Angular Observables Sensitive to NP in $\bar{B} \longrightarrow D^* l^- \bar{\nu}_l$

Kilian Lieret
Ludwig Maximilian University of Munich

Advisers: Prof. Dr. Thomas Kuhr, Dr. Martin Jung, Prof. Dr. Gerhard Buchalla

November 15, 2018

2018 WPI-next mini-workshop
"Hints for New Physics in Heavy Flavors"
Nagoya University
15-17 November 2018

Intro I

- Evidence for NP in $\bar{B} \longrightarrow D^* \tau^- \bar{\nu}_\tau$ is mounting
  $\Longrightarrow$ Next question: *What kind* of NP?
  $\Longrightarrow$ **New observables**! (also good as cross checks)
- Here we consider the **angular distribution** of $\bar{B} \longrightarrow D^*(\to D\pi) l^- \bar{\nu}_l$.
  Example: Forward-backward asymmetry:

$$A_\theta := \left[\int_{-1}^{0} - \int_{0}^{1}\right] \mathrm{d}\cos\theta_\ell \frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta_\ell}$$

- Generalization: Observables $\mathcal{O}_a$ built from *binned* measurements of $\frac{\mathrm{d}^3\Gamma}{\mathrm{d}\chi\,\mathrm{d}\theta_\ell\,\mathrm{d}\theta_{D^*}}$
  Similar to 12 observables constructed in 1602.03030[1] (using polarizations in addition to angles)

[1] Becirevic, Fajfer, Nisandzic, Tayduganov

Intro II

However (so far): Only proofs of concepts, little experimental considerations!

Aim:

+ Characterize general form of 12 observables $\mathcal{O}_a$
+ Introduce experimental error estimate
+ Minimize expected errors using d.o.f. in construction of $\mathcal{O}_a$
+ Minimal number of bins required for construction of $\mathcal{O}_a$
+ Optimal bin spacing for each $\mathcal{O}_a$

(the previous two items: **experimentally relevant!**)

+ Consider discriminatory power of observables on basis of operators corresponding to relevant NP mediators

The Differential Cross Section

Slightly adapted[2] from 1405.3719[3], we have[4] (with (pseudo)scalar, (axial) vector and tensor NP operators, no CP average)

$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}\chi\,\mathrm{d}\theta_\ell\,\mathrm{d}\theta_{D^*}} = \sum_a \overline{W}_a\, B_a(\chi, \theta_\ell, \theta_{D^*})$$

With **12** different

- Angle dependencies
  $B_a(\chi, \theta_\ell, \theta_{D^*})$ (known)
  e.g. $\cos(2\chi)\sin^3(\theta_{D^*})\sin^3(\theta_\ell)$
- NP dependent coefficients
  $\overline{W}_a$ (unknown)

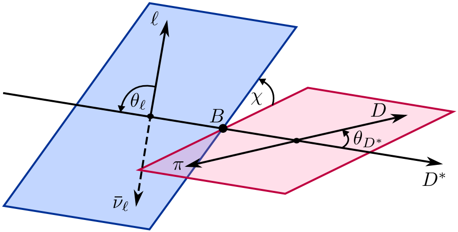


Fig. 1: The decay angles χ, θ_ℓ, θ_{D^*}

[2] In relation to the V_a from 1405.3719: $W_1^0 := V_1^0 + V_2^0$, $W_1^T := V_1^T + V_2^T$, $W_2^0 := -2V_2^0$, $W_2^T := -2V_2^T$; for the rest $W_a := V_a$. Furthermore $\overline{W}_a := \int \mathrm{d}q^2\, W_a(q^2)$.

[3] Duraisamy, Sharma, Datta

[4] the kinematic variable q^2 will not be considered, i.e. the relevant quantities are integrated over q^2

The Idea (I)

Experimental measurements of $\frac{d^3\Gamma}{d\chi\, d\theta_\ell\, d\theta_{D^*}}$ will be (published) **binned**:

bins: $U_i \subseteq \underset{\chi}{\underset{\Cup}{[-\pi, \pi]}} \times \underset{\theta_\ell}{\underset{\Cup}{[0, \pi]}} \times \underset{\theta_{D^*}}{\underset{\Cup}{[0, \pi]}}$

bin content:

$$\underbrace{\Gamma(i)}_{\text{measured binned CS}} := \int_{U_i} d\chi\, d\theta_\ell\, d\theta_{D^*} \frac{d^3\Gamma}{d\chi\, d\theta_\ell\, d\theta_{D^*}} = \sum_a \overline{W}_a \int_{U_i} d\chi\, d\theta_\ell\, d\theta_{D^*}\, B_a(\chi, \theta_\ell, \theta_{D^*}) =: \sum_a \underbrace{\overline{W}_a}_{\text{unknown NP-dependent coefficient}} \underbrace{\overline{B}_a(i)}_{\text{known “binned” angle-dep.}} \tag{1}$$

How to extract the $\overline{W}_a$?

‘ Linear algebra: Eq. (1): N linear equations with 12 unknowns

$\Longrightarrow$ there are **weights** $\omega_a(i)$ such that

$$\underbrace{\overline{W}_a}_{\text{unknown NP-dependent coefficient}} = \sum_i \underbrace{\omega_a(i)}_{\text{weights}} \underbrace{\Gamma(i)}_{\text{measured binned CS}}$$

The Idea (I)

Experimental measurements of $\dfrac{d^3\Gamma}{d\chi\, d\theta_\ell\, d\theta_{D^*}}$ will be (published) **binned**:

bins: $U_i \subseteq [-\pi, \pi] \times [0, \pi] \times [0, \pi]$, with $\chi \in [-\pi, \pi]$, $\theta_\ell \in [0, \pi]$, $\theta_{D^*} \in [0, \pi]$

bin content:

$$\underbrace{\Gamma(i)}_{\text{measured binned CS}} := \int_{U_i} d\chi\, d\theta_\ell\, d\theta_{D^*} \frac{d^3\Gamma}{d\chi\, d\theta_\ell\, d\theta_{D^*}} = \sum_a \overline{W}_a \int_{U_i} d\chi\, d\theta_\ell\, d\theta_{D^*}\, B_a(\chi, \theta_\ell, \theta_{D^*}) =: \sum_a \underbrace{\overline{W}_a}_{\text{unknown NP-dependent coefficient}} \underbrace{\overline{B}_a(i)}_{\text{known “binned” angle-dep.}} \qquad (1)$$

How to extract the $\overline{W}_a$?

‘ Linear algebra: Eq. (1): N linear equations with 12 unknowns

$\Longrightarrow$ there are **weights** $\omega_a(i)$ such that

$$\underbrace{\overline{W}_a}_{\text{unknown NP-dependent coefficient}} = \sum_i \underbrace{\omega_a(i)}_{\text{weights}} \underbrace{\Gamma(i)}_{\text{measured binned CS}}$$

The Idea (II)

The observables

$$\mathcal{O}_a\colon \underbrace{\Gamma}_{\text{measured binned CS}} \longmapsto \sum_i \underbrace{\omega_a(i)}_{\text{weights}}\, \Gamma(i) \quad \text{such that} \quad \mathcal{O}_a(\Gamma_{\text{theo}}) = \underbrace{\overline{W}_a}_{\text{unknown NP-dependent coefficient}}$$

Sensitivity

Expected uncertainty $\sqrt{\mathrm{Var}(\mathcal{O}_a)}$ as figure of merit $\rightarrow$ *the smaller the better*

(currently simplified setup with only statistical errors, studies of bin migration and flat systematics ongoing)

Degrees of freedom

If more bins than required

$\Longrightarrow$ DOFs in weights $\omega_a(i)$

$\Longrightarrow$ Use to improve sensitivity

Assumptions

1 "General bins": Arbitrary subsets $U_i \subseteq [-\pi, \pi] \times [0, \pi] \times [0, \pi]$

2a "Product bins": $U_{ijk} = U_i^{\chi} \times U_j^{\theta_\ell} \times U_k^{\theta_{D^*}}$

2b "Product weights": Product bins with weights in product form

Minimal number of bins ($l = \tau$)

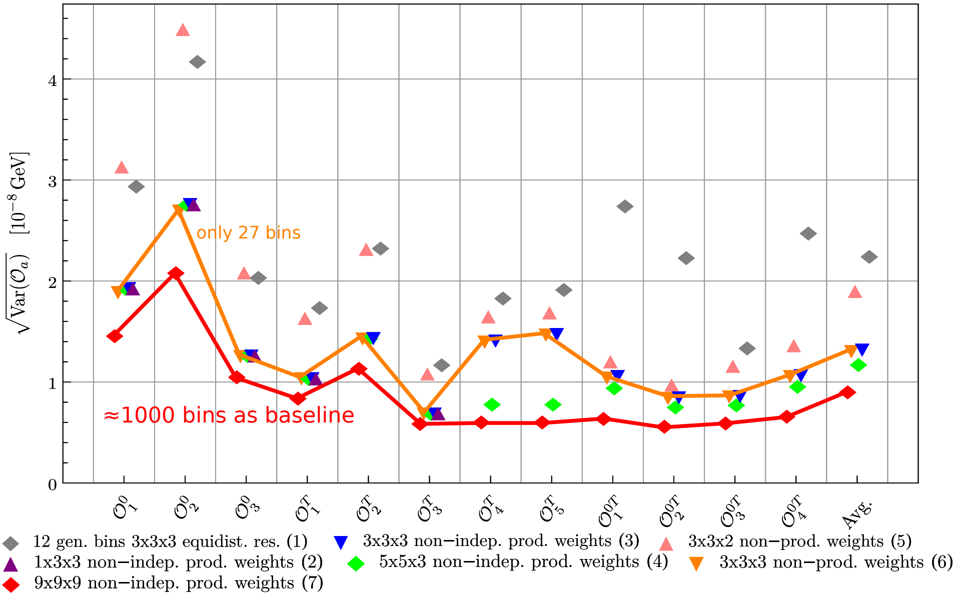
(under conservative assumptions for the binning)

Min. number of bins in $\chi \times \theta_\ell \times \theta_{D^*}$ required to construct $\mathcal{O}_a$

$\mathcal{O}_a$	$\mathcal{O}_1^0$, $\mathcal{O}_2^0$, $\mathcal{O}_3^0$, $\mathcal{O}_1^T$, $\mathcal{O}_3^T$	$\mathcal{O}_2^T$	$\mathcal{O}_4^T$, $\mathcal{O}_5^T$	$\mathcal{O}_1^{0T}$, $\mathcal{O}_2^{0T}$, $\mathcal{O}_3^{0T}$, $\mathcal{O}_4^{0T}$
Minimal # bins	$1 \times 3 \times 3$	$3 \times 3 \times 2$	$3 \times 1 \times 2$	$2 \times 2 \times 3$
	$1 \times 5 \times 2$		$3 \times 3 \times 1$	$2 \times 5 \times 1$
	$3 \times 3 \times 2$		$5 \times 1 \times 1$	$3 \times 2 \times 2$
				$3 \times 3 \times 1$
				$5 \times 2 \times 1$

All observables can be constructed for $\mathbf{3} \times \mathbf{3} \times \mathbf{2}$ binning.

Performance Comparison ($l = \tau$)



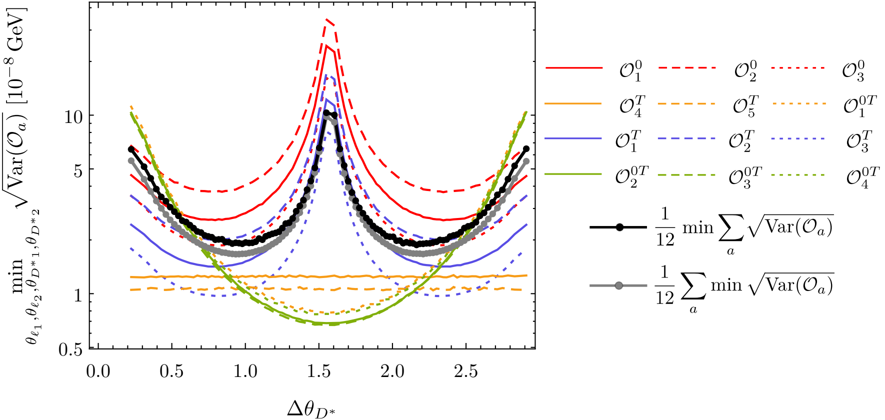
Finding optimal bin edges (example)

Example: 3 × 3 × 2 bins (prod bins, gen. weights) with edge points

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \underline{\Delta\theta_{D^*}}, \pi\}_{\theta_{D^*}}$$

$\Longrightarrow$ How does the optimal achievable sensitivity depend on $\Delta\theta_{D^*}$?

$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



What can we conclude about NP contributions? (I)

Suppose we can extract all $\overline{W}_a$ from the angular distribution

$$\underbrace{\Gamma(i)}_{\text{measured binned CS}} = \sum_a \underbrace{\overline{W}_a}_{\text{unknown NP-dependent coefficient}} \underbrace{\overline{B}_a(i)}_{\text{known "binned" angle-dep.}}$$

Split up $\overline{W}_a$:

$$\overline{W}_a = \sum_{i=1}^{13} \underbrace{\overline{W}_a^{(i)}}_{\text{known coeff}\in\mathbb{R}} \underbrace{c_i(g_A, g_V, g_S, g_P, T_L)}_{\text{unknown quadratic in NP coupl.}}$$

Want to extract c_i given $\overline{W}_a$:

- System of linear equations
- Sometimes $\overline{W}_a \propto c_i \implies$ easy!
- Generally hard $\implies$ additional assumptions for NP coefficients

---

Effective Lagrangian:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}} \Big[(1 + \mathcal{C}_{V_L}[\bar{c}\gamma_\mu P_L b][\bar{l}\gamma^\mu P_L \nu_l] + \mathcal{C}_{V_R}[\bar{c}\gamma^\mu P_R b][\bar{l}\gamma_\mu P_L \nu_l] + $$
$$+ \mathcal{C}_{S_L}[\bar{c}P_L b][\bar{l}P_L\nu_l] + \mathcal{C}_{S_R}[\bar{c}P_R b][\bar{l}P_L\nu_l] + T_L[\bar{c}\sigma^{\mu\nu}P_L b][\bar{l}\sigma_{\mu\nu}P_L\nu_l] \Big],$$

With $g_{V,A} = \mathcal{C}_{V_R} \pm \mathcal{C}_{V_L}$, $g_{S,P} = \mathcal{C}_{S_R} \pm \mathcal{C}_{S_L}$, $T_L = \mathcal{C}_T$, $P_{L/R} := (1 \mp \gamma_5)/2$, $\sigma_{\mu\nu} := \mathrm{i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$.

What can we conclude about NP contributions? (I)

Suppose we can extract all $\overline{W}_a$ from the angular distribution

$$\underset{\text{measured binned CS}}{\Gamma(i)} = \sum_a \underset{\text{unknown NP-dependent coefficient}}{\overline{W}_a} \; \underset{\text{known “binned” angle-dep.}}{\overline{B}_a(i)}$$

Split up $\overline{W}_a$:

$$\overline{W}_a = \sum_{i=1}^{13} \underset{\text{known coeff}\in\mathbb{R}}{\overline{W}_a^{(i)}} \; \underset{\text{unknown quadratic in NP coupl.}}{c_i(g_A, g_V, g_S, g_P, T_L)}$$

Want to extract c_i given $\overline{W}_a$:

- System of linear equations
- Sometimes $\overline{W}_a \propto c_i \implies$ easy!
- Generally hard $\implies$ additional assumptions for NP coefficients

---

Effective Lagrangian:

$$\mathcal{H}_{\text{eff}} = \frac{4 G_F V_{cb}}{\sqrt{2}} \Big[(1 + \mathcal{C}_{V_L} [\bar{c}\gamma_\mu P_L b][\bar{l}\gamma^\mu P_L \nu_l] + \mathcal{C}_{V_R} [\bar{c}\gamma^\mu P_R b][\bar{l}\gamma_\mu P_L \nu_l] + \\ + \mathcal{C}_{S_L} [\bar{c} P_L b][\bar{l} P_L \nu_l] + \mathcal{C}_{S_R} [\bar{c} P_R b][\bar{l} P_L \nu_l] + T_L [\bar{c}\sigma^{\mu\nu} P_L b][\bar{l}\sigma_{\mu\nu} P_L \nu_l] \Big],$$

With $g_{V,A} = \mathcal{C}_{V_R} \pm \mathcal{C}_{V_L}$, $g_{S,P} = \mathcal{C}_{S_R} \pm \mathcal{C}_{S_L}$, $T_L = \mathcal{C}_T$, $P_{L/R} := (1 \mp \gamma_5)/2$, $\sigma_{\mu\nu} := \mathrm{i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$.

What can we conclude about NP contributions? (II)

c_i	In terms of coupling constants	
c_1	$\lvert 1 - g_A \rvert^2$	
c_2	$\lvert g_V + 1 \rvert^2$	
c_3	$\lvert g_P \rvert^2$	
c_4	$\lvert T_L \rvert^2$	
c_5	$\mathrm{Re}((1 - g_A)(g_V^* + 1))$	
c_6	$\mathrm{Re}((1 - g_A)g_P^*)$	
c_7	$\mathrm{Re}((1 - g_A)T_L^*)$	
c_8	$\mathrm{Re}((g_V + 1)T_L^*)$	
c_9	$\mathrm{Re}(g_P T_L^*)$	
c_{10}	$\mathrm{Im}((1 - g_A)(g_V^* + 1))$	drop out in CP average
c_{11}	$\mathrm{Im}((1 - g_A)T_L^*)$	drop out in CP average
c_{12}	$\mathrm{Im}((g_V + 1)g_P^*)$	drop out in CP average
c_{13}	$\mathrm{Im}(g_P T_L^*)$	drop out in CP average

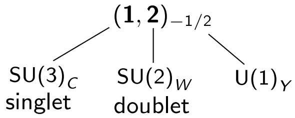
Fig. 2: $c_i \longrightarrow$ coupling constants

What can we conclude about NP contributions? (III)

Consider base of operators with[5]

- Renormalizable couplings relevant for $\bar{B} \longrightarrow D^* l^- \bar{\nu}_l$
- Dimension 4 or 6
- Non-flavor universal couplings $\Longrightarrow$ right-handed vector current not considered

Denote operators by transformation under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$:



[5] source: Freytsis, Ligeti, Ruderman 1506.08896

What can we conclude about NP contributions? (IV)

Considering base of operators corresponding to relevant mediators

	SM	All NP	$(\mathbf{1},\mathbf{3})_0$, $(\mathbf{3},\mathbf{3})_{2/3}$, $(\bar{\mathbf{3}},\mathbf{3})_{1/3}$	$(\mathbf{1},\mathbf{2})_{1/2}$, $(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	$(\mathbf{3},\mathbf{1})_{2/3}$	$(\mathbf{3},\mathbf{2})_{7/6}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$
$\mathbf{W_1^0}$		c_1, c_3, c_4, c_6, c_7	c_1	c_3, c_6	c_1, c_3, c_6	c_3, c_6	c_1, c_3, c_6
$\mathbf{W_2^0}$		c_1, c_4	c_1		c_1	c_3	c_1, c_3
W_3^0		c_1, c_6, c_7, c_9	c_1	c_6	c_1, c_6	c_6, c_3	c_1, c_6, c_3
$\mathbf{W_1^T}$		c_1, c_2, c_4, c_7, c_8	c_1		c_1	c_3, c_6	c_1, c_3, c_6
$\mathbf{W_2^T}$		c_1, c_2, c_4	c_1		c_1	c_3	c_1, c_3
W_3^T		c_4, c_5, c_7, c_8	c_1		c_1	c_3, c_6	c_3, c_1, c_6
W_4^T		c_1, c_2, c_4	c_1		c_1	c_3	c_1, c_3
(W_5^T)		c_{10}					
W_1^{0T}		c_1, c_4	c_1		c_1	c_3	c_1, c_3
W_2^{0T}		$c_1, c_4, c_5, c_6, c_7, c_8, c_9$	c_1	c_6	c_1, c_6	c_3, c_6	c_1, c_3, c_6
(W_3^{0T})		$c_{10}, c_{11}, c_{12}, c_{13}$		c_{12}	c_{12}	c_{11}	c_{11}
(W_4^{0T})		c_{10}					

Colors:					
	$W_a = 0$	W_a c_i-indep.	$W_a \propto c_i$	$(W_a - \text{const.}) \propto c_i$	Multiple c_i

Bold: Contributes to fully integrated CS; *Parenthesized*: Drops out in *CP* average

Fig. 3: All operators with dim. 4 or 6 and renormalizable non-flavor universal couplings relevant for $\bar{B} \longrightarrow D^* l^- \bar{\nu}_l$ (cf. 1506.08896) applied to observables for $l = \tau$.

Summary

$$\mathcal{O}_a : \underset{\substack{\text{measured}\\ \text{binned}\\ \text{CS}}}{\Gamma} \longmapsto \sum_i \underset{\text{weights}}{\omega_a(i)}\, \Gamma(i) \quad \text{such that} \quad \mathcal{O}_a(\Gamma_{\text{theo}}) = \underset{\substack{\text{unknown}\\ \text{NP-dependent}\\ \text{coefficient}}}{\overline{W}_a}$$

Some key points

- Can construct some of the observables even for very coarse binnings
  (especially relevant for the experimentally challenging $l = \tau$ case)
- The same strategies have been applied to $l = e, \mu$
  (both experimentally and theoretically easier, but less suspicious of NP so far)
- Can also consider the same observables with q^2 dependency
  (but will need a new figure of merit to optimize against and discriminating between NP models will be more complex)
- To cancel systematics: Consider ratios of observables or use a normalization mode

Summary

$$\mathcal{O}_a : \underset{\substack{\text{measured}\\\text{binned}\\\text{CS}}}{\Gamma} \longmapsto \sum_i \underset{\text{weights}}{\omega_a(i)}\, \Gamma(i) \quad \text{such that} \quad \mathcal{O}_a(\Gamma_{\text{theo}}) = \underset{\substack{\text{unknown}\\\text{NP-dependent}\\\text{coefficient}}}{\overline{W}_a}$$

Literature[6]

- Some simple expression of weights to extract (combinations of the) W_a (on a "proof of concept" base)
- Studied the NP dependency by considering the q^2 distribution of $W_a(q^2)$, considering only one of g_A, g_V, g_S, g_P, T_L to be non-vanishing

New results

- **Characterized** the construction of the observables $\mathcal{O}_a$ for several assumptions on binning and weights
- Determined the **min. number of bins** to construct each $\mathcal{O}_a$
- Introduced a **figure of merit** for the obtainable sensitivity
- Used degrees of freedom in the weights to **optimize sensitivity**
- Studied the **influence of bin widths** on the sensitivity
- Studied the **NP dependency** of $\overline{W}_a$ based on a base of operators *corresponding to relevant mediators* (better physical motivation)

[6] Becirevic, Fajfer, Nisandzic, Tayduganov 1602.03030 (ref. closest to our work to best knowledge)

Backup Slides

Backup slides overview

- General bins
- The angular functions B_a
- Correlation
- Plots $3 \times 3 \times 2$ edges
- Plots $3 \times 3 \times 3$ edges

General binning

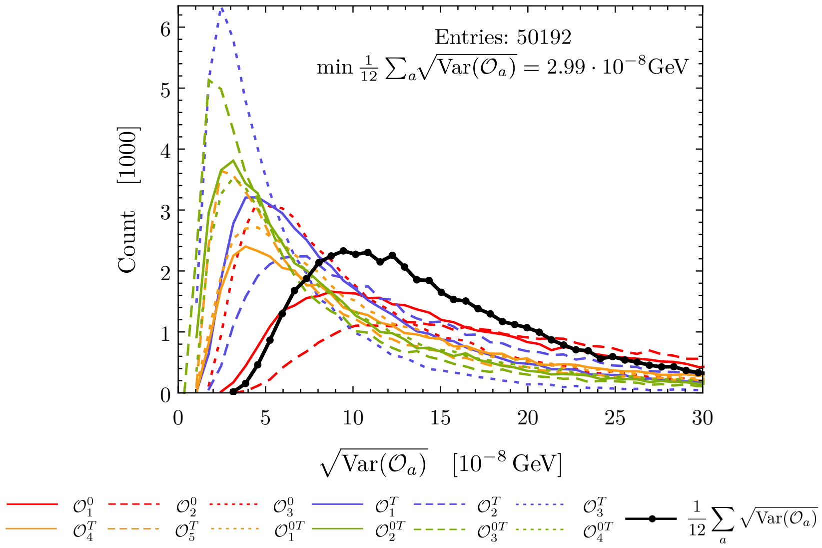


Fig. 4: 12 bins, $3 \times 3 \times 3$ cubes

General binning

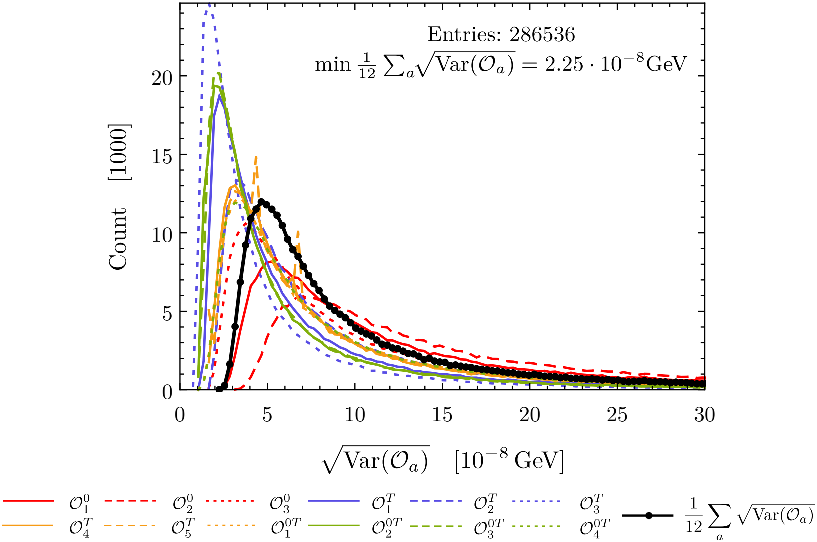


Fig. 4: 12 bins, $3 \times 3 \times 3$ cubes (equidistant)

General binning

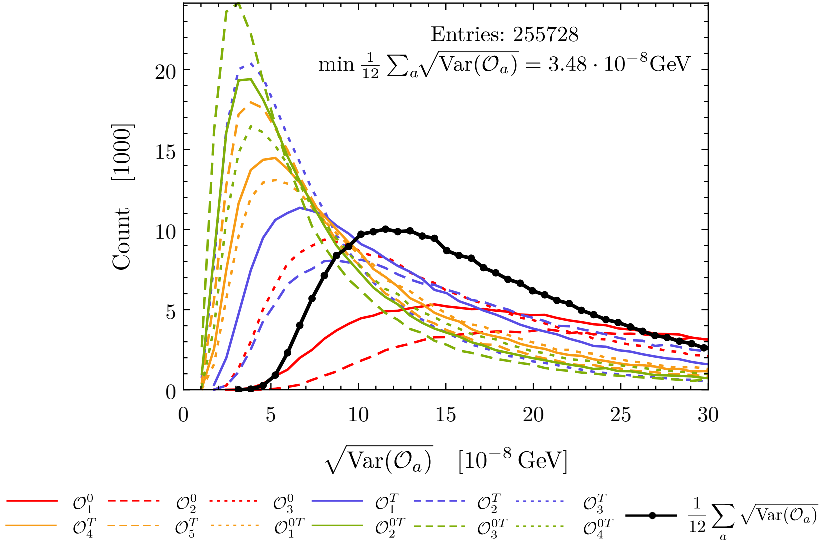


Fig. 4: 12 bins, 5 × 5 × 5 cubes

General binning

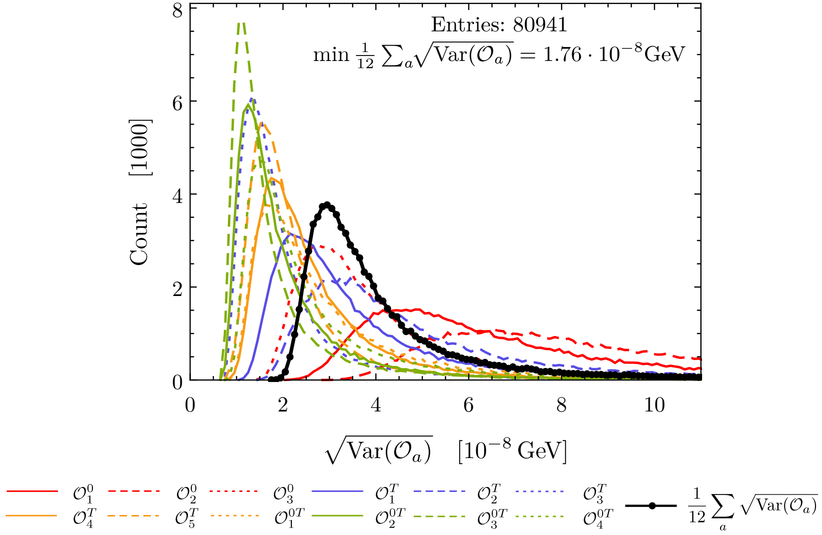


Fig. 4: 27 bins, 5 × 5 × 5 cubes

The angular functions B_a (I)

$$B_1^0 = \sin(\theta_{D^*})\cos^2(\theta_{D^*})\sin(\theta_\ell)$$
$$B_2^0 = \sin(\theta_{D^*})\cos^2(\theta_{D^*})\sin^3(\theta_\ell)$$
$$B_3^0 = \sin(\theta_{D^*})\cos^2(\theta_{D^*})\sin(\theta_\ell)\cos(\theta_\ell)$$

$$B_1^T = \sin^3(\theta_{D^*})\sin(\theta_\ell)$$
$$B_2^T = \sin^3(\theta_{D^*})\sin^3(\theta_\ell)$$
$$B_3^T = \sin^3(\theta_{D^*})\sin(\theta_\ell)\cos(\theta_\ell)$$
$$B_4^T = \cos(2\chi)\sin^3(\theta_{D^*})\sin^3(\theta_\ell)$$
$$B_5^T = \sin(2\chi)\sin^3(\theta_{D^*})\sin^3(\theta_\ell)$$

$$B_1^{0T} = \cos(\chi)\sin(\theta_{D^*})\sin(2\theta_{D^*})\sin(\theta_\ell)\sin(2\theta_\ell)$$
$$B_2^{0T} = \cos(\chi)\sin(\theta_{D^*})\sin(2\theta_{D^*})\sin^2(\theta_\ell)$$
$$B_3^{0T} = \sin(\chi)\sin(\theta_{D^*})\sin(2\theta_{D^*})\sin^2(\theta_\ell)$$
$$B_4^{0T} = \sin(\chi)\sin(\theta_{D^*})\sin(2\theta_{D^*})\sin(\theta_\ell)\sin(2\theta_\ell)$$

Table 1: The functions $B_a(\chi, \theta_\ell, \theta_{D^*})$

The angular functions B_a (II)

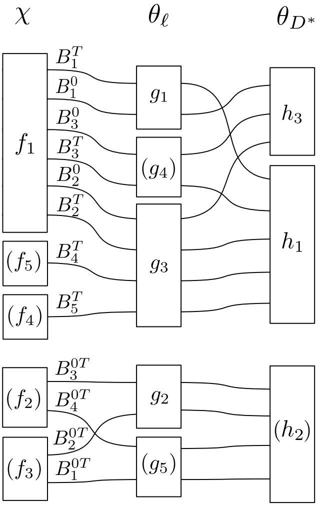


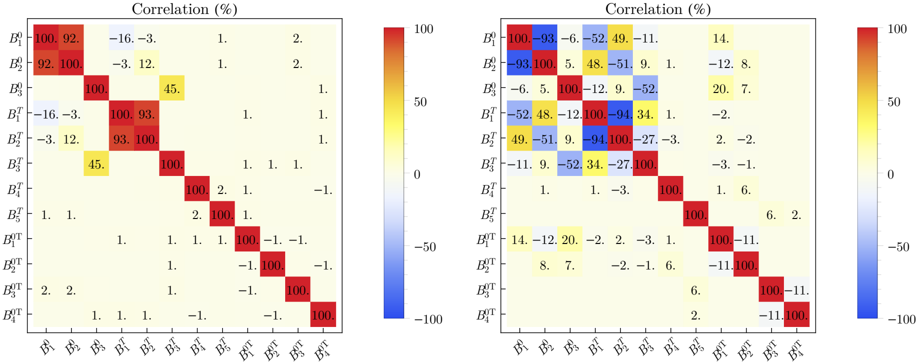
Fig. 5: Combinations of the angle functions.

$$f_1(\chi) := 1,$$
$$f_2(\chi) := \sin(\chi),$$
$$f_3(\chi) := \cos(\chi),$$
$$f_4(\chi) := \sin(2\chi),$$
$$f_5(\chi) := \cos(2\chi),$$

$$g_1(\theta_\ell) := \sin(\theta_l),$$
$$g_2(\theta_\ell) := \sin^2(\theta_l),$$
$$g_3(\theta_\ell) := \sin^3(\theta_l),$$
$$g_4(\theta_\ell) := \sin(\theta_l)\cos(\theta_l),$$
$$g_5(\theta_\ell) := \sin(\theta_l)\sin(2\theta_l),$$

$$h_1(\theta_{D^*}) := \sin^3(\theta_{D^*}),$$
$$h_2(\theta_{D^*}) := \sin(\theta_{D^*})\sin(2\theta_{D^*}),$$
$$h_3(\theta_{D^*}) := \sin(\theta_{D^*})\cos^2(\theta_{D^*}).$$

Correlation



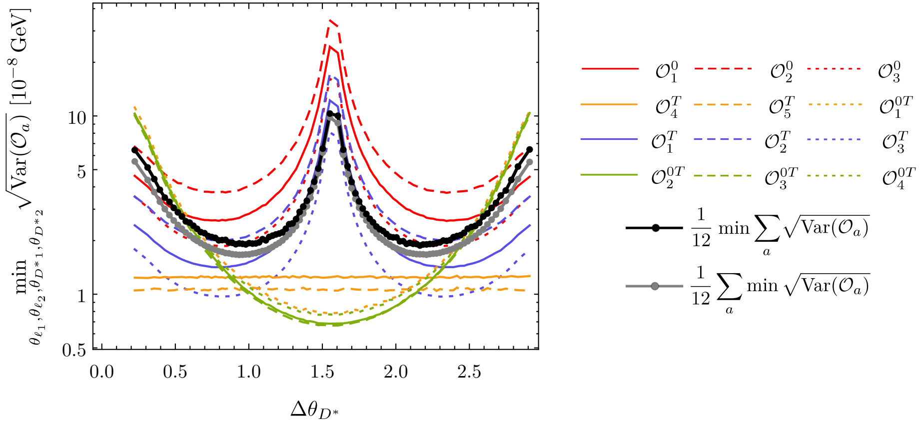
(a) Unbinned angle functions (b) Optimal observables $3 \times 3 \times 3$

Fig. 6: Correlation of angle functions resp. observables

$3 \times 3 \times 2$ (prod bins, gen. weights)

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \Delta\theta_{D^*}, \pi\}_{\theta_{D^*}}$$

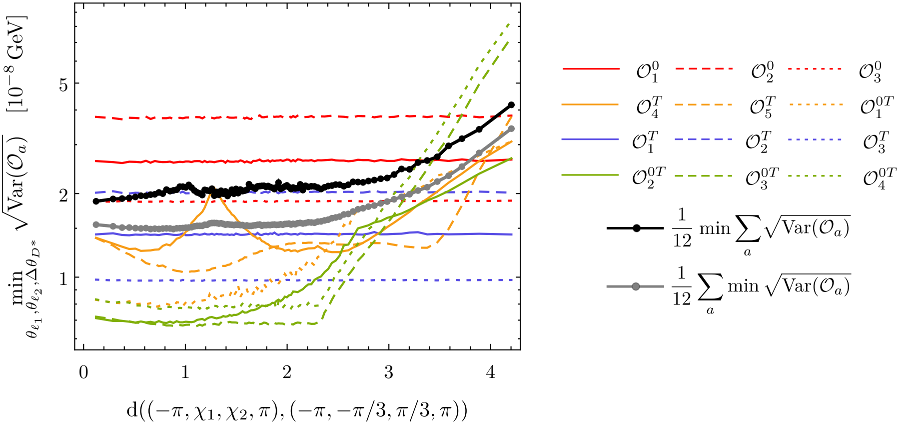
$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



$3 \times 3 \times 2$ (prod bins, gen. weights)

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \Delta\theta_{D^*}, \pi\}_{\theta_{D^*}}$$

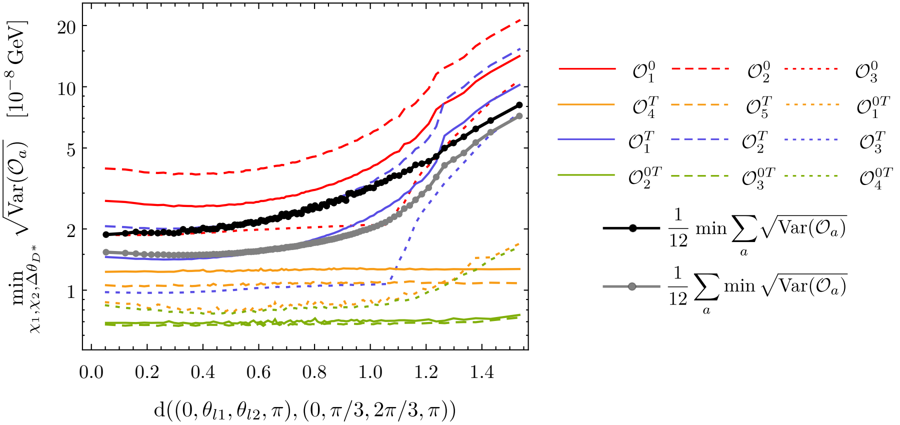
$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



$3 \times 3 \times 2$ (prod bins, gen. weights)

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \Delta\theta_{D^*}, \pi\}_{\theta_{D^*}}$$

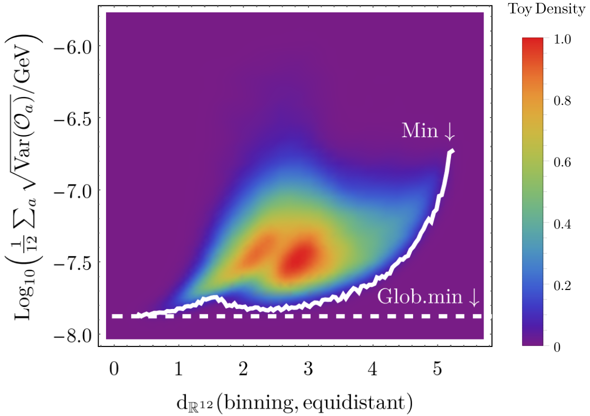
$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



$3 \times 3 \times 2$ (prod bins, gen. weights)

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \Delta\theta_{D^*}, \pi\}_{\theta_{D^*}}$$

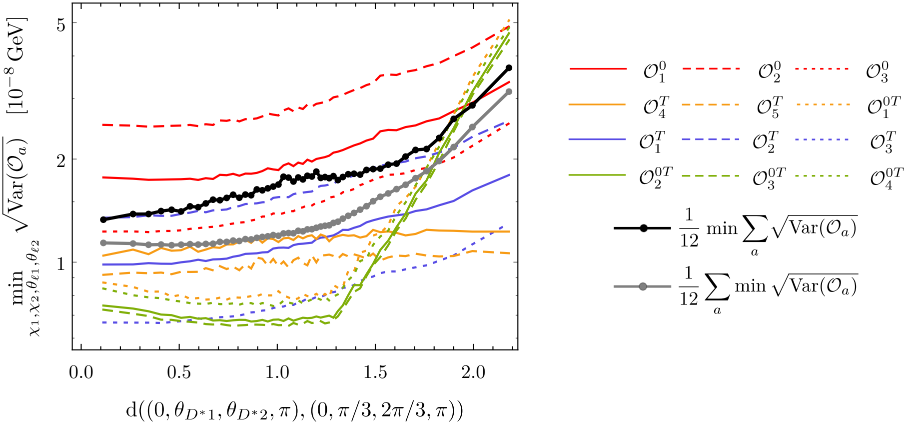
$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



$3 \times 3 \times 3$ (prod bins, gen. weights)

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \theta_{D^*1}, \theta_{D^*2}, \pi\}_{\theta_{D^*}}$$

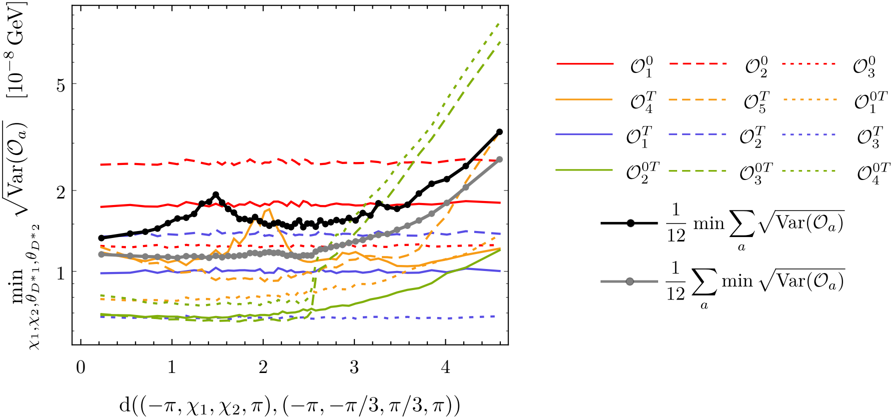
$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



$3 \times 3 \times 3$ (prod bins, gen. weights)

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \theta_{D^*1}, \theta_{D^*2}, \pi\}_{\theta_{D^*}}$$

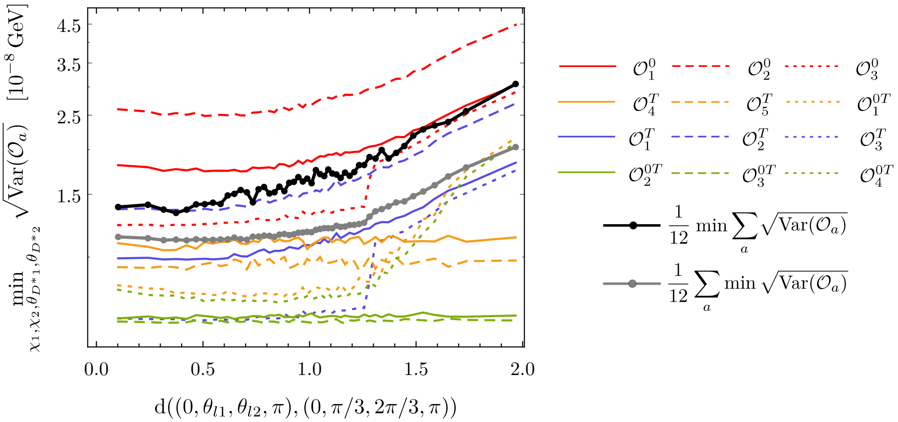
$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



$3 \times 3 \times 3$ (prod bins, gen. weights)

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \theta_{D^*1}, \theta_{D^*2}, \pi\}_{\theta_{D^*}}$$

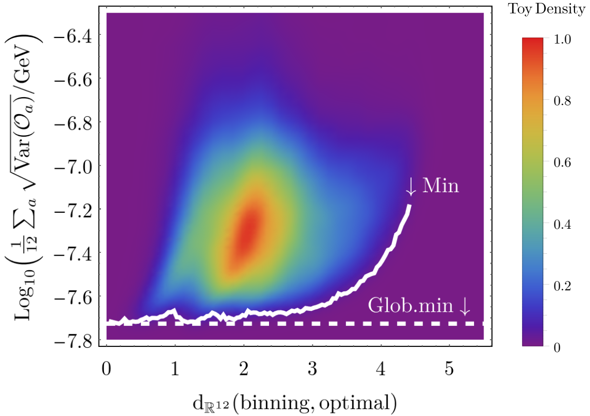
$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



$3 \times 3 \times 3$ (prod bins, gen. weights)

$$\{-\pi, \chi_1, \chi_2, \pi\}_\chi \times \{0, \theta_{\ell 1}, \theta_{\ell 2}, \pi\}_{\theta_\ell} \times \{0, \theta_{D^* 1}, \theta_{D^* 2}, \pi\}_{\theta_{D^*}}$$

$\longrightarrow$ Generate sample of **toy binnings** (i.e. random values $\chi_1, \chi_2, \theta_{\ell 1}, \theta_{\ell 2}, \Delta_{\theta_{D^*}}$)



Backup slides

Backup slides overview

- General bins
- The angular functions B_a
- Correlation
- Plots $3 \times 3 \times 2$ edges
- Plots $3 \times 3 \times 3$ edges