

Random Forests: Theory & Intuition

Bias-Variance Tradeoff

$$E[\text{mean squared error}] = \text{noise} + \text{bias}^2 + \text{variance}$$

bias: Difference between averaged prediction of model versus the Bayes model

variance: Variability of learned model conditioned upon the learning set.

noise: Irreducible error.

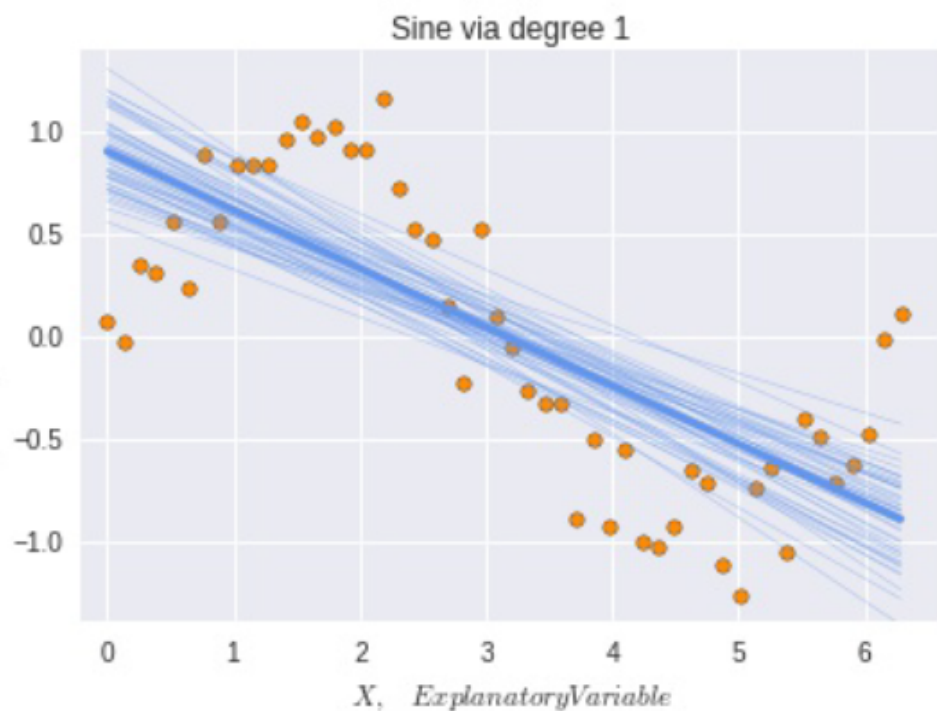
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Underfit model: **High Bias**, **Low Variance**.

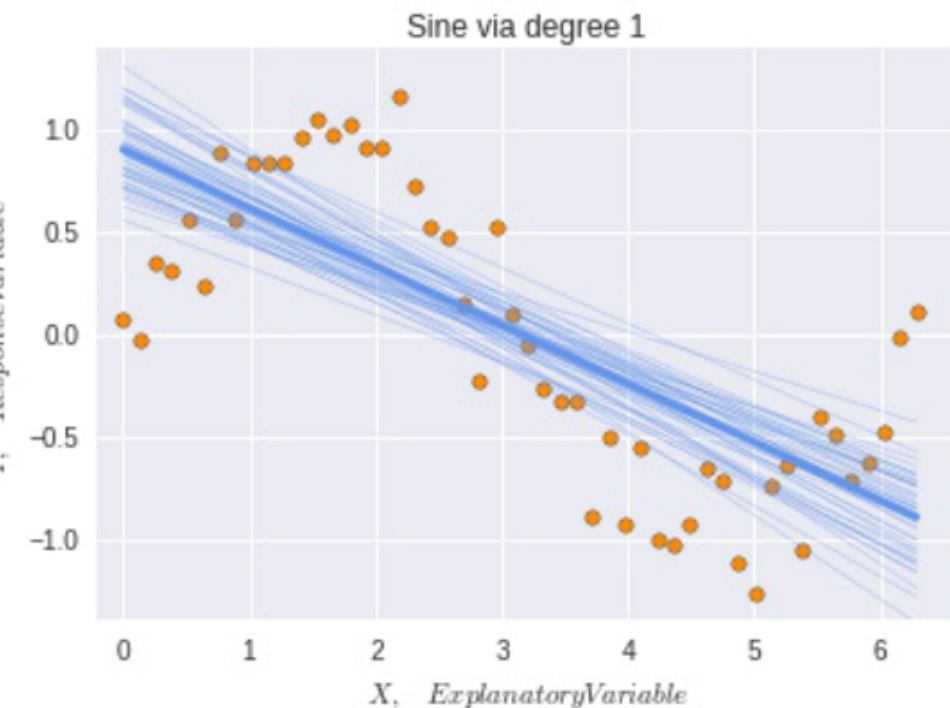
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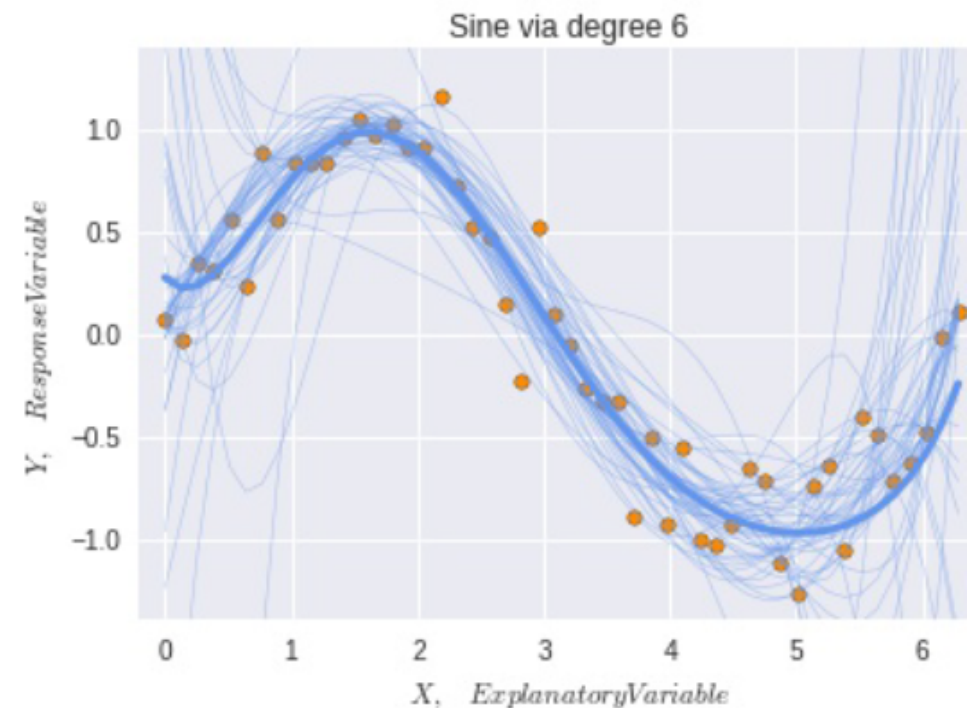
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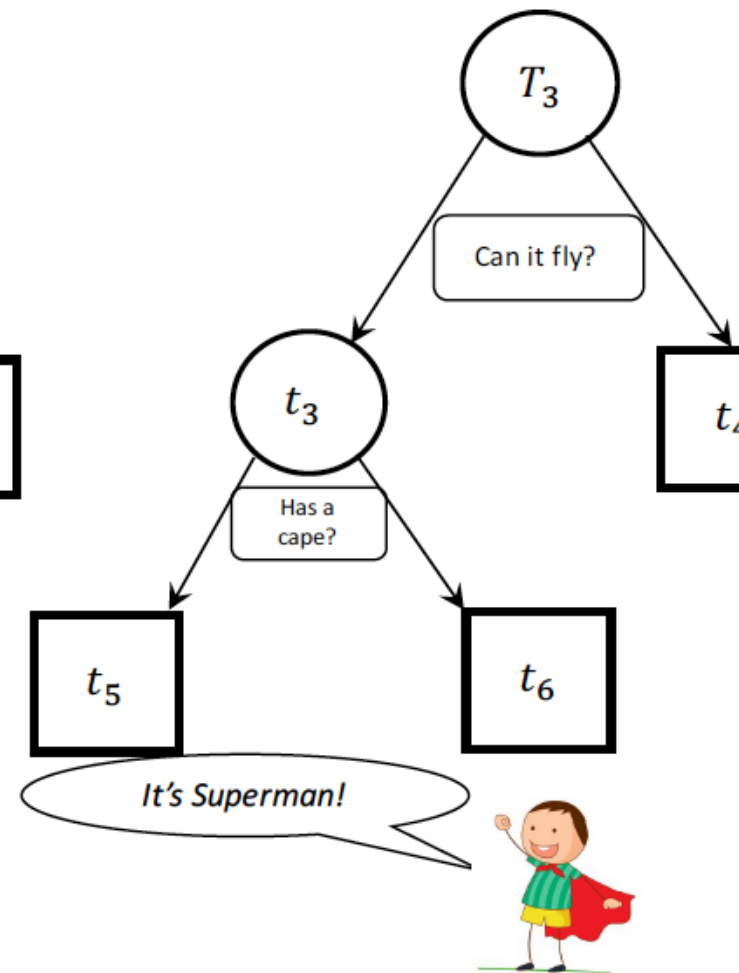
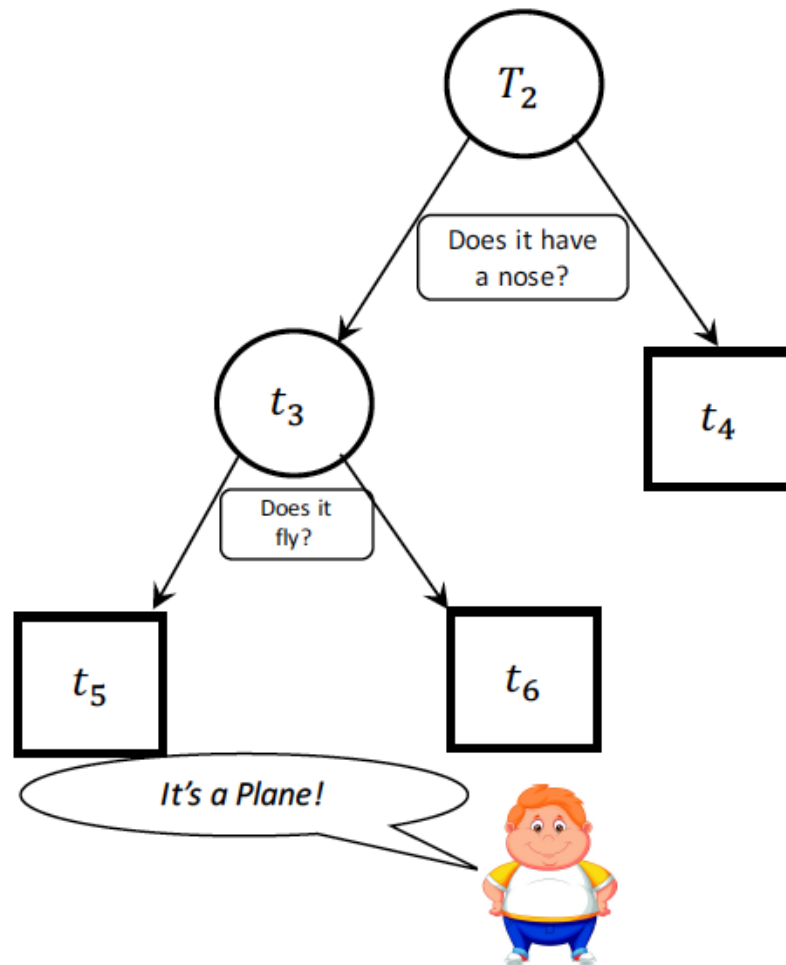
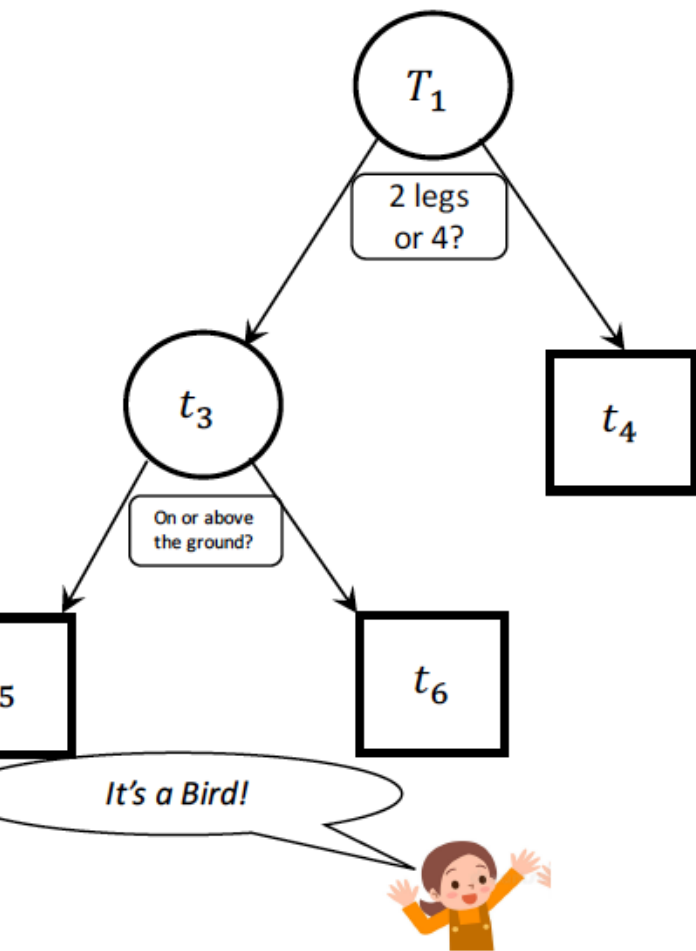
Noise: Irreducible error.



Underfit model: High Bias, Low Variance.



Overfit model: High Variance, Low Bias.



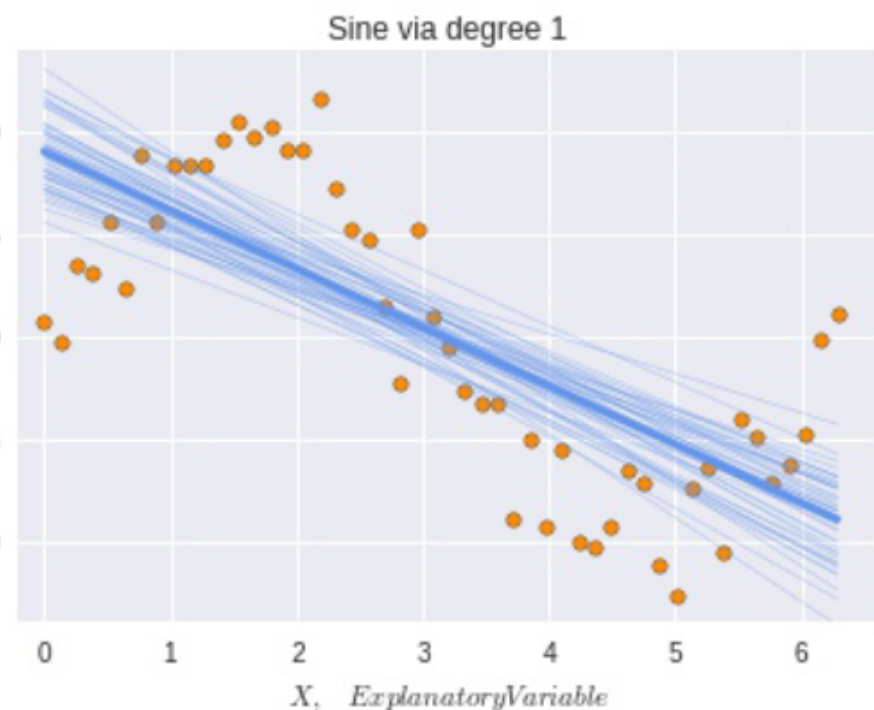
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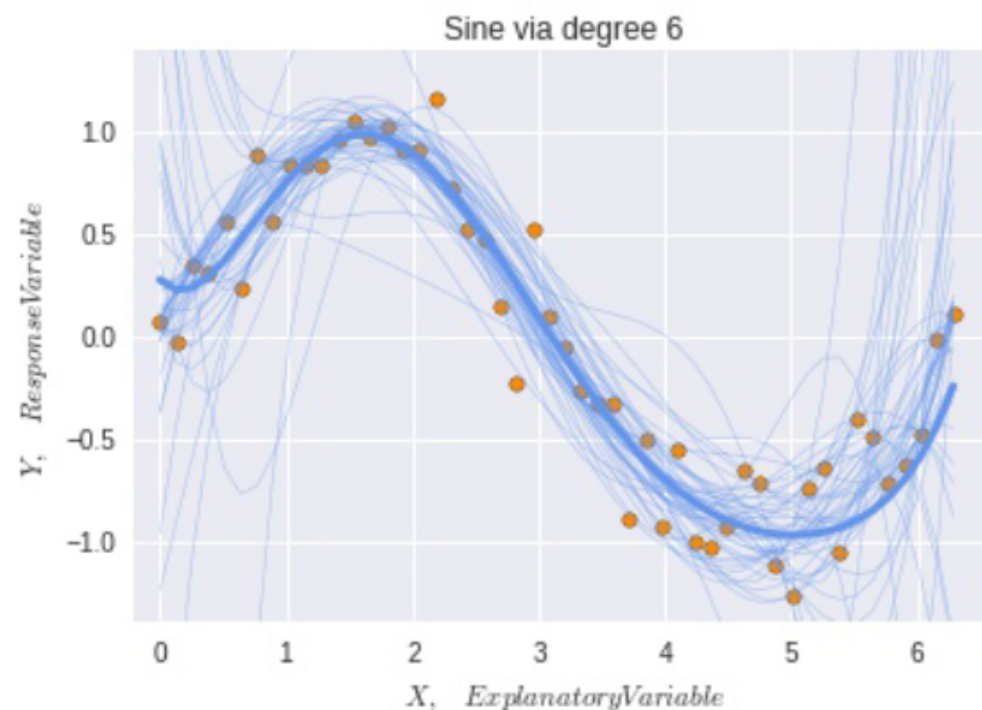
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Overfit model: **High Variance**, **Low Bias**.

Averaging over large ensembles of decorrelated models reduces variance, while maintaining bias.

Ensemble Learning

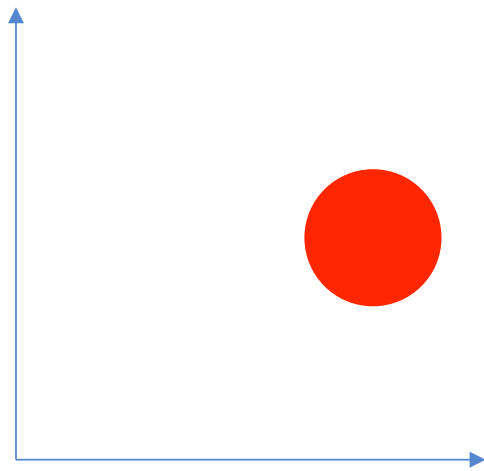
Condorcet's Jury Theorem: A group wants to arrive at the “correct” decision via majority vote, wherein each individual has a probability p of voting for the correct decision. What should the size of the group be for optimal performance?

Intuitive example: The performance of the “Ask The Audience” lifeline in *Who Wants To Be A Millionaire?* (92% accuracy vs 65% for “Phone A Friend”)

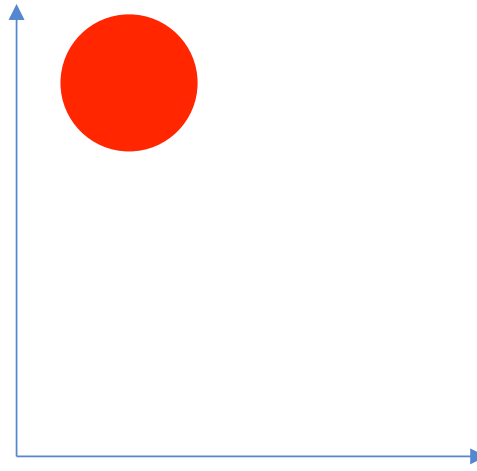
Aggregating randomized models decreases the variance of the ensemble.

Ensemble Prediction: Weather forecasting.

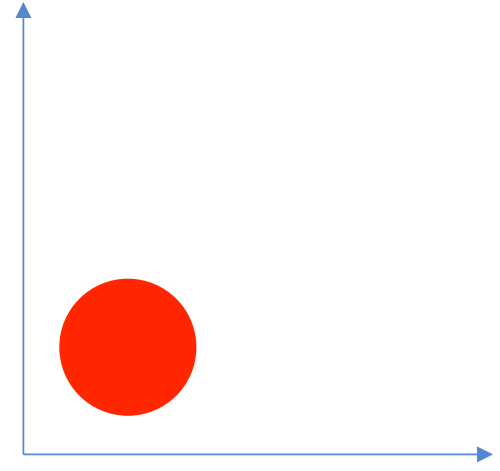
Ensemble Learning: Intuition



Model 1

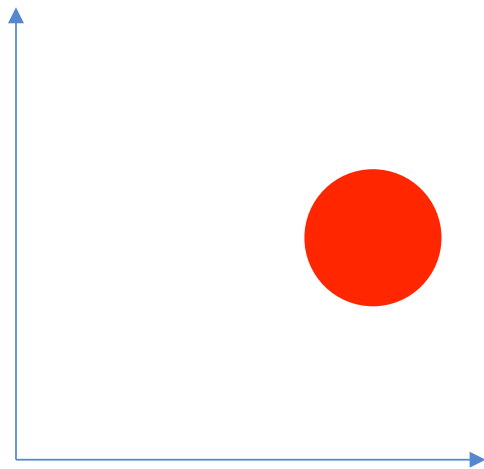


Model 2

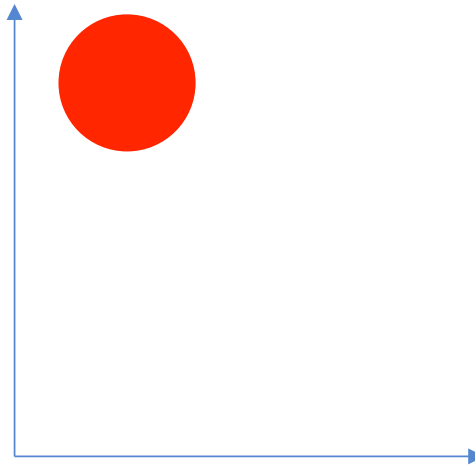


Model 3

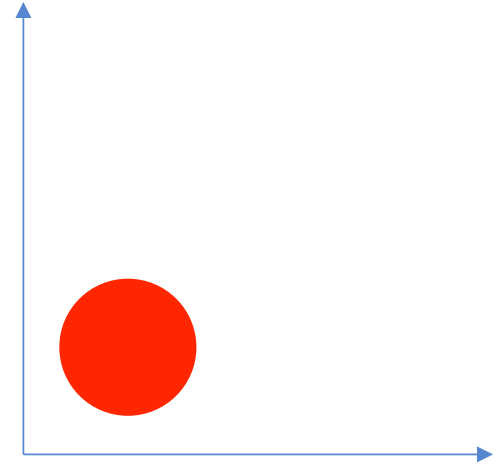
Ensemble Learning: Intuition



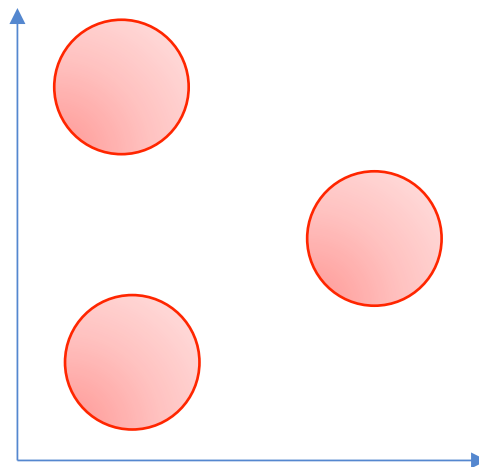
Model 1



Model 2



Model 3



Model Ensemble

Random Forests: Details

- Bagging: **B**ootstrap **a**ggregation
- Random Subsampling over features (aka “feature bagging”)

Random Forests: Bootstrapping

Original Toy Dataset

| X1 | X2 | X3 | X4 | Y |
|----|----|------|-----|-----|
| 0 | 0 | 2.7 | 314 | 17 |
| 0 | 0 | 3.14 | 516 | 42 |
| 1 | 1 | 3.14 | 400 | 31 |
| 1 | 1 | 2.3 | 300 | 99 |
| 0 | 0 | 2.3 | 200 | 157 |

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Bootstrapped Dataset 1

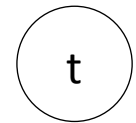
| X1 | X2 | X3 | X4 | Y |
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Random Forests: Feature Bagging

Bootstrapped Dataset 1

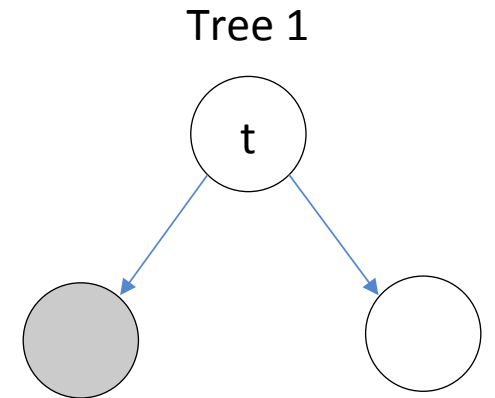
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Tree 1



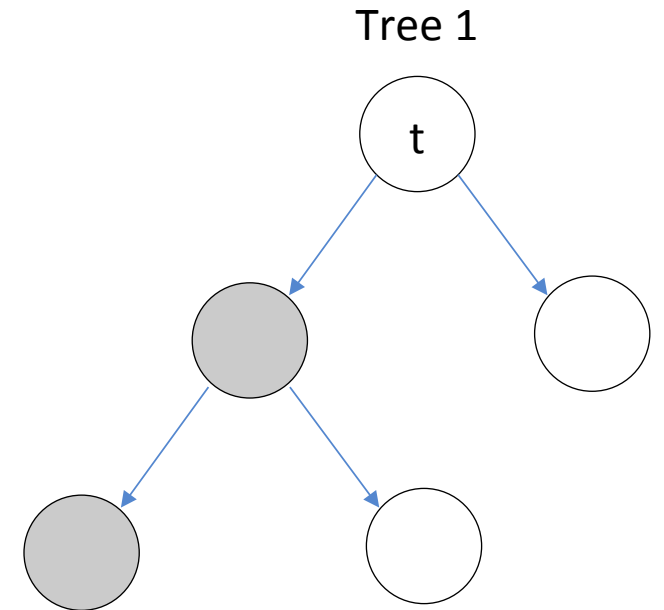
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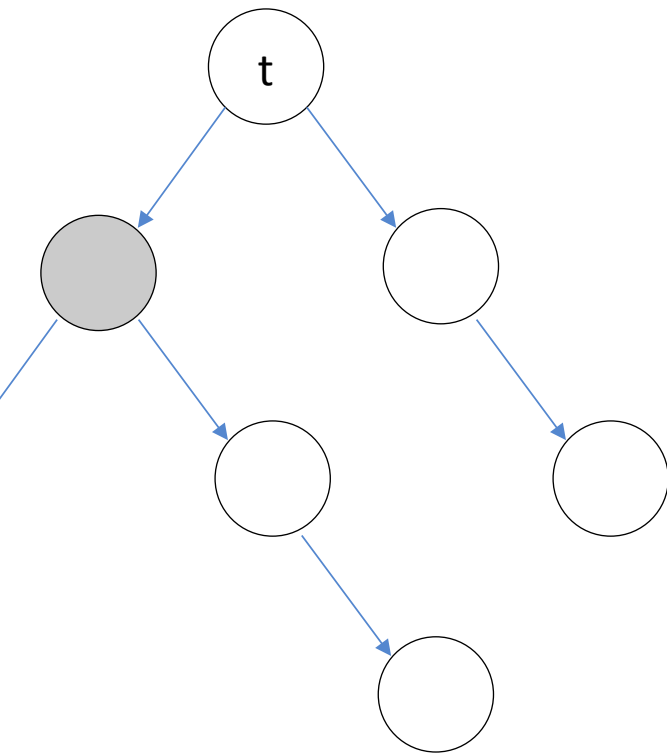
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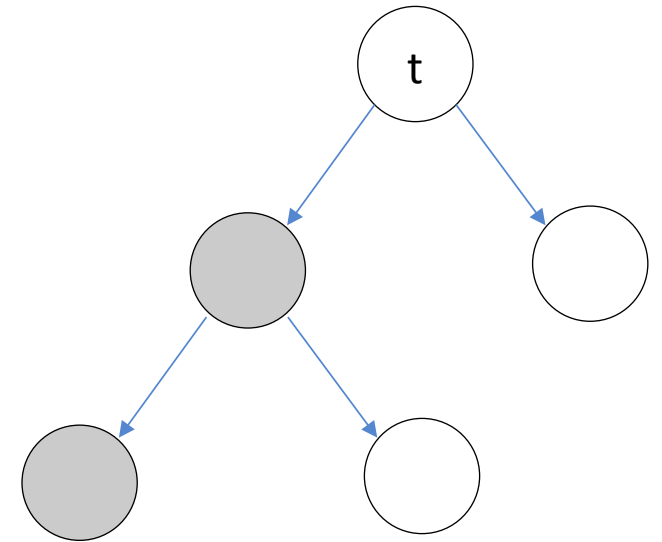
Random Forests: Ensemble

Tree 1

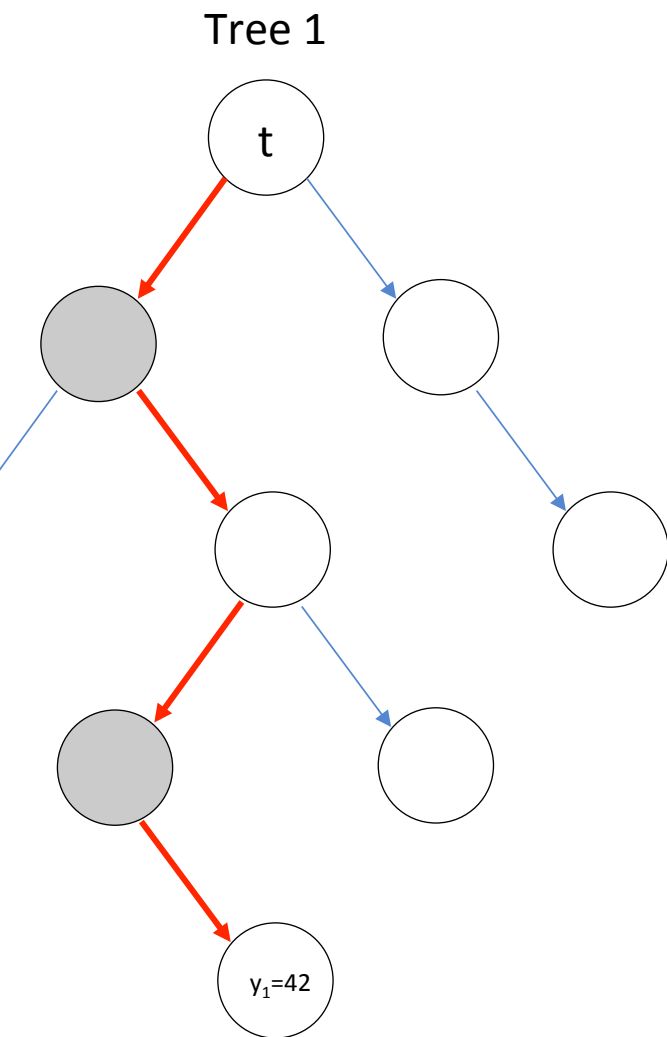


.....

Tree n



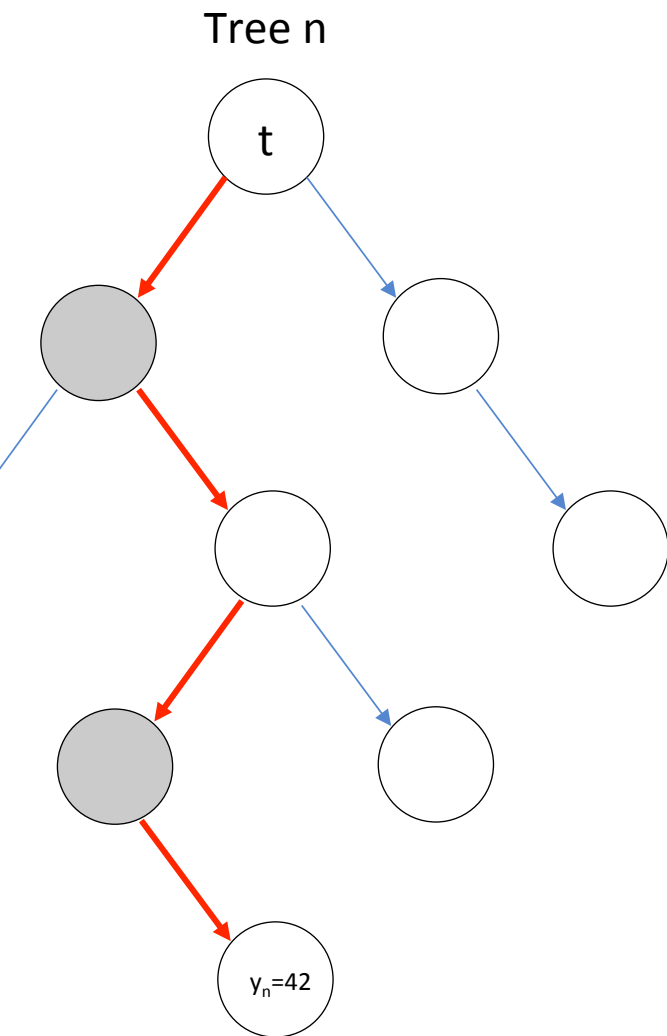
Random Forests: Predictions (Aggregation)



New Sample for Prediction

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Regression, $y_{\text{prediction}} = y_{\text{1}} + y_{\text{2}} + y_{\text{3}} + \dots + y_{\text{n}}$

Classification, $y_{\text{prediction}} = \text{majority}(y_{\text{1}}, y_{\text{2}}, y_{\text{3}}, \dots, y_{\text{n}})$

Random Forests: Easy Error Estimation, OOB estimates

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On average, about 32% of the samples will not end up in a specific Bootstrapped dataset.
These are referred to as Out Of Bag samples, or OOB samples.

Random Forests: Easy Error Estimation, OOB estimates

- Step 1: For every sample, determine the trees where it is an OOB sample.
- Step 2: Make predictions for this sample from these trees.
- Step 3: Calculate final averaged prediction for this sample.
- Step 4: Calculate error for this sample.
- Step 5: Repeat for all points to find OOB error.