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Flavor Problem,

SUSY Flavor Problem and

Flavor Symmetries

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Plan

I Motivation to Flavor (Family) Symmetry:
Solving (softening) Flavor problems

II Tow Examples: Q6 and D7 models

1. Predictions
2. The SUSY Flavor Problem

III Conclusion

The Flavor Problem

No principle to fix
the structure of the Yukawa sector

* Why Ng=3?
* What is the origin of
  the fermion masses hierarchy?

* Why $V_{\rm CKM} \not\simeq V_{\rm MNS}$?

and so on.

*(The most important to me) one is:

The problem of
the parameter redundancy
in the Yukawa sector

$$(Y_{ij} \times 2)$$

The total # of the free parameters = 36(36-8=28) in the quark sector.

But only 10

$$(m_q,\ q = u, d, \ldots, t \text{ and } V_{\rm CKM})$$

are observable!

36(28)-10=26(18) parameters can be anything they like.

This is the very nature of the SM

The Yukawa couplings in the basis of the flavor eigenstates are not measurable.

or equivalently,

$U_{uL},\ U_{dL}, U_{uR}, U_{dR}$ are not measurable in the SM.

But they may become physical parameters if the SM is extended.

In a naively extended model there will be more free parameters than in the SM

gauge field for R

$$\overline{\psi}\, q_R\, \phi \;,\; \overline{\psi}\, q_R\, W_R$$

making U_{uR}, U_{dR} observable.

The most famous example is the MSSM.

gaugino

THE SUSY flavor problem

A popular assumption is:

Unified Theory can solve the flavor problem.

Our standpoint is:

The problem can be partially solved by a low energy flavor symmetry.

The SUSY Flavor Problem

More than <u>100</u> soft parameters into the SM.

They induce FCNCs and $\not{CP}$ that are extremely suppressed in the SM.

$$\mu \to e + \gamma \ , \quad b \to s + \gamma \ ,$$

and EDMs.

$$\Delta M_K \ , \quad \Delta M_B \ , \quad \epsilon_K \ , \quad \epsilon'/\epsilon \ ,$$

Extreme fine-tuning of the soft ~~SUSY~~ parameters is needed to make the MSSM consistent.

The SUSY flavor problem

A popular assumption is:

Hidden Sector Scenario

Gravity, Gauge, Anomaly, Gaugino, etc

We use

low-energy flavor symmetry

to constrain the Yukawa sector, and simultaneously
to soften the SUSY flavor problem.

No hidden sector scenario

Testable predictions

II Two Examples (D7 and Q6)

- (D7,Q6) CONTINUOUS VS DISCRETE
- (D7,Q6) NON-ABELIAN VS ABELIAN
- (D7,Q6) SOFTLY OR SPONTANEOUSLY VS HARDLY BROKEN
- (D7,Q6) LOW ENERGY VS HIGH ENERGY
- (Q6) FOR THE QUARKS AND LEPTONS VS (D7) ONLY FOR THE QUARKS OR ONLY FOR THE LEPTONS
- (D7,Q6) RENORMALIZABLE VS NON-RENORMALIZABLE

g	
6	$D_3 \equiv S_3$
8	$D_4, Q = Q_4$
10	D_5
12	D_6, Q_6, T
14	D_7
16	$D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$
18	$D_9, Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T,$
	$Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

Few words on finite groups of lower orders

Classification of Finite Groups

(Frampton and Kephart, '01)

Twisted products $([Z_M\ ,\ Z_N] \neq 0)$

g	
16	$Z_2 \tilde{\times} Z_8$ (two, excluding D_8), $Z_4 \tilde{\times} Z_4$, $Z_2 \tilde{\times} (Z_2 \times Z_4)$ (two)
18	$Z_2 \tilde{\times} (Z_3 \times Z_3)$
20	$Z_4 \tilde{\times} Z_5$
21	$Z_3 \tilde{\times} Z_7$
24	$Z_3 \tilde{\times} Q, Z_3 \tilde{\times} Z_8, Z_3 \tilde{\times} D_4$
27	$Z_9 \tilde{\times} Z_3, Z_3 \tilde{\times} (Z_3 \times Z_3)$

Literature on Finite Groups: Landau and Lifschitz," Quantum mechanics"

Finite Groups

The classification of the finite groups has been completed 1981 (Gorenstein); about 100 years later than the case of the continues group.

g= order of a finite group
= # of the group elements

1. No non-abelian finite group exists for odd g.

2. For smaller g there exist only three types.

a) Permutation groups

$$S_N\ ,\ \ N=3,4,5,\ldots\ \ ,\ \ A_N\ ,\ \ N=4,5,\ldots$$

b) Dihedral groups

$$D_N\ ,\ \ N=3,4,5,\ldots$$

and

Binary dihedral (Dicyclic) group

$$Q_{2N}\ ,\ \ N=2,3,4,\ldots$$

c) Twisted products of

$$Z_M\times Z_N$$

3. The smallest non-abelian finite group is $S_3=D_3$.

D_7 model by Chen+Ma, hep-ph/0505064

$$D_7 : \mathbf{1}_+ \ , \ \mathbf{1}_- \ , \ \mathbf{2}_1 \ , \ \mathbf{2}_2 \ , \ \mathbf{2}_3$$

$$(u,d)_i \sim \mathbf{2}_1 + \mathbf{1}_+ \ , \ u_i^c \sim \mathbf{2}_2 + \mathbf{1}_+ \ , \ d_i^c \sim \mathbf{2}_1 + \mathbf{1}_+$$

$$H_i^u \sim \mathbf{2}_3 + \mathbf{1}_+ \ , \ H_i^d \sim \mathbf{2}_1 + \mathbf{1}_+$$

$$M_u = \text{diag.} \ , \ M_d = \begin{pmatrix} 0 & c & \xi b \\ c & 0 & b \\ \xi b' & b' & a \end{pmatrix}$$

a,b,b',c=real

ξ ~<H1>/<H2>=complex

6 parameters for md, ms, mb and Vckm

Q6 *model* by Babu+Kubo, hep-ph/0411226

$$Q_6 : \mathbf{1}_{+0}\ ,\ \mathbf{1}_{+2}\ ,\ \mathbf{1}_{-1}\ ,\ \mathbf{1}_{-3}\ ,\ \mathbf{2}_1\ ,\ \mathbf{2}_2$$

$$(u,d)_i \sim \mathbf{2}_1 + \mathbf{1}_{+2}\ ,\ u_i^c, d_i^c \sim \mathbf{2}_2 + \mathbf{1}_{-1}$$

$$(\nu,e)_i \sim \mathbf{2}_2 + \mathbf{1}_{+0}\ ,\ \nu_i^c \sim \mathbf{2}_2 + \mathbf{1}_{-3}\ ,\ e_i^c \sim \mathbf{2}_2 + \mathbf{1}_{+0}$$

$$H_i^u \sim \mathbf{2}_2 + \mathbf{1}_{-1}\ ,\ H_i^d \sim \mathbf{2}_2 + \mathbf{1}_{-1}$$

More singlet Higgses, spontaneous ~~CP~~

$$M_u\ ,\ M_d = \begin{pmatrix} 0 & c_{u,d} & b_{u,d} \\ c_{u,d} & 0 & b_{u,d} \\ b'_{u,d} & b'_{u,d} & a_{u,d} \end{pmatrix}\ ,\ M_D = \begin{pmatrix} -c_\nu & c_\nu & 0 \\ c_\nu & c_\nu & 0 \\ b'_\nu & b'_\nu & a_\nu \end{pmatrix}\ ,\ M_e = \begin{pmatrix} -c_e & c_e & b_e \\ c_e & c_e & b_e \\ b'_e & b'_e & 0 \end{pmatrix}$$

a, b, b', c=real

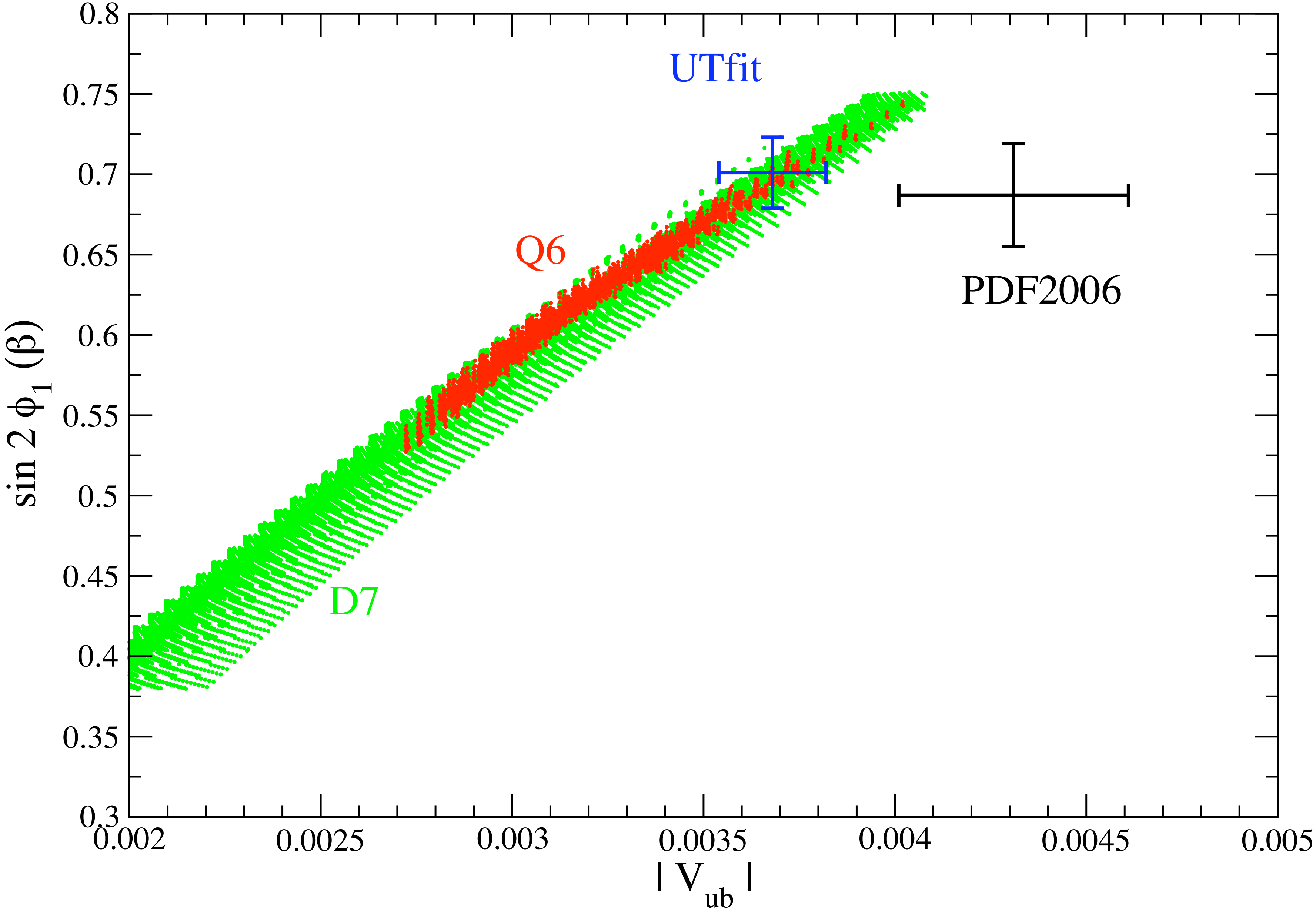
<H1> = <H2>

one phase for Vckm

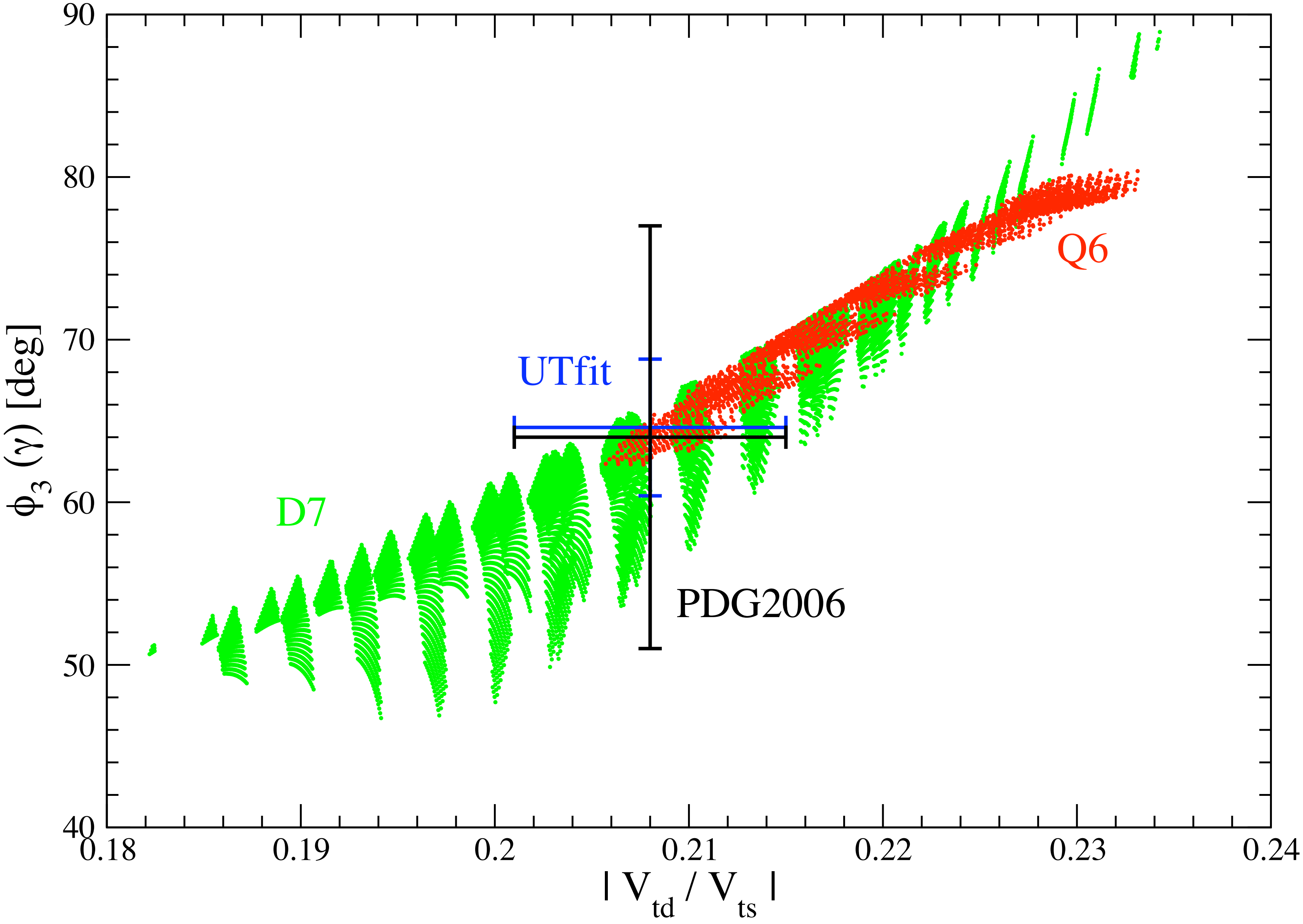
two phase for Vmns

9 parameters for m_quark, Vckm (10)

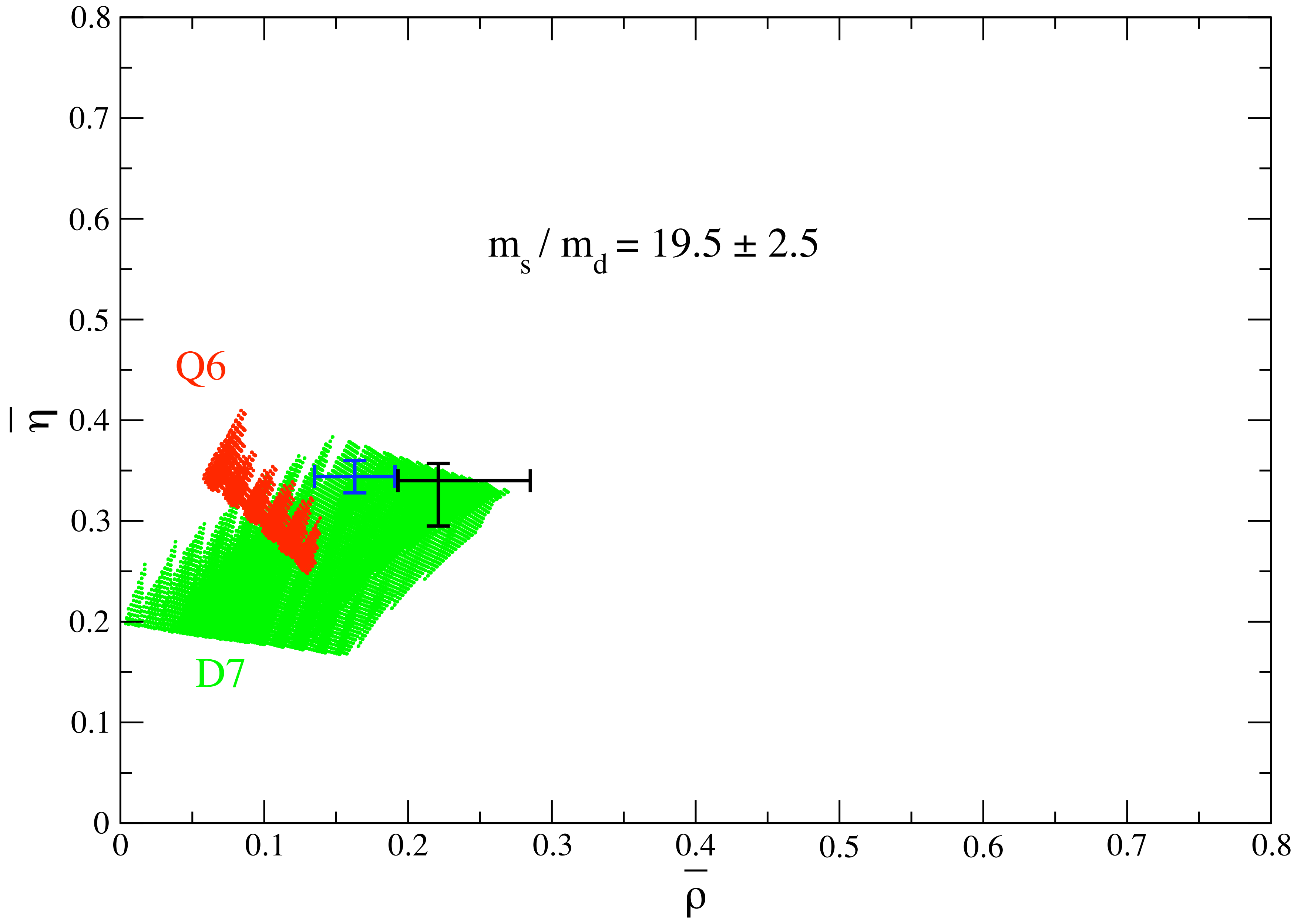
8 parameters for m_lepton, Vnms (12)



UTfit
Q6
D7
PDF2006
sin 2 φ1 (β)
| Vub |
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.002
0.0025
0.003
0.0035
0.004
0.0045
0.005



90
80
70
60
50
40
0.18
0.19
0.2
0.21
0.22
0.23
0.24
ϕ_3 (γ) [deg]
| V_{td} / V_{ts} |
UTfit
PDG2006
D7
Q6



$m_s / m_d = 19.5 \pm 2.5$
Q6
D7
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\bar{\eta}$
$\bar{\rho}$

u

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.5$ to 3.0 MeV [a]   Charge $= \frac{2}{3}\ e$ $I_z = +\frac{1}{2}$

$m_u/m_d = 0.3$ to 0.6

d

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 3$ to 7 MeV [a]

$m_s/m_d = 17$ to 22 → $19.5 \pm 2.5 \times 1.5$

$\overline{m} = (m_u + m_d)/2 = 2.5$ to 5.5 MeV

s

$I(J^P) = 0(\frac{1}{2}^+)$

Mass $m = 95 \pm 25$ MeV [a]   Charge $= -\frac{1}{3}\ e$ Strangeness $= -1$

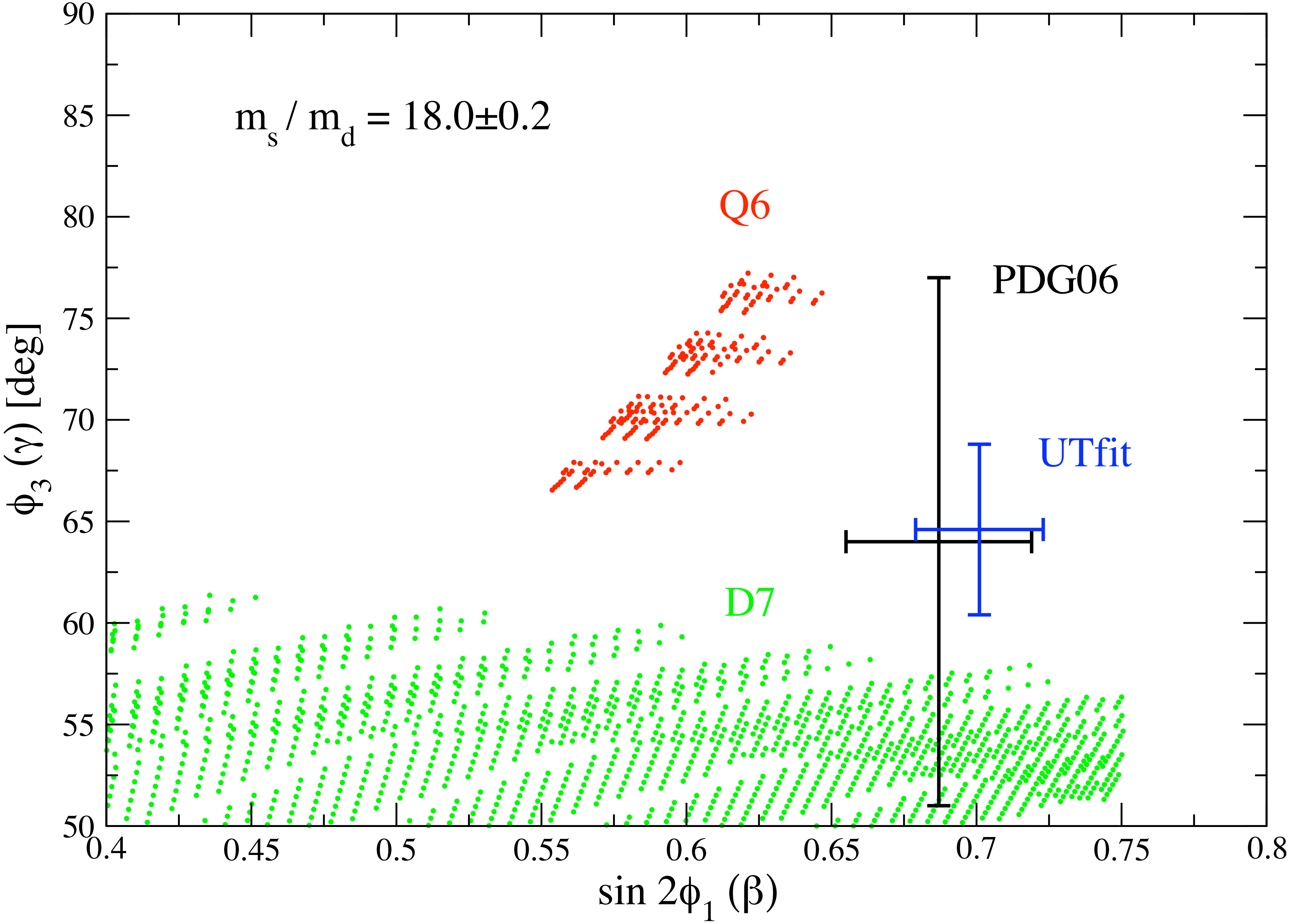
$(m_s - (m_u + m_d)/2)/(m_d - m_u) = 30$ to 50

c

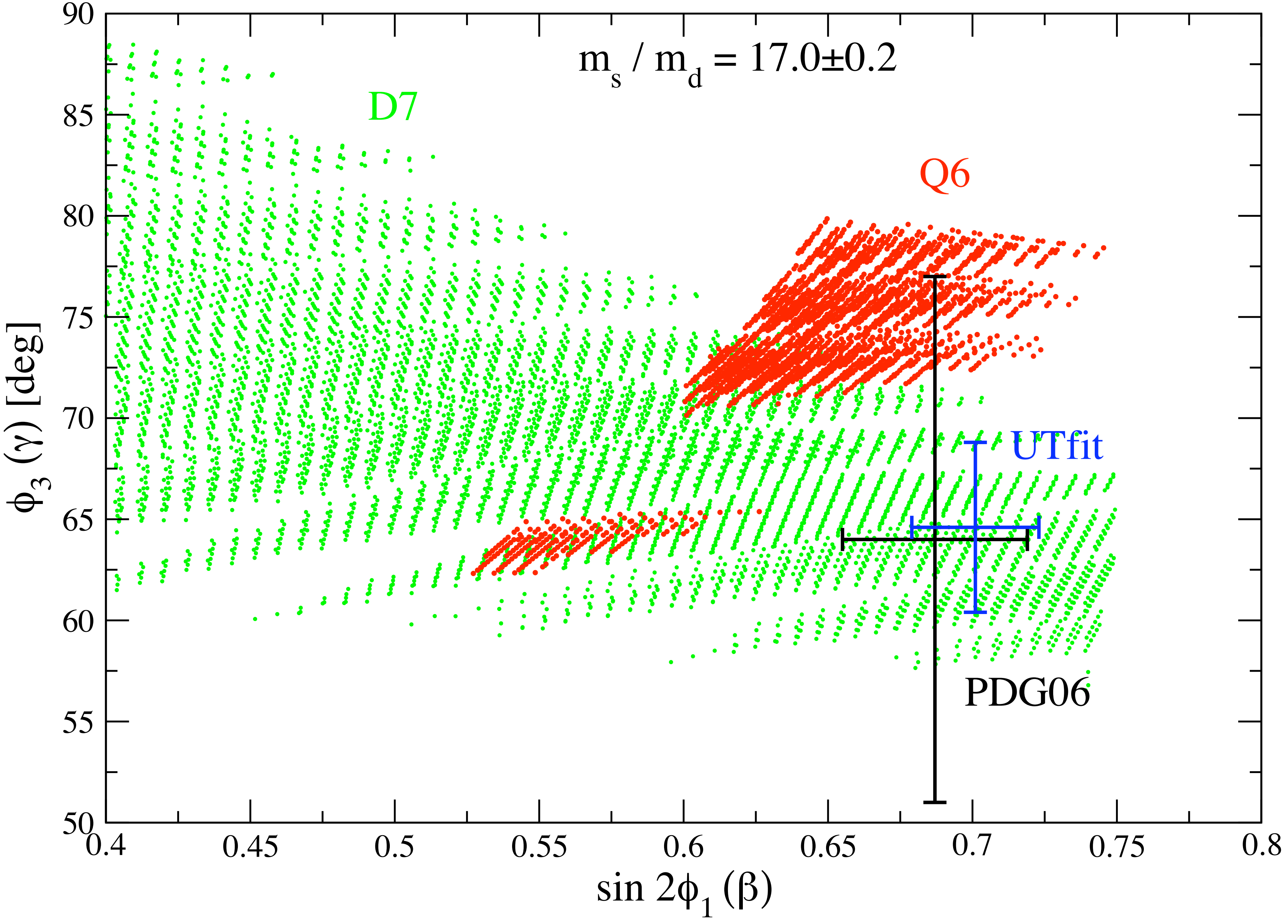
$I(J^P) = 0(\frac{1}{2}^+)$

Mass $m = 1.25 \pm 0.09$ GeV   Charge $= \frac{2}{3}\ e$ Charm $= +1$

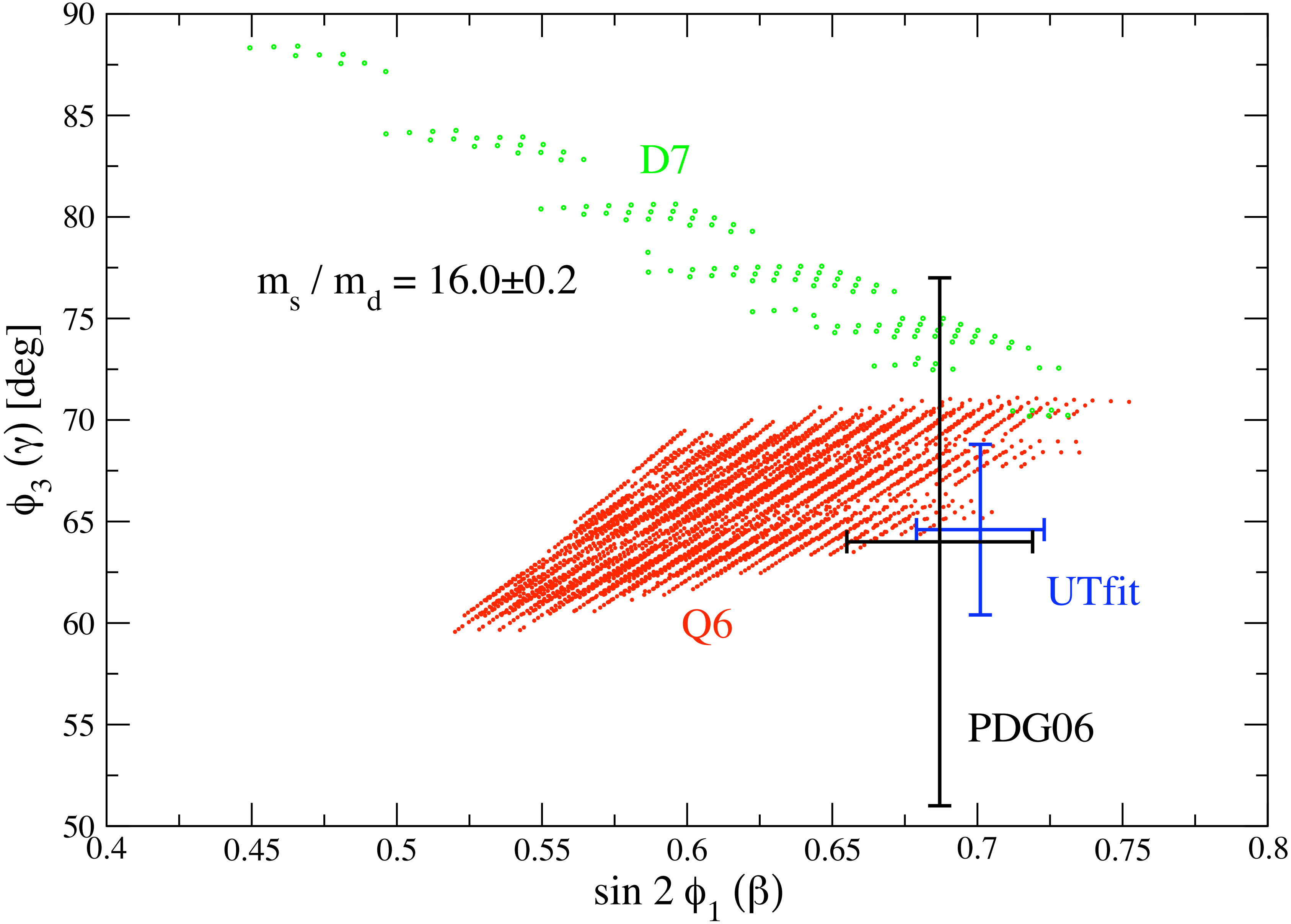
$$m_s/m_d = 19.25 \pm 2.5 \text{ (PDG2006)}$$



$m_s/m_d = 19.25 \pm 2.5$ (PDG2006)



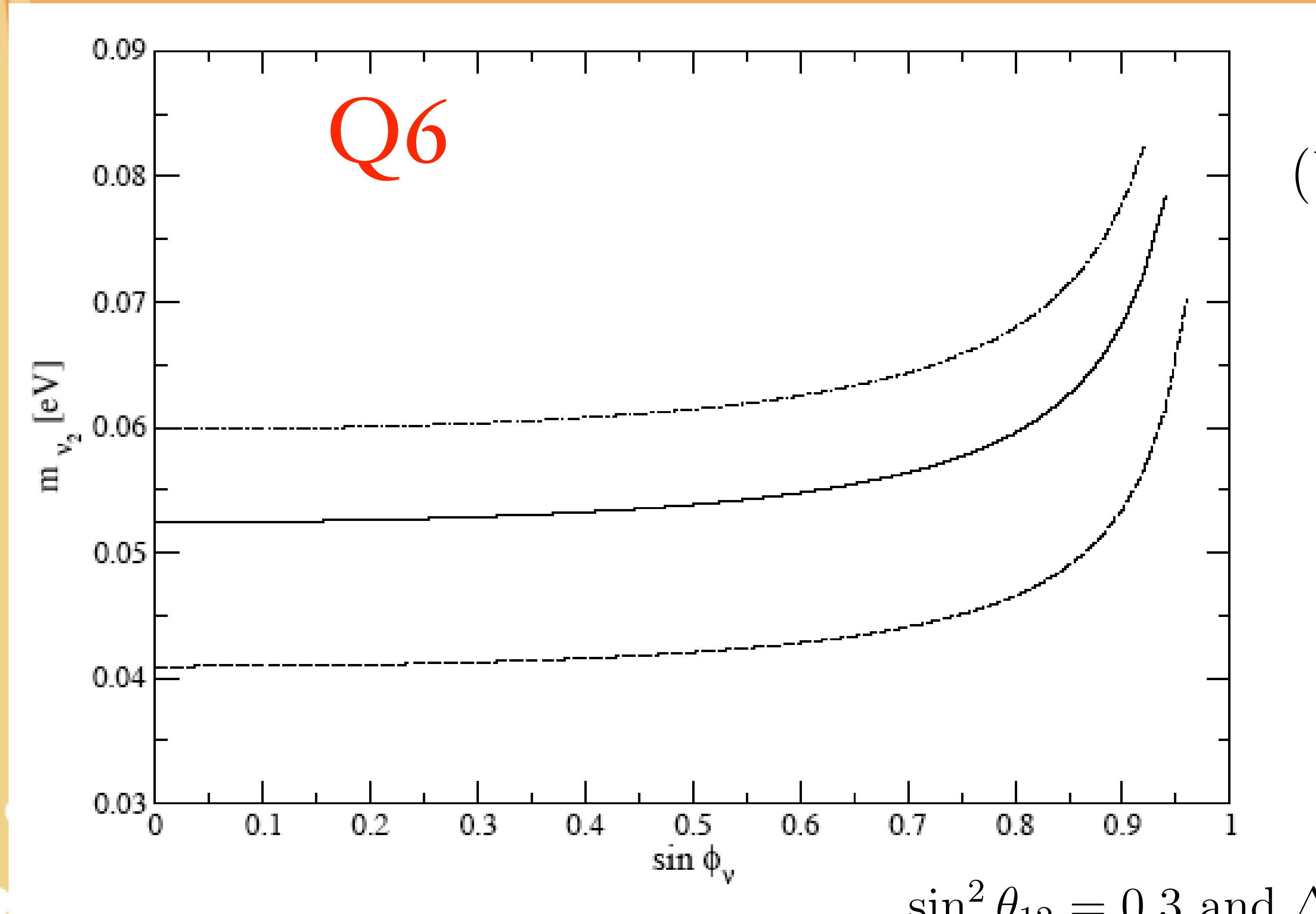
$m_s/m_d = 19.25 \pm 2.5$ (PDG2006)



1. Inverted neutrino mass spectrum, i.e., $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$

2. $m_{\nu_2}^2 / \Delta m_{23}^2 = \frac{(1+2t_{12}^2+t_{12}^4-rt_{12}^4)^2}{4t_{12}^2(1+t_{12}^2)(1+t_{12}^2-rt_{12}^2)\cos^2\phi_\nu} - \tan^2\phi_\nu$

$(r = \Delta m_{21}^2/\Delta m_{23}^2,\ t_{12} = \tan\theta_{12})$



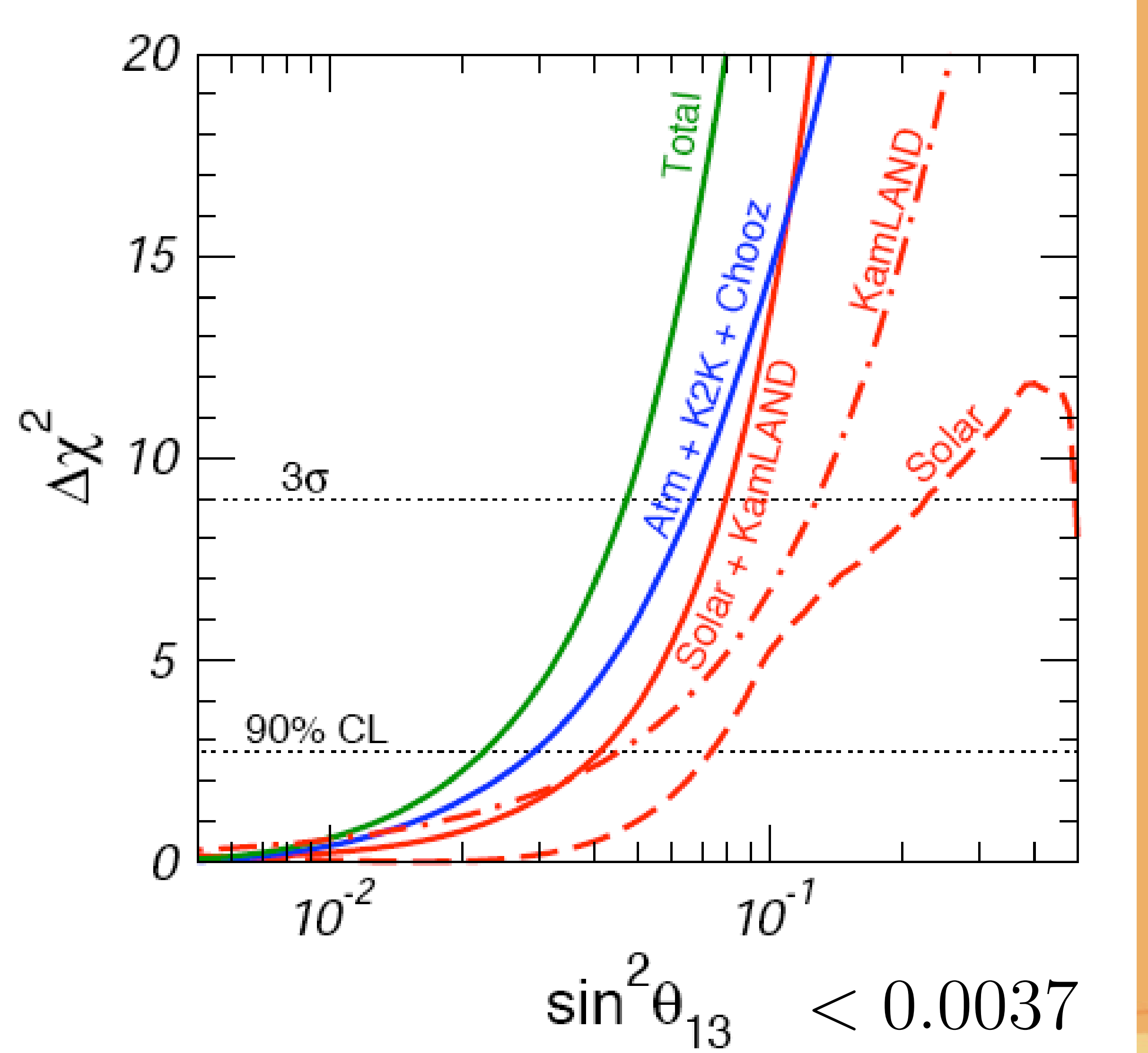
$(WMAP :\lesssim 0.2 \text{ eV})$

$\sin^2\theta_{12} = 0.3$ and $\Delta m_{21}^2 = 6.9\times 10^{-5}\ \text{eV}^2$

$\Delta m_{23}^2 = 1.4, 2.3$ and $3.0\times 10^{-3}\ \text{eV}^2$

3. $\sin^2\theta_{13} = \frac{1}{2}(m_e/m_\mu)^2 \simeq 10^{-6}$

$\sin^2\theta_{23} = \frac{1}{2} + O(m_e/m_\mu)^2) \simeq 10^{-6}$

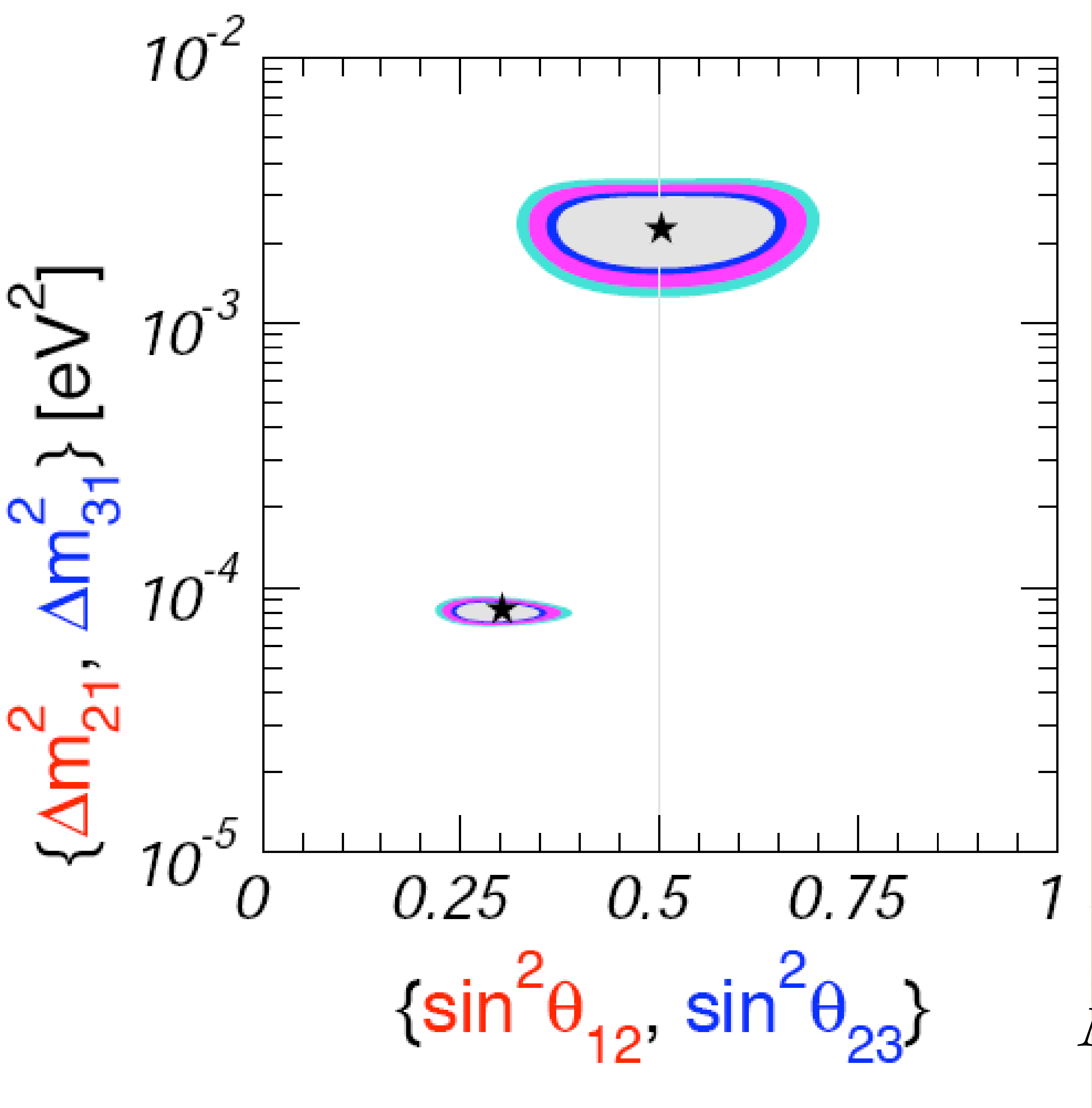


< 0.0037

Maltoni et al

3. $\sin^2\theta_{13} = \frac{1}{2}(m_e/m_\mu)^2 \simeq 10^{-6}$

$\sin^2\theta_{23} = \frac{1}{2} + O(m_e/m_\mu)^2) \simeq 10^{-6}$



Maltoni et al

3. The SUSY Flavor problem (Q6)

Q_6 →

$$\tilde{\mathbf{m}}^{\mathbf{2}}_{(q,\ell)LL} = m^2_{\tilde{q},\tilde{\ell}} \begin{pmatrix} a^{q,\ell}_L & 0 & 0 \\ 0 & a^{q,\ell}_L & 0 \\ 0 & 0 & b^{q,\ell}_L \end{pmatrix}$$

$$\tilde{\mathbf{m}}^{\mathbf{2}}_{aRR} = m^2_{\tilde{q},\tilde{\ell}} \begin{pmatrix} a^{a}_R & 0 & 0 \\ 0 & a^{a}_R & 0 \\ 0 & 0 & b^{a}_R \end{pmatrix} \quad (a = u, d, e)$$

$$(\tilde{\mathbf{m}}^2_{aLR})_{ij} = A^a_{ij}\ (\mathbf{m}^a_F)_{ij} \quad (a = u, d, e)$$

$$\Delta a^{q,\ell}_L = a^{q,\ell}_L - b^{q,\ell}_L, \ \Delta a^a_R = a^a_R - b^a_R$$

A parametrization of FCNCs and $\not{C}P$ in the superCKM basis:

$$\Delta^a_{LL,RR} = U^\dagger_{aL,R}\, \tilde{\mathbf{m}}^{\mathbf{2}}_{aLL,RR}\, U_{aL,R} \text{ and } \Delta^a_{LR} = U^\dagger_{aL}\, \tilde{\mathbf{m}}^{\mathbf{2}}_{aLR}\, U_{aR}$$

$$\delta^a_{LL,RR,LR} = \Delta^a_{LL,RR,LR}/m^2_{\tilde{a}} \quad (a = \ell, q)$$

$\mathrm{Im}\ \delta' s \rightarrow$ CP violations

Nondiag. $\delta' s \rightarrow$ FCNCs

A. CP violations induced by the soft terms

Q_6 *and spontaneous* $\not{CP}$

Phase alignment

$$\Delta_{LL,RR} = U^{\dagger}_{L,R}\, \tilde{\mathbf{m}}^{\mathbf{2}}_{LL,RR}\, U_{L,R} = \mathrm{real}$$

$$\Delta_{LR} = U^{\dagger}_{aL}\, \tilde{\mathbf{m}}^{\mathbf{2}}_{LR}\, U_{R} = \mathrm{real}$$

The most stringent constraints coming from EDMs are satisfied.

B. FCNCs induced by the soft terms

(Kobayashi, Terao and Kubo, '04)

Lepton sector

	Exp. bound	Q_6 Model
$\lvert(\delta^l_{12})_{LL}\rvert$	$4.0 \times 10^{-5}\ \tilde{m}^2_{\tilde{l}}$	$4.8 \times 10^{-3}\Delta_L$
$\lvert(\delta^l_{12})_{RR}\rvert$	$9 \times 10^{-4}\ \tilde{m}^2_{\tilde{l}}$	$8.4 \times 10^{-8}\Delta_e$
$\lvert(\delta^l_{13})_{LL}\rvert$	$2 \times 10^{-2}\ \tilde{m}^2_{\tilde{l}}$	$1.710^{-5}\Delta_L$
$\lvert(\delta^l_{23})_{LL}\rvert$	$2 \times 10^{-2}\ \tilde{m}^2_{\tilde{l}}$	$8.4 \times 10^{-8}\Delta_L$
$\lvert(\delta^l_{23})_{LL}(\delta^d_{13})_{LL}\rvert$	$1 \times 10^{-4}\ \tilde{m}^2_{\tilde{l}}$	$1.4 \times 10^{-12}\sqrt{\Delta_L\Delta_e}$
$\lvert(\delta^l_{23})_{LL}(\delta^l_{13})_{RR}\rvert$	$2 \times 10^{-5}\ \tilde{m}_{\tilde{q}}$	$5 \times 10^{-9}\sqrt{\Delta_L\Delta_e}$
$\lvert(\delta^l_{23})_{RR}(\delta^l_{13})_{RR}\rvert$	$9 \times 10^{-4}\ \tilde{m}^2_{\tilde{l}}$	$8.3 \times 10^{-8}\sqrt{\Delta_L\Delta_e}$
$\lvert(\delta^l_{23})_{RR}(\delta^l_{13})_{LL}\rvert$	$2 \times 10^{-5}\ \tilde{m}^2_{\tilde{l}}$	$2.4 \times 10^{-11}\sqrt{\Delta_L\Delta_e}$
$\lvert(\delta^l_{12})_{LR}\rvert$	$8.4 \times 10^{-7}\ \tilde{m}^2_{\tilde{l}}$	$\sim 10^{-6}\tilde{m}^{-1}_{\tilde{l}}$
$\lvert(\delta^l_{13})_{LR}\rvert$	$1.7 \times 10^{-2}\ \tilde{m}^2_{\tilde{l}}$	$\sim 10^{-7}\tilde{m}^{-1}_{\tilde{l}}$
$\lvert(\delta^l_{23})_{LR}\rvert$	$1 \times 10^{-2}\ \tilde{m}^2_{\tilde{l}}$	$\sim 10^{-9}\tilde{m}^{-1}_{\tilde{l}}$

Table 2: Experimental bounds on δ's, where the parameter $\tilde{m}_{\tilde{l}}$ denote $m_{\tilde{l}}/100$ GeV.

Quark sector

	Exp. bound	Q_6 Model
$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LL,RR} \rvert}$	$4.0 \times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$(LL) 1.2 \times 10^{-4} \Delta_Q, (RR) 1.7 \times 10^{-1} \Delta_d$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})_{LL} (\delta^d_{12})_{RR} \rvert}$	$2.8 \times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$4.5 \times 10^{-3} \sqrt{\Delta_Q \Delta_d}$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LR} \rvert}$	$4.4 \times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4} \tilde{m}_{\tilde{q}}^{-1}$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{13})^2_{LL,RR} \rvert}$	$9.8 \times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$(LL) 7.9 \times 10^{-3} \Delta_Q, (RR) 1.4 \times 10^{-1} \Delta_d$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{13})_{LL} (\delta^d_{13})_{RR} \rvert}$	$1.8 \times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$3.4 \times 10^{-2} \sqrt{\Delta_Q \Delta_d}$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{13})^2_{LR} \rvert}$	$3.3 \times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4} \tilde{m}_{\tilde{q}}^{-1}$
$\sqrt{\lvert \mathrm{Re}(\delta^u_{12})^2_{LL,RR} \rvert}$	$1.0 \times 10^{-1}\ \tilde{m}_{\tilde{q}}$	$(LL) 1.2 \times 10^{-4} \Delta_Q, (RR) 4.4 \times 10^{-4} \Delta_u$
$\sqrt{\lvert \mathrm{Re}(\delta^u_{12})_{LL} (\delta^u_{12})_{RR} \rvert}$	$1.7 \times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$2.3 \times 10^{-4} \sqrt{\Delta_Q \Delta_u}$
$\sqrt{\lvert \mathrm{Re}(\delta^u_{12})^2_{LR} \rvert}$	$3.1 \times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4} \tilde{m}_{\tilde{q}}^{-1}$
$\lvert (\delta^d_{23})_{LL,RR} \rvert$	$8.2\ \tilde{m}_{\tilde{q}}^2$	$(LL) 1.6 \times 10^{-2} \Delta_Q, (RR) 4.7 \times 10^{-1} \Delta_d$
$\lvert (\delta^d_{23})_{LR} \rvert$	$1.6 \times 10^{-2}\ \tilde{m}_{\tilde{q}}^2$	$\sim 10^{-2} \tilde{m}_{\tilde{q}}^{-1}$

Table 1: Experimental bounds on δ's, where the parameter $\tilde{m}_{\tilde{q}}$ denote $m_{\tilde{q}}/500$ GeV.

(Itou,Kajiyama, Kubo)

IV Conclusion

"Low-energy discrete Flavor Symmetry"

constrains the flavor structure of the SM,
reducing the number of the redundant parameters of the SM.

softens the flavor problem
and the SUSY flavor problem.

No assumption on the universality of the soft terms is needed.

$$
\begin{aligned}
W_H \;=\; & m_T(T_1^2 + T_2^2) + m_Y Y^2 + \lambda_S(S_1^2 + S_2^2)Y \\
& + \lambda_1(H_1^u S_2 + H_2^u S_1)H_3^d + \lambda_2(H_1^d S_2 + H_2^d S_1)H_3^u \\
& + \lambda_3\left[-(H_1^u H_1^d - H_2^u H_2^d)T_1 + (H_1^u H_2^d + H_2^u H_1^d)T_2\right]
\end{aligned}
$$