Dispersion Relation of $\bar{D}$ mesons in the Chiral Density Wave

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Abstract

We calculate the dispersion relation of $\bar{D}$ mesons in the Chiral Density Wave (CDW). CDW is an inhomogeneous phase that spatially varying chiral condensation occurs, and it is said that this phase may emerge in the finite density region near the chiral restoration point.. In this phase, the potentials of $\bar{D}$ mesons take the form of cosine or sine function, then we have to employ the Bloch's theorem for $\bar{D}$ mesons wave function to get dispersion relations. Thereby, dispersion of $\bar{D}$ mesons drastically changes from that in the vacuum, e.g., the existence of Brillouin zone and the emergence of collective modes. These modifications are signals of the existence of CDW.

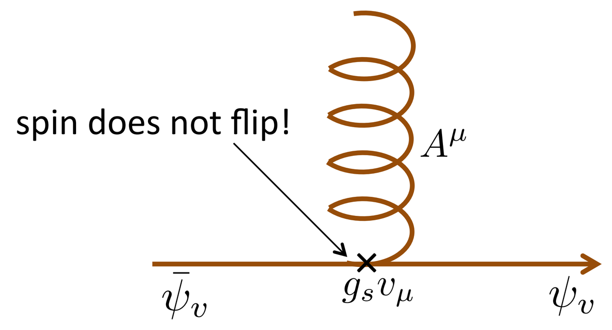
• Heavy Quark Symmetry

Heavy quarks have large masses compared with QCD scale $\Lambda_{\rm QCD}$, and hence magnetic gluon does not change the spin of heavy quarks:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - M_Q)\psi$$
$$= \bar{\psi}_v(\underline{iv\cdot D})\psi_v + O(1/M_Q)$$

no γ- matrices !

where $\psi_v = e^{iM_Q v\cdot x}\frac{1+\gamma^\mu v_\mu}{2}\psi$



Spin-up state and spin-down state is independent and equivalent
(Heavy Quark Symmtery)

When we regard that heavy hadron is a composite state of one heavy (anti-)quark and a cloud of light degrees of freedom **(Brown Muck)** , two states of $J_{\rm tot} = J_{\rm light} \pm \frac{1}{2}{}_{\rm heavy}$ have the same mass

heavy hadron ($J_{\rm tot}$)
($J_{\rm tot} = (J_{\rm light}-\frac{1}{2}, J_{\rm light}+\frac{1}{2})$ are degenerated)

For example...

$L_{\rm light}=0,\ S_{\rm light}=\frac{1}{2}$ ⇨ $J^P_{\rm tot} = (0^-, 1^-) = \underbrace{(\bar{D}, \bar{D}^*)}_{\text{degenerated}}$

$L_{\rm light}=1,\ S_{\rm light}=\frac{1}{2}$ ⇨ $J^P_{\rm tot} = (0^+, 1^+) = \underbrace{(\bar{D}_0^*, \bar{D}_1)}_{\text{degenerated}}$

Thanks to **Heavy Quark Symmetry** and description of **Brown Muck**, we can construct a concise lagrangian interacting with pion so easily.

• Chiral Partner Structure

The mass difference between opposite parity states comes from the breakdown of chiral symmetry **(Chiral Partner Structure)**

mass

parity-even $(\bar{D}_0^*, \bar{D}_1) \sim 2.40\ \mathrm{GeV}$

comes from chiral symmetry breaking

parity-odd $(\bar{D}, \bar{D}^*) \sim 1.97\ \mathrm{GeV}$

• Lagrangian

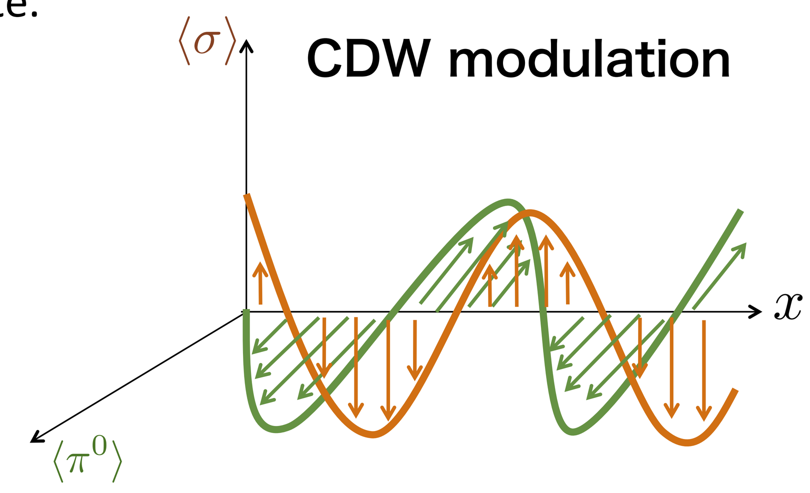
Lagrangian of $\bar{D}$ mesons interacting with pion which is based on the above ideas is

$$\begin{aligned}\mathcal{L} = {} & \partial_\mu\bar{D}_0^*\partial^\mu\bar{D}_0^{*\dagger} - M^2\bar{D}_0^*\bar{D}_0^{*\dagger} - \partial_\mu\bar{D}_{1\nu}\partial^\mu\bar{D}_1^{\dagger\nu} + \partial_\mu\bar{D}_{1\nu}\partial^\nu\bar{D}_1^{\dagger\mu} + M^2\bar{D}_{1\mu}\bar{D}_1^{\dagger\mu} \\ & +\partial_\mu\bar{D}\partial^\mu\bar{D}^\dagger - M^2\bar{D}\bar{D}^\dagger - \partial_\mu\bar{D}^*_\nu\partial^\mu\bar{D}^{*\dagger\nu} + \partial_\mu\bar{D}^*_\nu\partial^\nu\bar{D}^{*\dagger\mu} + M^2\bar{D}^*_\mu\bar{D}^{*\dagger\mu} \\ & -\frac{1}{2}M\Delta_M[\bar{D}_0^*(U+U^\dagger)\bar{D}_0^{*\dagger} - \bar{D}_{1\mu}(U+U^\dagger)\bar{D}_1^{\dagger\mu} - \bar{D}(U+U^\dagger)\bar{D}^\dagger + \bar{D}^*_\mu(U+U^\dagger)\bar{D}^{*\mu}] \\ & -\frac{1}{2}M\Delta_M[\bar{D}_0^*(U-U^\dagger)\bar{D}^\dagger - \bar{D}_{1\mu}(U-U^\dagger)\bar{D}^{*\dagger\mu} - \bar{D}(U-U^\dagger)\bar{D}_0^{*\dagger} + \bar{D}^{*\dagger}_\mu(U-U^\dagger)\bar{D}_1^{\dagger\mu}] \\ & -\frac{g}{2}M[\bar{D}_1^\mu(\partial_\mu U^\dagger - \partial_\mu U)\bar{D}_0^{*\dagger} - \bar{D}_0^*(\partial_\mu U^\dagger - \partial_\mu U)\bar{D}_1^{\dagger\mu} - \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\bar{D}'_{1\mu}(\partial_\nu U^\dagger - \partial_\nu U)i\partial_\sigma\bar{D}^\dagger_{1\rho}] \\ & +\frac{g}{2}M[\bar{D}^{*\mu}(\partial_\mu U^\dagger - \partial_\mu U)\bar{D}^\dagger - \bar{D}(\partial_\mu U^\dagger - \partial_\mu U)\bar{D}^{*\dagger\mu} - \frac{1}{M}\epsilon^{\mu\nu\rho\sigma}\bar{D}^*_\mu(\partial_\nu U^\dagger - \partial_\nu U)i\partial_\sigma\bar{D}^{*\dagger}_\rho] \\ & +\frac{g}{2}M[\bar{D}_1^\mu(\partial_\mu U^\dagger + \partial_\mu U)\bar{D} + \bar{D}(\partial_\mu U^\dagger + \partial_\mu U)\bar{D}_1^{\dagger\mu}] \\ & -\frac{g}{2}M[\bar{D}_0^*(\partial_\mu U^\dagger + \partial_\mu U)\bar{D}^{*\dagger\mu} + \bar{D}^{*\mu}(\partial_\mu U^\dagger + \partial_\mu U)\bar{D}_0^{*\dagger}] \\ & -\frac{g}{2}[\epsilon^{\mu\nu\rho\sigma}\bar{D}_{1\nu}(\partial_\rho U^\dagger + \partial_\rho U)i\partial_\sigma\bar{D}^{*\dagger}_\mu + \epsilon^{\mu\nu\rho\sigma}\bar{D}^*_\mu(\partial_\rho U^\dagger + \partial_\rho U)i\partial_\sigma\bar{D}^\dagger_{1\nu}]\end{aligned}$$

$M = 2185\ \mathrm{MeV}$ is averaged mass of $(\bar{D}_0^*, \bar{D}_1)$ and $(\bar{D}, \bar{D}^*)$.
$\Delta_M = 430\ \mathrm{MeV}$ is mass difference between $(\bar{D}_0^*, \bar{D}_1)$ and $(\bar{D}, \bar{D}^*)$.
$|g| = 0.50$ is estimated by the decay width of $\Gamma[D^{*+} \to D^0\pi^+]$.

• Chiral Density Wave

Chiral Density Wave (CDW) phase is an inhomogeneous phase where spatially varying chiral condensation occurs, especially scalar mode $\langle\bar{q}q\rangle$ and neutral pseudo-scalar mode $\langle\bar{q}i\gamma_5\tau^3 q\rangle$ condensate.

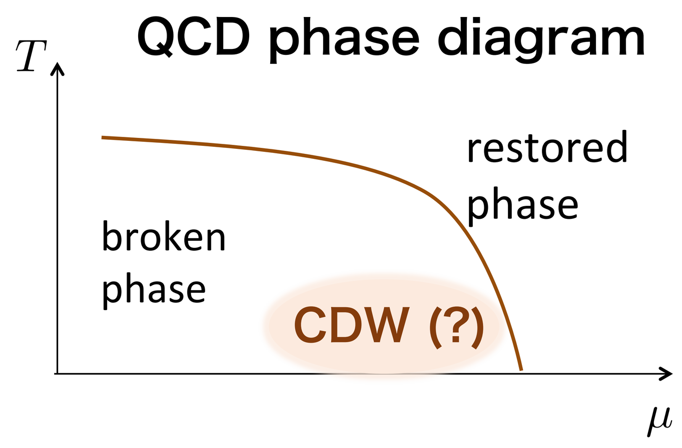


In this phase, pion is expressed as

$$U = \sigma + i\tau^a\pi^a$$

with $\langle\sigma\rangle = \cos(2fx)$ $\langle\pi^1\rangle = 0$
$\langle\pi^3\rangle = \sin(2fx)$ $\langle\pi^2\rangle = 0$

Some calculation suggest the existence of CDW phase near the chiral restoration point at low temperature



• EoMs in CDW

Equation of motion of $\bar{D}$mesons in CDW are

$$(\partial_\mu\partial^\mu + M^2)\bar{D} - M\Delta_M\cos(2fx)\bar{D} + iM\Delta_M\sin(2fx)\bar{D}_0^*\tau^3 + \cdots = 0$$
$$(\partial_\mu\partial^\mu + M^2)\bar{D}^{*i} - M\Delta_M\cos(2fx)\bar{D}^{*i} + iM\Delta_M\sin(2fx)\bar{D}_1^i\tau^3 \cdots = 0$$
$$(\partial_\mu\partial^\mu + M^2)\bar{D}_0^* + M\Delta_M\cos(2fx)\bar{D}_0^* - iM\Delta_M\sin(2fx)\bar{D}\tau^3 + \cdots = 0$$
$$(\partial_\mu\partial^\mu + M^2)\bar{D}_1^i + M\Delta_M\cos(2fx)\bar{D}_1^i - iM\Delta_M\sin(2fx)\bar{D}^{*i}\tau^3 + \cdots = 0$$

EoMs contain some periodic potentials with period π/f !

The Bloch's theorem

In a periodic potential with its period $a = 2\pi/K$, wave function is of the form

$$\psi_k(x) = e^{ikx}u_k(x) \quad \text{where} \quad u_k(x) \equiv \sum_{K'} C_{k-K'}e^{-iK'x} \quad (\ u_k(x+a) = u_k(x)\ )$$

k is so-called crystal momentum lies in the first Brillouin zone
and K' is the reciprocal lattice vector $K' = 0, \pm K, \pm 2K, \cdots$

and energy eigenvalue satisfies $E_k = E_{k\pm K} = E_{k\pm 2K} = \cdots$

We can restrict momentum in the region of $-K/2 \le k \le +K/2$ **(first Brillouin zone)**

Schrodinger equation $(H_0 + V)\psi = E\psi$ is reduced to

$$(E^0_{k-K} - E)C_{k-K} + \sum_{K'} V_{K'-K}C_{k-K'} = 0$$

All Fourier coefficients $\cdots, C_{k-K}, C_k, C_{k+K}, \cdots$ can mix each others

These correlations are the consequences of the Bloch's Theorem

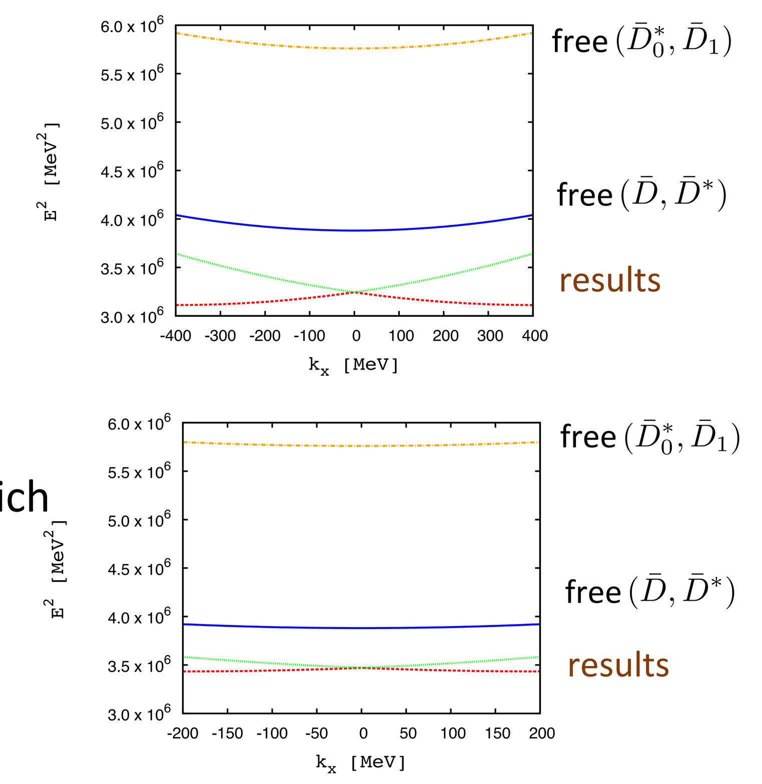
• Results

Result for $f = 400\ \mathrm{MeV}$ and $f = 200\ \mathrm{MeV}$ is plotted respectively. The lowest two levels are plotted in figures, however we can expect more higher levels.

The lowest two levels coincide at rest frame which is the consequence of chiral partner structure.

Energy gap between the lowest two levels is a few hundred MeV at most.

As the wavenumber f becomes smaller, energy gap becomes smaller.



Dispersion of $\bar{D}$ mesons for $f = 400\ \mathrm{MeV}$ and $f = 200\ \mathrm{MeV}$, with $k_\perp^2 = 0$. Blue, orange, red and green line describes the dispersion of free $(\bar{D}, \bar{D}^*)$ mesons, $(\bar{D}_0^*, \bar{D}_1)$ mesons, the lowest level and second level, respectively.

These modifications of dispersion of $\bar{D}$ mesons are the consequences of the existence of CDW phase !