



Lepton Flavour Violation: Theory Overview

Emilie Passemar Indiana University/Jefferson Laboratory

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- 1. Introduction and Motivation
- 2. Low energy probes: The reach and model discriminating power of
 - muon decays
 - tau decays
- 3. High energy probes: Higgs LFV
- 4. Conclusion and Outlook

1. Introduction and Motivation

Why study charged leptons?



 For some modes accurate calculations of hadronic uncertainties essential

Charged leptons very important to look for New Physics!

The Program



The Program



2. Charged Lepton-Flavour Violation

2.1 Introduction and Motivation

- Neutrino oscillations are the first evidence for lepton flavour violation
- How about in the charged lepton sector?
- In the SM with massive neutrinos effective CLFV vertices are tiny due to GIM suppression in unobservably small rates!

E.g.:
$$\mu \rightarrow e\gamma$$

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_{W}} \right|^2 < 10^{-54}$$



Petcov'77, Marciano & Sanda'77, Lee & Shrock'77...

$$\left[Br(\tau\to\mu\gamma)<10^{-40}\right]$$

2.1 Introduction and Motivation

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Petcov'77, Marciano & Sanda'77, Lee & Shrock'77...

$$\left[Br(\tau\to\mu\gamma)<10^{-40}\right]$$

• Extremely *clean probe of beyond SM physics*

2.1 Introduction and Motivation

In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin	$ au o \mu \gamma \ au o \ell \ell \ell$			
Lee, Shrock, PRD 16 (1977) 1444 SM + v mixing Cheng, Li, PRD 45 (1980) 1908			Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10-10	10-7	
SM + heavy Maj $v_{\rm R}$	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10-9	10-10	
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10-9	10-8	
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10-8	10-10	
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10-7	10-9	

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.2 CLFV processes: muon decays

• Several processes: $\mu \to e\gamma$, $\mu \to e\overline{e}e$, $\mu(A,Z) \to e(A,Z)$

10⁰ Original graph: Marciano-Mori, ARNPS 58, 315 (2009) $BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}$ Modified version 10⁻² 10^{-14} Talk by S. Mihara 10-4 90% – CL bound Posters by S. Ogawa, K. Oshida PSI/Mu3e 10-6 $BR(\mu \rightarrow eee) < 1.0 \times 10^{-12}$ 10-8 $10^{-15} - 10^{-16}$ 10-10 MEGA Talk by S. Mihara MEG Mu2e/COMET 10-12 Sindrum Sindrum-II $BR_{\mu-e}^{Ti} < 4.3 \times 10^{-12}$ 10-14 2020 1980 2000 1940 1960 $10^{-16} - 10^{-17}$ Mu2e / COMET Year DeeMe Talk by A. Sato Proposal for search of CLFV in $\mu^-e^- \rightarrow e^-e^-$

Koike et al'10 poster by Y. Uesaka

Posters by M. Roehrken, T. Wong, N. Yu, M. Wong, T. M, Nguyen, N.Teshima, T. Nagao

MEG'13

2.2 CLFV processes: tau decays

• Several processes: $\tau \to \ell \gamma, \ \tau \to \ell_{\alpha} \overline{\ell}_{\beta} \ell_{\beta}, \ \tau \to \ell Y$ $\swarrow P, S, V, P\overline{P}, ...$



48 LFV modes studied at Belle and BaBar

2.2 CLFV processes: tau decays

• Several processes: $\tau \to \ell \gamma, \ \tau \to \ell_{\alpha} \overline{\ell}_{\beta} \ell_{\beta}, \ \tau \to \ell Y$ $\swarrow P, S, V, P\overline{P}, ...$



• Expected sensitivity 10⁻⁹ or better at *LHCb*, *Belle II*?

2.3 Effective Field Theory approach



- Build all D>5 LFV operators:
 - ➢ Dipole

See e.g. Black, Han, He, Sher'02 Brignole & Rossi'04 Dassinger et al.'07 Matsuzaki & Sanda'08 Giffels et al.'08 Crivellin, Najjari, Rosiek'13 Petrov & Zhuridov'14 Cirigliano, Celis, E.P.'14

- Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector)
- Lepton-gluon (Scalar, Pseudo-scalar)
- 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector)
- Each UV model generates a *specific pattern* of them

2.3 Effective Field Theory approach

Integrating out heavy quarks generates gluonic operator:

$$\frac{1}{\Lambda^{2}} \overline{\mu} P_{L,R} \tau Q \overline{Q} \rightarrow \mathcal{L}_{eff}^{G} \supset -\frac{C_{G}}{\Lambda^{2}} m_{\tau} G_{F} \overline{\mu} P_{L,R} \tau G_{\mu\nu}^{a} G_{g}^{\mu\nu}$$

Importance of this operator emphasized in Petrov & Zhuridov'14

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/

2.3 Effective Field Theory approach

• 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector) :

• Summary table:

Cirigliano@Beauty2014

	$\mu \to 3e$	$\mu \to e \gamma$	$\mu \rightarrow e$ conversion
$O^{4\ell}_{S,V}$	✓	_	—
O_D	1	1	✓
O_V^q	—	_	✓
O_S^q	_	_	✓

- The notion of "*best probe*" (process with largest decay rate) is model dependent
- If observed, compare rate of processes
 key handle on *relative strength* between operators and hence on the *underlying mechanism*

Cirigliano@Beauty2014

• Summary table:



• $\mu \rightarrow e\gamma$ vs. $\mu \rightarrow 3e \implies$ relative strength between *dipole* and *4L* operators

$$\frac{\Gamma_{\mu \to 3e}}{\Gamma_{\mu \to e\gamma}} = \frac{\alpha}{4\pi} I_{\rm PS} \left(1 + \sum_{i} \frac{c_i^{\rm (contact)}}{c^{\rm (dipole)}} \right)$$

Cirigliano@Beauty2014

• Summary table:



• $\mu \rightarrow e\gamma$ vs. $\mu \rightarrow e$ conversion \implies relative strength between *dipole* and *quark* operators



Discriminating power of $\mu \rightarrow$ e conversion

For μ →e conversion, target dependence of the amplitude is different for V, D or S models
 Kitano, Koike, Okada'06



• Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu \gamma$	$\tau o \mu \pi^+ \pi^-$	$ au o \mu K \bar{K}$	$\tau \to \mu \pi$	$\tau \to \mu \eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	_	—	_	_	—
OD	✓	1	\checkmark	\checkmark	_	—
$O^{\mathbf{q}}_{\mathbf{V}}$	_	_	\checkmark (I=1)	$\checkmark(\mathrm{I=0,1})$	_	—
O_S^q	—	—	✓ (I=0)	$\checkmark(\mathrm{I=}0{,}1)$	—	—
O_{GG}	—	—	\checkmark	\checkmark	—	—
O^q_A	—	—	—	_	✓ (I=1)	✓ (I=0)
$\mathrm{O}_\mathrm{P}^\mathrm{q}$	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\widetilde{G}}$	—	—	—	—	—	✓

In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

• Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu \gamma$	$\tau o \mu \pi^+ \pi^-$	$ au o \mu K \bar{K}$	$\tau \to \mu \pi$	$\tau \to \mu \eta^{(\prime)}$
${ m O}_{{ m S},{ m V}}^{4\ell}$	1	_	—	_	_	_
OD	1	1	\checkmark	✓	_	_
O_V^q	_	_	✓ (I=1)	$\checkmark(\mathrm{I=}0{,}1)$	_	_
O_S^q	_	_	✓ (I=0)	$\checkmark(\mathrm{I=}0{,}1)$	_	—
O_{GG}	—	—	\checkmark	\checkmark	_	—
O^q_A	—	_	—	_	✓ (I=1)	✓ (I=0)
$\mathrm{O}_\mathrm{P}^\mathrm{q}$	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\widetilde{G}}$	—	_	—	_	—	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!
- But need reliable determinations of the hadronic part: form factors and *decay constants* (e.g. f_n , $f_{n'}$)

Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3 \mu$	$\tau \to \mu \gamma$	$\tau o \mu \pi^+ \pi^-$	$\tau \to \mu K \bar{K}$	$\tau \to \mu \pi$	$\tau \to \mu \eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	_	—	—	_	—
OD	✓	✓	\checkmark	✓	_	_
O_V^q	—	_	\checkmark (I=1)	\checkmark (I=0,1)	_	_
O_S^q	—	_	✓ (I=0)	\checkmark (I=0,1)	_	_
O _{GG}	_	_	\checkmark	\checkmark	_	_
O_A^q	_	_	—	_	\checkmark (I=1)	✓ (I=0)
O_P^q	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\widetilde{G}}$	_	_	_	_	_	1

Recent progress in $\tau \rightarrow \mu(e)\pi\pi$ using *dispersive techniques* Celis, Cirigliano, E.P.'14 •

- Hadronic part: $H_{\mu} = \langle \pi \pi | (V_{\mu} A_{\mu}) e^{iL_{QCD}} | 0 \rangle = (Lorentz \text{ struct.})^{i}_{\mu} F_{i}(s)$ with $s = (p_{\pi^{+}} + p_{\pi^{-}})^{2}$ •
- Form factors determined by solving 2-channel unitarity condition, with I=0 ٠ s-wave $\pi\pi$ and KK scattering data as input $\mathrm{Im}F_n(s) = \sum_{s} T^*_{nm}(s)\sigma_m(s)F_m(s)$

$$n=\pi\pi, K\overline{K}$$

- Two handles:

model M



Celis, Cirigliano, E.P.'14

 \blacktriangleright Spectra for > 2 bodies in the final state:

$$\frac{dBR(\tau \to \mu \pi^+ \pi^-)}{d\sqrt{s}} \quad \text{and} \quad dR_{\pi^+ \pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu \gamma)} \frac{d\Gamma(\tau \to \mu \pi^+ \pi^-)}{d\sqrt{s}}$$

• Studies in specific models

Buras et al.'10

ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\boxed{\frac{\operatorname{Br}(\mu^- \to e^- e^+ e^-)}{\operatorname{Br}(\mu \to e\gamma)}}$	0.021	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$	0.062.2
$\frac{\mathrm{Br}(\tau \to e^- e^+ e^-)}{\mathrm{Br}(\tau \to e\gamma)}$	0.040.4	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.07 \dots 2.2$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to \mu \gamma)}$	0.040.4	$\sim 2 \cdot 10^{-3}$	$0.06 \dots 0.1$	$0.06 \dots 2.2$
$\frac{\mathrm{Br}(\tau \to e^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to e\gamma)}$	0.040.3	$\sim 2 \cdot 10^{-3}$	$0.02 \dots 0.04$	$0.03 \dots 1.3$
$\frac{\mathrm{Br}(\tau^- \to \mu^- e^+ e^-)}{\mathrm{Br}(\tau \to \mu \gamma)}$	0.040.3	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04 \dots 1.4$
$\frac{\operatorname{Br}(\tau^- \to e^- e^+ e^-)}{\operatorname{Br}(\tau^- \to e^- \mu^+ \mu^-)}$	$0.8.\dots 2$	~ 5	$0.3. \ldots 0.5$	$1.5 \dots 2.3$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau^- \to \mu^- e^+ e^-)}$	0.71.6	~ 0.2	510	$1.4 \dots 1.7$
$\frac{\mathbf{R}(\mu \mathrm{Ti} \rightarrow e \mathrm{Ti})}{\mathrm{Br}(\mu \rightarrow e \gamma)}$	$10^{-3} \dots 10^2$	$\sim 5 \cdot 10^{-3}$	0.080.15	$10^{-12} \dots 26$



Disentangle the underlying dynamics of NP

2.7 Model discriminating of Spectra: $\tau \rightarrow \mu \pi \pi$

Celis, Cirigliano, E.P.'14



Celis, Cirigliano, E.P.'14



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Celis, Cirigliano, E.P.'14



3. Charged Lepton-Flavour Violation and Higgs Physics

3.1 Non standard LFV Higgs coupling

•
$$\Delta \mathcal{L}_{Y} = -\frac{\lambda_{ij}}{\Lambda^{2}} \left(\overline{f}_{L}^{i} f_{R}^{j} H \right) H^{\dagger} H \longrightarrow -Y$$

High energy : LHC

In the SM:
$$Y_{ii}^{h_{SM}} = \frac{m_i}{\delta_{ii}}$$

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 $\left(\overline{f}^{i}f^{j}\right)h$

Goudelis, Lebedev, Park'11 Davidson, Grenier'10 Harnik, Kopp, Zupan'12 Blankenburg, Ellis, Isidori'12 McKeen, Pospelov, Ritz'12 Arhrib, Cheng, Kong'12



Hadronic part treated with perturbative QCD



3.1 Non standard LFV Higgs coupling



3.2 Constraints in the µe sector

Constraints from Higgs decay (LHC) vs. low energy LFV and LFC observables



Harnik, Kopp, Zupan'12

Best constraints coming from *low* energy: $\mu \rightarrow e \gamma$

MEG'13

$$BR(\mu\to e\gamma) < 5.7 \ 10^{-13}$$

 $BR(h\to\mu e)<10^{-7}$

NB: Diagonal couplings set to SM value

3.3 Constraints in the $\tau\mu$ sector

• At low energy



3.3 Constraints in the $\tau\mu$ sector



3.4 Hint of New Physics in $h \rightarrow \tau \mu$?


Dorsner et al.'15

- If real what type of NP?
- If $h \rightarrow \tau \mu$ due to loop corrections:
 - extra charged particles necessary
 - $\tau \rightarrow \mu \gamma$ too large



• $h \rightarrow \tau \mu$ possible to explain if extra scalar doublet:

⇒ 2HDM of type III



- If real what type of NP?
- If $h \to \tau \mu$ due to loop corrections:
 - extra charged particles necessary
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Dorsner et al.'15

- $h \rightarrow \tau \mu$ possible to explain if extra scalar doublet \Rightarrow 2HDM of type III
- Type III 2HDMs with tree level FCNCs in lepton sector
 ⇒ explain also g-2



Omhura, Senaha, Tobe '15 See Poster by K.Tobe

- **2HDMs** with gauged $L_{\mu} L_{\tau} \Rightarrow$
 - $h \rightarrow \tau \mu$
 - $B \rightarrow K^* \mu \mu$
 - $R_K = B \rightarrow K \mu \mu / B \rightarrow K e e$

Z', explain anomalies in

Crivellin, D'Ambrosio, Heeck.'15

See also: Aristizabal-Sierra&Vicente'14, Lima et al'15

• **2HDMs** with gauged $L_{\mu} - L_{\tau}$

Crivellin, D'Ambrosio, Heeck.'15



•

- Sato & Yamanaka'15 See poster by M. Yamanaka
- In the muon sector: See poster by M. Yamana Ex: R-parity violating SUSY operators with no signals in $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$:



4. Conclusion and Outlook

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS penergy frontier
- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*
- *Charged leptons* offer an important spectrum of possibilities:
 - LFV measurement has SM-free signal
 - Current experiments and mature proposals promise orders of magnitude sensitivity improvements
 - We show how CLFV decays offer an excellent model discriminating tools giving indications on
 - the *mediator* (operator structure)
 - the source of flavour breaking (comparison $\tau \mu vs. \tau e vs. \mu e$)

Summary

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS penergy frontier
- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*
- Charged leptons offer an important spectrum of possibilities:
 - We show how CLFV decays offer an excellent model discriminating tools giving indications on
 - the *mediator* (operator structure)
 - the source of flavour breaking (comparison $\tau \mu vs. \tau e vs. \mu e$)
 - Interplay low energy and collider physics: LFV of the Higgs boson
 - Complementarity with LFC sector: EDMs, g-2 and colliders:

New physics models usually strongly correlate these sectors

5. Back-up

CLFV in see-saw models



- Observable CLFV if see-saw scale low (with protection of LN)
- Each model leads to specific CLFV pattern

 CLFV in Type II seesaw: tree-level 4L operator (D,V at loop) → 4-lepton processes most sensitive



• CLFV in Type III seesaw: tree-level LFV couplings of $Z \Rightarrow \mu \rightarrow 3e$ and $\mu \rightarrow e$ conversion at tree level, $\mu \rightarrow e\gamma$ at loop



Abada-Biggio-Bonnet-Gavela-Hambye '07, '08

 Ratios of 2 processes with same flavor transition are fixed

 $\begin{array}{lll} Br(\mu \to e\gamma) &=& 1.3 \cdot 10^{-3} \cdot Br(\mu \to eee) = 3.1 \cdot 10^{-4} \cdot R_{T_i}^{\mu \to e} \\ Br(\tau \to \mu\gamma) &=& 1.3 \cdot 10^{-3} \cdot Br(\tau \to \mu\mu\mu) \\ Br(\tau \to e\gamma) &=& 1.3 \cdot 10^{-3} \cdot Br(\tau \to eee) \end{array}$

2.4 Model discriminating power of muon processes

• Dependence: NP scale Λ versus ratio of two operators $\kappa = \frac{C_1}{C_2}$



DeGouvea & Vogel'13

2.5 Model discriminating power of Tau processes

- Two handles: ٠

model M

> Branching ratios: $R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_{M})}$ with F_{M} dominant LFV mode for

Celis, Cirigliano, E.P.'14

Spectra for > 2 bodies in the final state:

$$\frac{dBR(\tau \to \mu \pi^+ \pi^-)}{d\sqrt{s}} \text{ and } dR_{\pi^+ \pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu \gamma)} \frac{d\Gamma(\tau \to \mu \pi^+ \pi^-)}{d\sqrt{s}}$$

- Benchmarks: ٠
 - > Dipole model: $C_D \neq 0$, $C_{else} = 0$
 - > Scalar model: $C_S \neq 0$, $C_{else} = 0$
 - > Vector (gamma,Z) model: $C_V \neq 0$, $C_{else} = 0$
 - Gluonic model: $C_{GG} \neq 0$, $C_{else} = 0$

 $\mu \rightarrow e vs \mu \rightarrow e\gamma$

• Assume dipole dominance:



2.5 Model discriminating power of Tau processes

- Two handles: ٠

> Branching ratios: $R_{F,M} \equiv \frac{\Gamma(\tau \rightarrow F)}{\Gamma(\tau \rightarrow F_{M})}$

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with F_{M} dominant LFV mode for model M



- ρ (770) resonance (J^{PC}=1⁻⁻): cut in the $\pi^+\pi^-$ invariant mass: 587 MeV $\leq \sqrt{s} \leq$ 962 MeV
- $f_0(980)$ resonance (J^{PC}=0⁺⁺): cut in the $\pi^+\pi^-$ invariant mass: ٠ 906 MeV $\leq \sqrt{s} \leq 1065$ MeV

2.5 Model discriminating power of Tau processes

- Two handles:
 - Branching ratios:

$$R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}$$

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with F_{M} dominant LFV mode for model M

		$\mu\pi^+\pi^-$	μho	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	$0.26 imes10^{-2}$	$0.22 imes 10^{-2}$	$0.13 imes 10^{-3}$	$0.22 imes 10^{-2}$	1
	BR	$<1.1\times10^{-10}$	$<9.7\times10^{-11}$	$<5.7\times10^{-12}$	$<9.7\times10^{-11}$	$<4.4\times10^{-8}$
S	$R_{F,S}$	1	0.28	0.7	-	-
	BR	$<~2.1\times10^{-8}$	$<~5.9\times10^{-9}$	$<~1.47\times10^{-8}$	-	-
$V^{(\gamma)}$	$R_{F,V^{(\gamma)}}$	1	0.86	0.1	-	-
	BR	$<~1.4\times10^{-8}$	$<~1.2\times10^{-8}$	$<~1.4\times10^{-9}$	-	-
Z	$R_{F,Z}$	1	0.86	0.1	-	-
	BR	$<~1.4\times10^{-8}$	$<~1.2\times10^{-8}$	$<~1.4\times10^{-9}$	-	-
G ★	$R_{F,G}$	1	0.41	0.41	-	-
	BR	$<~2.1\times10^{-8}$	$< 8.6 imes 10^{-9}$	$<~8.6\times10^{-9}$	-	-
				7		
Benchmark		•	τ μ		τ μ →Q≁	τµ
			3		Ş	Ş

51

4.2 Prospects:



3.4.2 Dispersion relations: Method

Unitarity is the discontinuity of the form factor is known

$$\frac{1}{2i} disc \ \mathbf{F}_{\pi\pi}(s) = \operatorname{Im} F_{\pi\pi}(s) = \sum_{n} \mathbf{F}_{\pi\pi \to n} \left(\mathbf{T}_{n \to \pi\pi} \right)^{*}$$

• Only one channel $n = \pi \pi$

$$disc \left[\underbrace{1}_{2i} disc F_{I}(s) = Im F_{I}(s) = F_{I}(s) \sin \delta_{I}(s)e^{-i\delta_{I}(s)} \right]$$

$$\pi \pi \text{ scattering phase known from experiment }$$

$$Watson's theorem$$

3.4.2 Dispersion relations: Method

- Knowing the discontinuity of $F \implies$ write a dispersion relation for it
- Cauchy Theorem and Schwarz reflection principle

$$F(s) = \frac{1}{\pi} \oint \frac{F(s')}{s' - s} ds' \implies \frac{1}{2i\pi} \int_{M_{PP}^2}^{\infty} \frac{disc [F(s')]}{s' - s - i\varepsilon} ds'$$

• If *F* does not drop off fast enough for $|s| \rightarrow \infty$ subtract the DR

$$F(s) = P_{n-1}(s) + \frac{s^n}{\pi} \int_{M_{PP}^2}^{\infty} \frac{ds'}{s'^n} \frac{\operatorname{Im}[F(s')]}{(s'-s-i\varepsilon)}$$

 $P_{n-1}(s)$ polynomial

 $s_{th} \equiv M_{PP}^2$

3.4.2 Dispersion relations: Method

• Solution: Use analyticity to reconstruct the form factor in the entire space

$$ightarrow$$
 Omnès representation : $F_I(s) = P_I(s) \Omega_I(s)$
 $ightarrow N_I(s) = P_I(s) \Omega_I(s)$
 $ightarrow N_I(s) = P_I(s) \Omega_I(s)$
 $ightarrow N_I(s) = P_I(s) \Omega_I(s)$

Omnès function :
$$\Omega_{I}(s) = \exp\left[\frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{\delta_{I}(s')}{s'-s-i\varepsilon}\right]$$

 Polynomial: P_I(s) not known but determined from a matching to experiment or to ChPT at low energy • Vector form factor

Precisely known from experimental measurements

$$e^+e^- \rightarrow \pi^+\pi^-$$
 and $\tau^- \rightarrow \pi^0\pi^-\nu_{\tau}$ (isospin rotation)

> Theoretically: Dispersive parametrization for $F_V(s)$

Guerrero, Pich'98, Pich, Portolés'08 Gomez, Roig'13

$$F_{V}(s) = \exp\left[\lambda_{V}'\frac{s}{m_{\pi}^{2}} + \frac{1}{2}\left(\lambda_{V}'' - \lambda_{V}'^{2}\right)\left(\frac{s}{m_{\pi}^{2}}\right)^{2} + \frac{s^{3}}{\pi}\int_{4m_{\pi}^{2}}^{\infty}\frac{ds'}{s'^{3}}\frac{\phi_{V}(s')}{\left(s' + s - i\varepsilon\right)}\right]$$

Extracted from a model including 3 resonances $\rho(770)$, $\rho'(1465)$ and $\rho''(1700)$ fitted to the data

> Subtraction polynomial + phase determined from a *fit* to the Belle data $\tau^- \rightarrow \pi^0 \pi^- v_{\tau}$

3.4.3 Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.4.4 Determination of the form factors : $\Gamma_{\pi}(s)$, $\Delta_{\pi}(s)$, $\theta_{\pi}(s)$

- No experimental data for the other FFs → Coupled channel analysis up to √s~1.4 GeV Donoghue, Gasser, Leutwyler'90 Inputs: I=0, S-wave ππ and KK data Moussallam'99 Daub et al'13
- Unitarity:



3.4.4 Determination of the form factors : $\Gamma_{\pi}(s)$, $\Delta_{\pi}(s)$, $\theta_{\pi}(s)$

Inputs : $\pi\pi \rightarrow \pi\pi$, KK

Celis, Cirigliano, E.P.'14



- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree
- 3 inputs: $\delta_{\pi}(s)$, $\delta_{K}(s)$, η from *B. Moussallam* \Longrightarrow *reconstruct T matrix* Emilie Passemar

3.4.4 Determination of the form factors : $\Gamma_{\pi}(s)$, $\Delta_{\pi}(s)$, $\theta_{\pi}(s)$

• General solution:



• Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions X(s) = C(s), D(s)

$$\operatorname{Im} X_n^{(N+1)}(s) = \sum_{m=1}^2 \operatorname{Re} \left\{ T_{nm}^* \sigma_m(s) X_m^{(N)} \right\} \longrightarrow \operatorname{Re} X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \operatorname{Im} X_n^{(N+1)}(s) = \frac$$

Determination of the polynomial

General solution

$$\begin{pmatrix} F_{\pi}(s) \\ \frac{2}{\sqrt{3}}F_{K}(s) \end{pmatrix} = \begin{pmatrix} C_{1}(s) & D_{1}(s) \\ C_{2}(s) & D_{2}(s) \end{pmatrix} \begin{pmatrix} P_{F}(s) \\ Q_{F}(s) \end{pmatrix}$$

• Fix the polynomial with requiring $F_p(s) \rightarrow 1/s$ (Brodsky & Lepage) + ChPT: Feynman-Hellmann theorem: $\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d}\right) M_P^2$

$$\Delta P(0) = \left(\frac{m_u}{\partial m_u} + \frac{m_d}{\partial m_d} \right)^M$$
$$\Delta P(0) = \left(\frac{m_s}{\partial m_s} \right) M_P^2$$

• At LO in ChPT:

$$egin{aligned} M_{\pi^+}^2 &= (m_{ extsf{u}} + m_{ extsf{d}}) \, B_0 + O(m^2) \ M_{K^+}^2 &= (m_{ extsf{u}} + m_{ extsf{s}}) \, B_0 + O(m^2) \ M_{K^0}^2 &= (m_{ extsf{d}} + m_{ extsf{s}}) \, B_0 + O(m^2) \end{aligned}$$

$$P_{\Gamma}(s) = \Gamma_{\pi}(0) = M_{\pi}^{2} + \cdots$$

$$Q_{\Gamma}(s) = \frac{2}{\sqrt{3}}\Gamma_{K}(0) = \frac{1}{\sqrt{3}}M_{\pi}^{2} + \cdots$$

$$P_{\Delta}(s) = \Delta_{\pi}(0) = 0 + \cdots$$

$$Q_{\Delta}(s) = \frac{2}{\sqrt{3}}\Delta_{K}(0) = \frac{2}{\sqrt{3}}\left(M_{K}^{2} - \frac{1}{2}M_{\pi}^{2}\right) + \cdots$$

Determination of the polynomial

General solution

$$\begin{pmatrix} F_{\pi}(s) \\ \frac{2}{\sqrt{3}}F_{K}(s) \end{pmatrix} = \begin{pmatrix} C_{1}(s) & D_{1}(s) \\ C_{2}(s) & D_{2}(s) \end{pmatrix} \begin{pmatrix} P_{F}(s) \\ Q_{F}(s) \end{pmatrix}$$

• At LO in ChPT:

$$M_{\pi^{+}}^{2} = (m_{u} + m_{d}) B_{0} + O(m^{2})$$

$$M_{K^{+}}^{2} = (m_{u} + m_{s}) B_{0} + O(m^{2})$$

$$M_{K^{0}}^{2} = (m_{d} + m_{s}) B_{0} + O(m^{2})$$

$$M_{K$$

Problem: large corrections in the case of the kaons!
 Use lattice QCD to determine the SU(3) LECs

 $\Gamma_K(0) = (0.5 \pm 0.1) \ M_\pi^2$ $\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2M_\pi^2 \right)$

Dreiner, Hanart, Kubis, Meissner'13 Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

• General solution

$$\begin{pmatrix} F_{\pi}(s) \\ \frac{2}{\sqrt{3}}F_{K}(s) \end{pmatrix} = \begin{pmatrix} C_{1}(s) & D_{1}(s) \\ C_{2}(s) & D_{2}(s) \end{pmatrix} \begin{pmatrix} P_{F}(s) \\ Q_{F}(s) \end{pmatrix}$$

• For θ_P enforcing the asymptotic constraint is not consistent with ChPT The unsubtracted DR is not saturated by the 2 states

Relax the constraints and match to ChPT

$$\begin{aligned} P_{\theta}(s) &= 2M_{\pi}^{2} + \left(\dot{\theta}_{\pi} - 2M_{\pi}^{2}\dot{C}_{1} - \frac{4M_{K}^{2}}{\sqrt{3}}\dot{D}_{1}\right)s\\ Q_{\theta}(s) &= \frac{4}{\sqrt{3}}M_{K}^{2} + \frac{2}{\sqrt{3}}\left(\dot{\theta}_{K} - \sqrt{3}M_{\pi}^{2}\dot{C}_{2} - 2M_{K}^{2}\dot{D}_{2}\right)s\end{aligned}$$





Dispersion relations: Model-independent method, based on first principles that extrapolates ChPT based on data



3.5 Results



Emilie Passemar

Belle'08'11'12 except last from CLEO'97

2.5 Model discriminating power of Tau processes

• Depending on the UV model different correlations between the BRs



Interesting to study to determine the underlying dynamics of NP

4. CP-odd Higgs with LFV

4.1 Constraints from $\tau \rightarrow P$

• Tree level Higgs exchange



•
$$\mathbf{L}_{\mathbf{Y}} \longrightarrow \mathcal{L}_{eff} \simeq -\frac{A}{v} \left(\sum_{q=u,d,s} y_q^A m_q \, \bar{q} i \gamma_5 \, q - \sum_{q=c,b,t} y_q^A \frac{\alpha_s}{8\pi} \, G^a_{\mu\nu} \, \widetilde{G}^a_{\mu\nu} \right)$$

 $\widetilde{G}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \, G^a_{\alpha\beta}$

Mediate only one pseudoscalar meson very characteristic!

Tree level Higgs exchange
 ≻ η, η'

$$\Gamma\left(\tau \to \ell\eta^{(\prime)}\right) = \frac{\bar{\beta}\left(m_{\tau}^2 - m_{\eta}^2\right)\left(|Y_{\mu\tau}^A|^2 + |Y_{\tau\mu}^A|^2\right)}{256\,\pi\,M_A^4\,v^2\,m_{\tau}} \Big[(y_u^A + y_d^A)h_{\eta'}^q + \sqrt{2}y_s^Ah_{\eta'}^s - \sqrt{2}a_{\eta'}\sum_{q=c,b,t}\,y_q^A\Big]^2$$

with the decay constants :

$$\begin{split} \langle \eta^{(\prime)}(p) | \bar{q} \gamma_5 q | 0 \rangle &= -\frac{i}{2\sqrt{2}m_q} h^q_{\eta^{(\prime)}} \quad \langle \eta^{(\prime)}(p) | \bar{s} \gamma_5 s | 0 \rangle = -\frac{i}{2m_s} h^s_{\eta^{(\prime)}} \\ \langle \eta^{(\prime)}(p) | \frac{\alpha_s}{4\pi} G^{\mu\nu}_a \widetilde{G}^a_{\mu\nu} | 0 \rangle &= a_{\eta^{(\prime)}} \end{split}$$
$$\gg \pi : \Gamma(\tau \to \ell \pi^0) = \frac{f^2_\pi m^4_\pi m_\tau}{256\pi M^4_A v^2} \left(|Y^A_{\tau\mu}|^2 + |Y^A_{\mu\tau}|^2 \right) \left(y^A_u - y^A_d \right)^2 \end{split}$$

• $\tau \rightarrow \mu P$

Process	BR 90% CL	$M_A = 200 \text{ GeV}$	$M_A = 500 \text{ GeV}$	$M_A = 700 \text{ GeV}$
$\tau \to \mu \gamma$	$< 4.4 \times 10^{-8}$	Z < 0.018	Z < 0.040	Z < 0.055
$\tau \to \mu \mu \mu$	$< 2.1 \times 10^{-8}$	Z < 0.28	Z < 0.60	Z < 0.85
$(*) \ \tau \to \mu \pi$	$< 11 \times 10^{-8}$	Z < 41	Z < 257	Z < 503
$^{(*)} \tau \to \mu \eta$	$< 6.5 \times 10^{-8}$	Z < 0.52	Z < 3.3	Z < 6.4
$^{(*)} \tau \to \mu \eta'$	$< 13 \times 10^{-8}$	Z < 1.1	Z < 7.2	Z < 14.1
$\tau \to \mu \pi^+ \pi^-$	$< 2.1 \times 10^{-8}$	Z < 0.25	Z < 0.54	Z < 0.75
$\tau \to \mu \rho$	$< 1.2 \times 10^{-8}$	Z < 0.20	Z < 0.44	Z < 0.62

BaBar'06'10 , Belle'10'11'13

$$\boldsymbol{Z} = \sqrt{\left|\boldsymbol{Y}_{\mu\tau}^{A}\right|^{2} + \left|\boldsymbol{Y}_{\tau\mu}^{A}\right|^{2}}$$

(*) : No contribution from effective dipole operator or CP-even Higgs

N.B.: Diagonal couplings $|y_f^A| = 1$

• $\tau \rightarrow eP$

Process	BR 90% CL	$M_A = 200 \text{ GeV}$	$M_A = 500 \text{ GeV}$	$M_A = 700 \text{ GeV}$
$\tau \to e\gamma$	$< 3.3 \times 10^{8}$	Z < 0.016	Z < 0.034	Z < 0.05
$\tau \rightarrow eee$	$< 2.7 \times 10^8$	Z < 0.14	Z < 0.30	Z < 0.42
$^{(*)} \tau \to e\pi$	$< 8 \times 10^{8}$	Z < 35	Z < 219	Z < 430
$^{(*)}\tau \to e\eta$	$< 9.2 \times 10^{8}$	Z < 0.6	Z < 3.9	Z < 7.6
$^{(*)} \tau \to e\eta'$	$< 16 \times 10^8$	Z < 1.3	Z < 8	Z < 15.6
$\tau \to e \pi^+ \pi^-$	$< 2.3 \times 10^{8}$	Z < 0.26	Z < 0.56	Z < 0.80
$\tau \to e \rho$	$< 1.8 \times 10^{8}$	Z < 0.25	Z < 0.54	Z < 0.76

BaBar'06'10 , Belle'10'11'13

$$\boldsymbol{Z} = \sqrt{\left|\boldsymbol{Y}_{e\tau}^{A}\right|^{2} + \left|\boldsymbol{Y}_{\tau e}^{A}\right|^{2}}$$

- (*) : No contribution from effective dipole operator or CP-even Higgs
- N.B.: Diagonal couplings $|y_f^A| = 1$
4.3 Prospects at LHC

• Decay width :
$$\Gamma(A \to \tau^+ \mu^- + \tau^- \mu^+) \equiv \Gamma(A \to \tau \mu) = \frac{M_A \left(|Y_{\tau\mu}^A|^2 + |Y_{\mu\tau}^A|^2 \right)}{8\pi}$$

Assumption : only SM channels $(A \rightarrow gg, b\bar{b}, c\bar{c}, \tau\tau...)$ are important

• Large BR for $A \rightarrow \tau \mu$ can be expected since A does not couple to WW, ZZ at tree level. Results :







3.5 What if $\tau \rightarrow \mu(e)\pi\pi$ observed? Reinterpreting Celis, Cirigliano, E.P'14

Talk by J. Zupan @ KEK-FF2014FALL

- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}!$
- $Y_{u,d,s}$ poorly bounded



- For $Y_{u,d,s}$ at their SM values : $Br(\tau \to \mu \pi^+ \pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu \pi^0 \pi^0) < 4.6 \times 10^{-12}$ $Br(\tau \to e \pi^+ \pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e \pi^0 \pi^0) < 6.9 \times 10^{-11}$
- But for $Y_{u,d,s}$ at their upper bound:

 $Br(\tau \to \mu \pi^+ \pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu \pi^0 \pi^0) < 1.5 \times 10^{-8}$ $Br(\tau \to e\pi^+ \pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0 \pi^0) < 2.1 \times 10^{-7}$

below present experimental limits!

If discovered among other things upper limit on Y_{u,d,s}!
Interplay between high-energy and low-energy constraints!