Istituto Nazionale di Fisica Nucleare
SEZIONE DI TORINO

New physics and other exotica in bottomonium annihilations

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Hints for New Physics in Heavy Flavors and related topics
Nagoya University, November 17th 2018

Lessons learned

What did we learn of new about bottomonium in the last 15 years?

→ $b\bar{b}$ is not a bad model all

→ Quite nice Quark model/data matching

→ $b\bar{b}$ is not the whole story, at least near the thresholds

→ Zb's, triangular contributions, anomalous transitions...

→ The light degrees of freedom matter a lot

→ Hadronic annihilations are very peculiar

Bottom line: quite some QCD exotic and new effects

NP signals in bottomonium

A comprehensive summary of the signals of new physics in quarkonia

Outline

Why? It look likes we paid little or no attention to these analyses

Rare decay

→ two-lepton decays: flavor violation and lepton universality tests

→ Invisible decays: direct dark matter searches

Hadronic annihilations

→ stable exaquarks (aka dibaryons)

New Physics! New Physics!

(The rare decays)

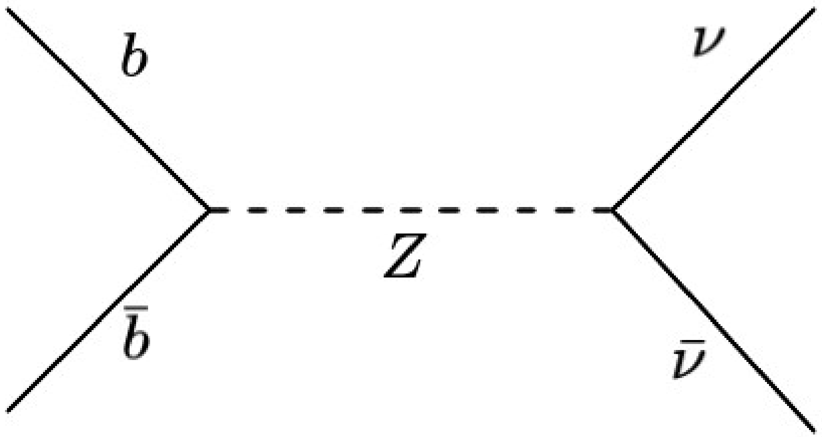
Υ(1S), Υ(2S), Υ(3S) are among the few resonances decaying in $\tau\tau$

$\Upsilon(1S) \to$ *invisible*

$\Upsilon(1S) \to$ invisible is well calculable in the SM

$$\frac{BR(\Upsilon(1S)\to\nu\bar{\nu})}{BR(\Upsilon(1S)\to e^+e^-)} = \frac{27G^2M^4_{\Upsilon(1S)}}{64\pi^2\alpha^2}\left(-1+\frac{4}{3}\sin^2\theta_W\right)^2 = 4.14\times10^{-4}$$

$$BR(\Upsilon(1S)\to\nu\bar{\nu}) \sim 9.9\times10^{-6}$$



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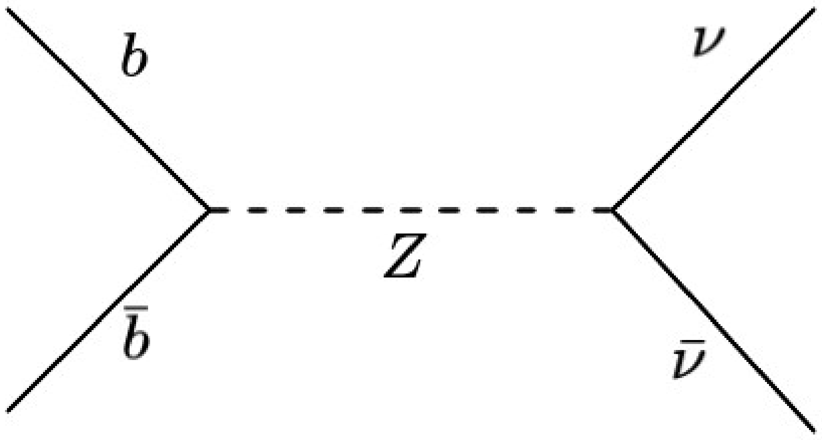
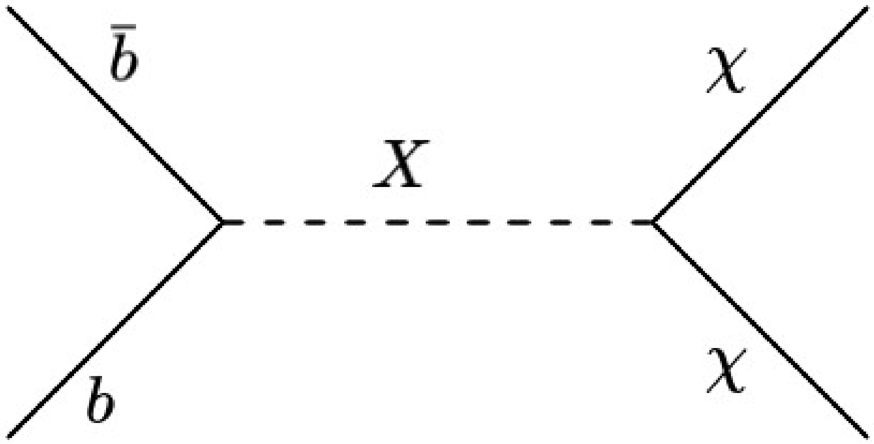
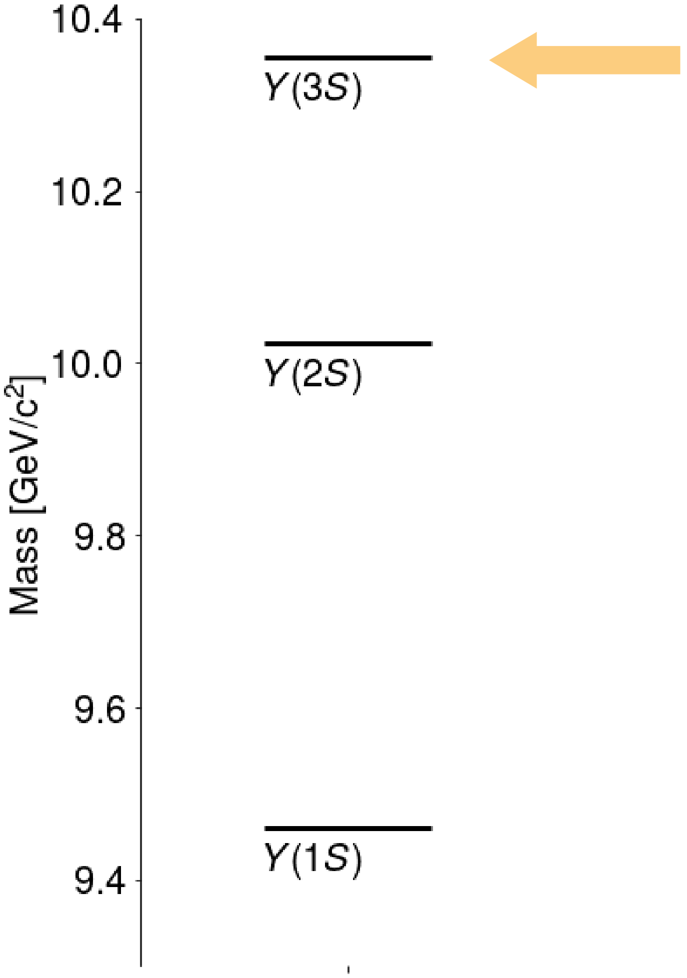


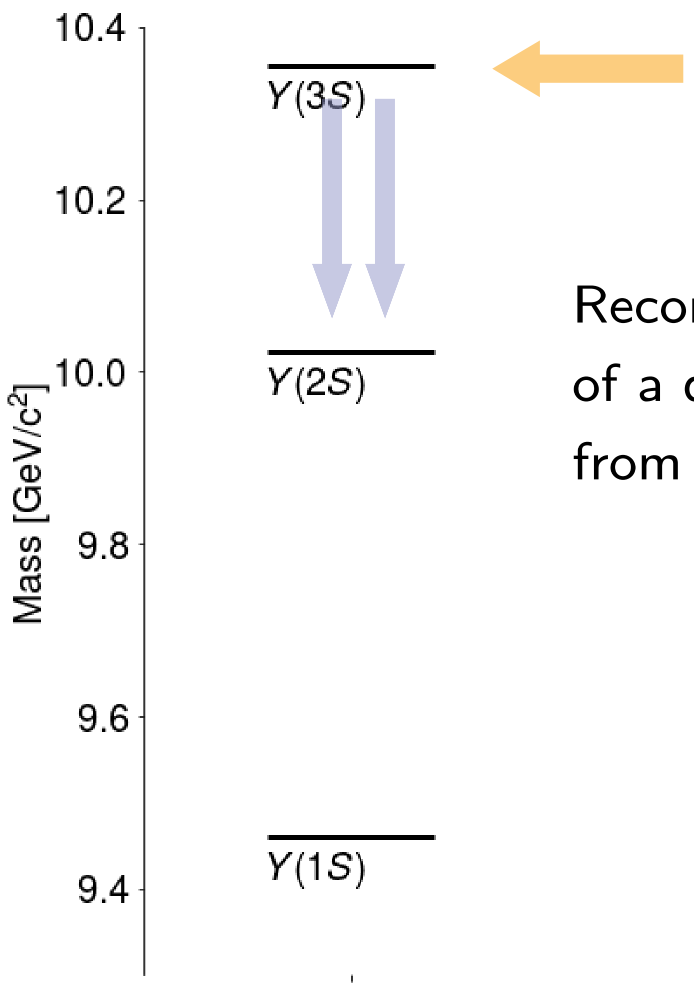


How to see the invisible



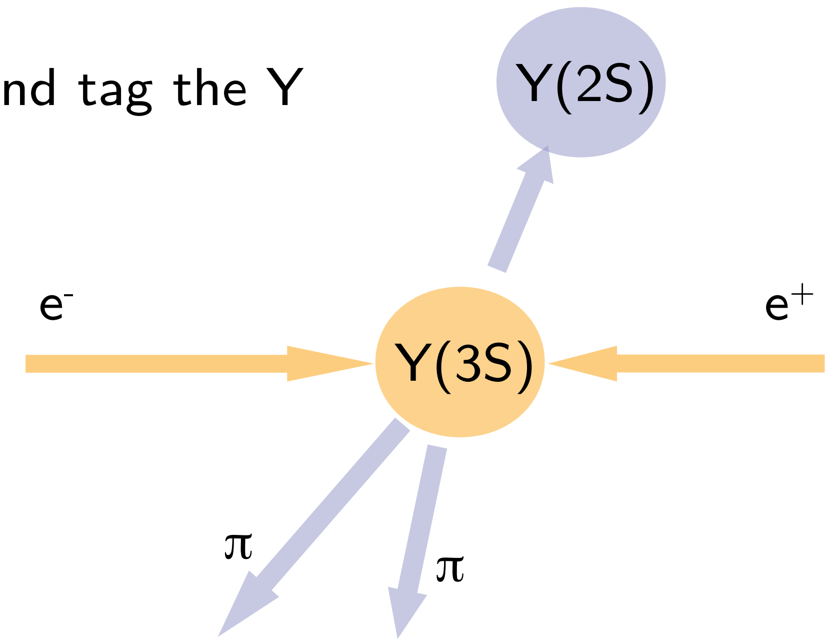
Record e^+e^- collision at this energy

How to see the invisible

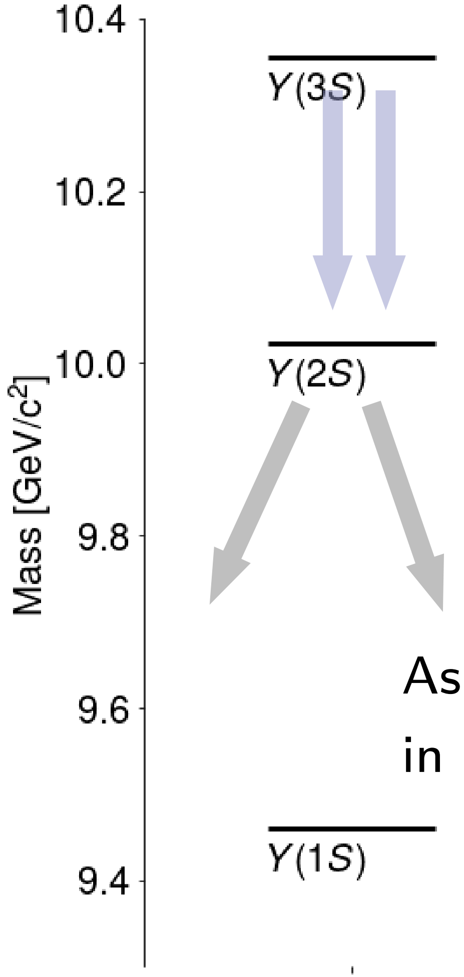


Record e^+e^- collision at this energy

Reconstruct the pions
of a di-pion transition and tag the Υ
from the recoil mass



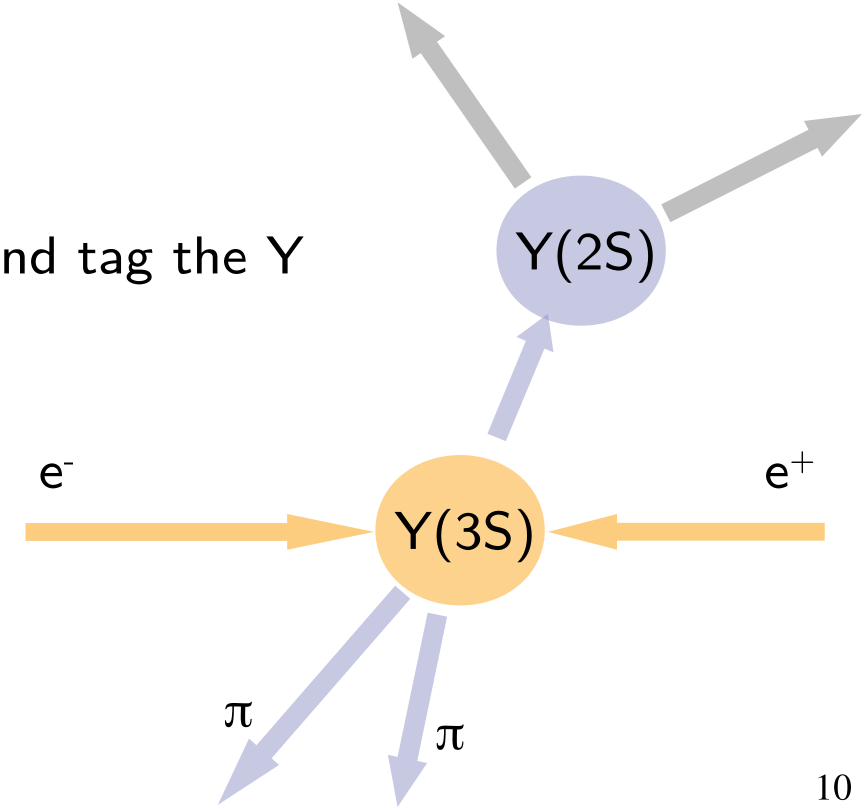
How to see the invisible



Record e^+e^- collision at this energy

Reconstruct the pions
of a di-pion transition and tag the Y
from the recoil mass

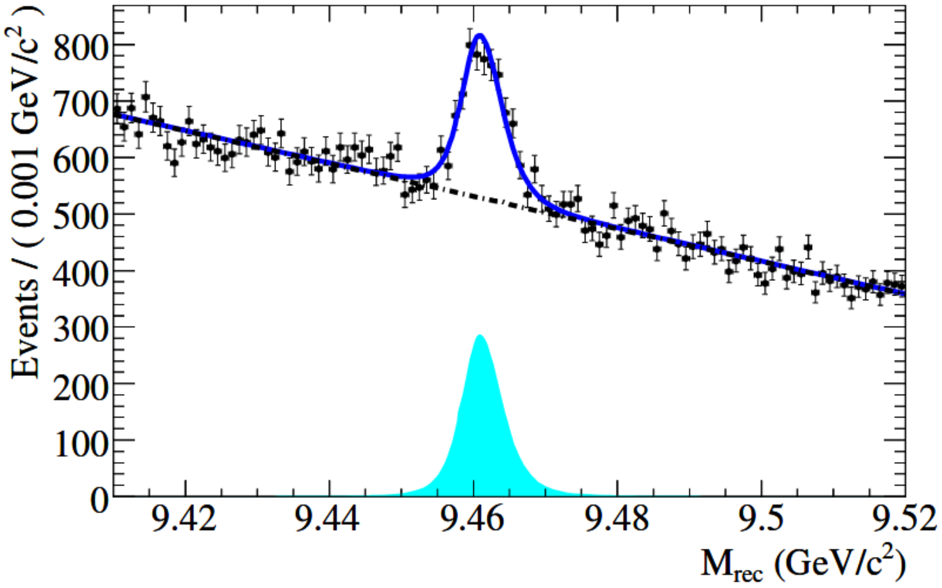
Ask for **no other signal**
in the detector



Υ(1S) → invisible: where do we stand

BaBar, Phys. Rev. Lett. 103, 251801 (2009)

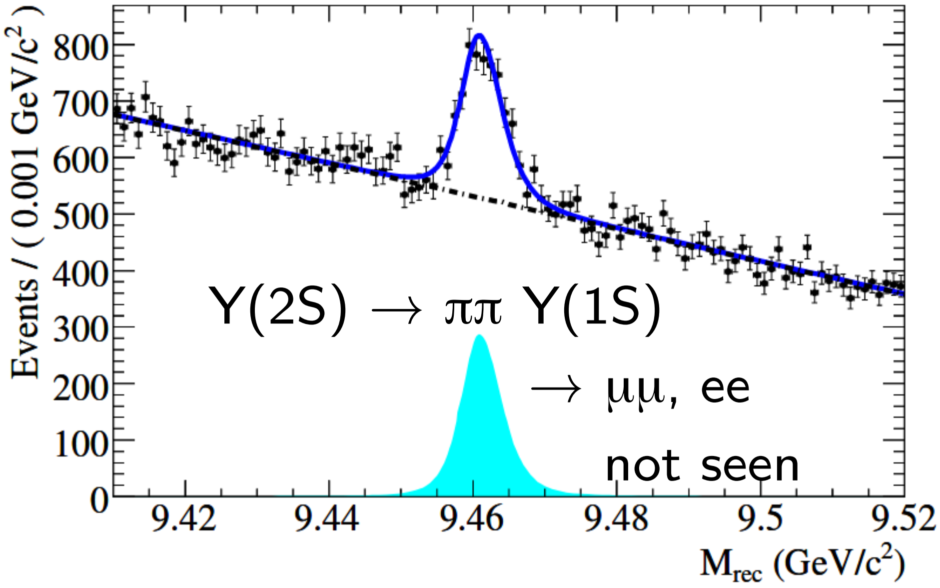
Υ(2S) → ππ nothing



$\Upsilon(1S) \rightarrow$ invisible: where do we stand

BaBar, Phys. Rev. Lett. 103, 251801 (2009)

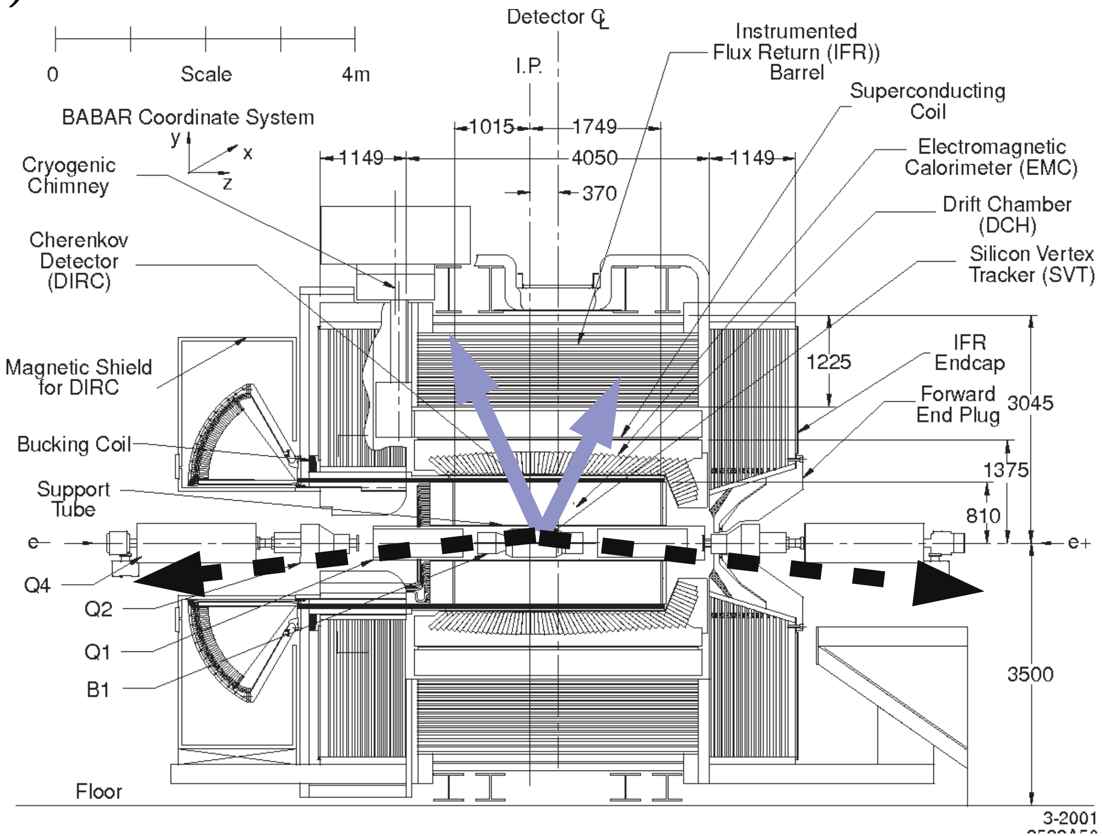
$\Upsilon(2S) \rightarrow \pi\pi$ nothing



$\Upsilon(2S) \rightarrow \pi\pi\ \Upsilon(1S)$

$\rightarrow \mu\mu$, ee

not seen



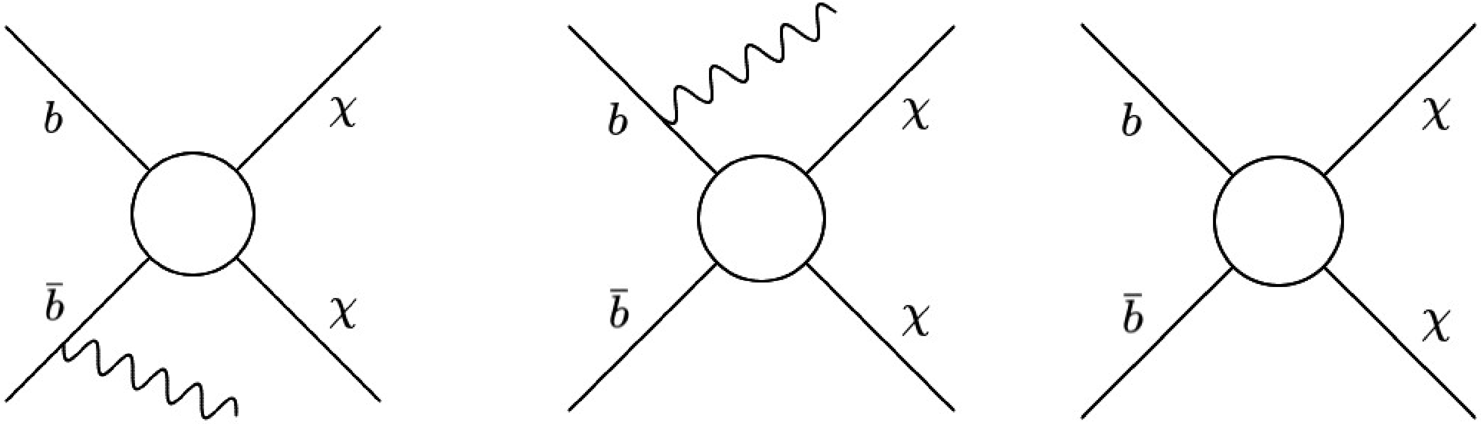
Interpreting Υ(1S) → DM DM (γ)

Υ(1S) → γ invisible in terms of DM limits

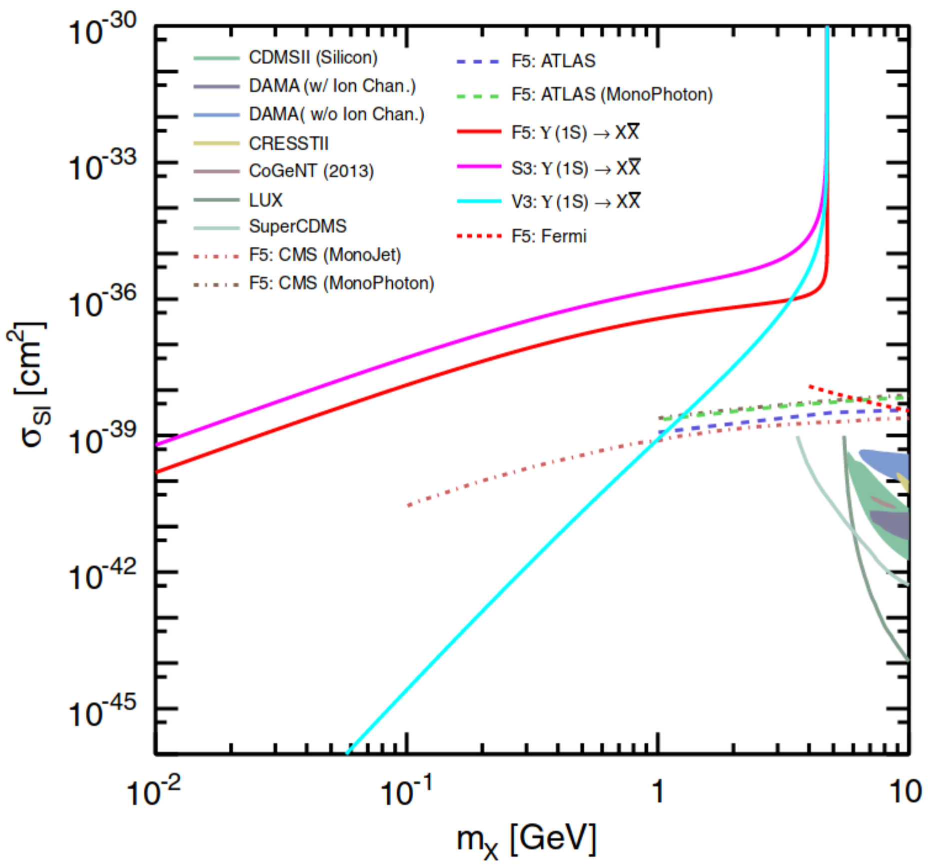
Fernandez, Seong, Stengel, PRD93, 054023 (2016)

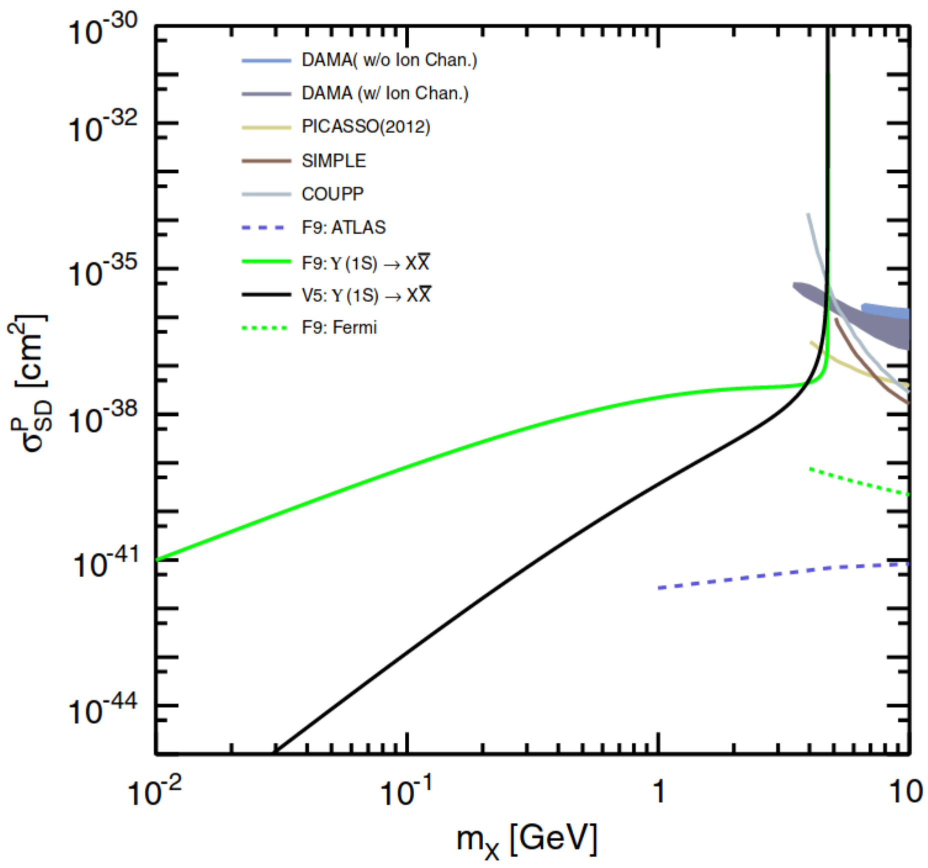
Fernandez, Kumar, Seong, Stengel, PRD90, 015029 (2014)

Name	Interaction structure	Annihilation	Scattering
F5	$(1/\Lambda^2)\bar{X}\gamma^\mu X\bar{q}\gamma_\mu q$	Yes	SI
F6	$(1/\Lambda^2)\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu q$	No	No
F9	$(1/\Lambda^2)\bar{X}\sigma^{\mu\nu}X\bar{q}\sigma_{\mu\nu}q$	Yes	SD
F10	$(1/\Lambda^2)\bar{X}\sigma^{\mu\nu}\gamma^5 X\bar{q}\sigma_{\mu\nu}q$	Yes	No
S3	$(1/\Lambda^2)\imath\,\mathrm{Im}(\phi^\dagger\partial_\mu\phi)\bar{q}\gamma^\mu q$	No	SI
V3	$(1/\Lambda^2)\imath\,\mathrm{Im}(B_\nu^\dagger\partial_\mu B^\nu)\bar{q}\gamma^\mu q$	No	SI
V5	$(1/\Lambda)(B_\mu^\dagger B_\nu - B_\nu^\dagger B_\mu)\bar{q}\sigma^{\mu\nu}q$	No	SD
V7	$(1/\Lambda^2)B_\nu^{(\dagger)}\partial^\nu B_\mu\bar{q}\gamma^\mu q$	No	No
V9	$(1/\Lambda^2)\varepsilon^{\mu\nu\rho\sigma}B_\nu^{(\dagger)}\partial_\rho B_\sigma\bar{q}\gamma_\mu q$	No	No

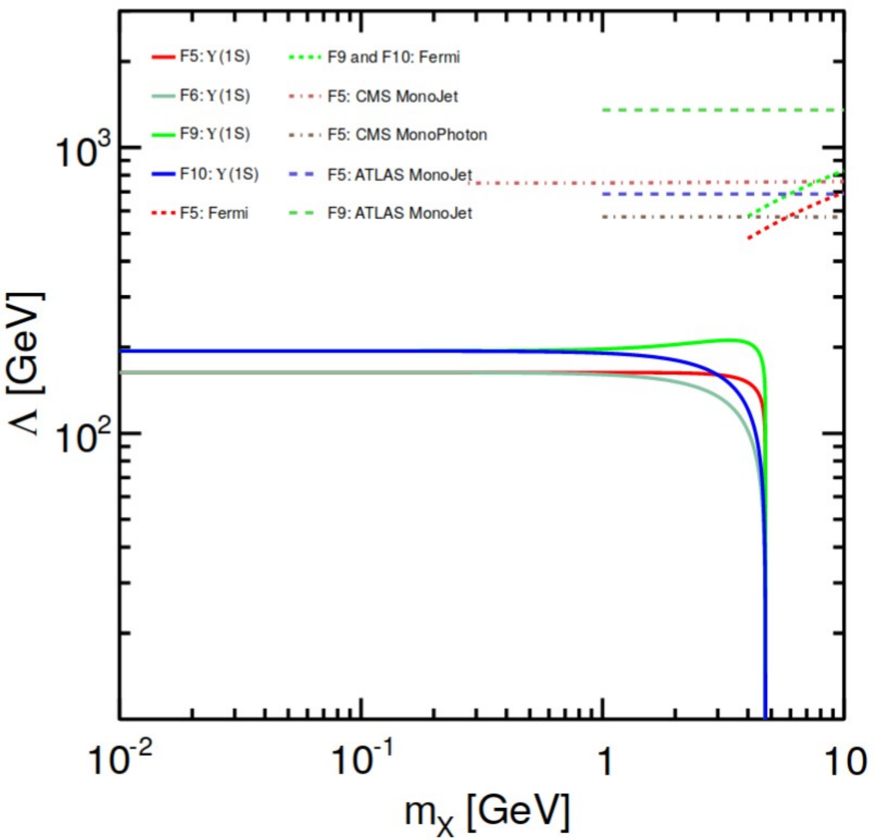


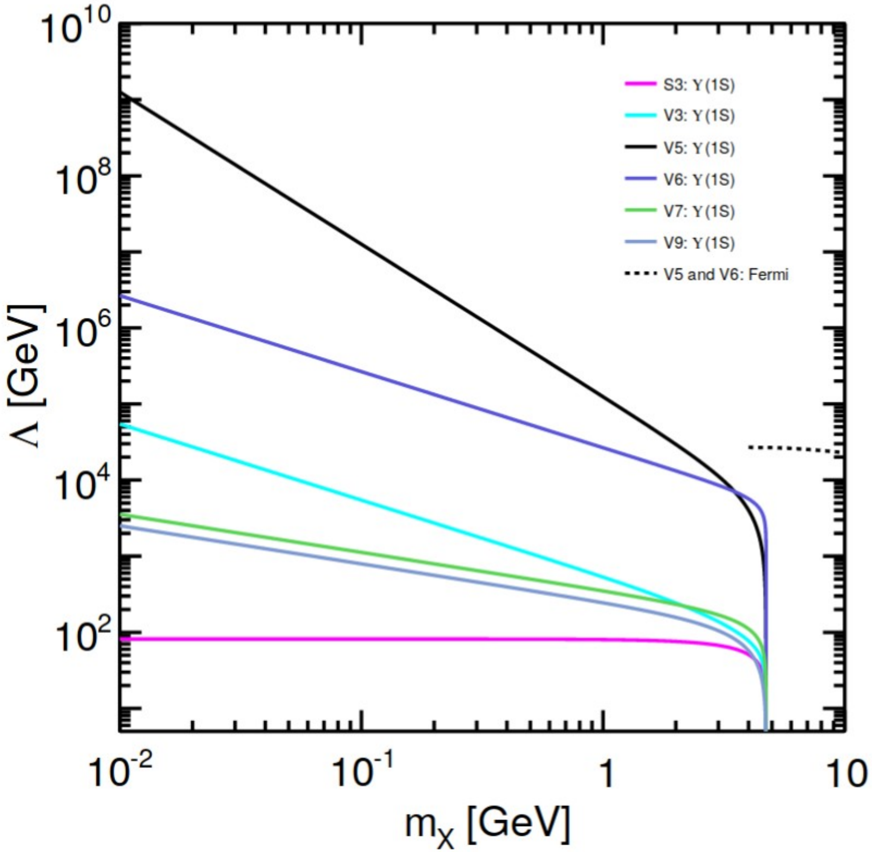
$\Upsilon(1S) \to DM\ DM$



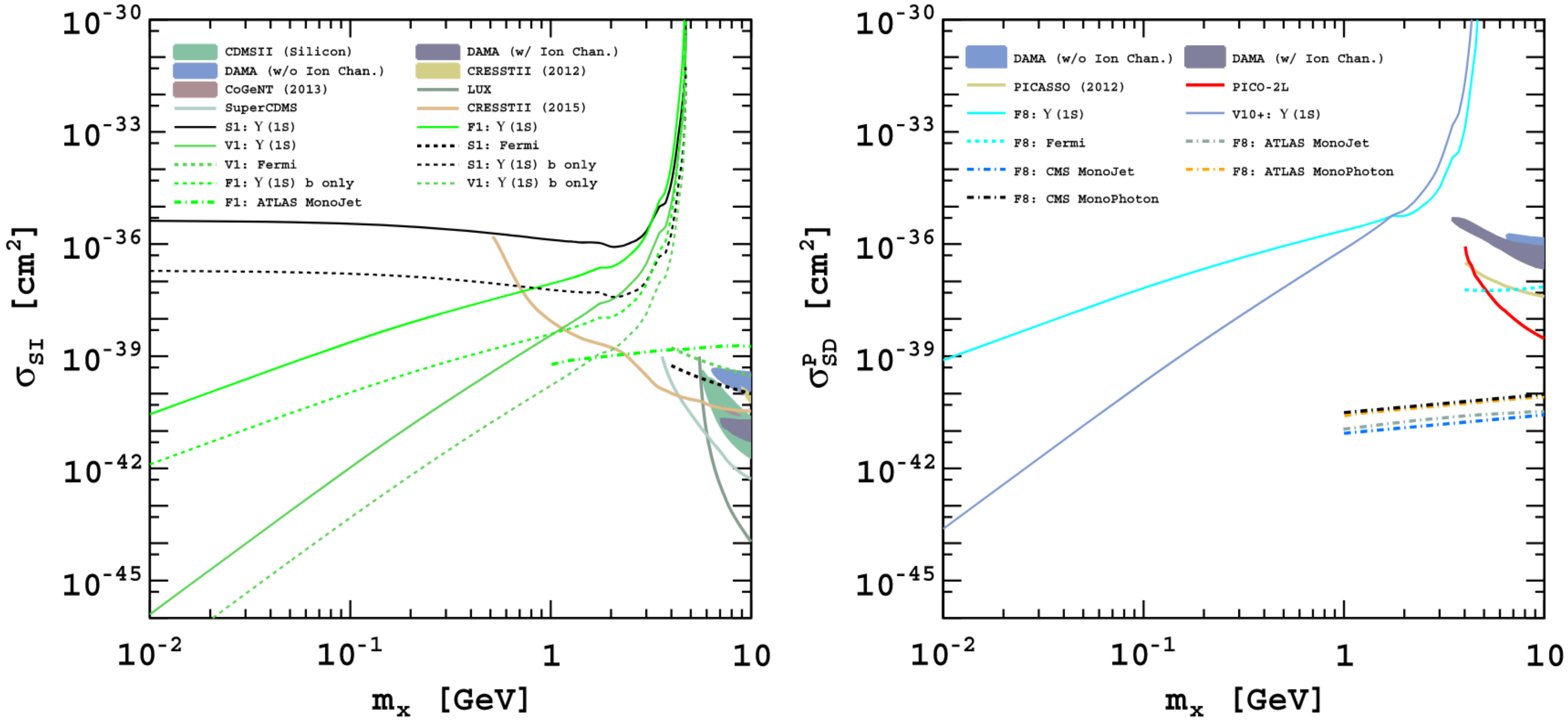


$\Upsilon(1S) \to DM\ DM$, mediator scale

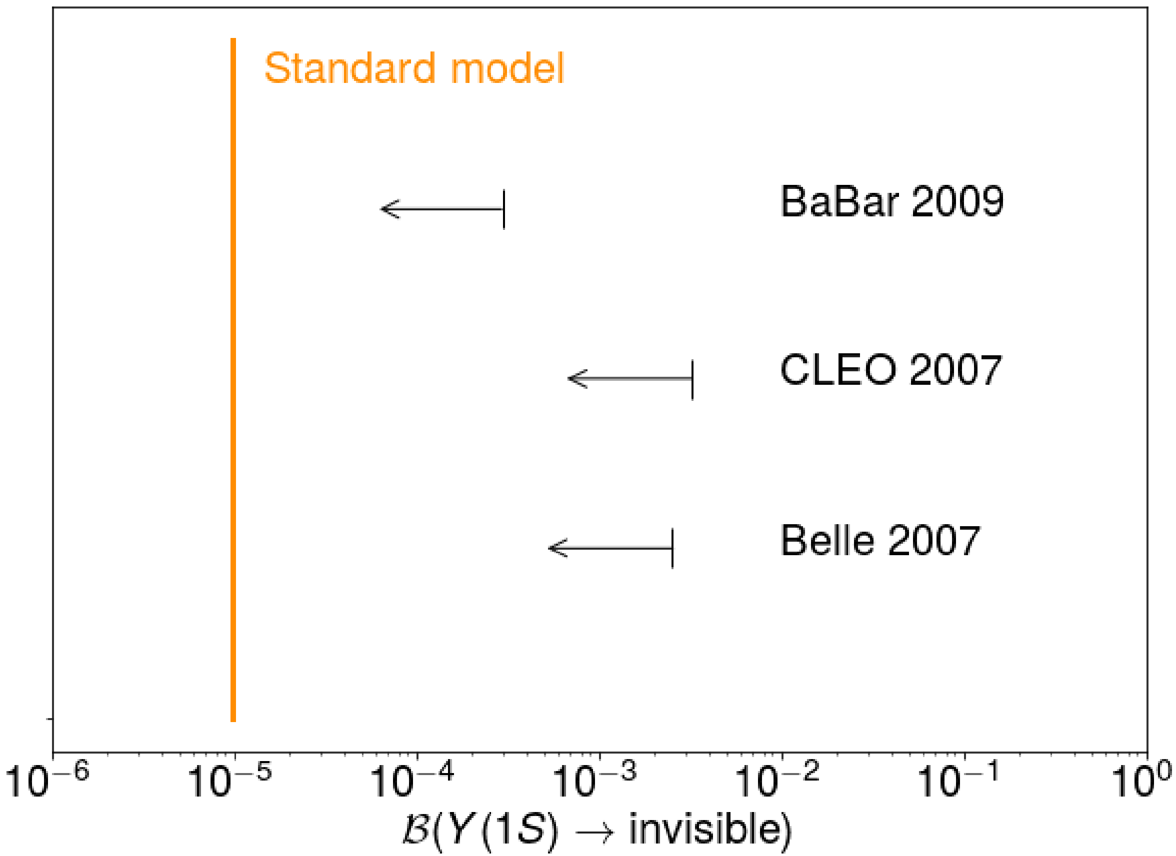




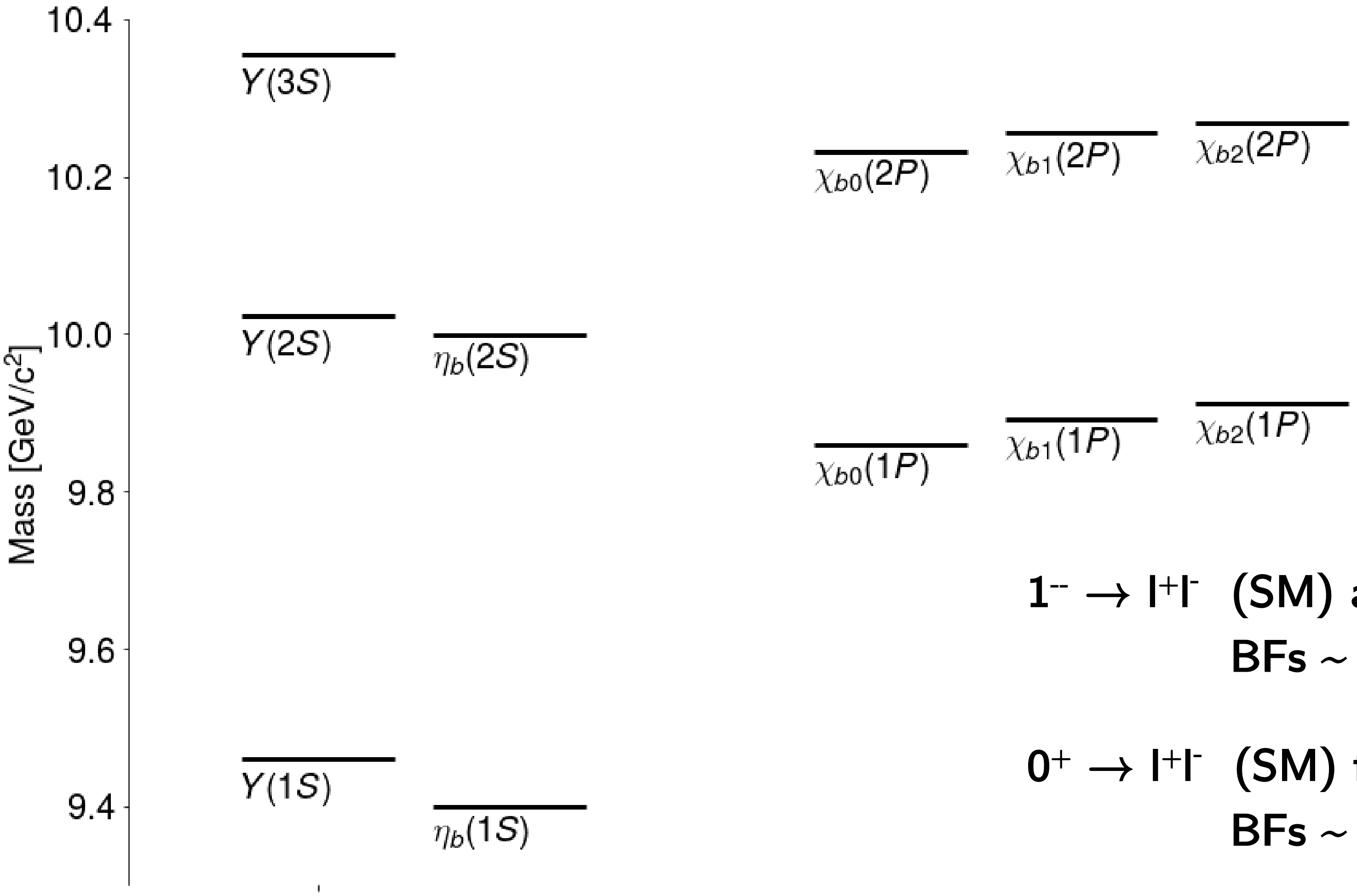
$\Upsilon(1S) \to DM\ DM\ \gamma$



$\Upsilon(1S) \to$ *invisible: where do we stand*



Rare leptonic decays



$1^{--} \to l^+l^-$ (SM) allowed tree-level
BFs ~ 2 %

$0^{+} \to l^+l^-$ (SM) forbidden at tree-level
BFs ~ 10^{-10}

Rare leptonic decays

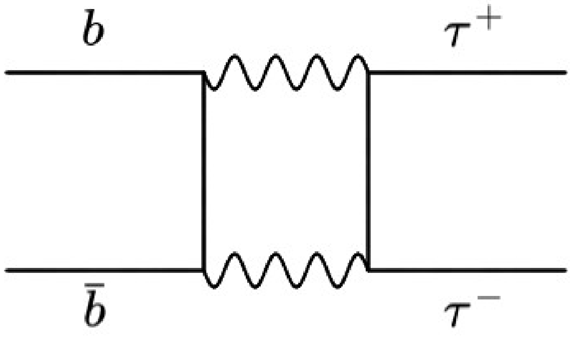
Study of $\chi_{b0} \to \tau\tau$ in the Type II 2HDM model, *Godfrey and Logan [PRD 93, 055014 (2016)]*

QED contribution:

$$\Gamma^{2\gamma}(\chi_0 \to \ell^+\ell^-) \simeq \frac{\alpha^2}{2\beta_\ell}\left[\frac{m_\ell}{M_{\chi_0}}\ln\frac{(1+\beta_\ell)}{(1-\beta_\ell)}\right]^2 \Gamma(\chi_0 \to \gamma\gamma)$$

$$\mathrm{BR}^{2\gamma}(\chi_{b0}(1P) \to \tau^+\tau^-) \simeq 1 \times 10^{-9}$$

$$\mathrm{BR}^{2\gamma}(\chi_{b0}(2P) \to \tau^+\tau^-) \simeq 6 \times 10^{-9}$$

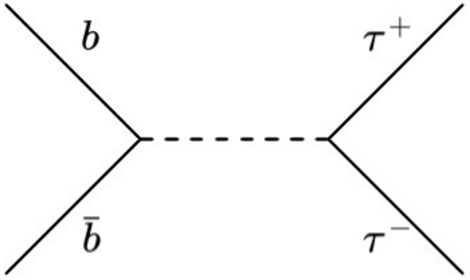


SM higgs contribution:

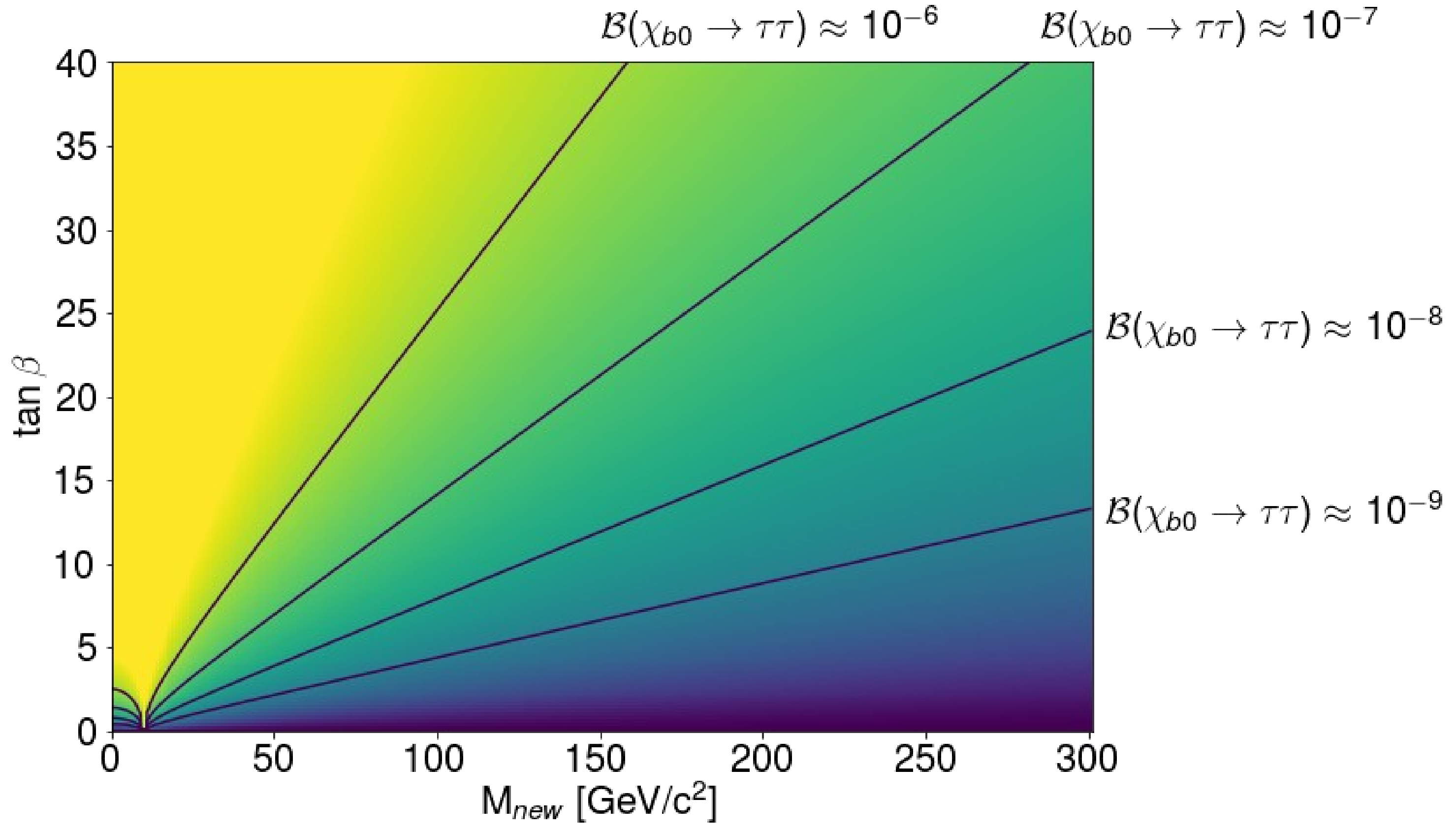
$$\Gamma^H(\chi_0 \to \ell^+\ell^-) = \frac{M_{\chi_0}}{8\pi}\left[1 - \frac{4m_\ell^2}{M_{\chi_0}^2}\right]^{3/2}\left(\frac{m_q m_\ell}{v^2 M_H^2}\right)^2 f_{\chi_0}^2.$$

$$\mathrm{BR}^H(\chi_{b0}(1P) \to \tau^+\tau^-) = 3.1 \times 10^{-13},$$

$$\mathrm{BR}^H(\chi_{b0}(2P) \to \tau^+\tau^-) = (1.9 \pm 0.5) \times 10^{-12}$$

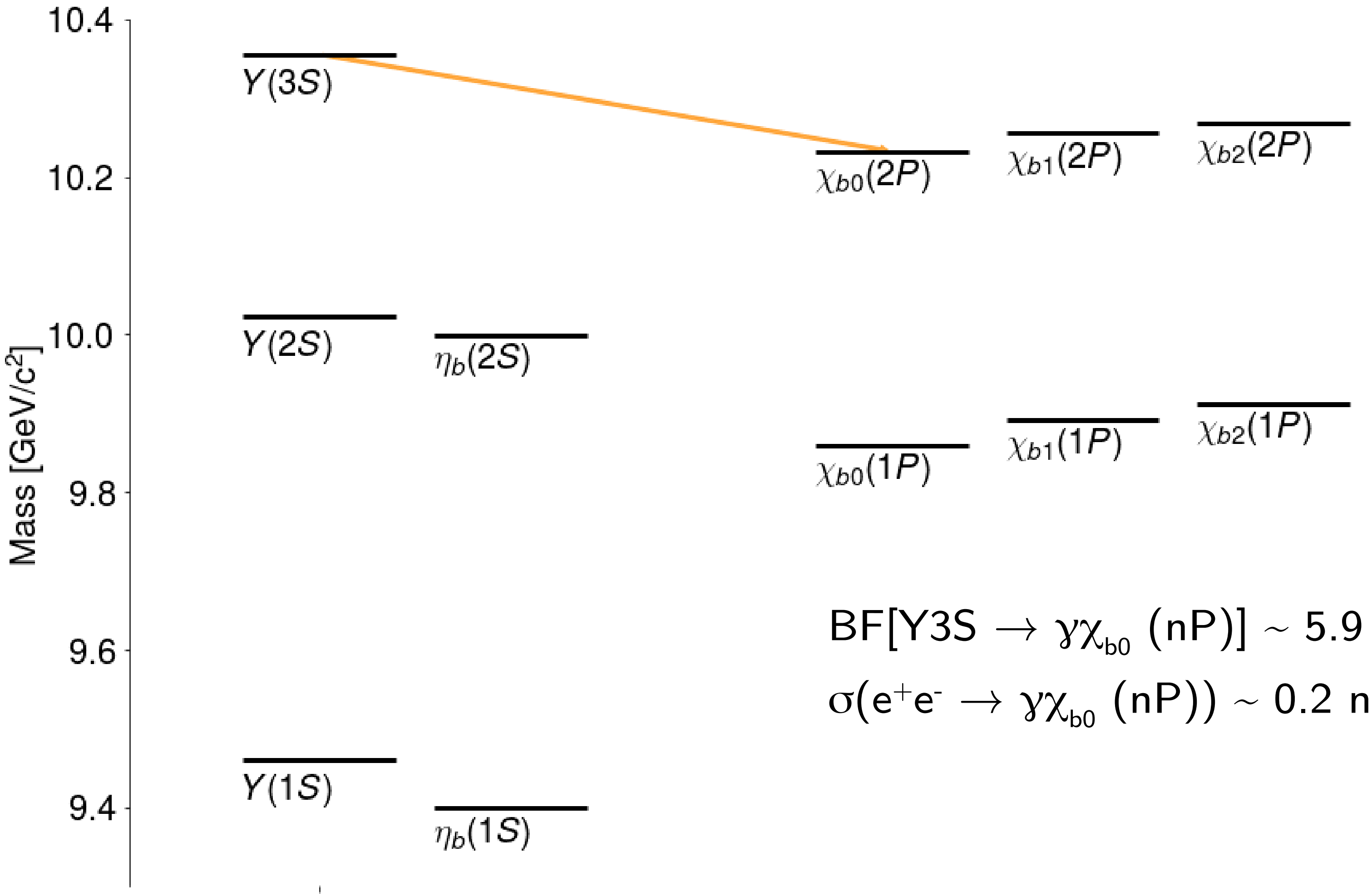


Rare leptonic decays



$$\Gamma^H(\chi_0 \to \ell^+\ell^-) = \frac{M_{\chi_0}}{8\pi}\left[1-\frac{4m_\ell^2}{M_{\chi_0}^2}\right]^{3/2}\left(\frac{m_q m_\ell}{v^2 M_H^2}\right)^2 f_{\chi_0}^2 . \times \left[1+\frac{M_H^2}{M_{\rm new}^2 - M_{\chi_{b0}}^2}\tan^2\beta\right]^2$$

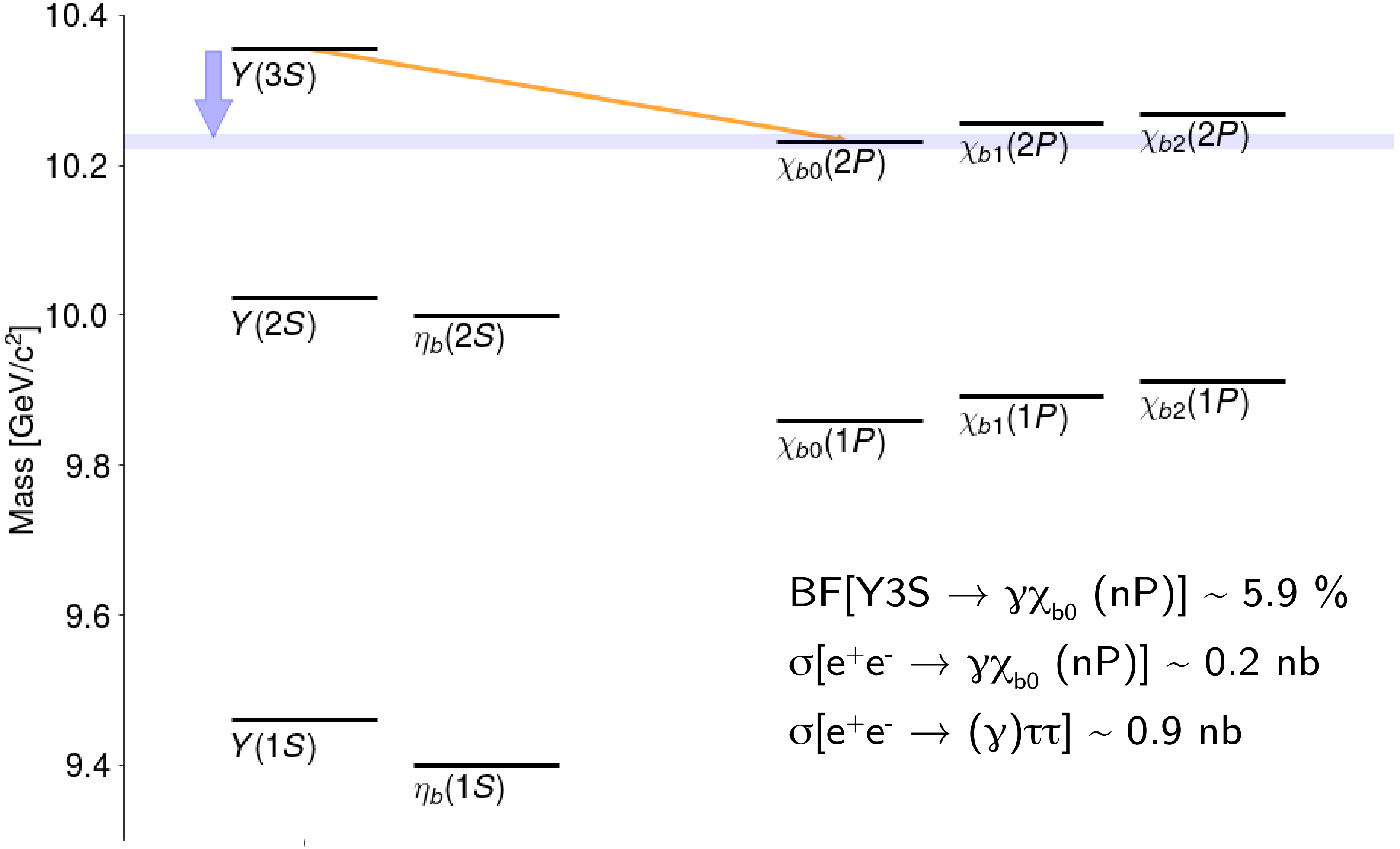
Experiment



BF[Y3S $\rightarrow \gamma\chi_{b0}$ (nP)] ~ 5.9 %

$\sigma(e^+e^- \rightarrow \gamma\chi_{b0}$ (nP)) ~ 0.2 nb

Experiment

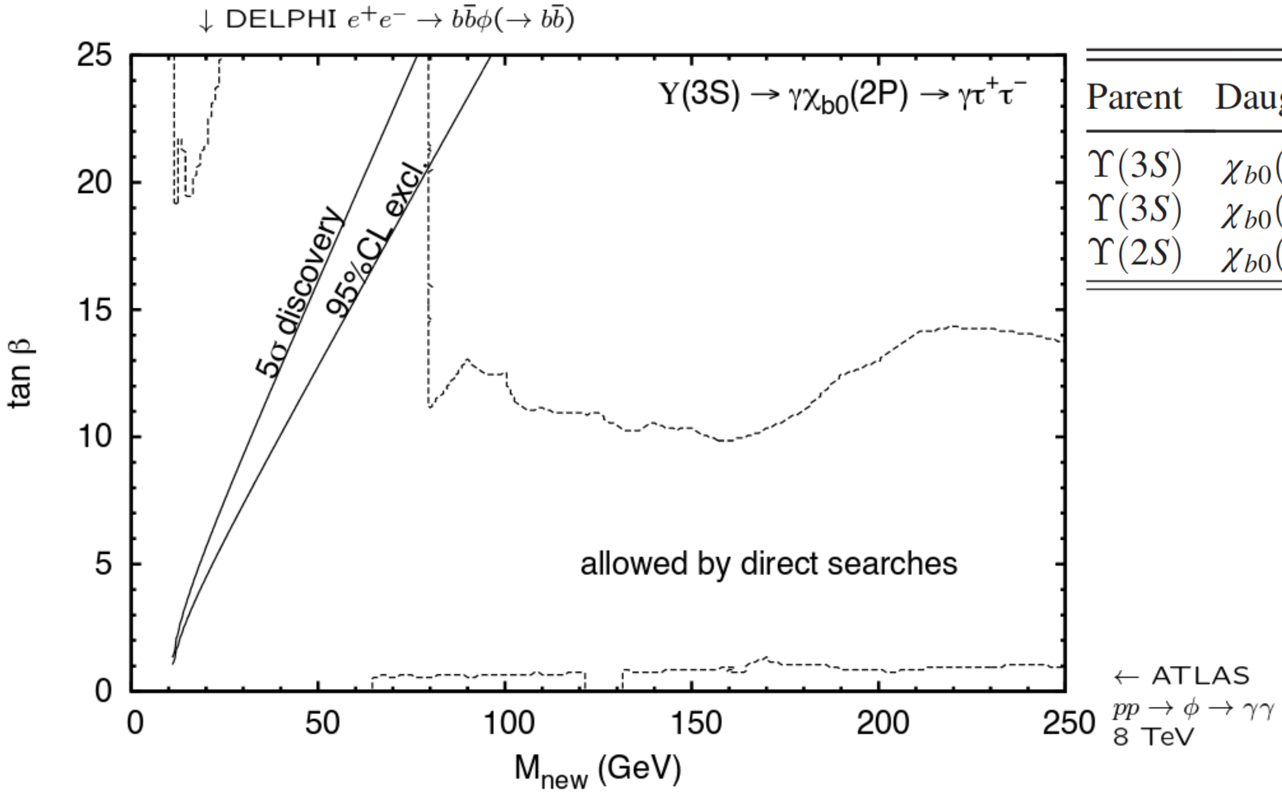


BF[Y3S $\rightarrow \gamma\chi_{b0}$ (nP)] ~ 5.9 %

σ[e^+e^- $\rightarrow \gamma\chi_{b0}$ (nP)] ~ 0.2 nb

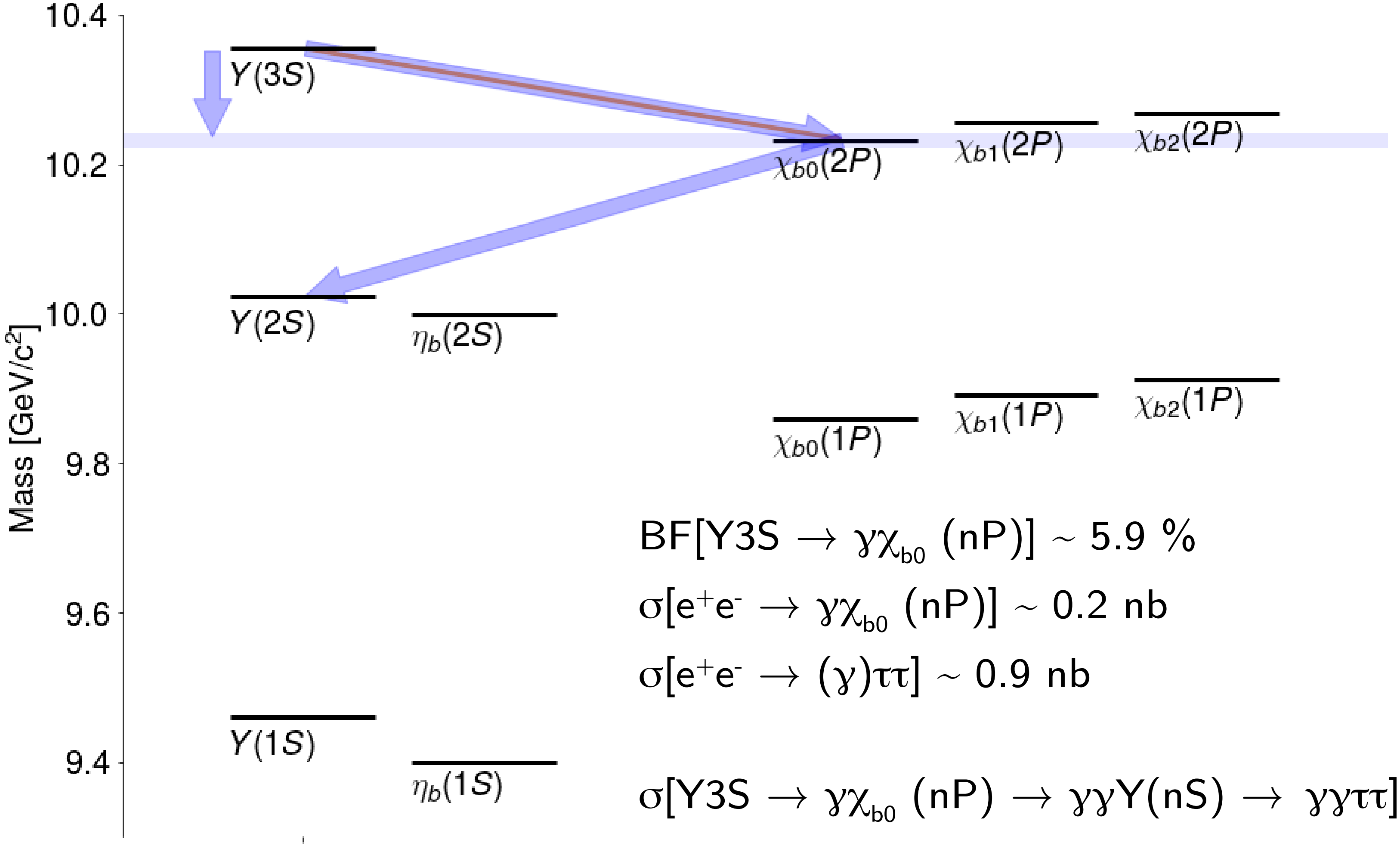
σ[e^+e^- $\rightarrow$ (γ)$\tau\tau$] ~ 0.9 nb

$\Upsilon(3S)$: rare χ_b decays



Parent	Daughter	E_γ	δE_γ	$d\sigma_B/dE_\gamma$	N_B
$\Upsilon(3S)$	$\chi_{b0}(2P)$	122 MeV	0.24 MeV	36 fb/MeV	4320
$\Upsilon(3S)$	$\chi_{b0}(1P)$	484 MeV	1.3 MeV	8.8 fb/MeV	5720
$\Upsilon(2S)$	$\chi_{b0}(1P)$	163 MeV	1.3 MeV	30 fb/MeV	19500

Experiment



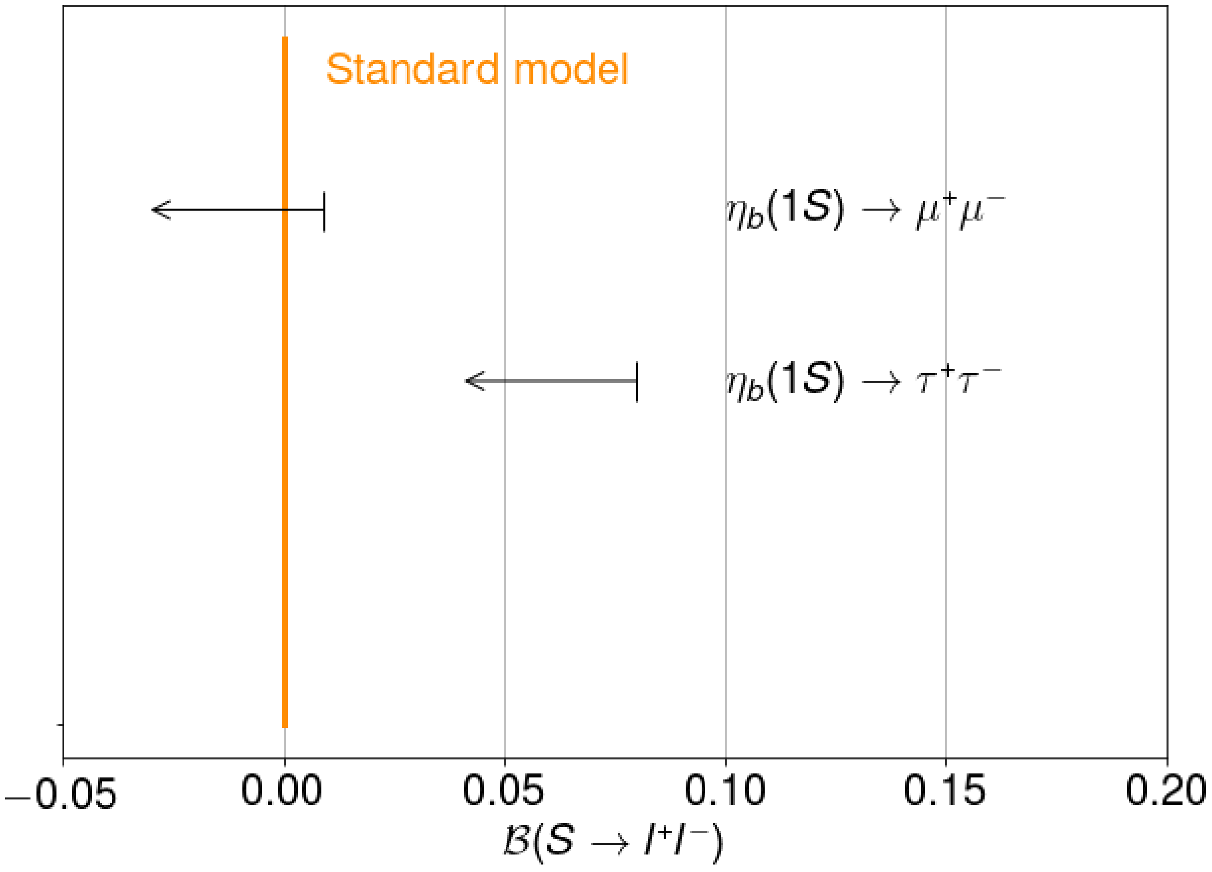
$BF[Y3S \rightarrow \gamma\chi_{b0}\ (nP)] \sim 5.9\ \%$

$\sigma[e^+e^- \rightarrow \gamma\chi_{b0}\ (nP)] \sim 0.2$ nb

$\sigma[e^+e^- \rightarrow (\gamma)\tau\tau] \sim 0.9$ nb

$\sigma[Y3S \rightarrow \gamma\chi_{b0}\ (nP) \rightarrow \gamma\gamma Y(nS) \rightarrow \gamma\gamma\tau\tau] \sim 8 \times 10^{-5}$ nb

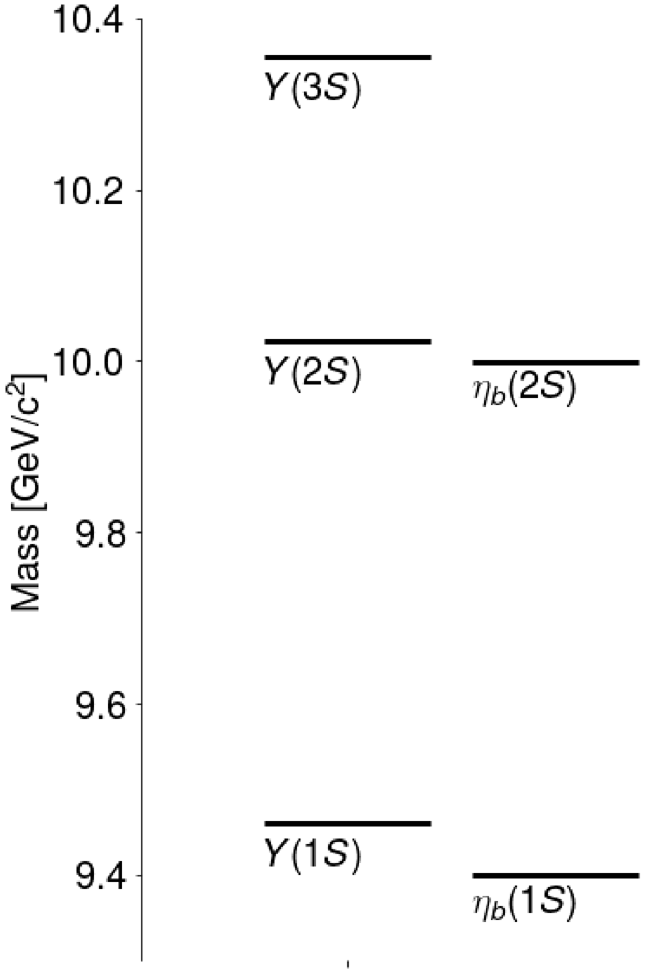
Scalar $\to$ leptons: where we stand



Very old limits only for the most challenging channel

Predictions:
Rachid, Duraisamy, Datta, PRD82,054031 (2010)

Lepton universality violation in Υ decays



LUV from "untagged" $\eta_b(1S) \to \tau\tau$ decays

$\to$ 2HDM(II) light higgs at ~ 10 GeV

$\to \Upsilon(1S) \to \gamma\, \eta^*_b(1S)$, $\eta^{(*)}_b(1S)$ mixes with A^0

$\to \eta^{(*)}_b(1S)$ gives an extra contribution to $\Upsilon(1S) \to \tau\tau$

Sanchis-Lozano,
Modern Physics Letters A, Vol. 17, No. 34 (2002) 2265-2276

Lepton universality violation in Υ decays

Aloni, Efrati,Grossman, Nir, JHEP06 (2017) 019

Υ and ψ leptonic decays as probes of solutions to the $R\left(D^{(*)}\right)$ puzzle

---

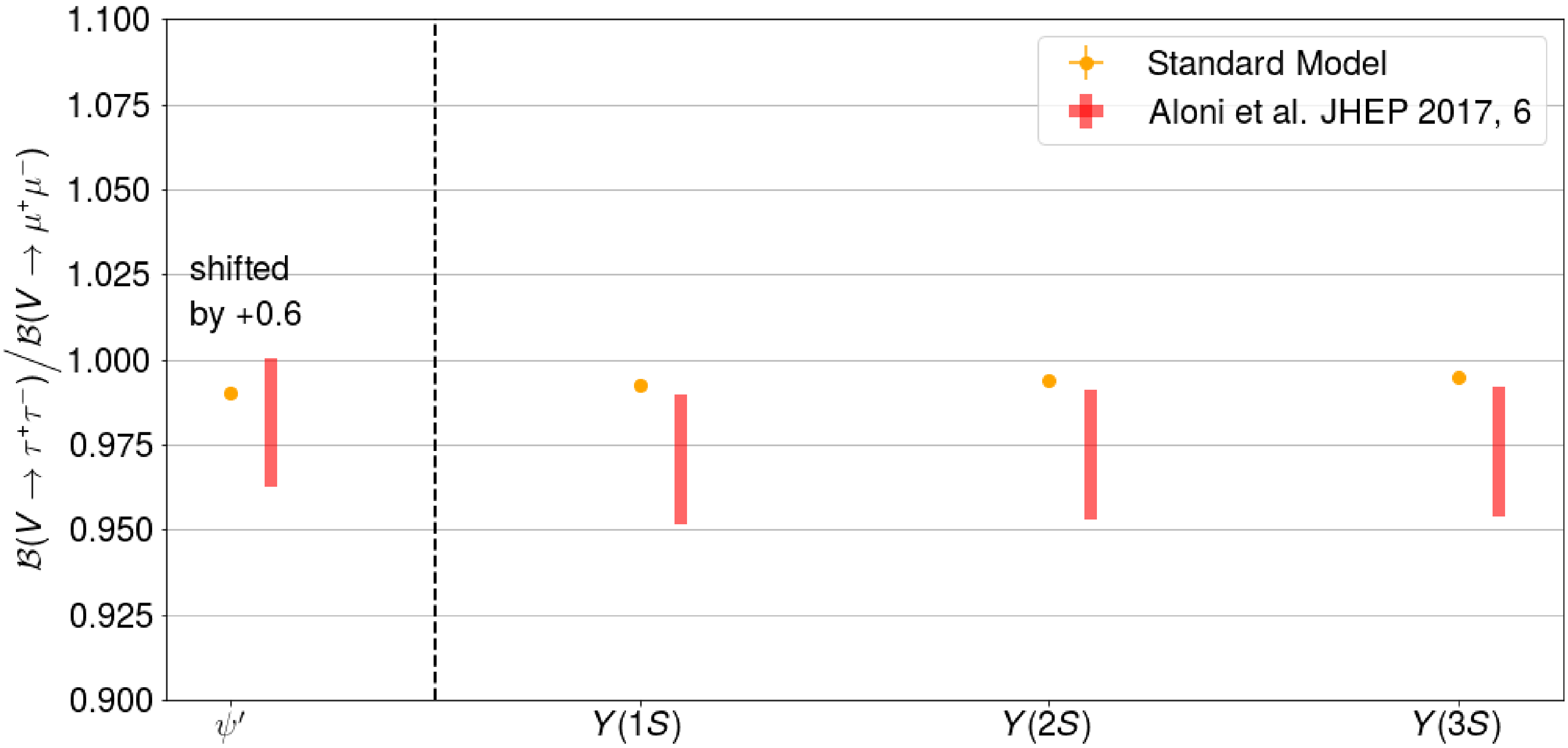
$\rightarrow$ Write the Υ(nS) leptonic widths in EFT (SM)

$\rightarrow$ Add 4-fermion operators for new contributions

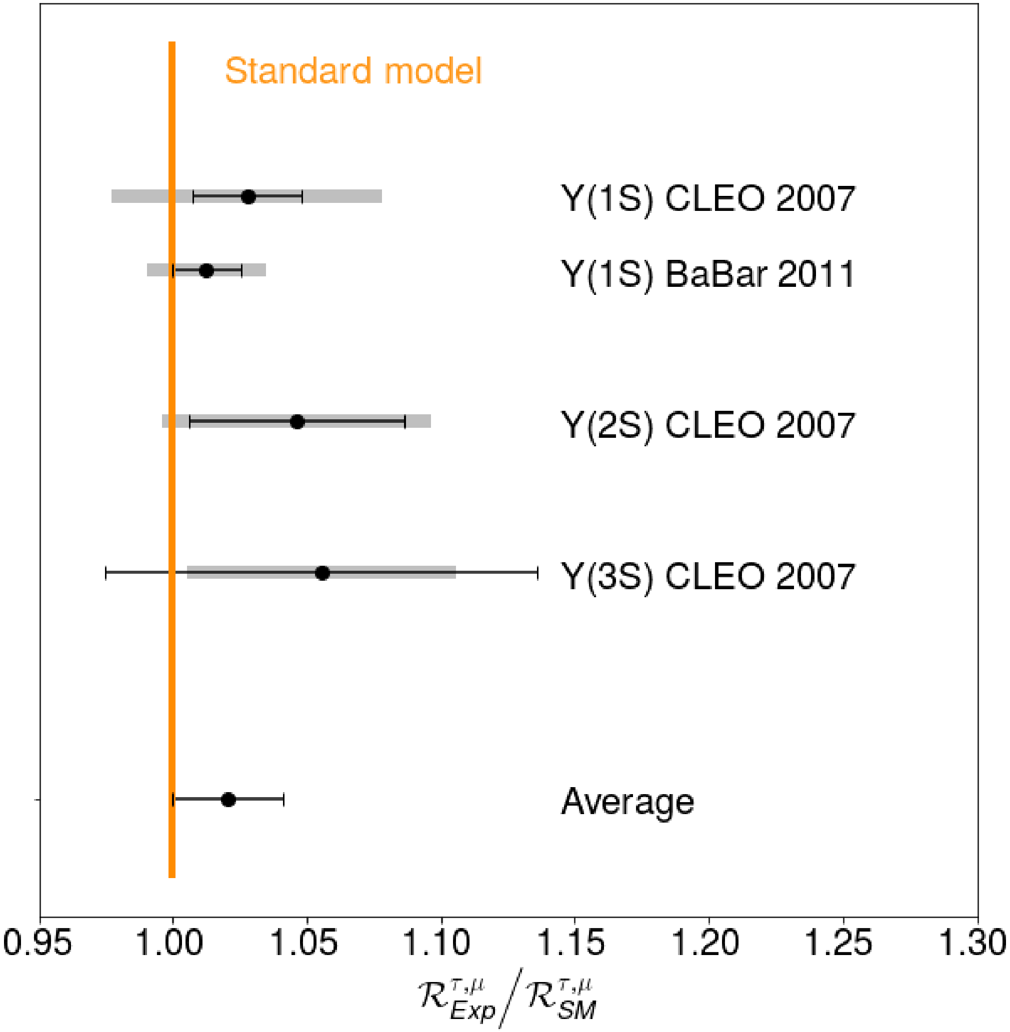
$\rightarrow$ Tune the Wilson coefficients to reproduce R(D*)

Lepton universality violation in Υ decays

Aloni, Efrati,Grossman, Nir, JHEP06 (2017) 019



Lepton Universality: where do we stand



CLEO:

count the tt pairs

BaBar:

$\Upsilon(3S) \to \pi\pi\, \Upsilon(1S) \to \pi\pi\, (\tau\tau, \mu\mu)$

Dominant uncertainties:

→ tracking efficiency

→ event shapes

Can we do better?

Lepton flavor violation

Direct LFV: Υ(1S) → μ τ

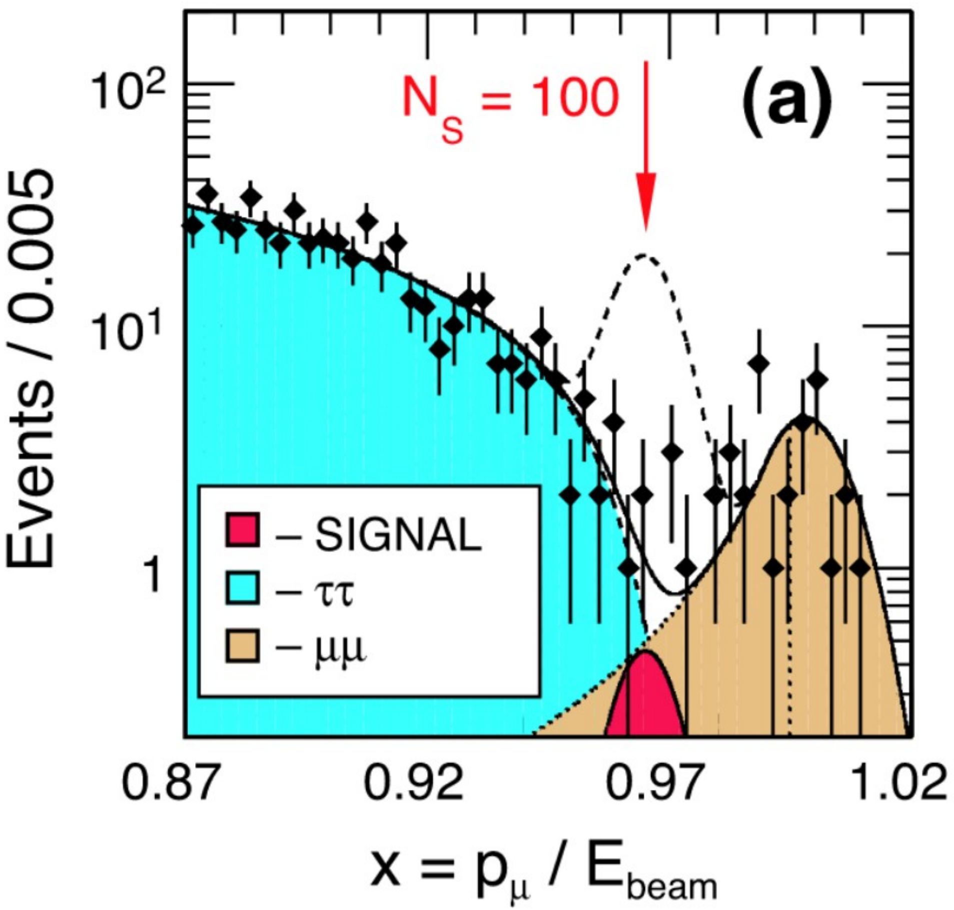
Untagged:

→ Sit on the resonance, reconstruct the muon and measure its momentum

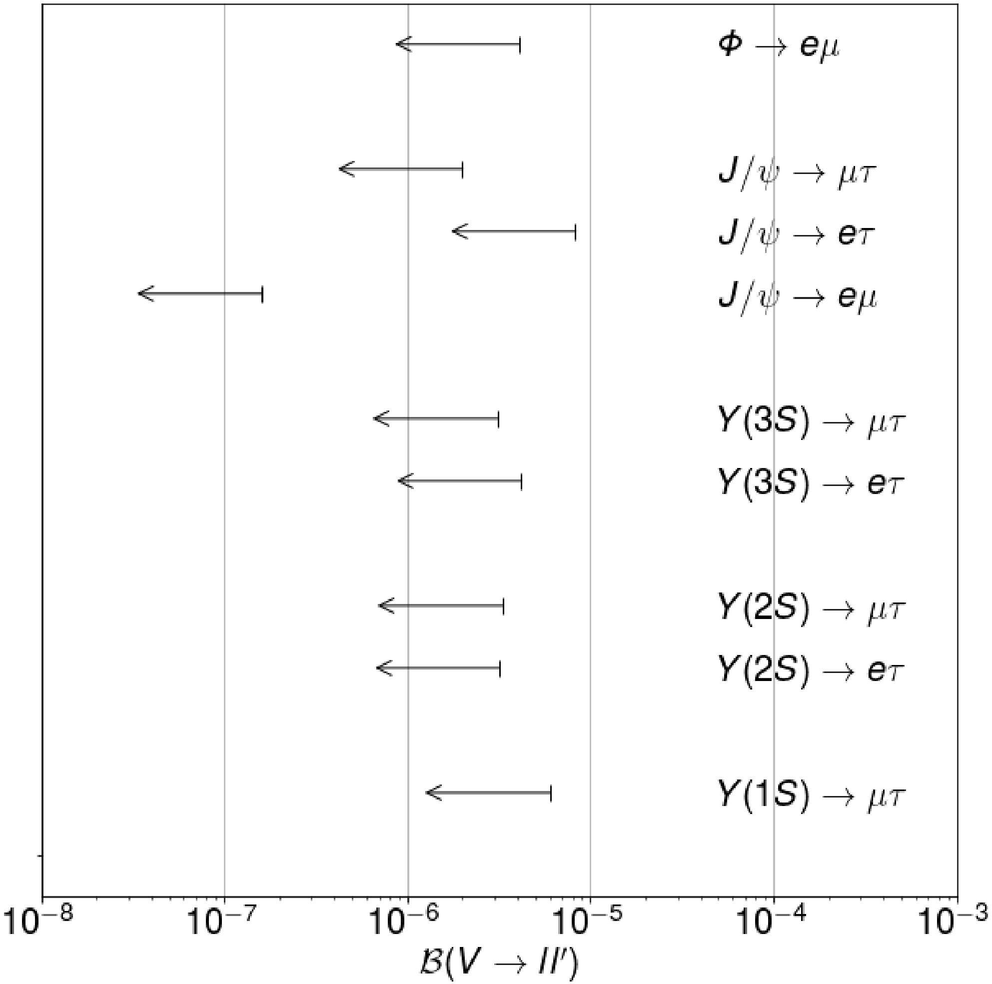
Tagged:

→ As for the invisible

CLEO Υ(1S)



LFV: where do we stand



Most of these limits are still from CLEO...

LFV: where do we stand

Hazard, Petrov, PRD 94,074023 (2016)

Most stringent limits on the Wilson coefficients from the bottomonium

Wilson coefficient [GeV^{-2}]	Leptons	Initial state (quark)				
	$\ell_1\ell_2$	$\Upsilon(1S)(b)$	$\Upsilon(2S)(b)$	$\Upsilon(3S)(b)$	$J/\psi(c)$	$\phi(s)$
$\lvert C_{VL}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	FPS
	$e\tau$	...	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	FPS
	$e\mu$	...	...	...	1.0×10^{-5}	2×10^{-3}
$\lvert C_{VR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	FPS
	$e\tau$	...	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	FPS
	$e\mu$	...	...	...	1.0×10^{-5}	2×10^{-3}
$\lvert C_{TL}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	FPS
	$e\tau$	...	3.3×10^{-2}	3.2×10^{-2}	2.4	FPS
	$e\mu$	...	...	...	4.8	1×10^{4}
$\lvert C_{TR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	FPS
	$e\tau$	...	3.3×10^{-2}	3.2×10^{-2}	2.4	FPS
	$e\mu$	...	...	...	4.8	1×10^{4}

Non-exotic exotica
(Hadronic Annihilations)

~90% of the $\Upsilon(1S)$ decays are $\Upsilon(1S) \rightarrow ggg \rightarrow$ hadrons (10-20 of them)

Loosely bound dibaryons

Jaffe,

Phys.Rev.Lett. 38 (1977) 195-198, Erratum:

Phys.Rev.Lett. 38 (1977) 617 SLAC-PUB-1828

$$|H\rangle = \sqrt{\frac{1}{8}}|\Lambda\Lambda\rangle + \sqrt{\frac{4}{8}}|N\Xi\rangle - \sqrt{\frac{3}{8}}|\Sigma\Sigma\rangle$$

SLAC-PUB-1828
October 1976
(T/E)

PERHAPS A STABLE DIHYPERON*

R. L. Jaffe**
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

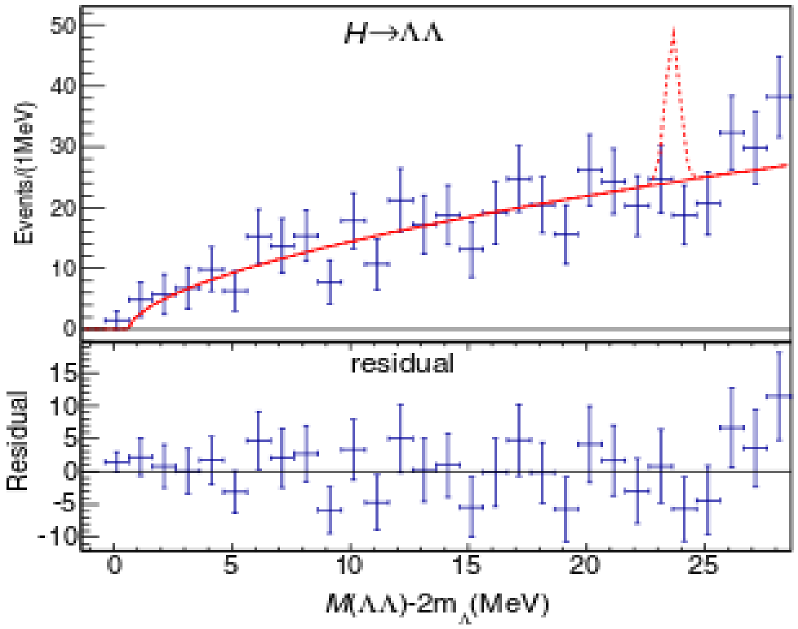
and

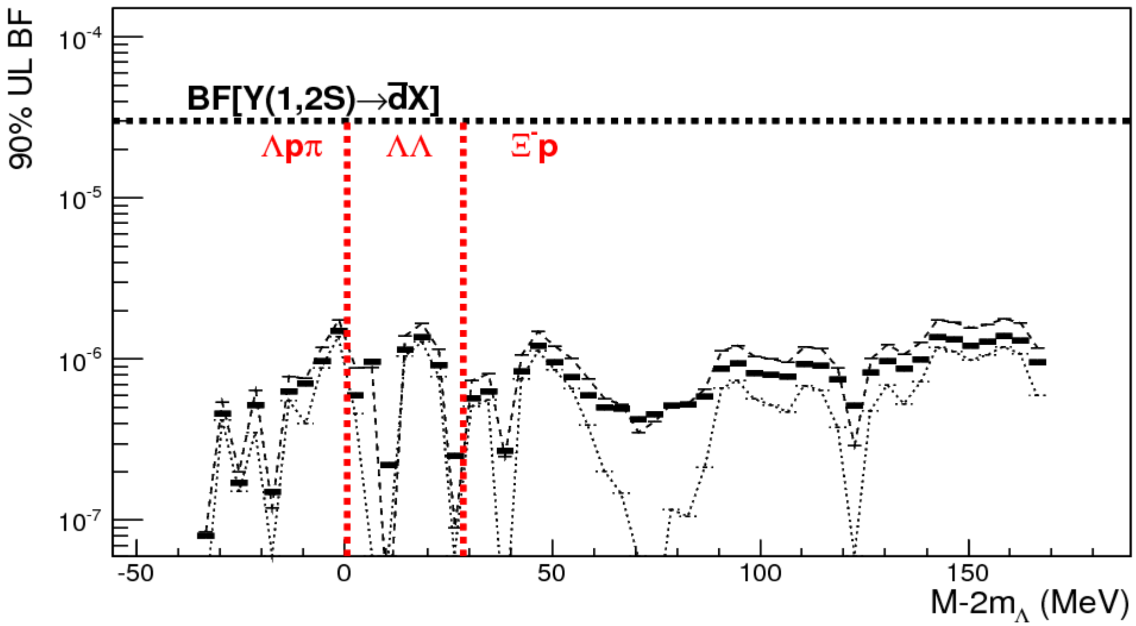
Department of Physics and Laboratory of Nuclear Science†
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

The doubly-strange dibaryons

Strangeness production + anti-deuteron production = H dibaryon?

Belle PRL 110, 222002 (2013)





The loosley bound version of the H seems not to be there...

Baryonic dark matter (?)

What if Jaffe's di-baryon is **very, very bound?**

Kochelev JETP Lett. 70 (1999) 491-494

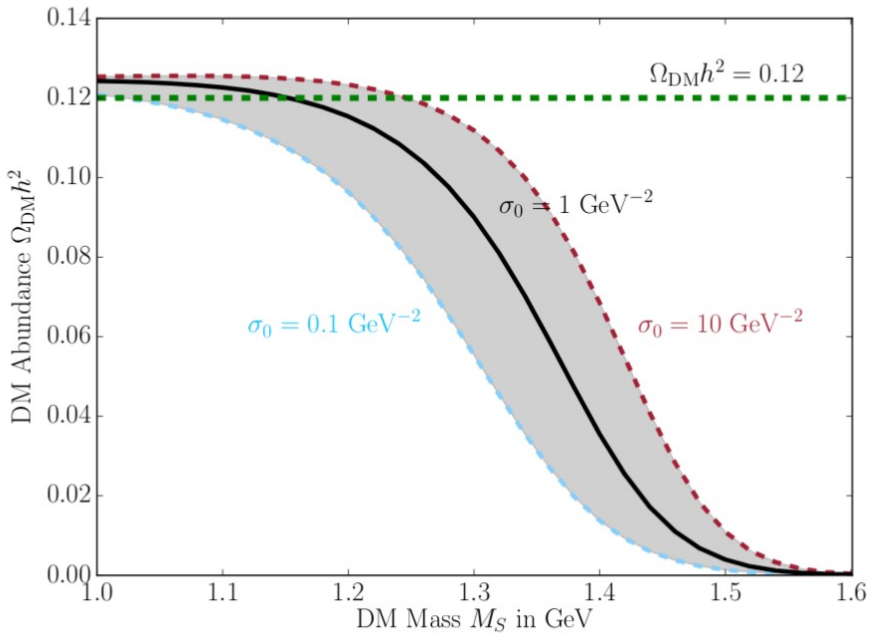
H with mass $\sim$ 1.7 GeV to explain the GZK cutoff

Farrar arXiv:1708.08951 [hep-ph]

Very light H as dark matter candidate

Must be compact to avoid photo-disintegration

Gross, Polosa et a., PRD 98 (2018) no.6, 063005



Some comments…

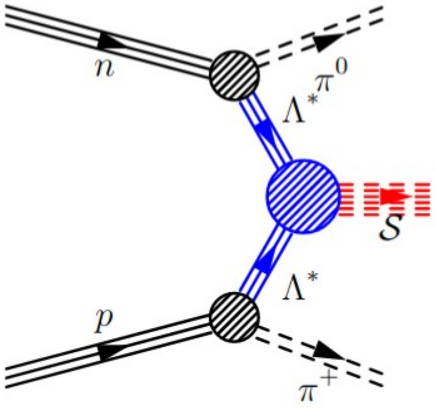
2. **A Deeply Bound Dibaryon is Incompatible with Neutron Stars and Supernovae**
   Samuel D. McDermott, Sanjay Reddy, Srimoyee Sen. Sep 18, 2018. 2 pp.
   FERMILAB-PUB-18-490-A
   e-Print: **arXiv:1809.06765 [hep-ph] |** PDF
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   ADS Abstract Service
   Detailed record

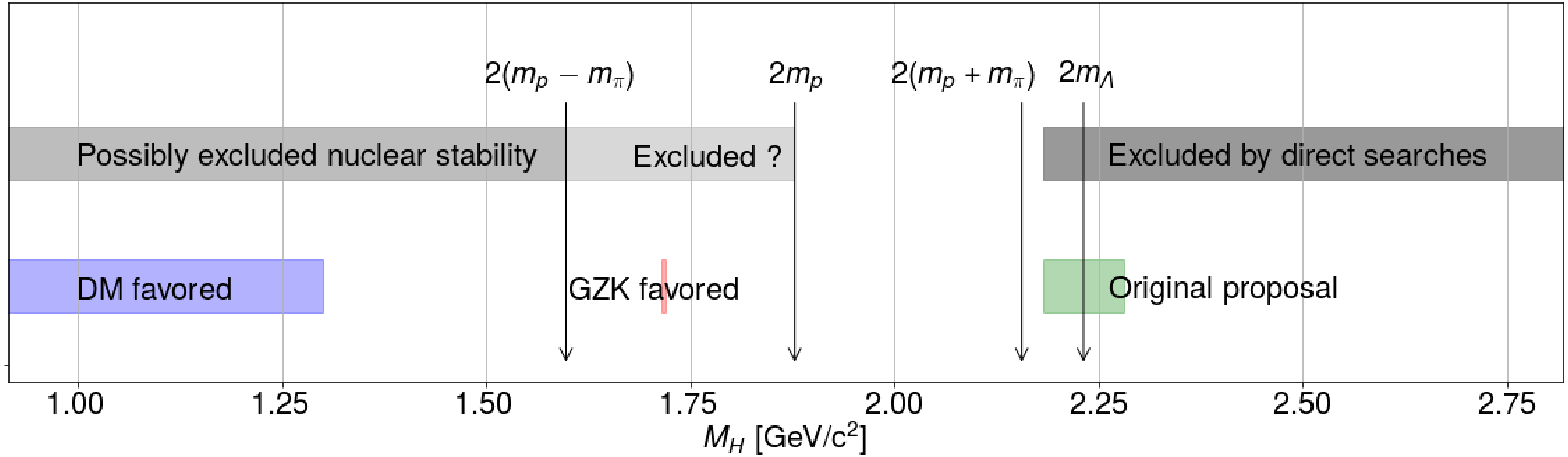
3. **Dibaryons cannot be the dark matter**
   Edward W. Kolb, Michael S. Turner (Chicago U., EFI & Chicago U., KICP). Sep 16, 2018. 11 pp.
   e-Print: **arXiv:1809.06003 [hep-ph] |** PDF
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   ADS Abstract Service
   Detailed record - Cited by 2 records

Baryonic dark matter (?)

A very light H seems to be problematic
for oxigen stability

Gross, Polosa et a., PRD 98 (2018) no.6, 063005





Υ(nS) annihilations

Similarities between hadronic collisions and bottomonium annihilations

1) High density

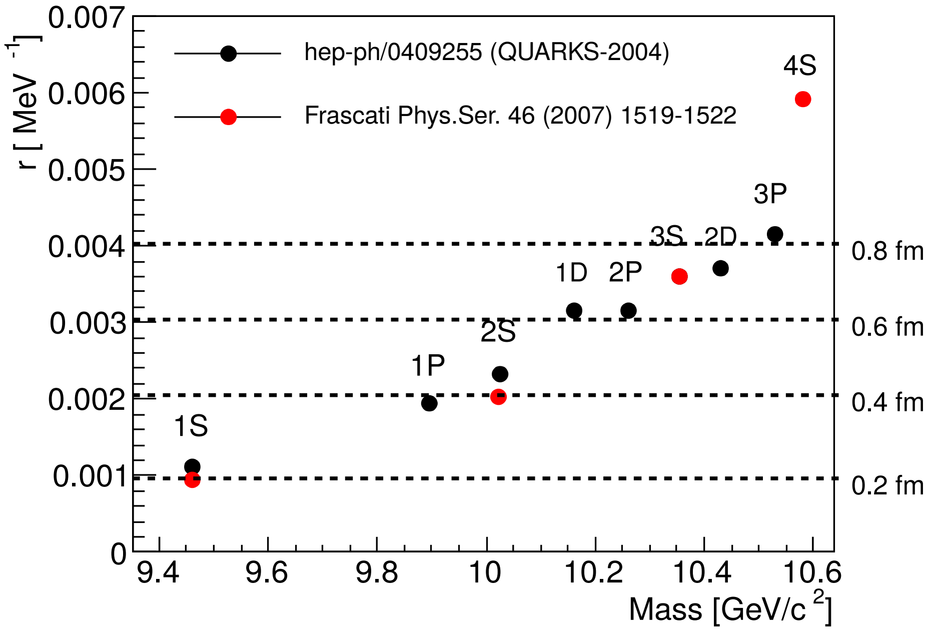
   Frascati Phys. Ser. (2007) 1519-1522

2) Baryon and strangeness enhancement

   PRD76 012005 (2007)

3) Production of nuclei

   Phys.Rev. D89 (2014) no.11, 111102



Strangeness enhancement

Z.Phys. C62 (1994) 367-370

The results on the inclusive production of the Λ in direct ϒ(1s) decays

Experiment	$\langle n_\Lambda(Y_{dir})\rangle$
CLEO(85) [1]	0.19±0.02
ARGUS(88) [2]	0.228±0.003±0.021
this experiment	0.194±0.018±0.017

Table 5

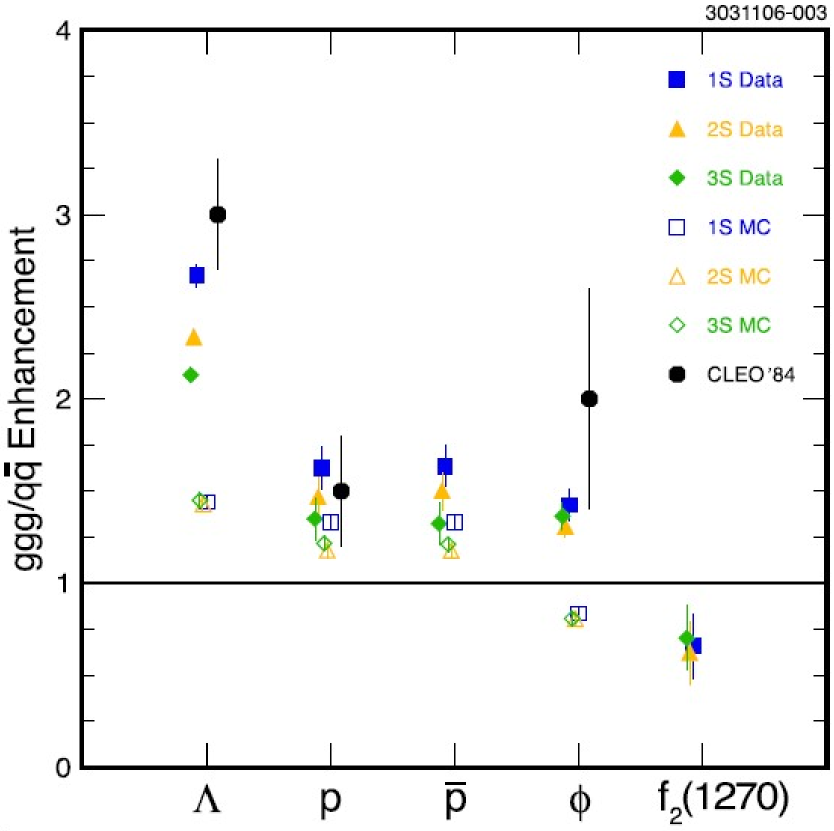
The results on the inclusive production of the Λ in the continuum

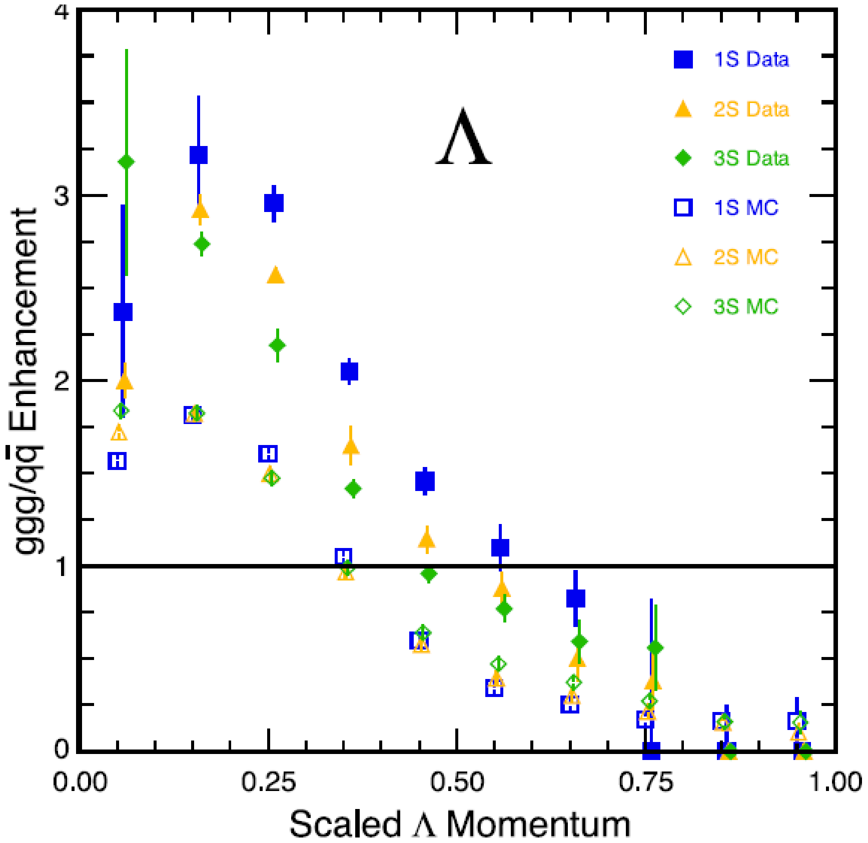
Experiment	the cms range, GeV	$\langle n_\Lambda(\text{continuum})\rangle$
CLEO(85) [1]	10.4-10.6	0.066±0.010
ARGUS(88) [2]	9.4-10.6	0.092±0.003±0.008
this experiment	7.2-10.0	0.076±0.018±0.015
	7.2-9.4	0.070±0.027±0.020

Gluon-strangeness affinity *CLEO Phys.Rev.D76, 012005*

$$\frac{\sigma[e^+e^- \to \Upsilon(nS) \to \mathcal{B} + X]}{\sigma[e^+e^- \to q\bar{q} \to \mathcal{B} + X]}$$

Enhancement for baryon $\mathcal{B}$:





Lessons learned (cont'd)

What did we learn of new about bottomonium in the last 15 years?

→ $b\bar{b}$ is not a bad model all

→ Quite nice Quark model/data matching

→ $b\bar{b}$ is not the whole story, at least near the thresholds

→ Zb's, triangular contributions, anomalous transitions...

→ The light degrees of freedom matter a lot

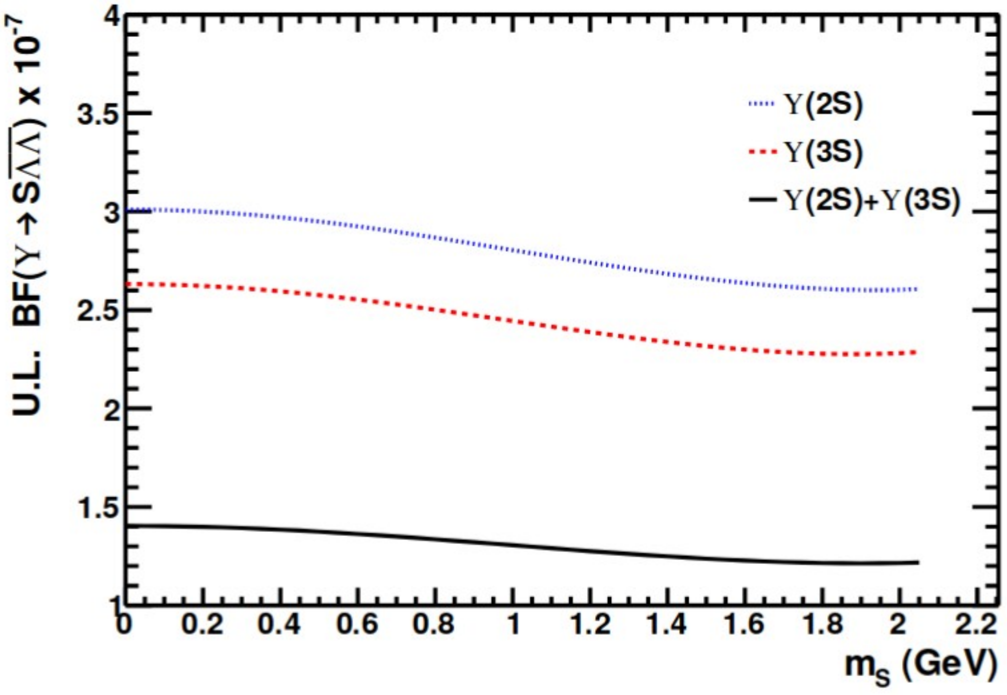
→ Hadronic annihilations are very peculiar

→ If you are looking for strange (exotic) baryons, you should look at the Υ(nS) annihilations

Stable H: where are we?

BaBar arXiv:1810.04724 [hep-ex]

$B(\Upsilon(3S, 2S) \rightarrow \Lambda\Lambda$ + invisible) < 1.2 x 10^{-7}



Stable H: where are we?

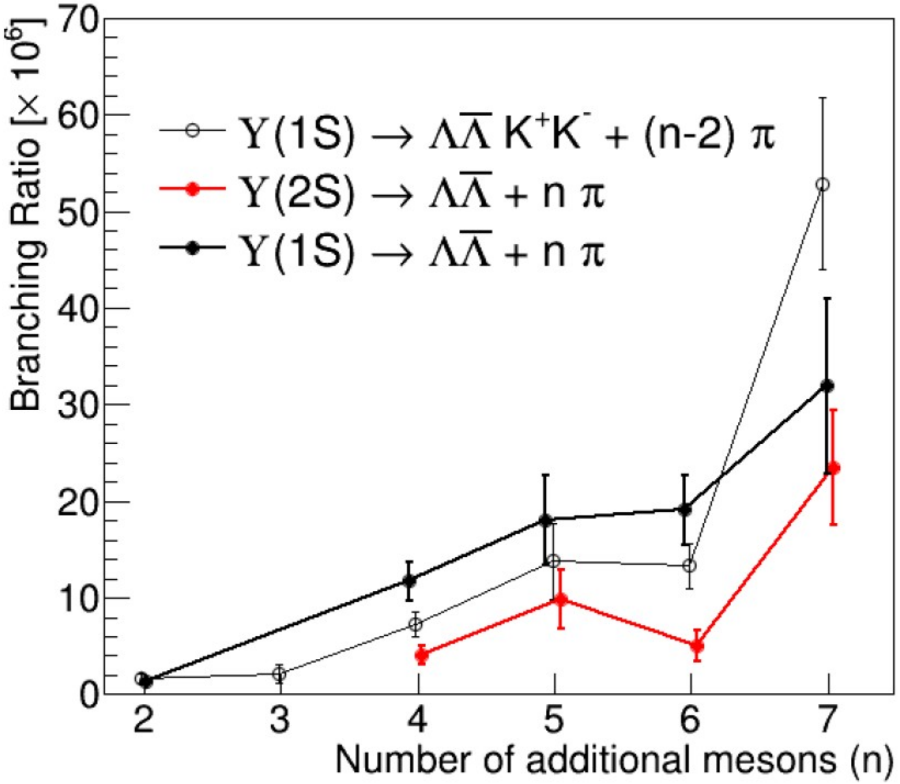
BaBar arXiv:1810.04724 [hep-ex]

$B(\Upsilon(3S, 2S) \rightarrow \Lambda\Lambda + \text{invisible}) < 1.2 \times 10^{-7}$

Belle preliminary

$B(\Upsilon(2S) \rightarrow \Lambda\bar{\Lambda}\, p\bar{p}) < 1.8 \times 10^{-7}$

Few body reactions are largely suppressed.



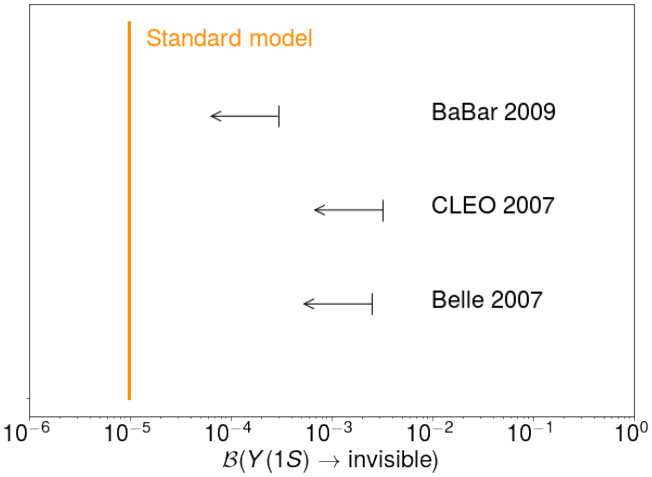
Conclusions

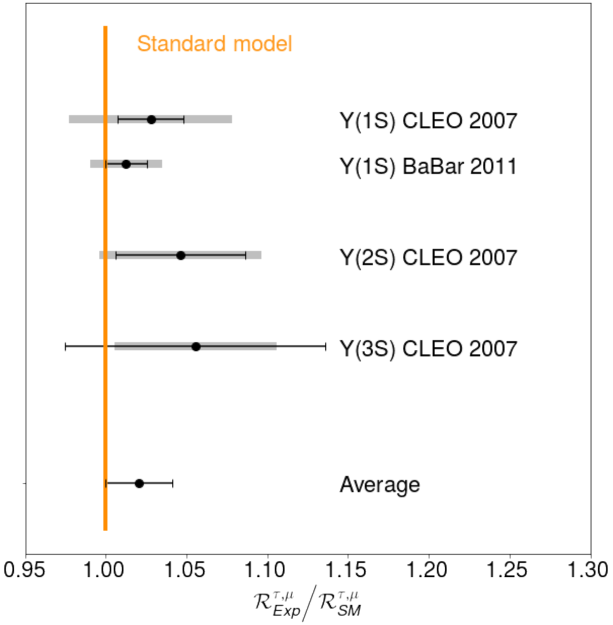
We didn't see hints of new physics in bottomonium decays, but we barely looked for them

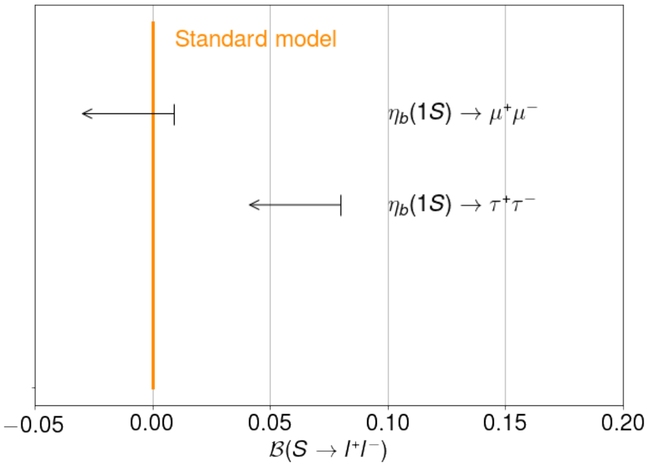
→ Analyses not been updated

→ Few theoretical papers (?)

→ Orthogonal communities?







Hadronic decays are the perfect place for strangeness studies.

Backup

Charmonium

Charmonium is experimentally easy and accessible

→ Direct production in e^+e^- collisions

→ Production in $B \to K\, c\bar{c}$

→ Photon-photon scattering $\gamma\gamma^* \to (c\bar{c})$

→ Double Charmonium $e^+e^- \to (c\bar{c})(c\bar{c})$

→ Prompt production

→ Direct production in $p\bar{p}$ (???)

Bottom line: Charmonium will still be fully covered in the next 15 yrs.

Pentaquarks, multi-charm baryons...

→ Direct production in e^+e^- collisions

→ Production in B and Λ_b decays

→ Direct production in $p\bar{p}$ (???)

Bottom line: well covered by LHCb, little room for other experiments

Bottomonium

Bottomonium is much less accessible

→ Direct production in e^+e^- collisions

→ Prompt production

Bottom line: after Belle II, only the LHC experiments will cover bottomonia with strong limitations

Production is not everything

e^+e^- machines

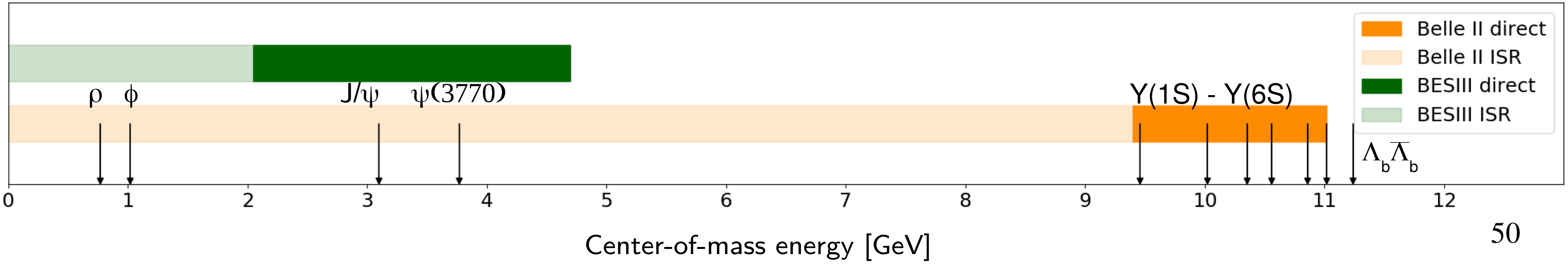
→ Triggers are quite open

→ High efficiency / Sensitive to very low momentum

→ Unique measurements (double charmonium, $\gamma\gamma^* \rightarrow c\bar{c}$)

→ Initial states is always a 1^{--} quarkonium or a B meson

→ CM energy is a limiting factor



The players: Belle II

Current samples in fb^{-1} (millions of events), and the proposal for Belle II

Experiment	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$	$\Upsilon(5S)$	$\Upsilon(6S)$	$\frac{\Upsilon(nS)}{\Upsilon(4S)}$
CLEO	1.2 (21)	1.2 (10)	1.2 (5)	16 (17.1)	0.1 (0.4)	-	23%
BaBar	-	14 (99)	30 (122)	433 (471)	R_b scan	R_b scan	11%
Belle	6 (102)	25 (158)	3 (12)	711 (772)	121 (36)	5.5	23%
BelleII	-	-	300 (1200)	5×10^4 (5.4×10^4)	1000 (300)	100+400(scan)	3.6%

→ Bottomonium program is alternative to the B-physics one (special runs)

→ Supported by the Collaboration, seen fully as part of the Belle II physics program

→ Still, external support is very welcome!

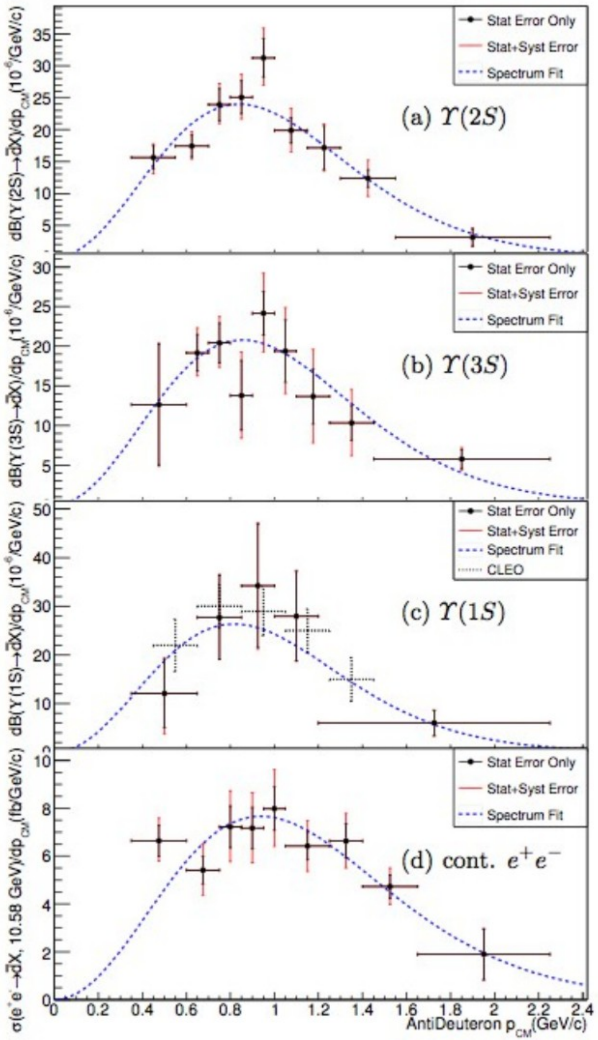
→ Sensible plan: one (or two) special runs / year starting from 2021

Idea nr. 2: nucleon coalescence

With no dedicated PID or tracking, BaBar measured the $\bar{d}$ spectrum *Phys.Rev. D89 (2014) no.11, 111102*

Process	**Rate**
$\mathcal{B}(\Upsilon(3S) \to \bar{d}X)$	$(2.33 \pm 0.15^{+0.31}_{-0.28}) \times 10^{-5}$
$\mathcal{B}(\Upsilon(2S) \to \bar{d}X)$	$(2.64 \pm 0.11^{+0.26}_{-0.21}) \times 10^{-5}$
$\mathcal{B}(\Upsilon(1S) \to \bar{d}X)$	$(2.81 \pm 0.49^{+0.20}_{-0.24}) \times 10^{-5}$
$\sigma(e^+e^- \to \bar{d}X)$ $[\sqrt{s} \approx 10.58\,\text{GeV}]$	$(9.63 \pm 0.41^{+1.17}_{-1.01})$ fb
$\dfrac{\sigma(e^+e^- \to \bar{d}X)}{\sigma(e^+e^- \to \text{Hadrons})}$	$(3.01 \pm 0.13^{+0.37}_{-0.31}) \times 10^{-6}$

Deuteron production ~ 10 x more likely in Y(nS) than in $q\bar{q}$

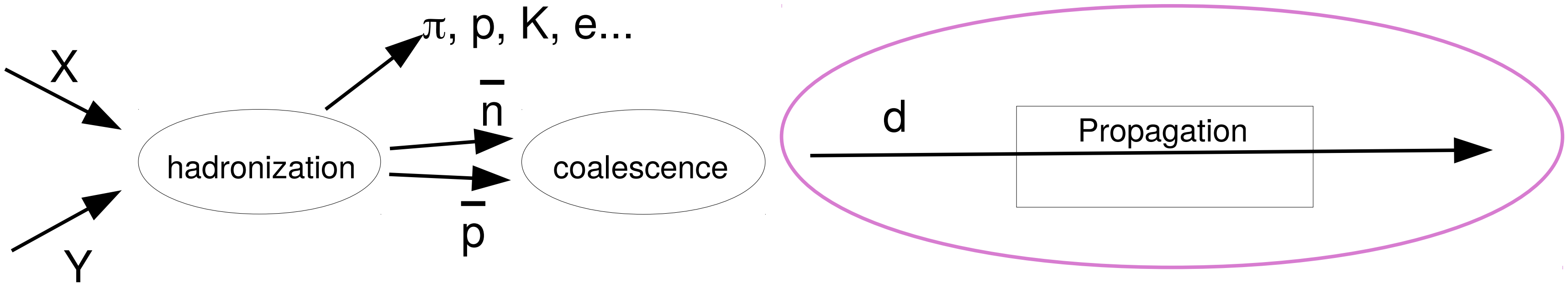


Idea nr. 2: nucleon coalescence

$\bar{d}$ detection in cosmic rays is considered since long a probe for low or intermediate mass WIMPs

Donato, Fornengo, Salati, PRD 62, 043003 (2000)
Aramaki et al. Phys. Rept. 618 (2016) 1-37

→ it's kinematically easier to produce a d from $\chi\chi$ annihilation than from SM processes



Nuclear uncertainties

→ $\bar{p}$ and $\bar{n}$ production rates

rel. uncertainty ~ 10

→ $\bar{d}$ production model

rel. uncertainty ~ 50 - 200

Astrophysical uncertainties

→ Galactic density profile

rel. uncertainty ~ 20

→ Transport models

rel. uncertainty ~ 500

Idea nr. 2: nucleon coalescence

Two versions of the coalescence model:

$$\frac{\mathrm{d}N_{\bar{d}}}{\mathrm{d}T_{\bar{d}}} = \frac{p_0^3}{6} \frac{m_{\bar{d}}}{m_{\bar{n}} m_{\bar{p}}} \frac{1}{\sqrt{T_{\bar{d}}^2 + 2 m_{\bar{d}} T_{\bar{d}}}} \frac{\mathrm{d}N_{\bar{n}}}{\mathrm{d}T_{\bar{n}}} \frac{\mathrm{d}N_{\bar{p}}}{\mathrm{d}T_{\bar{p}}}$$

$$E_i \frac{\mathrm{d}^3 N_i}{\mathrm{d}p_i^3} = B_A \left(E_\mathrm{p} \frac{\mathrm{d}^3 N_\mathrm{p}}{\mathrm{d}p_\mathrm{p}^3} \right)^A$$