Relativistic Stars in dRGT Massive Gravity

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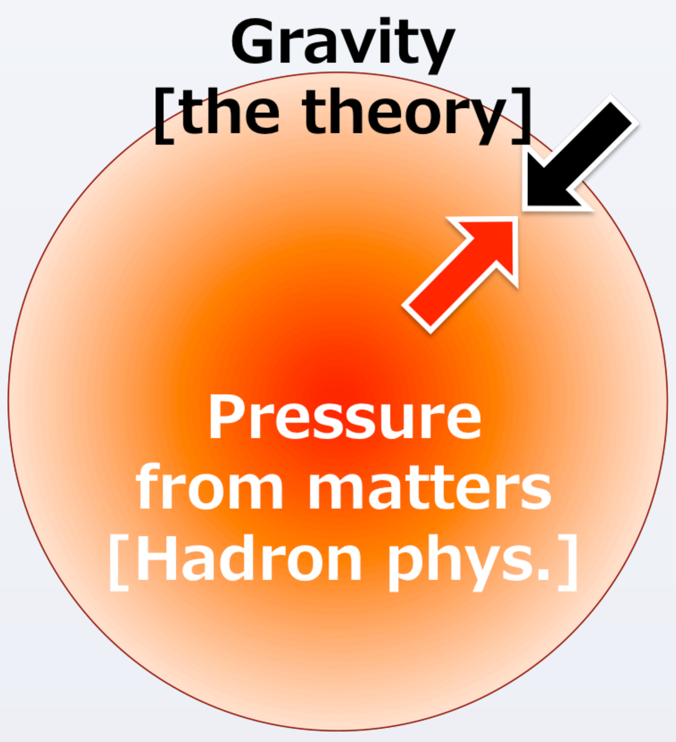
INTRODUCTION

Modified gravity, which is alternative theories of general relativity, is considered for explaining late-time accelerating universe without cosmological constant.
On the other hand, **the short range properties** should also be considered for its consistency. **Maximum mass of neutron stars** is one of phenomena that is influenced strongly by the short range properties.
The maximum mass is determined by its gravity and pressure from inner matters. Therefore gravity theory and hadron physics model determine the maximum mass.



From observation, a neutron star whose mass is **about 2M$_\odot$** is discovered. To get such a solution in modified gravity, we also have to consider **the hydrostatic equilibrium** (modified TOV eq.).
If some screening mechanism are active, the result will be similar to general relativity solutions.
If the mechanisms are not active, the result should be that the mass is **larger than 2M$_\odot$** because of the consistency. The result show whether the theory is suitable for astrophysics.

de Rham-Gabadadze-Tolley (dRGT) MASSIVE GRAVITY

The theory describe a ghost-free massive spin-2 field and **the mass** can be seen as **cosmological constant**. The action is made from the Einstein-Hilbert action plus mass and interaction terms.

$$S_{\mathrm{dRGT}} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\det(g)}\left[R - 2m_0^2\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)\right] + S_{\mathrm{matter}}$$

$$\to G_{\mu\nu} + m_0^2 I_{\mu\nu} = \kappa^2 T_{\mu\nu},\ \nabla_\mu I^{\mu\nu} = 0$$

- $g_{\mu\nu},\ f_{\mu\nu}$: dynamical and reference metrics
- $\kappa^2 = 8\pi G$: gravitational coupling
- $\beta_n,\ m_0$(graviton mass) : free parameters
- $e_n(X)$: n-th order polynomials of X

The condition for having flat spacetime in entirely vacuum restricts the free parameters. Moreover redefinition of these are reduced to **3 effective free parameters**.

$$\tilde{m}_0^2 := m_0^2\beta_0,\quad \tilde{\beta}_1 := \frac{\beta_1}{\beta_0},\quad \tilde{\beta}_2 := \frac{\beta_2}{\beta_0},\quad \tilde{\beta}_3 = -1 - 3\left(\tilde{\beta}_1 + \tilde{\beta}_2\right)$$

$$m_0^2\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right) = m_0^2\beta_0\sum_{n=0}^{4}\frac{\beta_n}{\beta_0}e_n\left(\sqrt{g^{-1}f}\right).\qquad m_0^2 I_{\mu\nu} \equiv \tilde{m}_0^2\tilde{I}_{\mu\nu}.$$

Our Ansatz and Modified TOV Equations

For our purpose, we assume **static and spherical dynamical metric** and the graviton mass is taken as cosmological constant $m_0^2 = \Lambda$. For simplicity, the reference metric is taken as **Minkowski metric**. We consider perfect fluid as the matter.

$$\begin{cases} g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -e^{2\nu(r)}\mathrm{d}t^2 + e^{2\lambda(r)}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right), \\ f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + \left(\chi'(r)\right)^2\mathrm{d}r^2 + \chi(r)^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right). \end{cases}$$

$$T^\mu{}_\nu = (-\rho(r), p(r), p(r), p(r)),$$

$$\nabla_\mu T^\mu{}_\nu = 0 \Rightarrow \nu'(r) = -\frac{p'(r)}{p(r)+\rho(r)}.$$

From these assumption, we obtain modified TOV equations. **The red letters** are modification from general relativity.

$$e^{-2\lambda(r)} \equiv 1 - \frac{2GM(r)}{r},$$

$$GM' = 4\pi G\rho r^2 + \frac{\tilde{m}_0^2}{2}r^2\tilde{I}^t{}_t(\chi,\chi',\lambda),$$

$$p' = -\frac{\left(4\pi Gpr^3 + GM - \tilde{m}_0^2 r^3\tilde{I}^r{}_r(\chi,\nu)\right)(p+\rho)}{r(r-2GM)},$$

$$\kappa^2 p = \left(\nu'' + \nu'^2 + \frac{\nu'}{r}\right)\left(1 - \frac{2GM}{r}\right) + \frac{1}{2}\left(\frac{1}{r} + \nu'\right)\left(1 - \frac{2GM}{r}\right)' + \tilde{m}_0^2\tilde{I}^\theta{}_\theta(\chi,\chi',\lambda,\nu).$$

The additional constraint $\nabla_\mu I^\mu{}_r = 0$ determines the additional degree of freedom $\chi(r)$.

4th order for χ

$$\begin{aligned}\nabla_\mu I^\mu{}_r =& \frac{\chi'}{r^2}e^{-\lambda-\nu}\Big\{\left(1 + 3\left(\tilde{\beta}_1 + \tilde{\beta}_2\right)\right)\chi(2 - 2e^\lambda + \chi\nu' e^\nu) \\ & - \tilde{\beta}_1\left[\left(\frac{2}{r} + \nu'\right)e^{-\lambda} - \frac{2}{r}\right]r^2 e^{\lambda+\nu} \\ & -2\tilde{\beta}_2\left[r(1 - e^\lambda) + \chi(1 - e^\lambda + r\nu')e^\nu\right]\Big\} \\ =&0\end{aligned}$$

$$-\frac{p'}{p+\rho} = \nu' = \frac{G}{r^2}\frac{4\pi pr^3 + M - \tilde{m}_0^2 r^3\tilde{I}^r{}_r}{1 - \dfrac{2GM}{r}}$$

2nd order for χ

Existence of the solution means existence of usual stars in dRGT massive gravity.
As a first step, we consider **the minimal model** that the constraint is reduced to a linear equation and can be solved uniquely.

$$\tilde{\beta}_2 = \tilde{\beta}_3 = 0 \Rightarrow \tilde{\beta}_1 = -\frac{1}{3}.$$

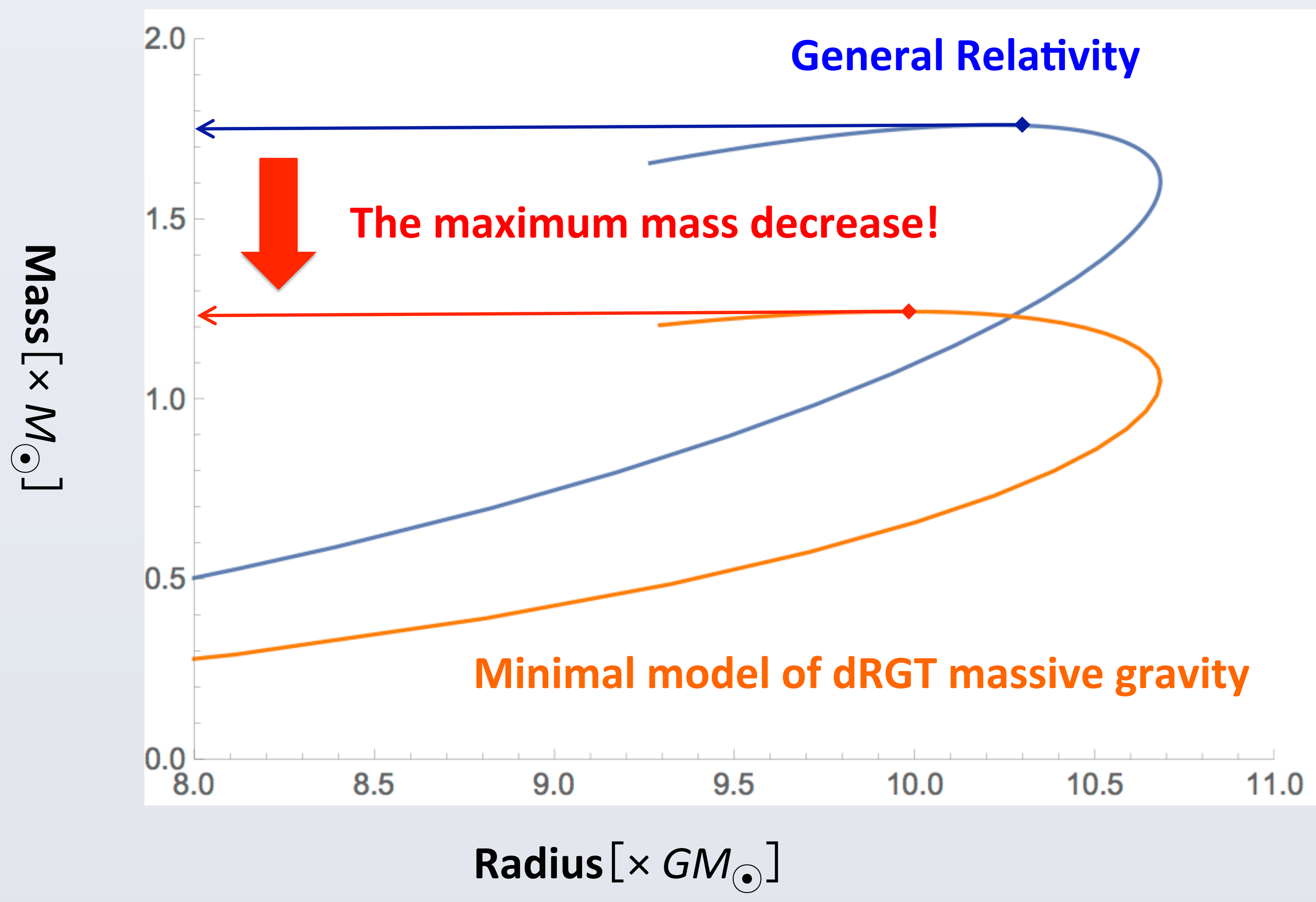
$$m_0^2 = \Lambda,\quad \beta_0 = 3,\quad \beta_1 = -1$$

NUMERICAL ANALYSIS

◎ Quark star cases (MIT bag model)

$P(\rho(r)) = c(\rho(r) - 4B)$

- c : It depends on chosen mass of strange quark. We used $c = 0.28$.
- B : bag constant. We use $B = 60\mathrm{MeV/fm}^3$.



It is known that **the screening mechanism (the Vainshtein mechanism) does not work** in the minimal model. If the mechanism is active, the additional attracting force is screened and the result becomes similar to general relativity case.

We **can also see the same result in SLy model** (one of the traditional star model).
The minimal model have a tendency that the maximum mass become smaller.

SUMMARY AND DISCUSSION

We derive modified TOV equations in dRGT massive gravity. The additional degree of freedom is determined by **additional 4th order algebraic equation**. The discriminant shows whether the dRGT massive gravity is suitable for astrophysics.

The minimal model does not have enough massive neutron stars because of absence of screening mechanism. We also use the same discussions in the other models if the solutions exist. We can check whether the screening mechanism works by checking the maximum mass of neutron stars.

There are future works as following.

- **Considering additional algebraic equation's discriminant** and existence of the solutions in several effective free parameters
- **Classifying the solution's branch** if the equation can have real solutions
- **Calculating the maximum mass** and comparing with the case of general relativity

Our final purpose is **restricting the effective parameter regions** that have consistent solutions as astrophysics phenomenology.