

# Effective Field Theory of inflation in Horava Lifshitz gravity

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## Abstract

We study the evolution of scalar perturbations in inflationary epoch with a single Lifshitz scalar in the context of the BPSH theory, which generalizes the original non-projectable Horava Lifshitz gravity. In the previous studies, power spectrum of the curvature in the fixed de-Sitter background, the evolution in the IR regime and the evolution the isocurvature mode in the Einstein Aether theory have been obtained. In our study, we consistently solve the evolution of the coupled scalar graviton and the inflaton fluctuation and we obtain only the adiabatic perturbation is generated because the scalar graviton become massive Hubble crossing. Moreover, we show that the detection of primordial gravitational waves can verify LV in inflationary epoch.

## 1. Introduction

**Horava Lifshitz gravity (HL gravity)** is the one of the modified theory of gravity which is **power-counting renormalizable** in the UV regime (P. Horava 2009).

$$\text{GR} \quad S_{EH} = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} R \rightarrow \frac{M_{\text{pl}}^2}{2} \int d^4x \{ (\partial h)^2 + h(\partial h)^2 + \dots \}$$

$\dim h = 1$     irrelevant term

$$\text{HL gravity} \quad S_{HL} = \frac{M_{\text{pl}}^2}{2} \int dt d^3x \sqrt{\gamma} N \{ (K_{ij} K^{ij} - \lambda K^2) + R^{(3)} + \mathcal{V} \}$$

anisotropic scaling     $\Delta R, R^2, R_{ij} R^{ij}, (\nabla_i R)^2, (\nabla_i R_{jk})^2$

$$t \rightarrow b^z t, \mathbf{x} \rightarrow b\mathbf{x} \Rightarrow h \rightarrow b^{(3-z)/2} h \quad \dim h = \frac{3-z}{2} \leq 0 \quad (z \geq 3)$$

But the anisotropic scaling law **violates the local Lorentz symmetry** in the UV regime. Then a new scalar degree of freedom appears called "**scalar graviton**" or "**khronon**".

## Purposes of our study

- Investigate consistent growth of the scalar graviton and the inflaton fluctuation in the Lifshitz regime.
- Derive the curvature perturbation and isocurvature perturbation.
- Compare the curvature & isocurvature power spectra with current CMB observation to constrain to LV scale and modulations of higher derivatives.

## 2. Formulation

We introduce the general action of the BPSH theory including the transition energy scale **M**, where the **local Lorentz symmetry is broken**.

$$\text{Gravity sector} \quad S_{HL} = \int dt d^3x N \sqrt{\gamma} \frac{M_*^2}{2} \left[ \frac{1}{\alpha_1} K_{ij} K^{ij} - \frac{1}{\alpha_2} K^2 + \frac{1}{\alpha_3} R + a_i a^i \right]$$

$$- \frac{1}{2} \left[ \frac{R_{ij} R^{ij}}{\beta_1} + \frac{R^2}{\beta_2} - \frac{R \nabla_i a^i}{\beta_3} + \frac{a_i \Delta a^i}{\beta_4} \right]$$

$$- \frac{1}{2M_*^2} \left[ \frac{(\nabla_i R_{jk})^2}{\gamma_1} + \frac{(\nabla_i R)^2}{\gamma_2} - \frac{\Delta R \nabla_i a^i}{\gamma_3} - \frac{a_i \Delta^2 a^i}{\gamma_4} \right]$$

$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$   
 $K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$   
 $a_i = \partial_i \ln N$  non-projectable condition  
 $R_{ij}$  3-dimensional Ricci tensor  
 $\Delta$  3-dimensional Laplacian

$$\text{Inflaton sector} \quad S_{\Phi} = \frac{1}{2\kappa^2} \int dt d^3x N \sqrt{g} \left[ \frac{(\dot{\Phi} - N^i \partial_i \Phi)^2}{2N^2} + \kappa_1 \frac{\Phi \Delta \Phi}{2} - \kappa_2 \frac{\Phi \Delta^2 \Phi}{2M_*^2} + \kappa_3 \frac{\Phi \Delta^3 \Phi}{2M_*^4} - V(\Phi) \right].$$

Inflaton is a single Lifshitz scalar field and thus higher derivative terms appear in the UV regime.

Then we take the metric ansatz of perturbation

### Metric ansatz

$$N = e^{\phi}, N_i = \partial_i B, \gamma_{ij} = a^2 e^{2\mathcal{R}} \delta_{ij}$$

➔  $\mathcal{R}$ : scalar graviton     $\phi$ : inflaton fluctuation

momentum & energy constraints

Then we compute the dynamics of 2 scalars that interact each other.

## 3. Results

### 3.1 initial density fluctuations

In some energy scale higher than **M**, the scalar graviton become massive due to non-locality of Hamiltonian constraint with its mass as

$$m_K = \frac{1}{\sqrt{\alpha}} H \gg H$$

Since the mass is much larger than Hubble scale, an adiabatic perturbation can be generated as

$$\zeta \equiv \mathcal{R} - \frac{H}{\dot{\phi}} \varphi \sim -\frac{H}{\dot{\phi}} \varphi$$

with its power spectrum as

$$\mathcal{P}_{\zeta} \propto \frac{\alpha_1}{\varepsilon} \left( \frac{H_p}{M_*} \right)^{3/z-1}$$

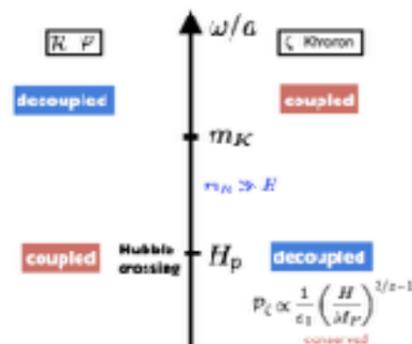


Figure 1: EFT picture of the HL gravity

### 3.2 Smoking gun for Lorentz Violation (LV)

Inflation models with 4D Diffs invariance, the consistency relation between scalar and tensor fluctuations are generally given by

$$n_t = -\frac{r}{8c_s}$$

which is **always negative nonzero value**. On the contrary in Horava Lifshitz gravity, the consistency relation is modified due to LV as

$$n_t \propto -\frac{3-z}{z} \frac{r}{16} \Rightarrow n_t = 0 \quad (z = 3)$$

which means **we can obtain a direct evidence of LV** if we detect primordial gravitational waves with totally scale invariant spectrum.

## 4. Summary & Future works

We have obtained the solutions of the scalar graviton and inflaton fluctuation. But our analysis has yet to complete among the regime where the curvature fluctuation conserves. Therefore we will have to **calculate the perturbations in the whole regimes** and obtain the curvature power spectrum. Then we will obtain **constraints of the LV scale and modulations of higher derivatives** by using the PLANCK data. Moreover, **the detection of primordial gravitational waves can verify LV in inflationary epoch**.

## 5. References

- P. Horava, Phys. Rev. D 79, 084008 (2009) [arXiv:0901.3775 [hep-th]]. P. Creminelli, J. Gleyzes, J. Norena and F. Vernizzi, Phys. Rev. Lett. 113, no. 23, 231301 (2014)[arXiv:1407.8439 [astro-ph.CO]]. D. Blas, O. Pujolas and S. Sibiryakov, JHEP 1104, 018 (2011) [arXiv:1007.3503 [hep-th]]. C. Armendariz-Picon, N.F. Sierra, J. Garriga. JCAP 007.010,2010[arXiv:1003.1283 [hep-th]]