 1998

$$T^{(k,m)}_{(Q\bar{q})(i,j)} = 2\sqrt{2} G_F (c_V^2 - c_A^2) \sqrt{\frac{N_c}{\pi}} c_k^{(qi)} \left[\sum_{n=0} \frac{[(-1)^k q^2 + (-1)^n M_n^2] c_n^{(Qj)}}{q^2 - M_n^2} F_{nm} + (-1)^{k+1} q^2 \mathcal{C}_m + m_Q m_j \mathcal{D}_m \right],$$

F_{nm}, $\mathcal{C}_m$, $\mathcal{D}_m$: overlap integrals of meson wave functions

Inclusive result for $D^0 - \bar{D}^0$

$$\Gamma_{12} = C_A < \overline{D^0} \,|\, (\bar{u}^\alpha \gamma^\mu \gamma_5 c^\alpha)(\bar{u}^\beta \gamma_\mu \gamma_5 c^\beta) \,|\, D^0 > \; + C_P < \overline{D^0} \,|\, (\bar{u}^\alpha i\gamma_5 c^\alpha)(\bar{u}^\beta i\gamma_5 c^\beta) \,|\, D^0 >$$

$$\begin{cases} C_A = + \dfrac{2G_F^2}{M_{D^0}} (c_V^2 - c_A^2) V_{ci}^* V_{uj} [(c_V^2 - c_A^2)(F_{ij}^{(\mathrm{th})} + 2G_{ij}^{(\mathrm{th})}) - (c_V^2 + c_A^2)(H_{ij}^{(\mathrm{th})} + H_{ji}^{(\mathrm{th})})] \\ C_P = - \dfrac{2G_F^2}{M_{D^0}} (c_V^2 - c_A^2) V_{ci}^* V_{uj} [(c_V^2 - c_A^2)(G_{ij}^{(\mathrm{th})} + 2H_{ij}^{(\mathrm{th})}) + (c_V^2 + c_A^2)(H_{ij}^{(\mathrm{th})} + H_{ji}^{(\mathrm{th})})] \end{cases}$$

generalized weak vertex: $\dfrac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^\mu (c_V + c_A \gamma_5)$

the standard model $c_V = \dfrac{1}{2}, \quad c_A = -\dfrac{1}{2}$

Matrix elements are calculated in the large-N_c factorization.

$F_{ij}^{(\mathrm{th})} = \sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}$, : 4D-like phase space

$G_{ij}^{(\mathrm{th})} = \dfrac{z_i + z_j - (z_i - z_j)^2}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}}$: 2D-specific phase space

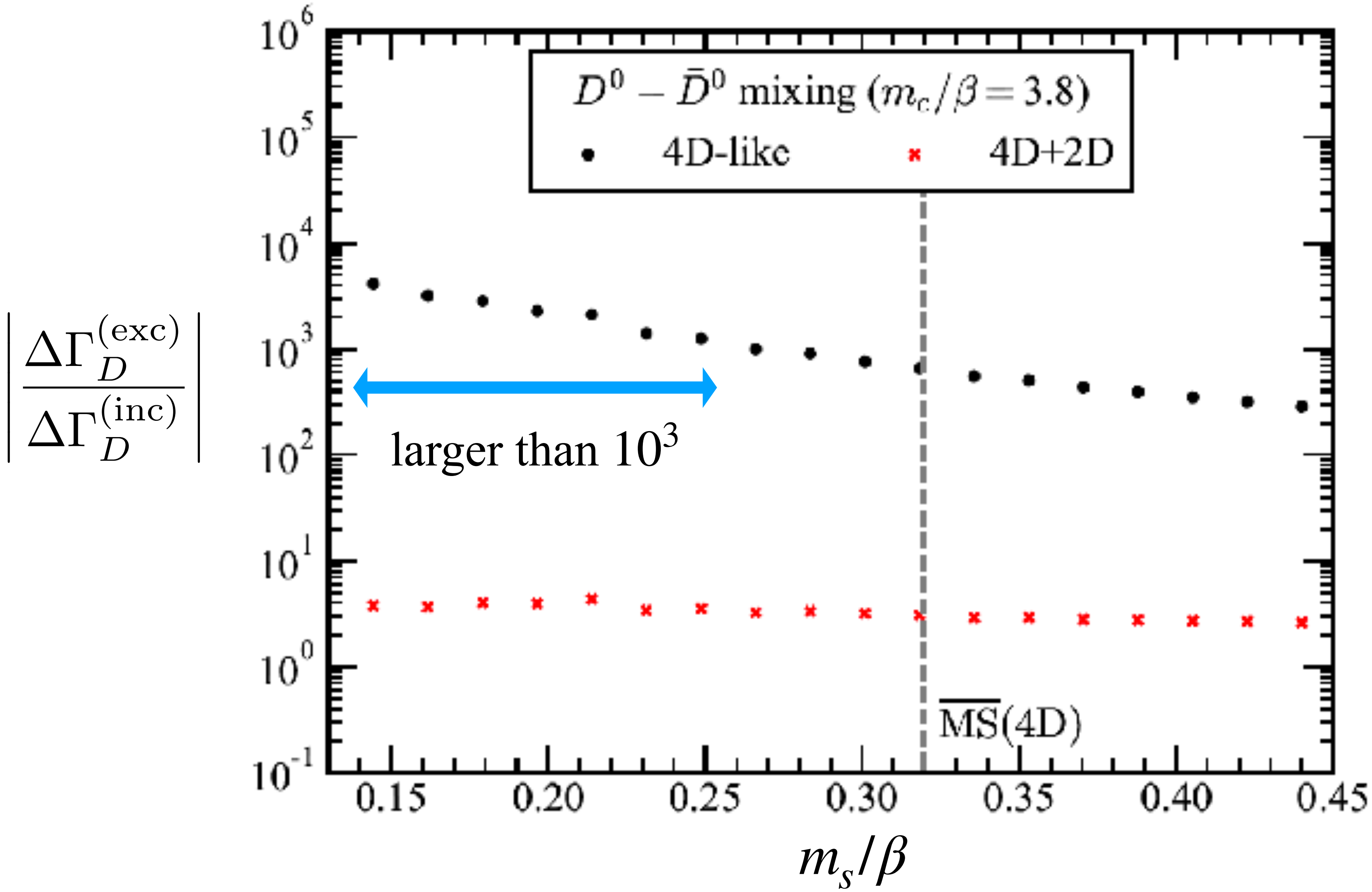
$H_{ij}^{(\mathrm{th})} = \dfrac{\sqrt{z_i z_j}}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}}$: 2D-specific phase space

$$\begin{cases} z_d = m_d^2 / m_c^2 \\ z_s = m_s^2 / m_c^2 \end{cases}$$

expansion parameter

We present the two cases:
- (1) only 4D-like term
- (2) pure 2D result (all of the 4D-like and 2D-specific terms are included)

Numerical result: $D^0 - \bar{D}^0$ mixing



$$m_c = m_c^{\overline{\mathrm{MS}},4\mathrm{D}}$$

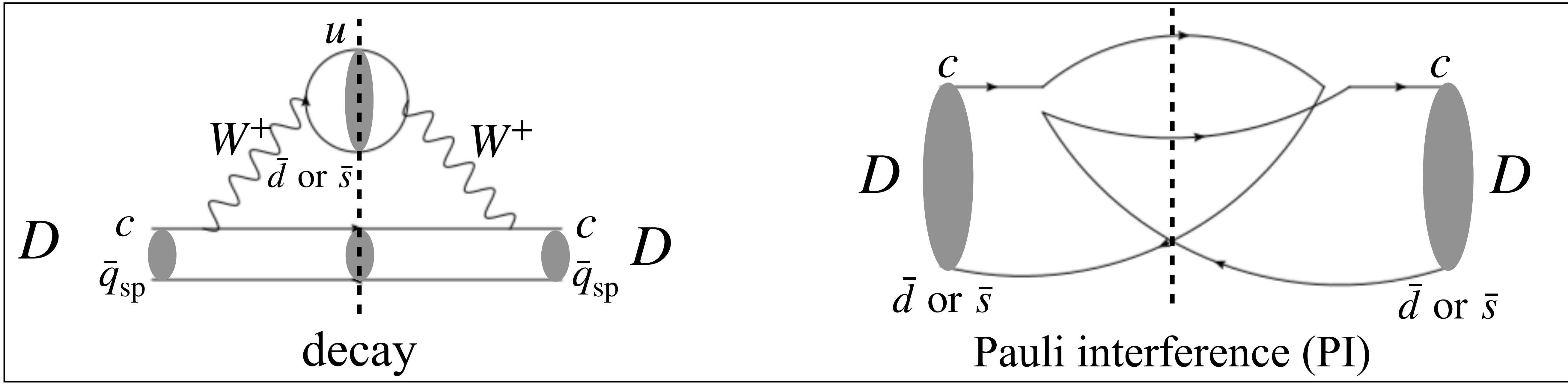
points: based only on the 4D-like phase space
crosses: based on the 4D-like phase space + 2D-specific one

◎ The exclusive rate is enhanced by more than 10^3, confirmed for $0.14 < m_s/\beta < 0.25$, when only the 4D-like phase space function is used.

Lifetimes of D mesons

$1/m_c$ correction

$$\Gamma = \Gamma_{\text{dec}} + \Gamma_{\text{PI}}$$



charged current neutral current

$$\mathcal{L}_{|\Delta C|=1} = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} [a_1 (\bar{s}^\alpha \gamma_\mu c^\alpha)(\bar{u}^\beta \gamma_\mu d^\beta) + a_2 (\bar{u}^\alpha \gamma_\mu c^\alpha)(\bar{s}^\beta \gamma_\mu d^\beta)]$$

Fiertz rearranged

Weak interaction parameters are taken by the corresponding 4D values

$a_1 \approx 1.2,\ a_2 \approx -0.38$ CKM: from PDG

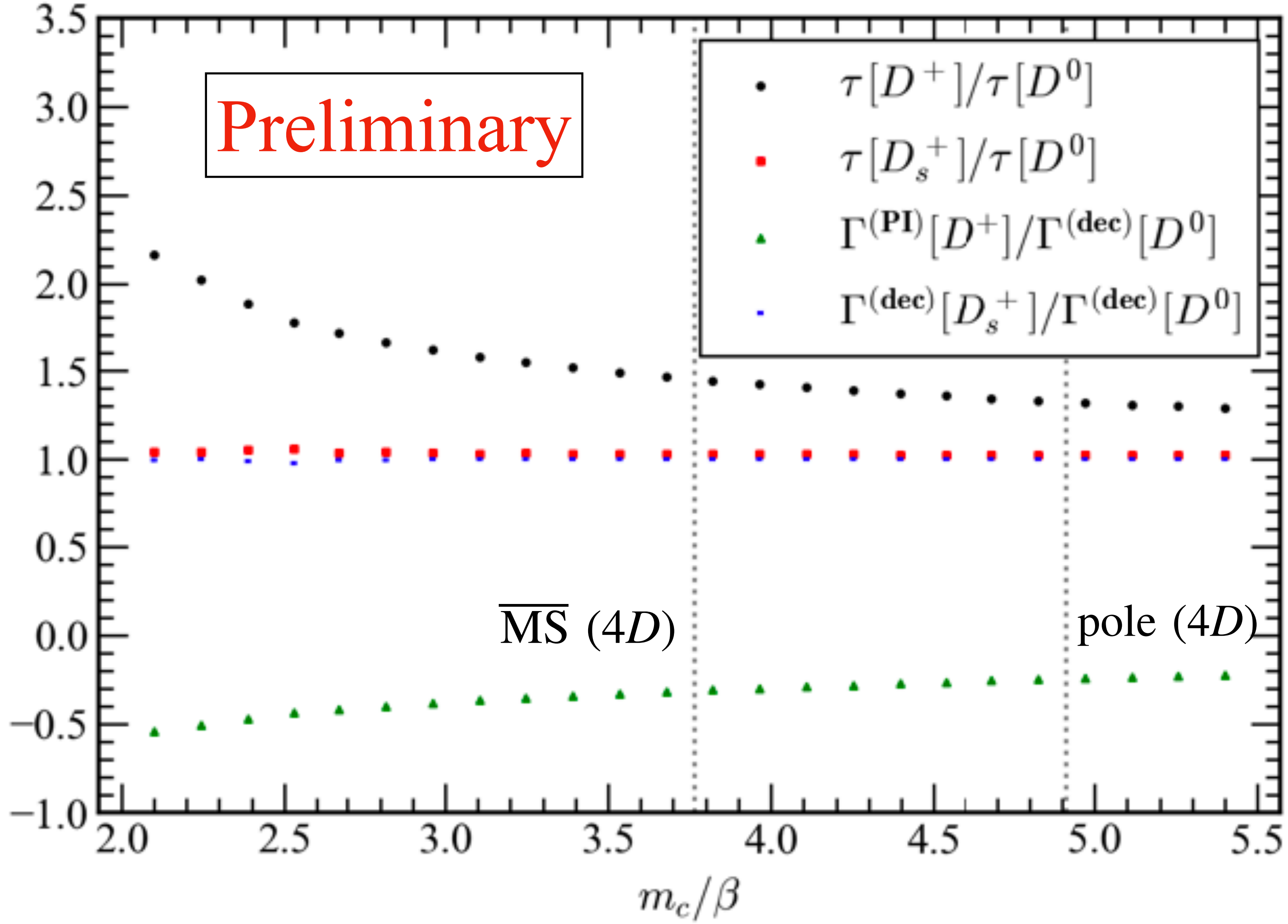
exclusive

$$\begin{cases} \Gamma^{(\mathbf{dec},\,\text{hadron})}_{H,\,(q_1,q_2,q_3)} = a_1^2 \sum_{k,m} \dfrac{\left| T^{(k,m)}_{(Q\bar{q})(\bar{q}_1,q_2,q_3)} \right|^2}{4M_H^2 |p_{km}|} + a_2^2 \sum_{k,m} \dfrac{\left| T^{(k,m)}_{(Q\bar{q})(\bar{q}_1,q_3,q_2)} \right|^2}{4M_H^2 |p_{km}|}, \\ \Gamma^{(\mathbf{PI},\,\text{hadron})}_{H,\,(q_1,q_2,q_3)} = a_1 a_2 \sum_{k,m} (-1)^{k+m+1} \dfrac{T^{(k,m)}_{(Q\bar{q})(\bar{q}_1,q_2,q_3)} T^{(m,k)*}_{(Q\bar{q}_1)(\bar{q},q_3,q_2)}}{2M_H^2 |p_{km}|}, \end{cases}$$

inclusive → ongoing

T: color-allowed tree diagram

Numerical result: lifetimes for D mesons



$$\begin{cases} \tau[D^+]/\tau[D^0] = 1.4 - 1.5, \quad \tau[D_s]/\tau[D^0] \approx 1.03 \quad (\text{for } m_c = m_c^{\overline{\mathrm{MS}}}) \\ \tau[D^+]/\tau[D^0] \approx 1.3, \quad \tau[D_s]/\tau[D^0] \approx 1.02 \quad (\text{for } m_c = m_c^{\mathrm{pole}}) \end{cases}$$

- The convergence is better in 2D than in 4D, because of no enhancement for the phase space factor in PI.
- Quark-hadron duality will be checked.

Summary

◎ We have studied quark-duality for $D^0 - \bar{D}^0$ mixing on the basis of one certain dynamical mechanism.

— For the $D^0 - \bar{D}^0$ mixing, the order of magnitude for $\Delta\Gamma_D$ is enhanced by more than 10^3, confirmed for $0.14 < m_s/\beta < 0.25$, if the phase space function is given by 4D-like one.

$\longrightarrow$ The result indicates that large duality violation in $D^0 - \bar{D}^0$ mixing may be still possible.

◎ The lifetime of charmed mesons in the 't Hooft model are calculated by varying charm quark mass.

$$\begin{cases} \tau[D^+]/\tau[D^0] = 1.4 - 1.5, & \tau[D_s]/\tau[D^0] \approx 1.03 \quad (\text{for } m_c = m_c^{\overline{\text{MS}}}) \\ \tau[D^+]/\tau[D^0] \approx 1.3, & \tau[D_s]/\tau[D^0] \approx 1.02 \quad (\text{for } m_c = m_c^{\text{pole}}) \end{cases}$$

— Numerical check of quark-hadron duality including PI is ongoing.

Backup

How duality violation appears?

$$\Gamma_{\rm inc} \overset{?}{\neq} \sum \Gamma_{\rm exc}$$

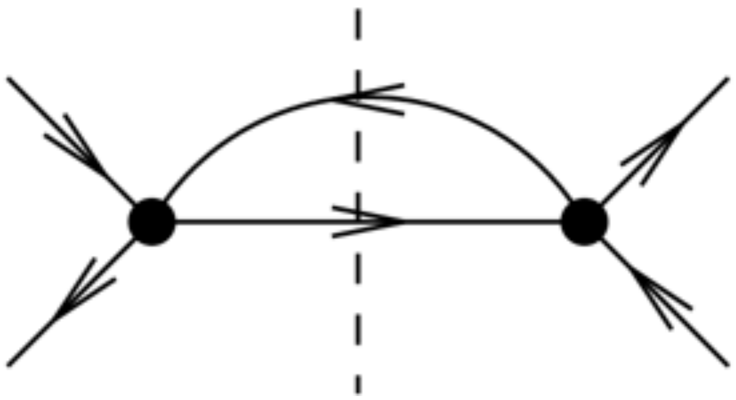
- Accuracy of the HQE is limited up to non-perturbative effects in perturbation theory.

$$\text{divergences} \begin{cases} \text{(1) Proliferation of Feynman diagrams} \\ \text{(2) Renormalons} \\ \text{(3) } 1/m_Q \text{ series } ✔? \end{cases}$$

- Duality violation is modeled by
  - (a) Instanton-based approach
    - — takes accout of the effect of (fixed sized) background instanton
    - — may represent the trancated effects for (3).
    - — considered more or less as an orientation.
  - (b) Resonance-based approach ✔
    - — large-N_c + linear Regge tragectory.
    - — factorial divergence (sign-flipping) for (3) is captured.

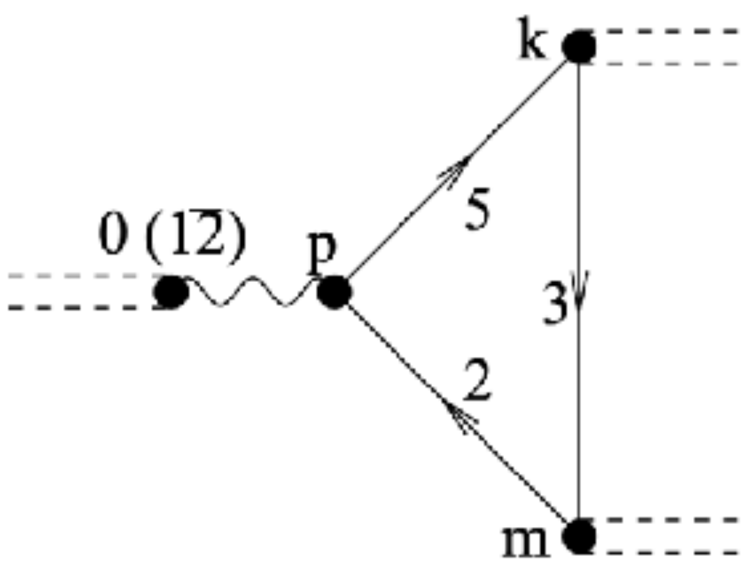
Weak annihilation in the ’t Hooft model

Inclusive:



$$\Gamma_{\rm part} = N_c^2 G^2 \left[c_V^2 - c_A^2\left(1+\xi\mu_0^2\right)\right]^2 \frac{2m^2c_0^2}{\pi\sqrt{\mu_0^2-4m^2}}.$$

Exclusive:



$$\mathcal{M}_A = Gc_0\sqrt{\frac{N_c}{\pi}}\sum_p\Big[\left[(c_V^2-c_A^2)\left(1+(-1)^p\right)\right] - \xi p^2 c_A\left[(c_V+c_A)(-1)^p-(c_V-c_A)\right]\Big]$$
$$\cdot\frac{c_p\mu_p^2}{(p^2-\mu_p^2+i\mu_p\Gamma_p)}F_{plcm}(\omega_0),$$

$$\Gamma = \frac{1}{4\mu_0^2|\mathbf{p}|}|\mathcal{M}_A|^2 \propto N_c?$$

The quark-hadron duality may not be captured in the large-N_c limit.

finite-N_c + smearing with the Gausian weight

Grinstein, Lebed [9805404]

Inclusive analysis for $D^0 - \bar{D}^0$ mixing

$$\Gamma_{12} = \lambda_d^2 \Gamma_{dd}^{(D,\mathrm{inc})} + 2\lambda_s\lambda_d \Gamma_{ds}^{(D,\mathrm{inc})} + \lambda_s^2 \Gamma_{ss}^{(D,\mathrm{inc})} . \qquad \lambda_k = V_{ck}V_{uk}^*$$

$$\Gamma_{ij}^{(D,\ \mathrm{inc})} \propto F_{ij}^{(\mathrm{th})}, G_{ij}^{(\mathrm{th})}, H_{ij}^{(\mathrm{th})} . \qquad i, j = d \text{ or } s$$

three types of phase space functions

$$\begin{cases} F_{ij}^{(\mathrm{th})} = \sqrt{1-2(z_i+z_j)+(z_i-z_j)^2}, & \text{: 4D-like phase space} \\ G_{ij}^{(\mathrm{th})} = \dfrac{z_i+z_j-(z_i-z_j)^2}{\sqrt{1-2(z_i+z_j)+(z_i-z_j)^2}} & \text{: 2D-specific phase space} \\ H_{ij}^{(\mathrm{th})} = \dfrac{\sqrt{z_i z_j}}{\sqrt{1-2(z_i+z_j)+(z_i-z_j)^2}} & \text{: 2D-specific phase space} \end{cases}$$

$$\begin{cases} z_d = m_d^2/m_c^2 = 0 \\ z_s = m_s^2/m_c^2 \ll 1 \end{cases}$$

expansion parameter

$\boxed{\text{CKM unitarity} + \lambda_b \to 0 \text{ limit}}$ $\Rightarrow$ $\Gamma_{12} \simeq \lambda_s^2 \Gamma_{(\mathrm{GIM},1)}^{(D,\mathrm{inc})}$, $\quad \Gamma_{(\mathrm{GIM},1)}^{(D,\mathrm{inc})} = \Gamma_{dd}^{(D,\ \mathrm{inc})} + \Gamma_{ss}^{(D,\ \mathrm{inc})} - 2\Gamma_{sd}^{(D,\ \mathrm{inc})}$

inclusive observable: $\begin{cases} \Gamma_{(\mathrm{GIM},1)}^{(D,\mathrm{inc})}\big|_{\mathrm{4D-like}} \propto z_s^2 \\ \Gamma_{(\mathrm{GIM},1)}^{(D,\mathrm{inc})}\big|_{\mathrm{4D+2D}} \propto z_s \end{cases}$

suppressed more strongly for large m_c

The order of magnitude for $\left|\Gamma^{(\mathrm{exc})}/\Gamma^{(\mathrm{inc})}\right|$ strongly depends on the two cases.

$\Rightarrow$ Both cases are presented in what follows.

Formulas of D and B decays

charged-current neutral-current

$$\mathcal{L}_{|\Delta C|=1} = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}[a_1(\bar{s}^\alpha \gamma_\mu c^\alpha)(\bar{u}^\beta \gamma_\mu d^\beta) + a_2(\bar{u}^\alpha \gamma_\mu c^\alpha)(\bar{s}^\beta \gamma_\mu d^\beta)]$$

(Fiertz-transformed)

inclusive

$$\Gamma^{(\mathbf{dec},\ \mathrm{quark})}_{H,\ (q_1,q_2,q_3)} = (a_1^2 + a_2^2)\frac{G_F^2}{4}\frac{m_Q^2 - m_q^2}{\sqrt{m_Q^2 - \beta^2}}\frac{\langle H|QQ|H\rangle}{2M_H},$$

to be improved by including mass dependences

$$\Gamma^{(\mathbf{PI},\ \mathrm{quark})}_{H,\ (q_1,q_2,q_3)} = -a_1 a_2 G_F^2 \left[(F_{23}^{(\mathrm{th})} + 2G_{23}^{(\mathrm{th})})\frac{\langle H|(Q\gamma_\mu\gamma_5 q)(q\gamma^\mu\gamma_5 Q)|H\rangle}{2M_H} + (G_{23}^{(\mathrm{th})} + 2H_{23}^{(\mathrm{th})})\frac{\langle H|(Qi\gamma_5 q)(qi\gamma_5 Q)|H\rangle}{2M_H}\right];$$

exclusive

$$\Gamma^{(\mathbf{dec},\ \mathrm{hadron})}_{H,\ (q_1,q_2,q_3)} = a_1^2 \sum_{k,m}\frac{\left|T^{(k,m)}_{(Q\bar{q})(\bar{q}_1,q_2,q_3)}\right|^2}{4M_H^2|p_{km}|} + a_2^2 \sum_{k,m}\frac{\left|T^{(k,m)}_{(Q\bar{q})(\bar{q}_1,q_3,q_2)}\right|^2}{4M_H^2|p_{km}|},$$

$$\Gamma^{(\mathbf{PI},\ \mathrm{hadron})}_{H,\ (q_1,q_2,q_3)} = a_1 a_2 \sum_{k,m}(-1)^{k+m+1}\frac{T^{(k,m)}_{(Q\bar{q})(\bar{q}_1,q_2,q_3)} T^{(m,k)*}_{(Q\bar{q}_1)(\bar{q},q_3,q_2)}}{2M_H^2|p_{km}|},$$

Exclusive formula is written by color-allowed tree diagram

$$T^{(k,m)}_{(Q\bar{q})(\bar{q}_1,q_2,q_3)} = \frac{G_F}{\sqrt{2}}\sqrt{\frac{N_c}{\pi}}c^{(k)}_{(q_3\bar{q}_1)}[(-1)^{k+1}q^2\mathcal{C}^{(m)}_{(Q\bar{q})(q_2)} + m_Q m_{q_2}\mathcal{D}^{(m)}_{(Q\bar{q})(q_2)}],$$

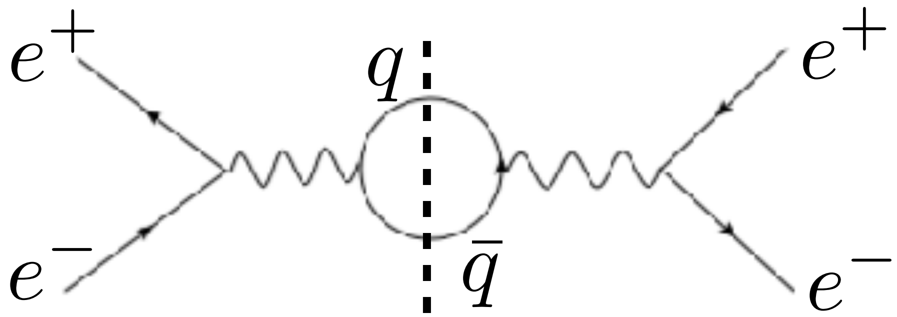
$\mathscr{C}$, $\mathscr{D}$: overap integrals

Inclusive and exclusive processes

Quark | hadron

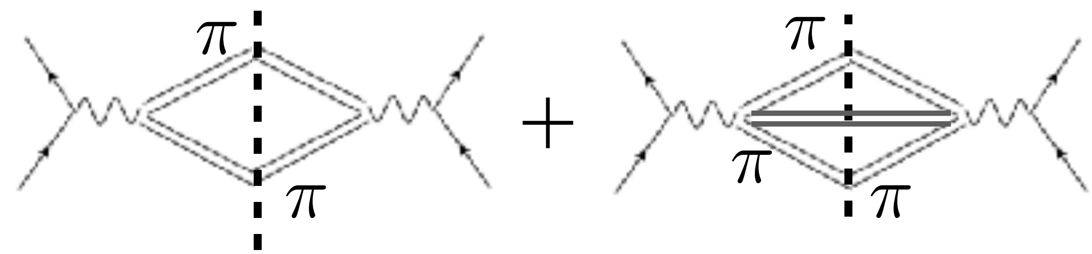
$e^+e^- \to$ hadrons

$\sigma^{(\mathrm{inc})}(e^+e^- \to q\bar{q})$



+ (condensates)

$\sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 3\pi) + \cdots$



Poggio, Quinn and Weinberg, 1976 for the case with smearing

Quark-hadron duality: inclusive = sum of exclusive

(one definition of)
Duality violation: inclusive $\neq$ sum of exclusive

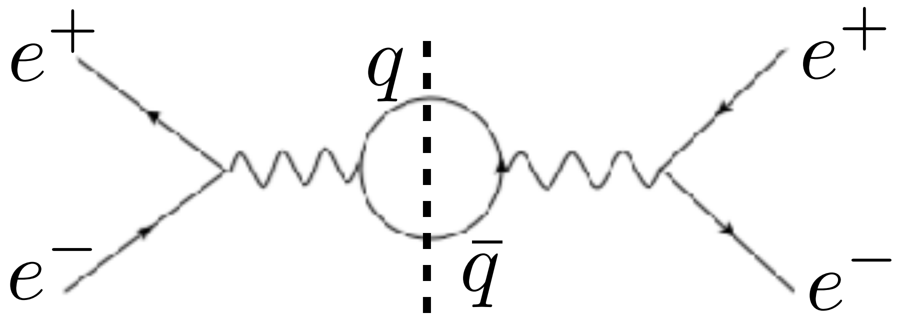
Inclusive and exclusive processes

Quark

hadron

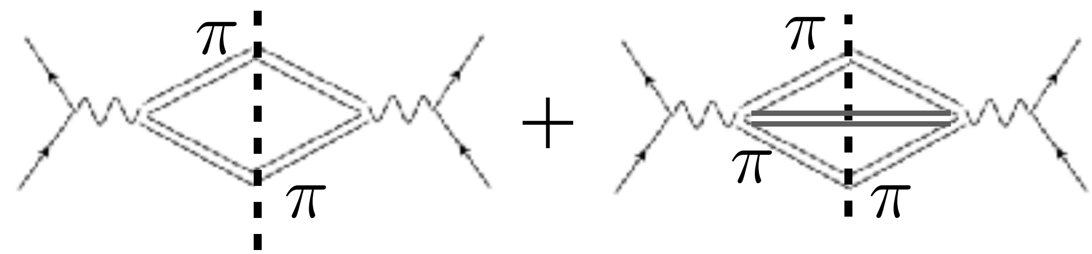
$$e^+e^- \to \text{hadrons}$$

$\sigma^{(\mathrm{inc})}(e^+e^- \to q\bar{q})$



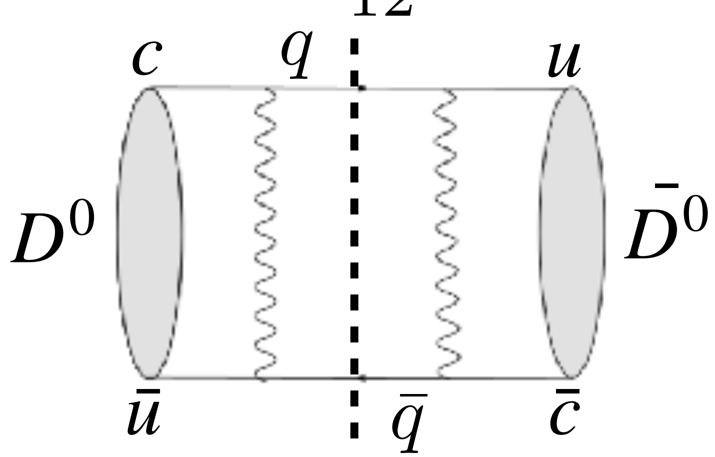
+ (condensates)

$\sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 3\pi) + \cdots$



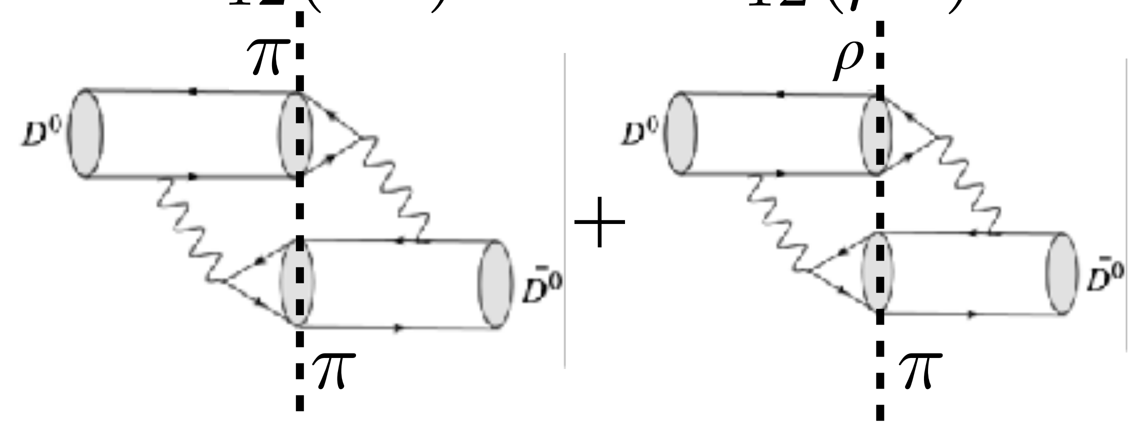
$$D^0 - \bar{D}^0$$

$\Gamma_{12}^{(\mathrm{inc})}$



+ (higher dimensional operators)

$\Gamma_{12}(\pi\pi) \quad + \quad \Gamma_{12}(\rho\pi) + \cdots$



+ (all the other exclusive states)

Non-trivial point

✔Does duality violation possibly give a large correction to the box diagram? This talk

- **<u>Charm quark mass is a unique scale</u>.**

$$m_c \approx 1.3 - 1.7 \ \mathrm{GeV}$$

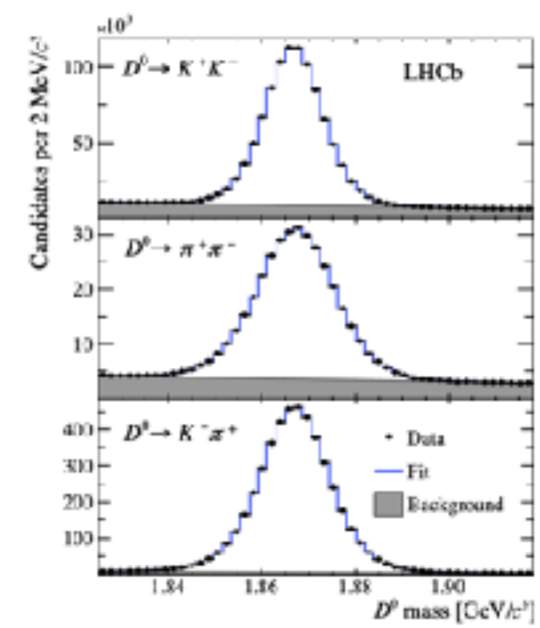
— too heavy for ChPT

— too light for $\Lambda_{\rm QCD}/m_c$ expansion?

Theoretically challenging

- **<u>High statistics data are provided</u>.**

— in a good stage to test theories



LHCb [1810.06874]

- **<u>$D^0 - \bar{D}^0$ mixing</u>.**

— $\Lambda_{\rm QCD}/m_c$ expansion is not successful

— experimental data are not quantitatively reproduced yet

perhaps, quark-hadron duality is violated?

Previous works for heavy meson ***decays***

◉ non-leptonic decays (color-allowed tree)

[1] B. Grinstein and R. F. Lebed, Phys. Rev. D**57**, 1366-1378 (1998) [arXiv:hep-ph/9708396 [hep-ph]].

◉ semi-leptonic decay and also non-leptonic decay

[2] I. I. Y. Bigi, M. A. Shifman, N. Uraltsev and A. I. Vainshtein, Phys. Rev. D**59**, 054011 (1999) [arXiv:hep-ph/9805241 [hep-ph]].

◉ non-leptonic decay (weak annhilation)

[3] B. Grinstein and R. F. Lebed, Phys. Rev. D**59**, 054022 (1999) [arXiv:hep-ph/9805404 [hep-ph]].

◉ non-leptonic decays (weak annhilation, Pauli interference)

[4] I. I. Y. Bigi and N. Uraltsev, Phys. Rev. D60, 114034 (1999) [arXiv:hep-ph/9902315 [hep-ph]]; Phys. Lett. B**457**, 163-169 (1999) [arXiv:hep-ph/9903258 [hep-ph]].

◉ semi-leptonic decay

[5] R.~F. Lebed and N. G. Uraltsev, Phys. Rev. D**62**, 094011 (2000) [arXiv:hep-ph/0006346 [hep-ph]].

This work → heavy meson **mixings**

・Mixing is suppressed by the GIM cancellation.

➡ Tiny duality violation is possibly enlarged after cancellation.

$D^0 - \bar{D}^0$ mixing

time evolution

$\bar{D}^0 \rightarrow D^0$

Time evolution Eq. $$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

CP-conserving limit

Mass eigenstate $|D_{1,2}\rangle = |D^0\rangle \pm |\bar{D}^0\rangle$

observables
$$\begin{cases} x = (M_1 - M_2)/\Gamma = 2M_{12}/\Gamma & \text{\underline{mass difference}} \\ y = (\Gamma_1 - \Gamma_2)/2\Gamma = \Gamma_{12}/\Gamma & \text{\underline{width difference}} \end{cases}$$

Γ : total width

$D^0 - \bar{D}^0$ mixing: theory

Two methods

◉Exclusive

Hadronic-level analysis

Hard to calculate $\begin{cases} \Gamma[D \to \pi\pi] \\ \Gamma[D \to KK] \end{cases}$

↓

Data are used

✔◉Inclusive

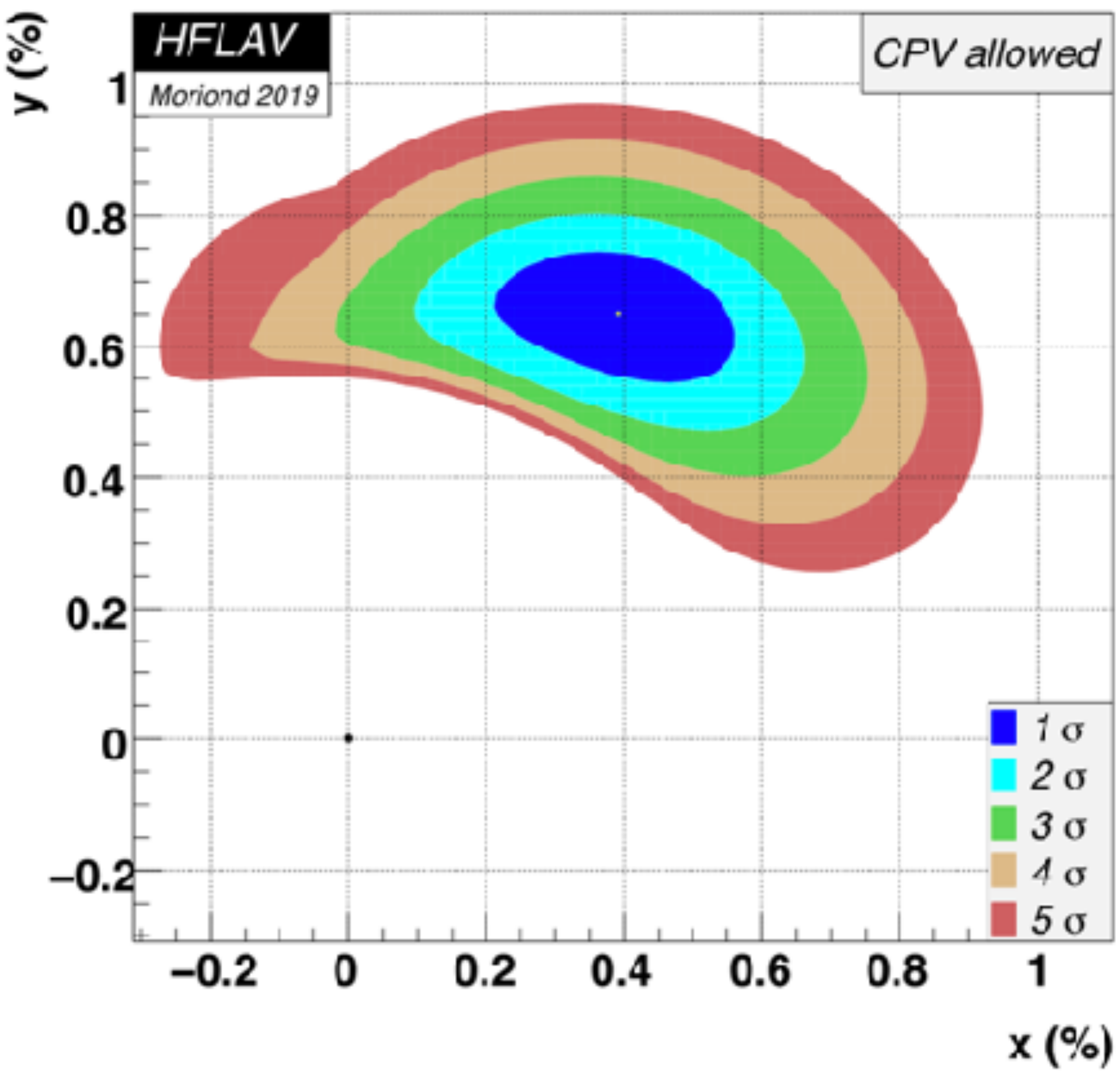
Quark-level analysis

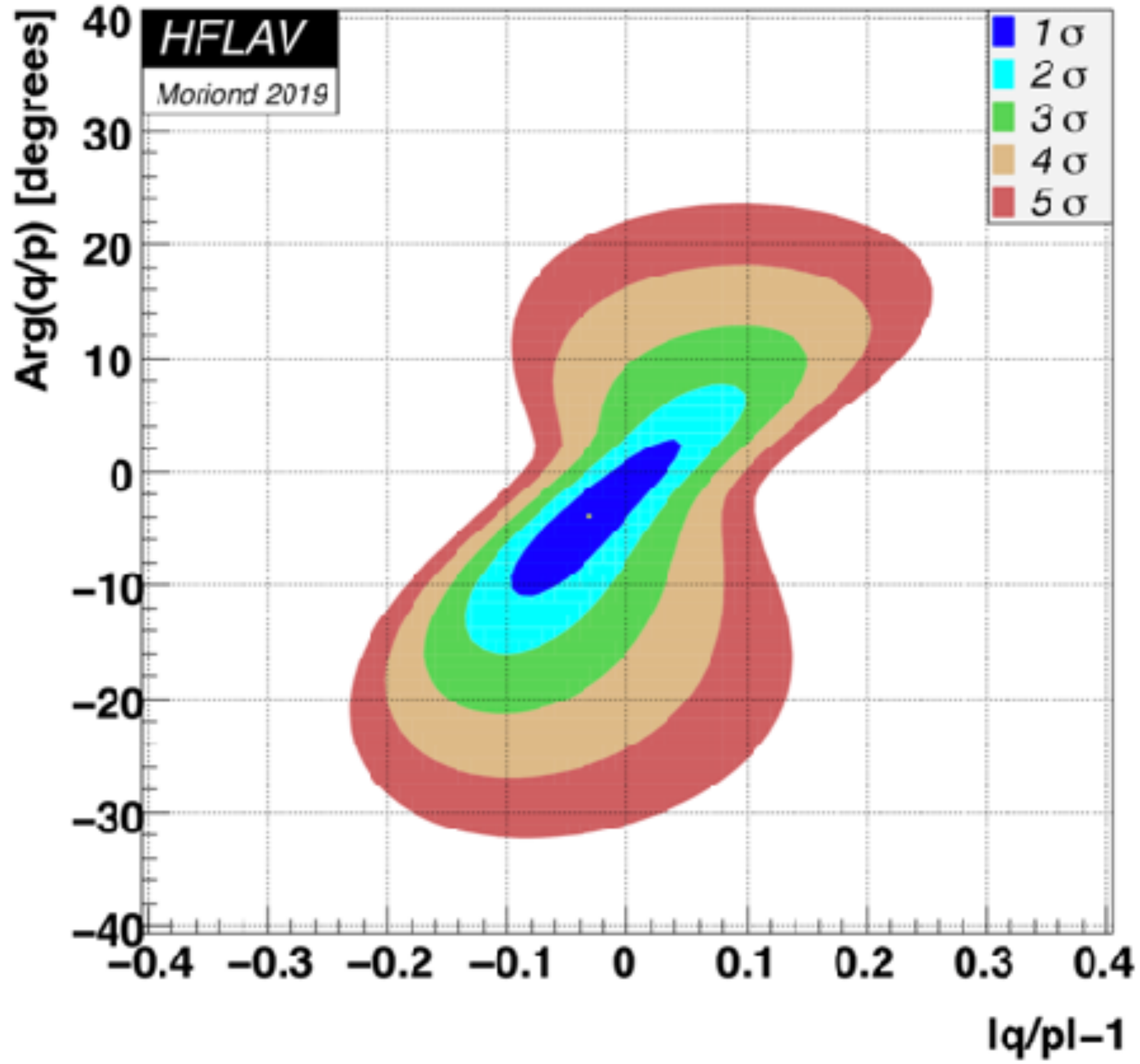
without data

purely theoretical method

(quark-hadron duality is assumed)

$D^0 - \bar{D}^0$ mixing: experiment





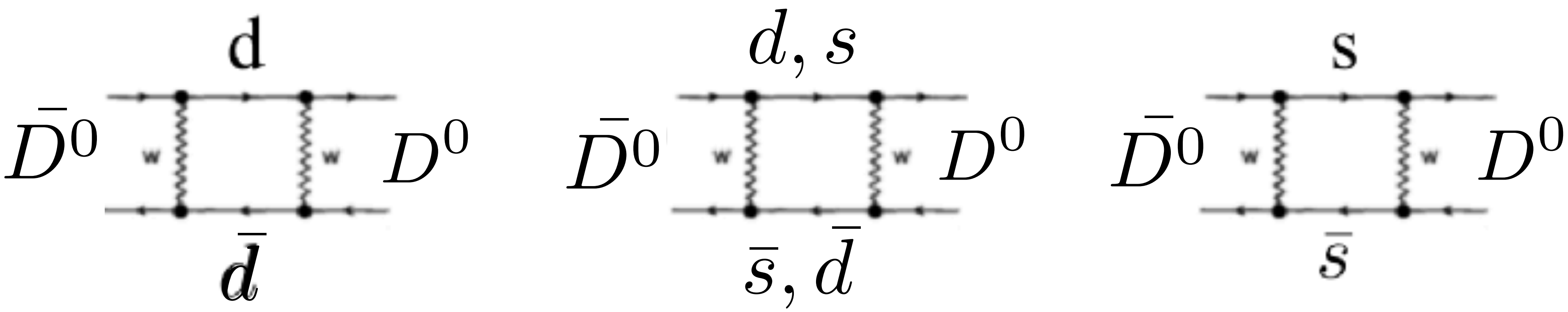
$(x, y) = (0, 0)$ is excluded by >>11.5σ

$(|q/p| - 1, \mathrm{Arg}(q/p)) = (0, 0)$ is allowed

➡ No signal for CP violation

Contributions

$\lambda_i = V_{ci} V_{ui}^*$



CKM suppressed

~~+ bottom loop~~

neglect

For $m_s = m_d$

CKM unitarity

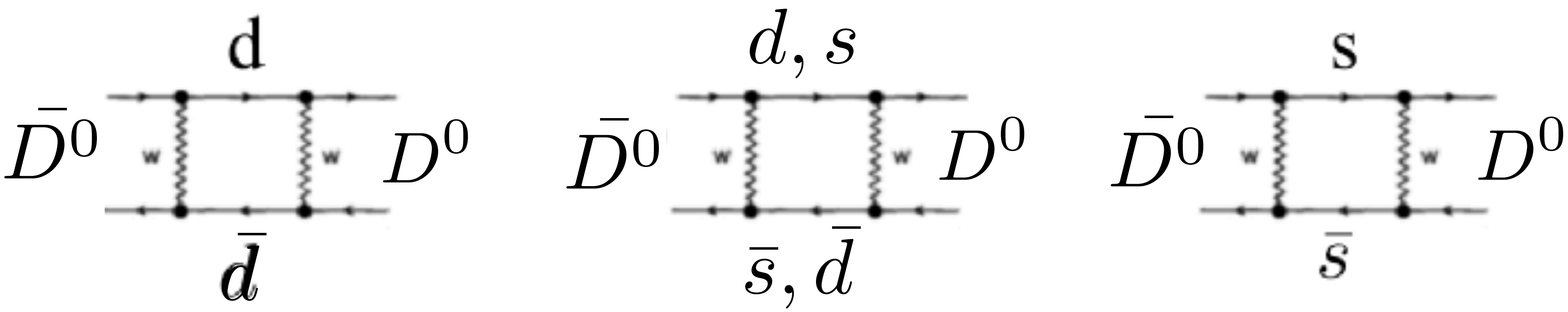
$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$

neglect

summation $\propto \lambda_d^2 + 2\lambda_d \lambda_s + \lambda_s^2$

Contributions

$\lambda_i = V_{ci}V_{ui}^*$



CKM suppressed

~~+ bottom loop~~

neglect

For $m_s = m_d$

CKM unitarity

$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$

neglect

summation $\propto \lambda_d^2 + 2\lambda_d\lambda_s + \lambda_s^2 = 0$

Suppressed by the GIM mechanism.

non-zero contributions to $D^0 - \bar{D}^0$ mixing $\propto$ SU(3) breaking: $\left(\frac{m_s^2 - m_d^2}{m_c^2}\right)^n$

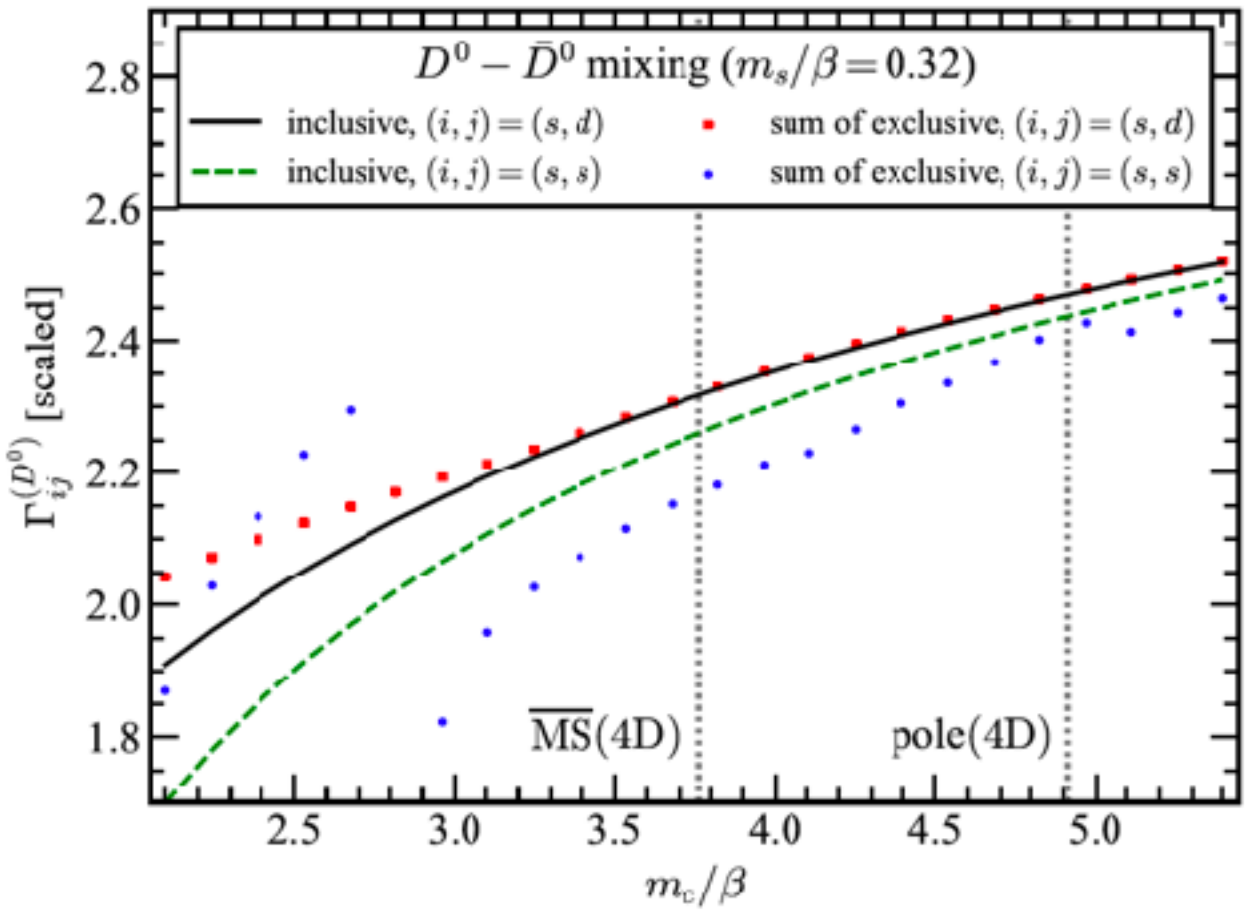
(1) $D^0 - \bar{D}^0$ mixing for individual flavors

$D^0 \to K^- \pi^+ \to \bar{D}^0$

solid line: inclusive
squares: sum of exclusive

$D^0 \to K^- K^+ \to \bar{D}^0$

dashed line: inclusive
points: sum of exclusive



Vertical dotted lines:
reference values corresponding to masses in 4D

β is fixed by 340 MeV.
(ansatz for fitting the string tension in QCD_4)

m_s/β is fixed by 0.32.
($\overline{\text{MS}}$ mass at the scale of charm quark mass)

The vertical axis is normalized by $4G_F^2(c_V^2 - c_A^2)^2 \beta N_c/\pi$.

◎ For large m_c, inclusive/exclusive agrees with each other.

◎ For small m_c, certain difference between inclusive/exclusive appears.

◎ The agreement beween inclusive/exclusive is better for $K^-\pi^+$ than K^-K^+.

◎ Obvious spikes are observed for K^-K^+ whereas it is not seen for $K^-\pi^+$.

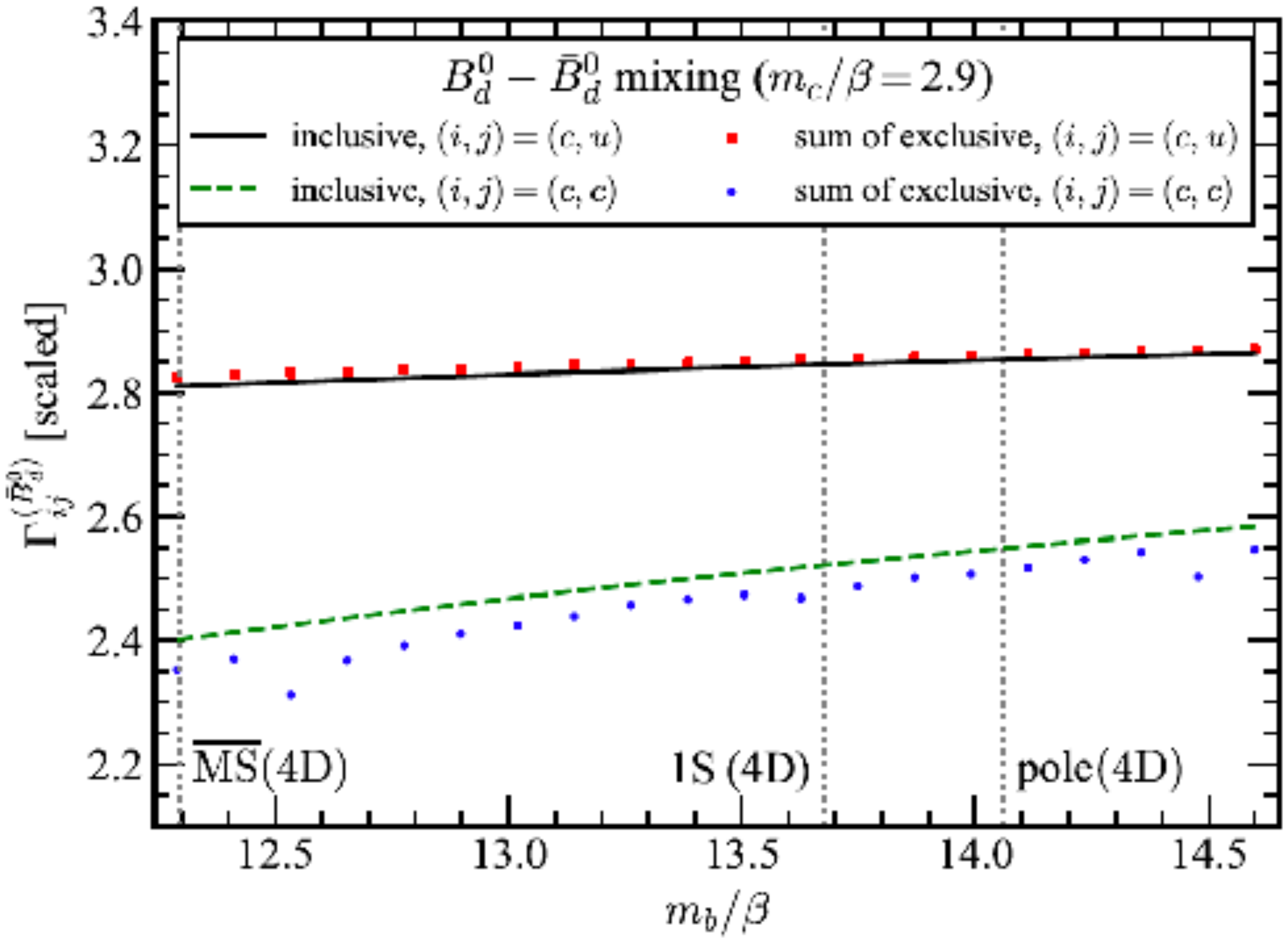
(2) $B_q^0 - \bar{B}_q^0$ $(q = d, s)$ mixing for individual flavors

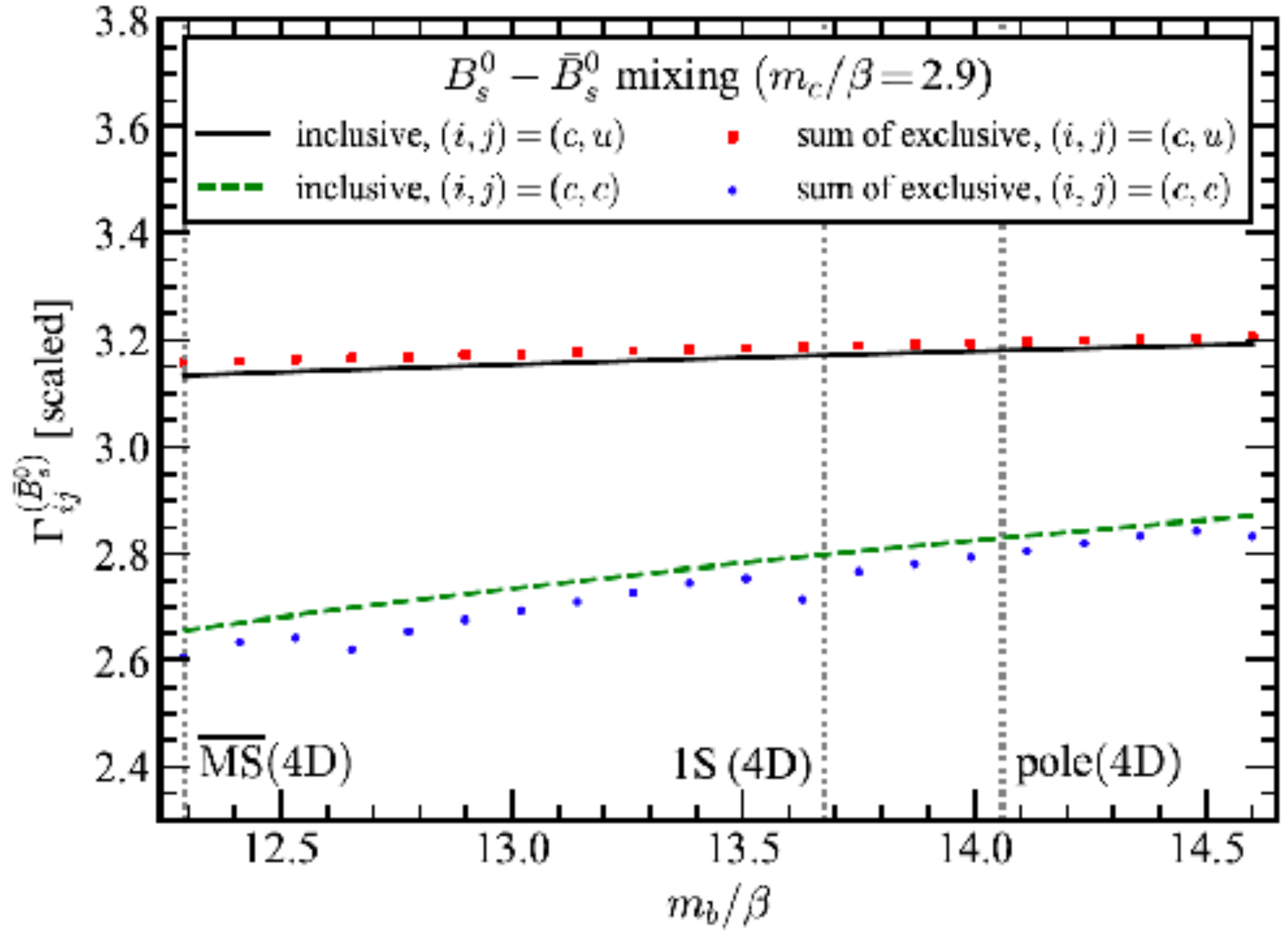
$$B_d^0 \to D^- \pi^+ \to \bar{B}_d^0$$
$$B_d^0 \to D^- D^+ \to \bar{B}_d^0$$

{ solid line: inclusive
squares: sum of exclusive

$$B_s^0 \to D_s^- K^+ \to \bar{B}_s^0$$
$$B_s^0 \to D_s^- D_s^+ \to \bar{B}_s^0$$

{ dashed line: inclusive
points: sum of exclusive





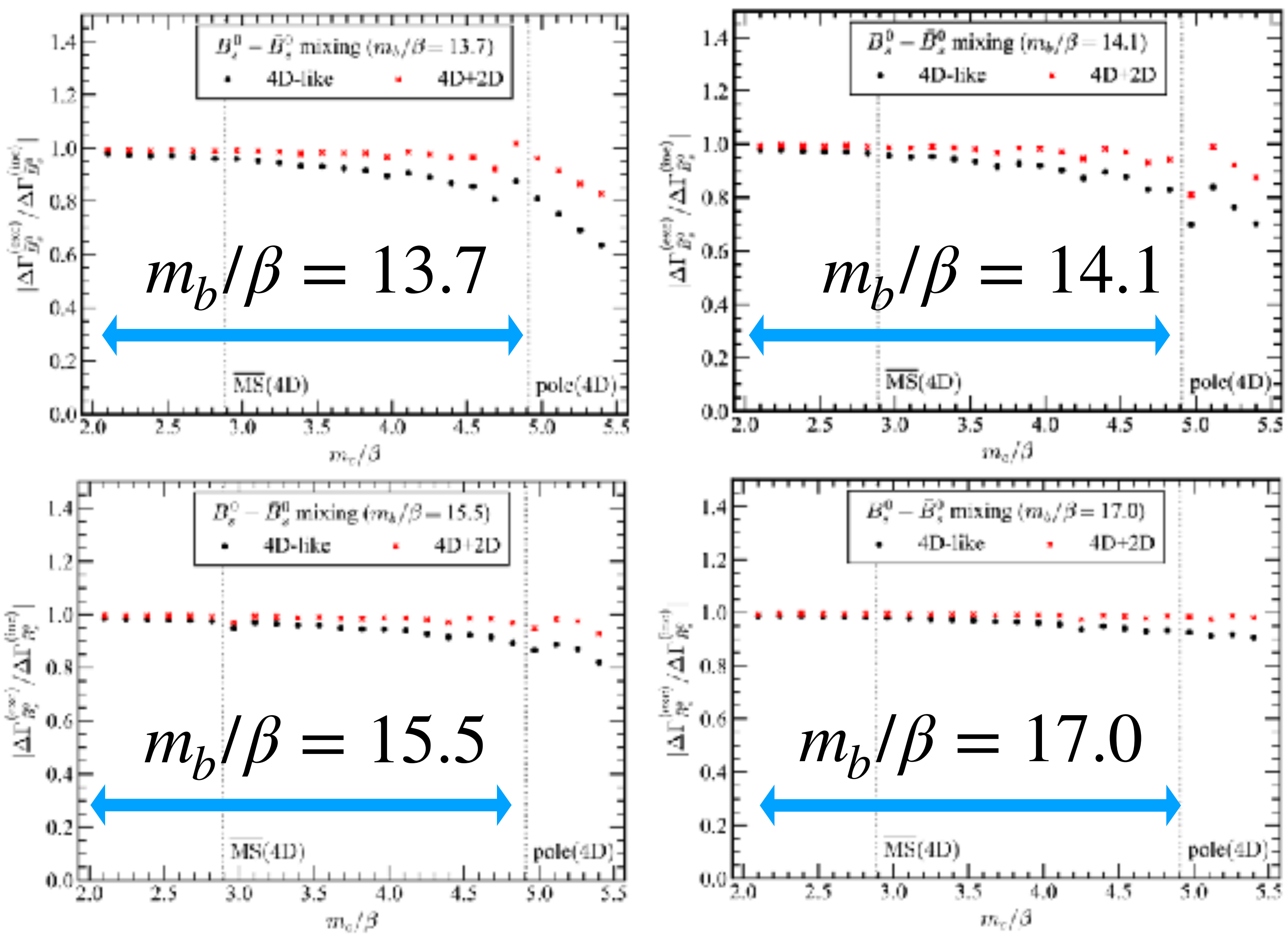
◎ The patters are similar for the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings.

◎ The disagreement between inclusive/exclusive is larger for $B_q^0 \to DD \to \bar{B}_q^0$.

$B_s^0 - \bar{B}_s^0$ mixing

points: based only on the 4D-like phase space
crosses: based on the 4D-like phase space + 2D-specific one

If the domain of $m_c < m_c^{\rm pole,4D}$ is considered:



◎ The corrections are up to (20 % , 18 % , 11 % , 8%) for $m_b/\beta = (13.7,\ 14.1,\ 15.5,\ 17.0)$.

→ The result is consistent with 4D within 1σ for the latter two.

(The ratio of the HFLAV data to the HQE gives $\Delta\Gamma_{B_s}^{\rm (ex)}/\Delta\Gamma_{B_s}^{\rm (th)} = 0.99 \pm 0.15$.)

A. Lenz and G. Tetlalmatzi-Xolocotzi, 2020

Analytical check of local duality

generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^\mu (c_\mathrm{V} + c_\mathrm{A}\gamma_5)$

the standard model

$c_\mathrm{V} = \frac{1}{2}, \quad c_\mathrm{A} = -\frac{1}{2}$

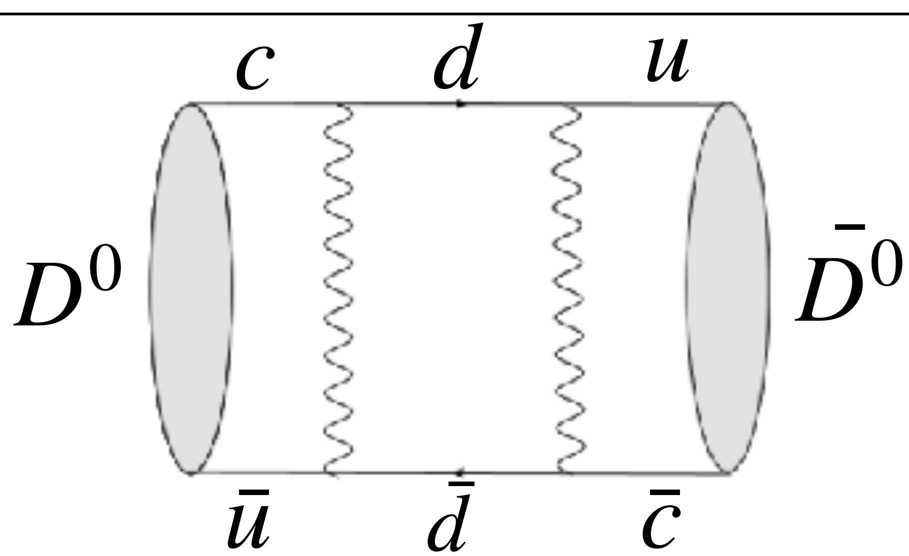
◉ Inclusive width difference

$$\Gamma_{12} = C_A < \overline{D^0} | (\bar{u}^\alpha \gamma^\mu \gamma_5 c^\alpha)(\bar{u}^\beta \gamma_\mu \gamma_5 c^\beta) | D^0 > \; + C_P < \overline{D^0} | (\bar{u}^\alpha i\gamma_5 c^\alpha)(\bar{u}^\beta i\gamma_5 c^\beta) | D^0 >$$

$$\begin{cases} C_A = + \dfrac{2G_F^2}{M_{D^0}} (c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_A^2)(F_{dd}^{(\mathrm{th})} + 2G_{dd}^{(\mathrm{th})}) - (c_V^2 + c_A^2)(I_{dd}^{(\mathrm{th})} + I_{dd}^{(\mathrm{th})})] \\ C_P = - \dfrac{2G_F^2}{M_{D^0}} (c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_A^2)(G_{dd}^{(\mathrm{th})} + 2H_{dd}^{(\mathrm{th})}) + (c_V^2 + c_A^2)(I_{dd}^{(\mathrm{th})} + I_{dd}^{(\mathrm{th})})] \end{cases}$$

$F_{dd}^{(\mathrm{th})}, G_{dd}^{(\mathrm{th})}, H_{dd}^{(\mathrm{th})}, I_{dd}^{(\mathrm{th})}$: phase space functions

$F_{dd}^{(\mathrm{th})} = \sqrt{1 - 4m_d^2/m_c^2}$



large-N_c factorization:

$R = [M_H/(m_Q + m_q)]^2$

$$\begin{cases} \dfrac{\langle \bar{H} | (\bar{q}^\alpha \gamma^\mu \gamma_5 Q^\alpha)(\bar{q}^\beta \gamma_\mu \gamma_5 Q^\beta) | H \rangle}{2M_H} = f_H^2 M_H \\ \dfrac{\langle \bar{H} | (\bar{q}^\alpha i\gamma_5 Q^\alpha)(\bar{q}^\beta i\gamma_5 Q^\beta) | H \rangle}{2M_H} = f_H^2 M_H R \end{cases}$$

down quark massless limit: $\Gamma_{12} \to 4(c_\mathrm{V}^2 - c_\mathrm{A}^2)^2 V_{cd}^* V_{ud} G_F^2 f_{D^0}^2 M_{D^0}$

Analytical check of local duality

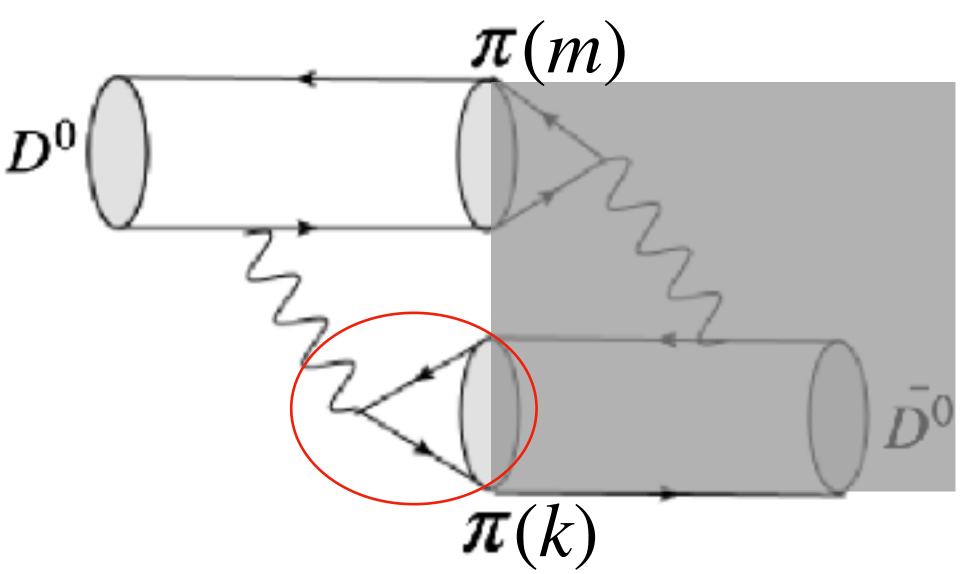
generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^{\mu} (c_{\mathrm{V}} + c_{\mathrm{A}} \gamma_5)$

the standard model
$c_{\mathrm{V}} = \frac{1}{2}, \quad c_{\mathrm{A}} = -\frac{1}{2}$

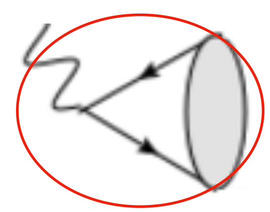
◉ Sum of exclusive width difference

massless limits for u and d quark for $\pi(u\bar{d})$

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|}$$



$k = 0,\ m = 0$: ground states



$\propto f_{\pi}^{(k)} p_{\mu} \qquad f_{\pi}^{(k)} = \sqrt{\frac{N_c}{\pi}} \int_0^1 \phi_k(x) dx$

$$\begin{cases} \text{(a) exact solution, } \phi_0(x) = 1. \\ \text{(b) completeness: } \sum_{k=0}^{\infty} \phi_k(x) \phi_k^*(y) = \delta(x - y) \end{cases}$$

$$\Rightarrow f_{\pi}^{(k)} = \begin{cases} \sqrt{N_c/\pi} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

Analytical check of local duality

generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\rm CKM}\gamma^\mu(c_V + c_A\gamma_5)$

the standard model $c_V = \frac{1}{2},\quad c_A = -\frac{1}{2}$

◉ Sum of exclusive width difference

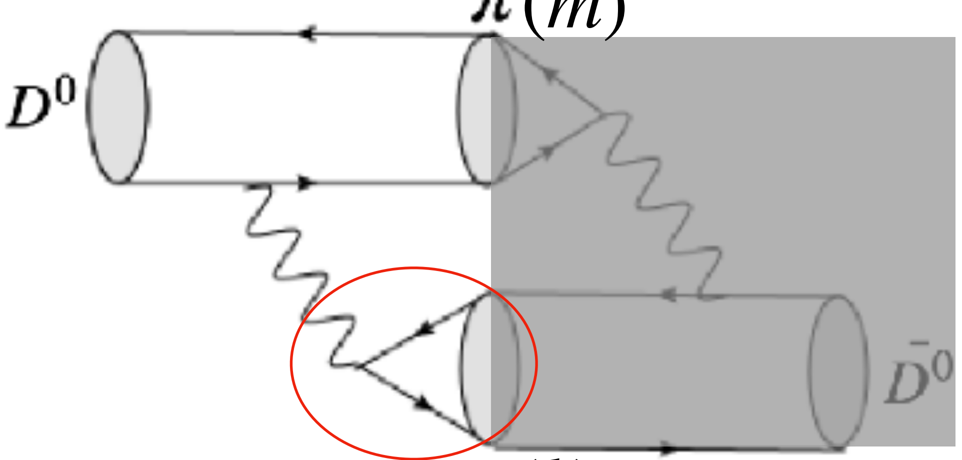
massless limits for u and d quark for $\pi(u\bar{d})$

analog of the Pauli interference (Bigi and Uraltsev, 1999)

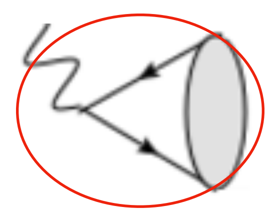
$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{k+m} \frac{T^{(k,m)}T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|} \to \frac{T^{(0,0)}T^{(0,0)*}}{4M_{D^0}^2 |p_{00}|} = 4(c_V^2 - c_A^2)^2 G_F^2 V_{cd}^* V_{ud} M_{D^0} \frac{N_c}{\pi}\left(\int_0^1 \phi_{D^0}(x)\phi_\pi(x)\right)^2$$

$$= 4(c_V^2 - c_A^2)^2 G_F^2 V_{cd}^* V_{ud} f_{D^0}^2 M_{D^0}$$

agrees with the inclusive result



$k = 0,\ m = 0$: ground states



$\propto f_\pi^{(k)} p_\mu \qquad f_\pi^{(k)} = \sqrt{\frac{N_c}{\pi}} \int_0^1 \phi_k(x)dx$

(a) exact solution, $\phi_0(x) = 1.$

(b) completeness: $\sum_{k=0}^{\infty} \phi_k(x)\phi_k^*(y) = \delta(x-y)$

$$\Rightarrow\ f_\pi^{(k)} = \begin{cases} \sqrt{N_c/\pi} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

Numerical evaluation of local duality

Motivations

(1) Check whether local duality exists for massive final states.

(2) Check the net size of observables in the presence of the GIM cancellation. (main motivation)

— We calculate $D \to \pi\pi, D \to K\pi, D \to KK$ and implement the cancellation.

Numerical method to solve the 't Hooft equation

$$M_n^2 \phi_n^{q_1\bar{q}_2}(x) = \left(\frac{m_1^2 - \beta^2}{x} + \frac{m_2^2 - \beta^2}{1-x} \right) \phi_n^{q_1\bar{q}_2} - \beta^2 \mathrm{Pr} \int_0^1 \mathrm{d}y \frac{\phi_n^{q_1\bar{q}_2}(y)}{(x-y)^2}$$ for $q_1\overline{q_2}$ bound state

based on the BSW-improved Multhopp technique Brower, Spence and Weis, 1979

$$\phi = \sum_{k=1}^{N} a_k \sin(k\theta), \quad x = \frac{1+\cos\theta}{2}$$

eigenvalue problem: $M^2 a_i = (H_0 + V)_{ij} a_j$

M^2: eigenvalue

a_i: eigenvector

Definition of amplitude and overlap integrals

Grinstein, Lebed 1997, Bigi, Uraltsev 1999

Amplitude
$$T^{(k,m)}_{(Q\bar q)(i,j)} = 2\sqrt{2}G_F(c_{\rm V}^2 - c_{\rm A}^2)\sqrt{\frac{N_c}{\pi}}\,c_k^{(q\bar i)}\left[\sum_{n=0}\frac{[(-1)^k q^2 + (-1)^n M_n^2]c_n^{(Q\bar j)}}{q^2 - M_n^2}F_{nm} + (-1)^{k+1}q^2\mathcal{C}_m + m_Q m_j \mathcal{D}_m\right],$$

Overlap int.
$$\begin{cases} F_{nm} = \omega(1-\omega)\displaystyle\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\,\frac{\phi_n^{(Q\bar j)}(x)\phi_m^{(j\bar q)}(y)}{[\omega(1-x)+(1-\omega)y]^2} \\ \qquad \times\{\phi_0^{(Q\bar q)}(\omega x) - \phi_0^{(Q\bar q)}[1-(1-\omega)(1-y)]\}, \\ \mathcal{C}_m = -\dfrac{1-\omega}{\omega}\displaystyle\int_0^1 \mathrm{d}x\,\phi_0^{(Q\bar q)}[1-(1-\omega)(1-x)]\phi_m^{(j\bar q)}(x), \\ \mathcal{D}_m = -\omega\displaystyle\int_0^1 \mathrm{d}x\,\frac{\phi_0^{(Q\bar q)}[1-(1-\omega)(1-x)]}{1-(1-\omega)(1-x)}\,\frac{\phi_m^{(j\bar q)}(x)}{x}, \end{cases}$$

Kinematical val.
$$\omega = \frac{1}{2}\left[1 + \left(\frac{q^2 - M_m^2}{M_0^2}\right) - \sqrt{1 - 2\left(\frac{q^2 + M_m^2}{M_0^2}\right) + \left(\frac{q^2 - M_m^2}{M_0^2}\right)^2}\right]$$

$q^2 = M_k^2$ for on-shell amplitudes

Expressions in the SM

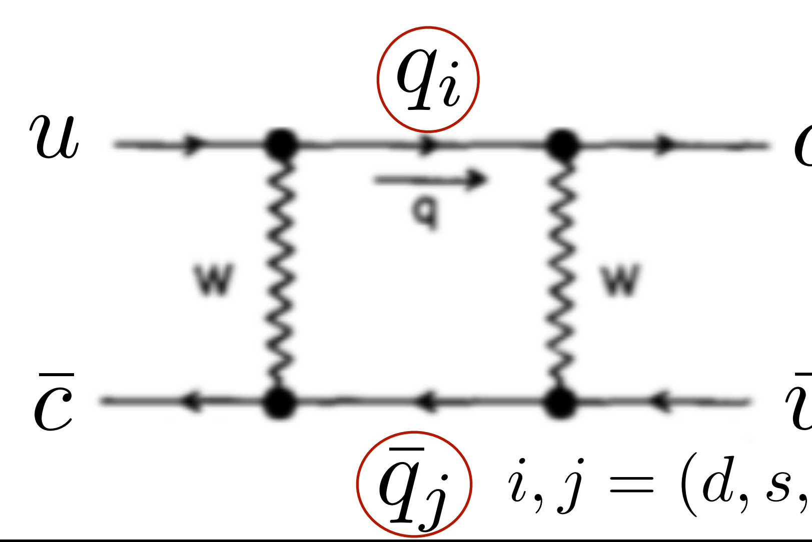
$$M_{12} - \frac{i}{2}\Gamma_{12} = B\,\langle D^0|\mathcal{O}_1|\bar{D}^0\rangle + C\,\langle D^0|\mathcal{O}_2|\bar{D}^0\rangle$$

Two contributions $\begin{cases} \mathcal{O}_1 = (\bar{c}u)_{\mathrm{V-A}}(\bar{c}u)_{\mathrm{V-A}} \\ \mathcal{O}_2 = (\bar{c}u)_{\mathrm{S-P}}(\bar{c}u)_{\mathrm{S-P}} \end{cases}$

Flavor sum

$\lambda_i = V_{ci}V_{ui}^*$

Explicit formula
$$\begin{cases} M_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H \displaystyle\sum_{i,j}^{d,s,b} \lambda_i\lambda_j \left[\xi_1 B_1(\mu) B_{ij}^{(d)}(s) + \xi_2 R(s) B_2(\mu) C_{ij}^{(d)}(s)\right], \\ \Gamma_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H \displaystyle\sum_{i,j}^{d,s,b} \lambda_i\lambda_j \left[\xi_1 B_1(\mu) B_{ij}^{(a)}(s) + \xi_2 R(s) B_2(\mu) C_{ij}^{(a)}(s)\right], \end{cases}$$



$i, j = (d, s, b)$

CKM unitarity

$\lambda_d + \lambda_s + \lambda_b = 0 \quad \longleftrightarrow \quad \lambda_d = -\lambda_s - \lambda_b$

$$\begin{cases} M_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H (\lambda_s^2 U_{ss}^{(d)} + 2\lambda_s\lambda_b U_{sb}^{(d)} + \lambda_b^2 U_{bb}^{(d)}) \\ \Gamma_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H (\lambda_s^2 U_{ss}^{(a)} + 2\lambda_s\lambda_b U_{sb}^{(a)} + \lambda_b^2 U_{bb}^{(a)}) \end{cases}$$

U : loop function

Buras, Slominski and Steger, Nucl. Phys. B245, 369 (1984)

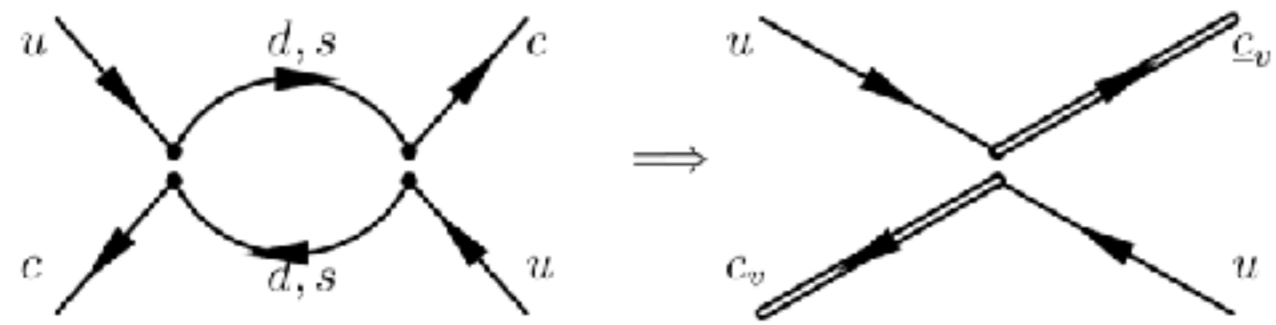
Structure of higher-dimensional operators

Ohl, Ricciardi and Simmons [9301212]

D=6

Leading in $1/m_q$

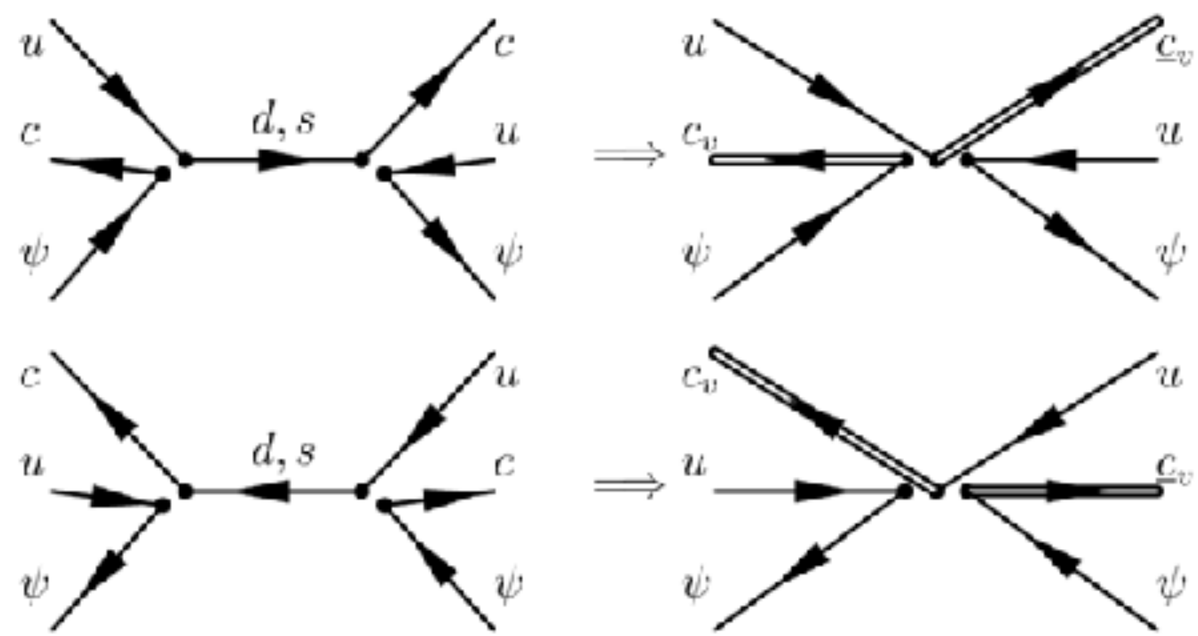
$(\bar{c}_v \Gamma_1 u)\,(\underline{\bar{c}}_v \Gamma_2 u)$



D=9

Subleading in $1/m_q$

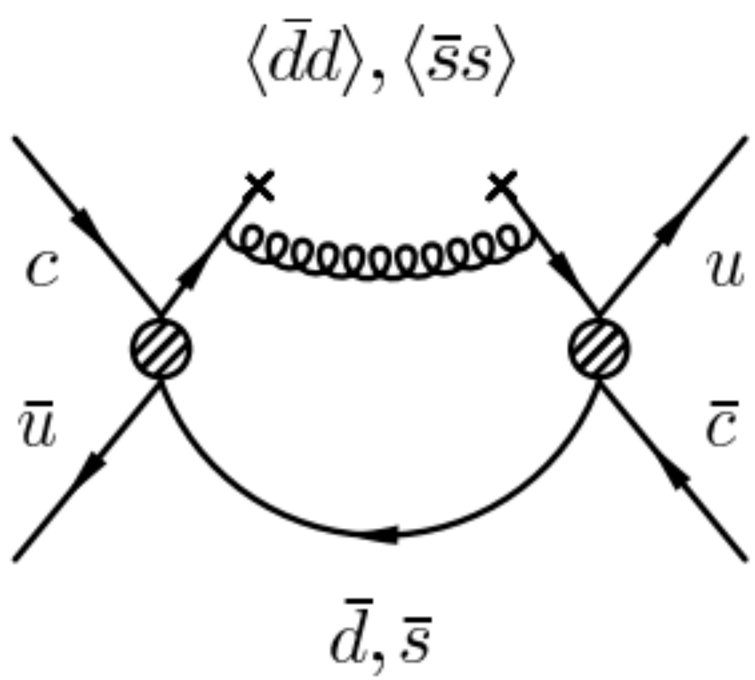
$\left(\bar{\psi}\Gamma_1 u\right)\,(\bar{c}_v \Gamma_2 \psi)\,(\underline{\bar{c}}_v \Gamma_3 u)$



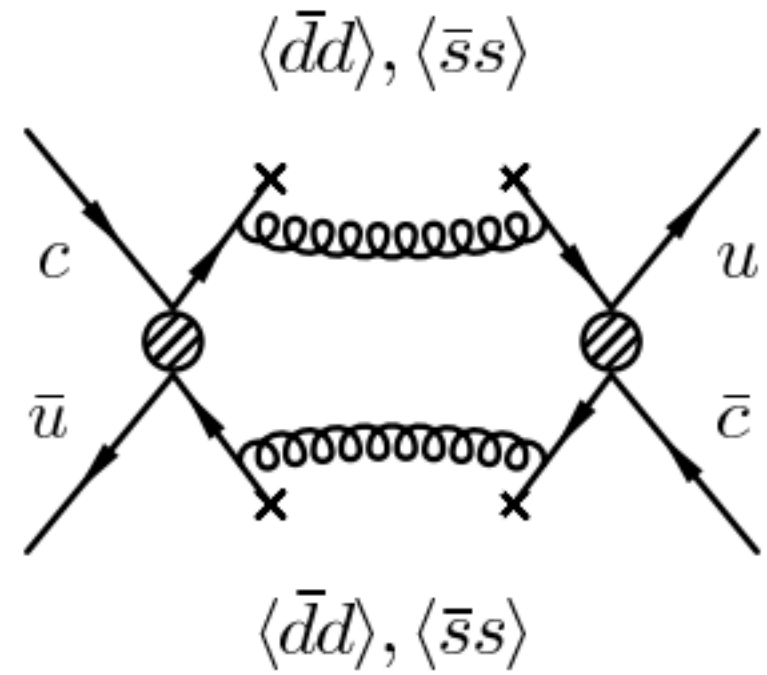
Γ_i : color/Dirac structure

Higer-dimensional operators with condensates

Bobrowski, Lenz and Riedl [1002.4794]



D=9



D=12

$\mathcal{O}(\alpha_s(4\pi)\langle\bar{q}q\rangle/m_c^3)$

$\mathcal{O}(\alpha_s^2(4\pi)^2\langle\bar{q}q\rangle^2/m_c^6)$

y	no GIM	with GIM
$D = 6, 7$	$2 \cdot 10^{-2}$	$5 \cdot 10^{-7}$
$D = 9$	$5 \cdot 10^{-4}$	?
$D = 12$	$2 \cdot 10^{-5}$	?

Motivations to study duality violation

✔

(1) For the $D^0 - \bar{D}^0$ mixing, an inclusive calculation, based on the leading operator in HQE, disagrees with data.

[0506185]

$$y \simeq 6 \cdot 10^{-7}$$

HFLAV 2019

$$y = (6.51^{+0.63}_{-0.69}) \times 10^{-3}$$

(2) For, e.g., $\bar{B} \to X_q l \bar{\nu}$ $(q = c, u)$, one can get the systematic uncertainty in the OPE.

(3) ~~Crucial to explain the short lifetime of Λ_b?~~ data updated.

before 2003

Year	Exp	Decay	$\tau(\Lambda_b)$ [ps]	$\tau(\Lambda_b)/\tau(B_d)$
1998	OPAL	$\Lambda_c l$	1.29 ± 0.25	0.85 ± 0.16
1997	ALEPH	$\Lambda_c l$	1.21 ± 0.11	0.80 ± 0.07
1995	ALEPH	$\Lambda_c l$	1.02 ± 0.24	0.67 ± 0.16
1992	ALEPH	$\Lambda_c l$	1.12 ± 0.37	0.74 ± 0.24

after 2003

Year	Exp	Decay	$\tau(\Lambda_b)$ [ps]	$\tau(\Lambda_b)/\tau(B_d)$
2010	CDF	$J/\psi\Lambda$	1.537 ± 0.047	1.020 ± 0.031
2009	CDF	$\Lambda_c + \pi^-$	1.401 ± 0.058	0.922 ± 0.038
2007	D0	$\Lambda_c \mu\nu X$	1.290 ± 0.150	0.849 ± 0.099
2007	D0	$J/\psi\Lambda$	1.218 ± 0.137	0.802 ± 0.090
2006	CDF	$J/\psi\Lambda$	1.593 ± 0.089	1.049 ± 0.059
2004	D0	$J/\psi\Lambda$	1.22 ± 0.22	0.87 ± 0.17

[1405.3601]

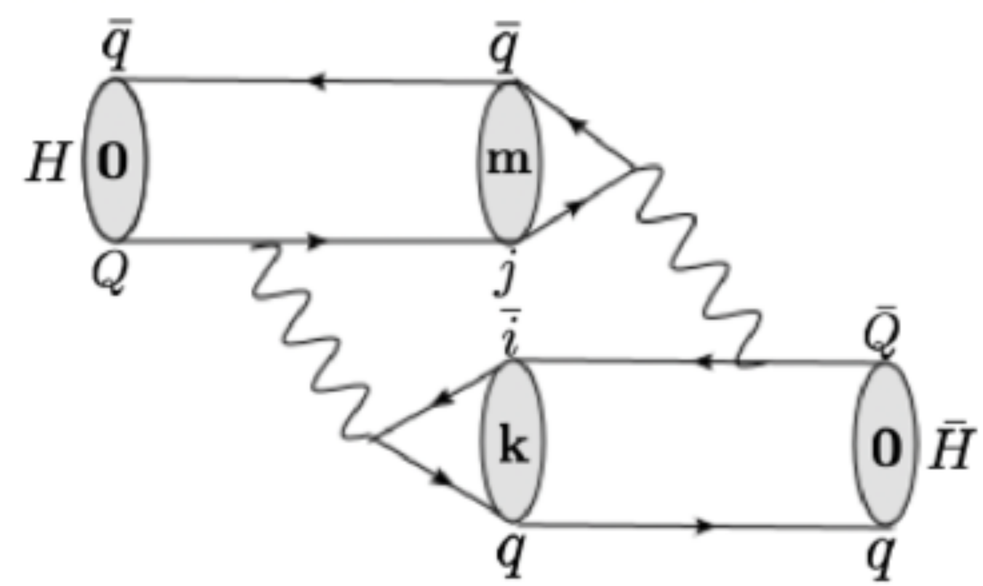
Topological amplitudes in the $1/N_c$ counting

naive countings: $\begin{cases} T \propto N_c^{1/2} \\ C, E, P, PA \propto N_c^{-1/2} \\ PE \propto N_c^{-3/2} \end{cases}$

(A does not contribute to the neutral meson mixings)

◉ Even in the presence of intermediate resonances, the color-allowed tree diagram is still dominant. [9805404]

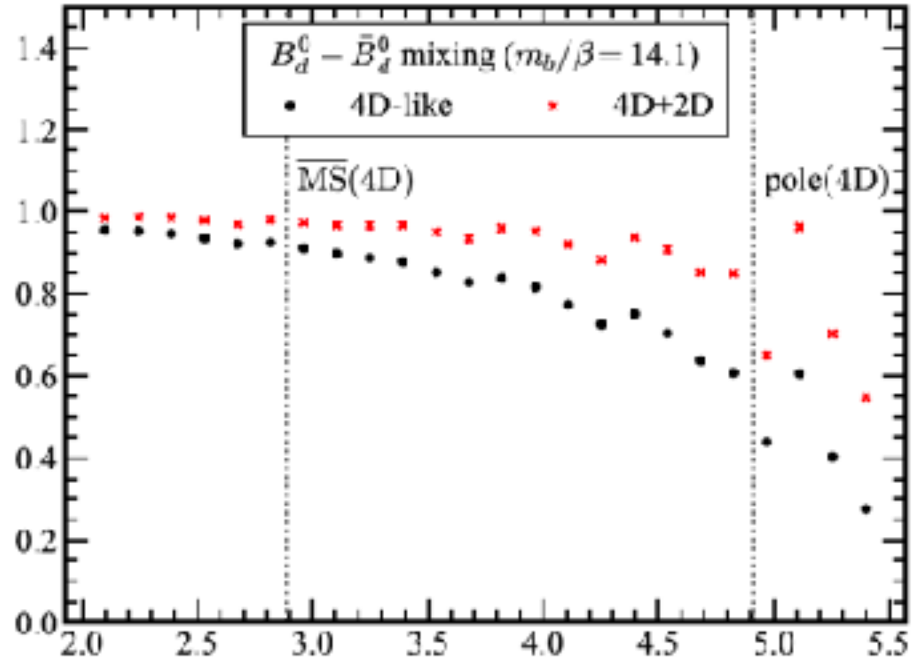
➡ dominant topology in large-N_c limit:



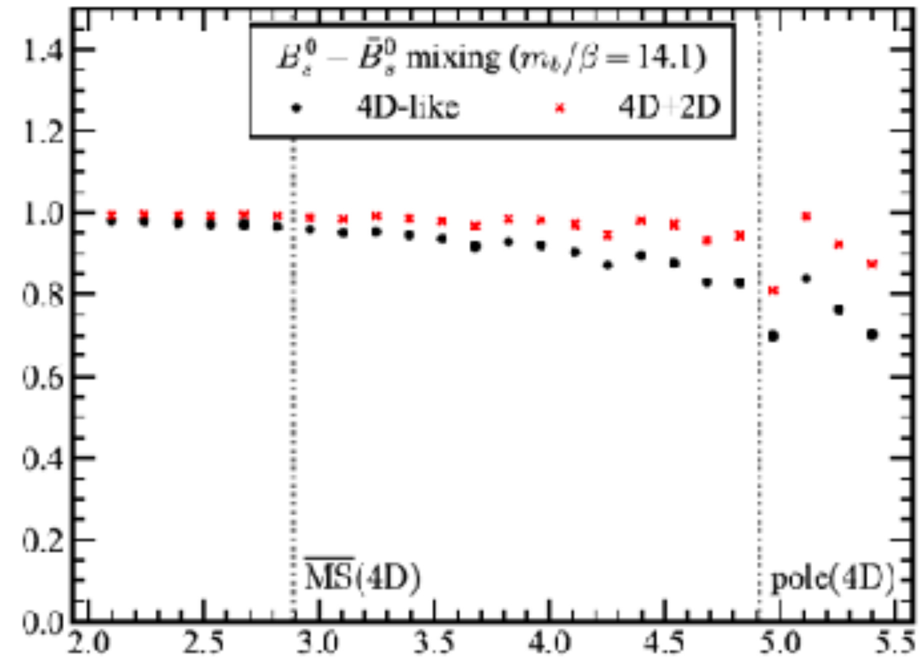
Numerical result: $B_q^0 - \bar{B}_q^0$ $(q = d, s)$

$B_d^0 - \bar{B}_d^0$ $\qquad$ $B_s^0 - \bar{B}_s^0$

$\left|\dfrac{\Delta\Gamma_q^{(\mathrm{exc})}}{\Delta\Gamma_q^{(\mathrm{inc})}}\right|$



m_c/β



m_c/β

$m_b = m_b^{\mathrm{pole,4D}}$

For $m_c < m_c^{\mathrm{pole,4D}}$

◎ For the $B_d^0 - \bar{B}_d^0$ mixing, the correction to the inclusive rate up to 40% is observed.

→ The correction of this size is not excluded yet.

(The experimental data for $\Delta\Gamma_{B_d}/\Gamma_{B_d}$ is still consistent with zero.)

◎ For the $B_s^0 - \bar{B}_s^0$ mixing, the correction to the inclusive rate up to 18% is observed.

→ The result is consistent with what is currently indicated in 4D.

(The ratio of the HFLAV data to the HQE gives $\Delta\Gamma_{B_s}^{(\mathrm{ex})}/\Delta\Gamma_{B_s}^{(\mathrm{th})} = 0.99 \pm 0.15$.)

A. Lenz and G. Tetlalmatzi-Xolocotzi, 2020