Precise calculation of muon g-2 based on lattice QCD

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(RBC&UKQCD collaboration)

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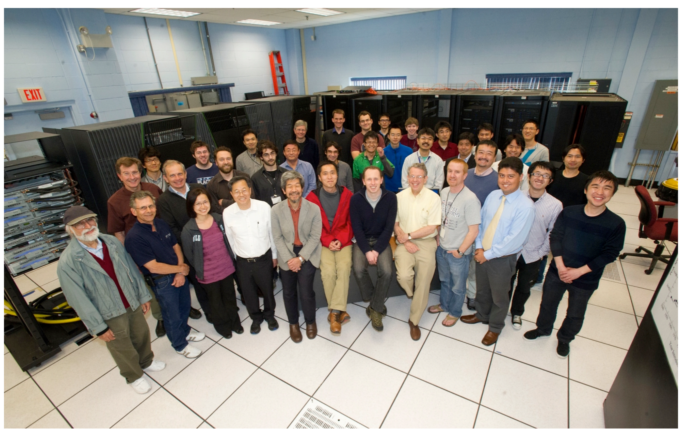
Antonin Portelli (Edinburgh)
Aaron Meyer (BNL)



Part of related calculation are done by resources from
USQCD (DOE), XSEDE, ANL BG/Q Mira (DOE, ALCC), Edinburgh BG/Q,
BNL BG/Q, RIKEN BG/Q and Cluster (RICC, HOKUSAI)

Support from RIKEN, JSPS, US DOE, and BNL

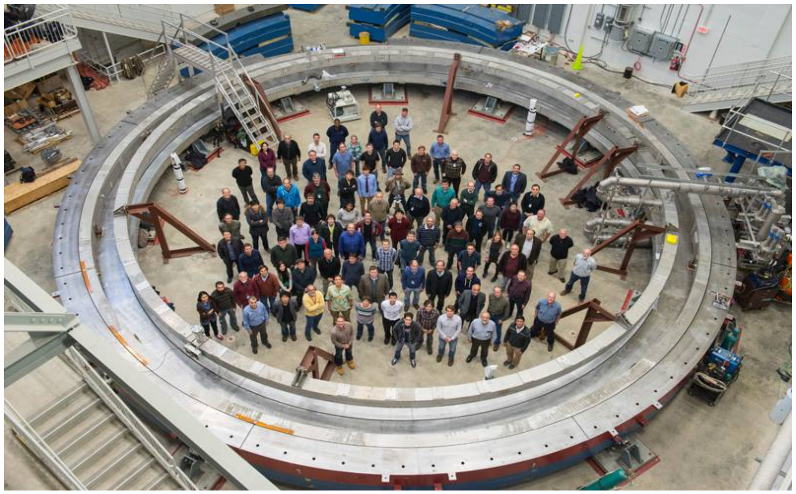
Reference

- g-2 HVP
  Phys. Rev. Lett. 121 (2018) 022003

- g-2 Hadronic Light-by-Light (HLbL)
  Phys. Rev. D96 (2017) 034515
  Phys. Rev. Lett. 118 (2017) 022005

- Tau input for g-2
  PoS Lattice 2018 (2018) 135

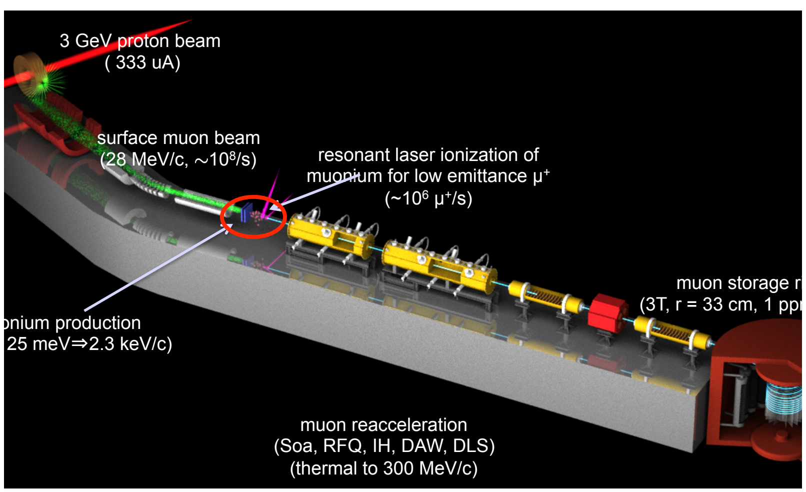
muon anomalous magnetic moment



BNL g-2 till 2004 : ~ 3.7 σ larger than SM prediction

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

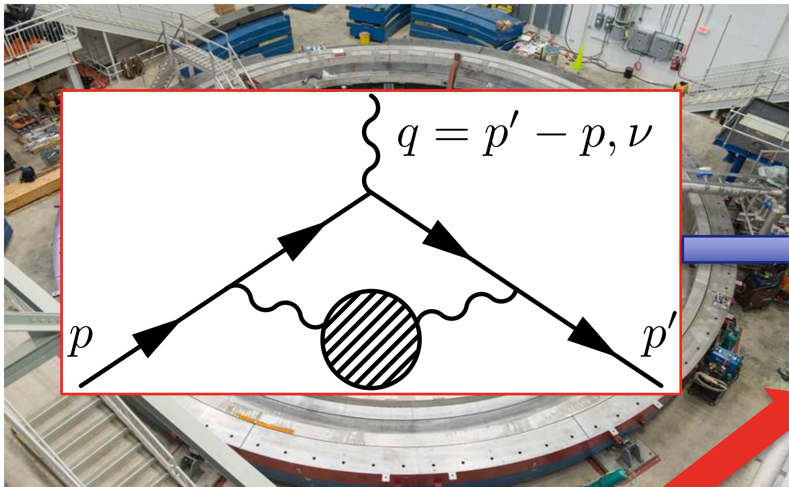
$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = 27.4\, \underbrace{(2.7)}_{\mathrm{HVP}}\, \underbrace{(2.6)}_{\mathrm{HLbL}}\, \underbrace{(0.1)}_{\mathrm{other}}\, \underbrace{(6.3)}_{\mathrm{EXP}} \times 10^{-10}$$



FNAL E989 (began 2017-)
- move storage ring from BNL
- x4 more precise results, 0.14ppm

J-PARC E34
- ultra-cold muon beam
- 0.37 ppm then 0.1 ppm, also EDM

muon anomalous magnetic moment



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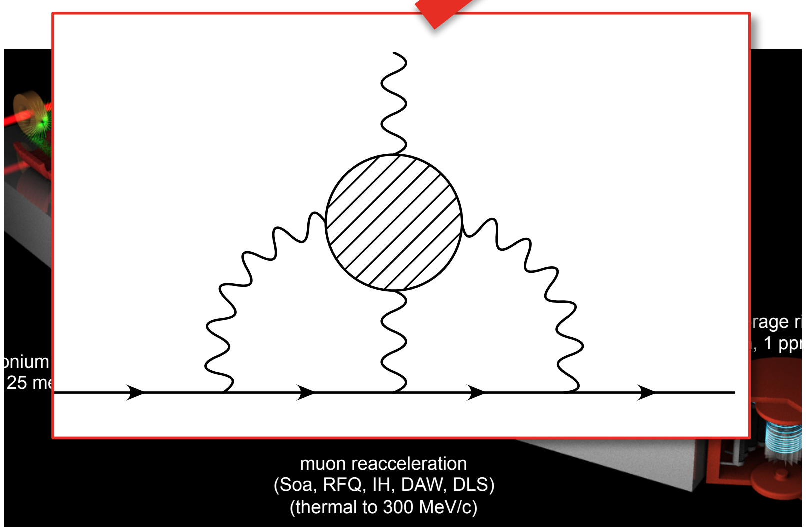

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[Luchang Jin's analogy]

Precession of Mercury and GR

discrepancy recognized since 1859

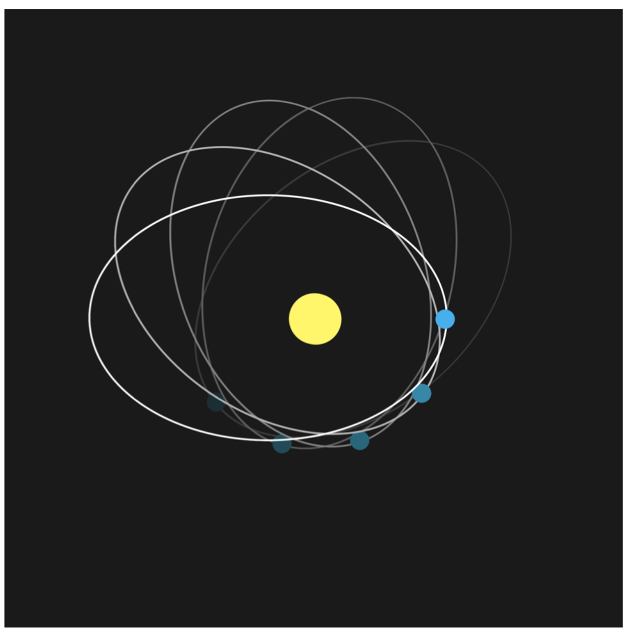
Amount (arc-sec/century)	Cause
5025.6	Coordinate (due to precession of equinoxes)
531.4	Gravitational tugs of the other planets
0.0254	Oblateness of the sun (quadrupole moment)
42.98±0.04	General relativity
5600.0	Total
5599.7	Observed

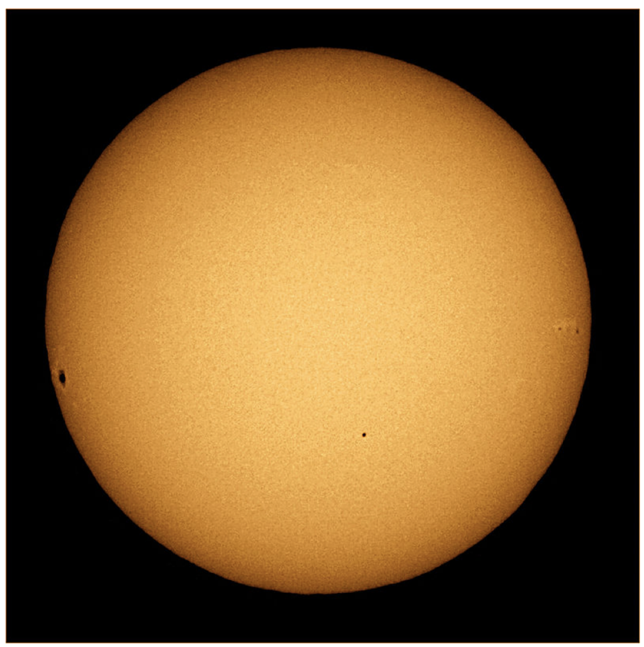
Known physics

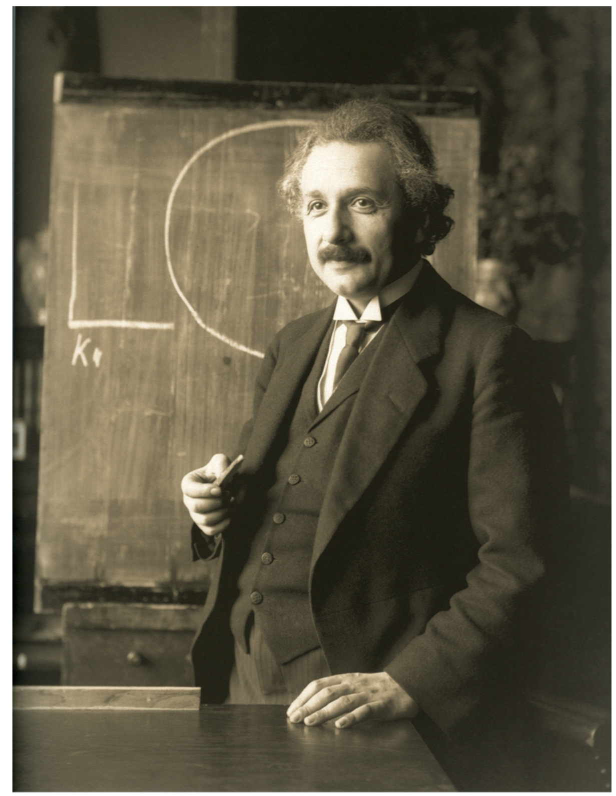
1915 by-then New physics
GR revolution

http://worldnpa.org/abstracts/abstracts_6066.pdf

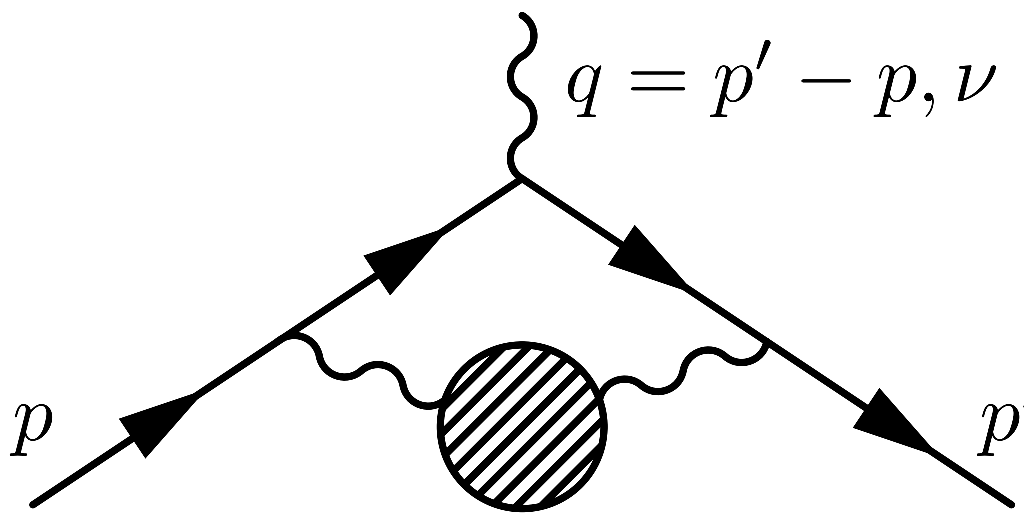
precession of perihelion





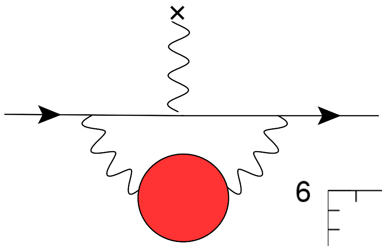


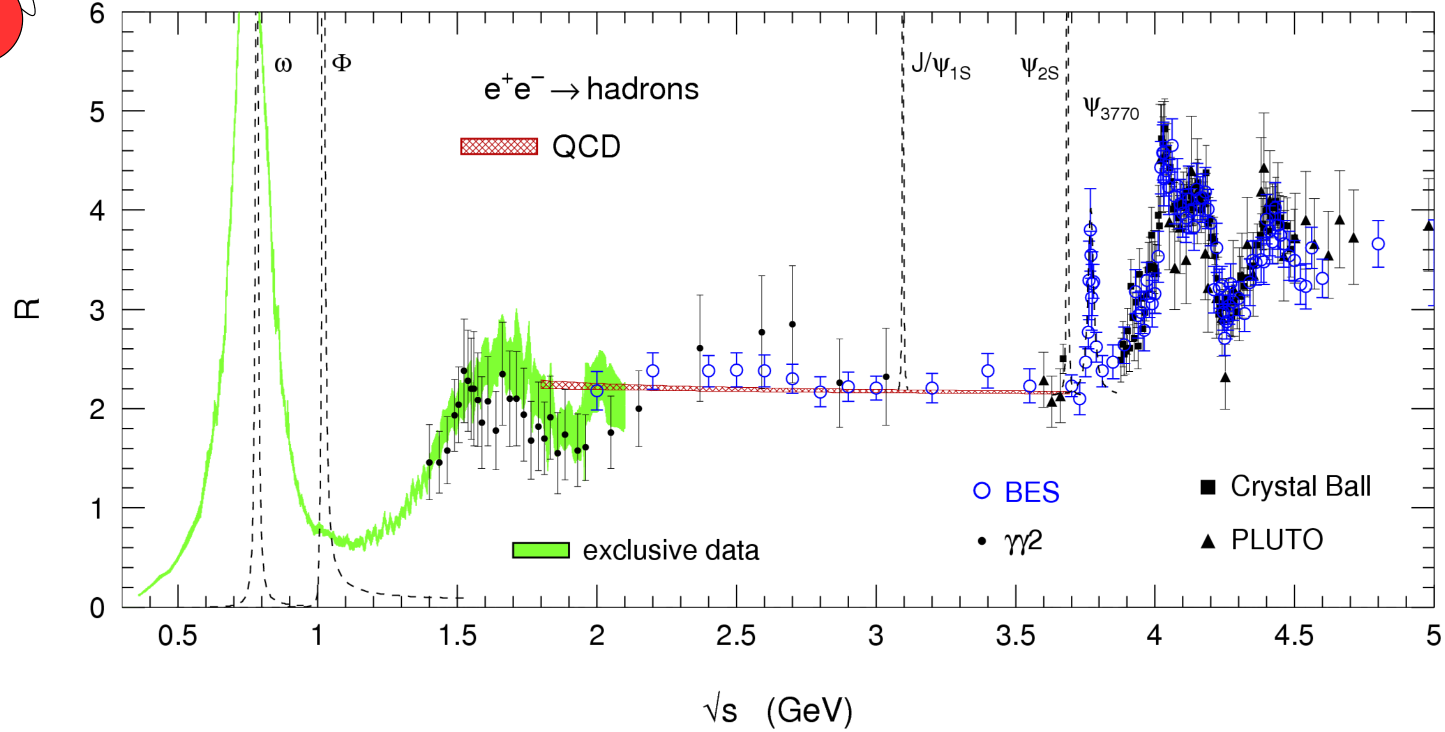
Hadronic Vacuum Polarization (HVP) contribution to g-2



g-2 from R-ratio

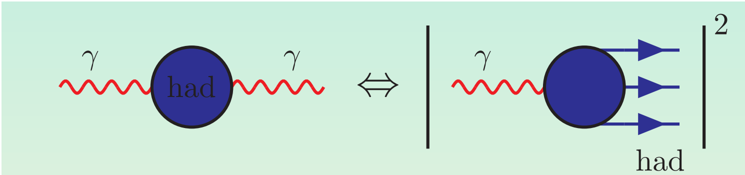
[Y. Maeda's talk]
[D. Nomura's talk]





- From experimental e+ e- inclusive hadron decay cross section $\sigma_{total}(s)$ in time-like $s = q^2 > 0$, and dispersion relation, optical theorem

$$a_\mu^{\mathrm{HVP}} = \frac{1}{4\pi^2}\int_{s_{th}}^{\infty} ds K(s)\sigma_{\mathrm{total}}(s)$$



Dispersive methods 2018

[D. Nomura's talk]

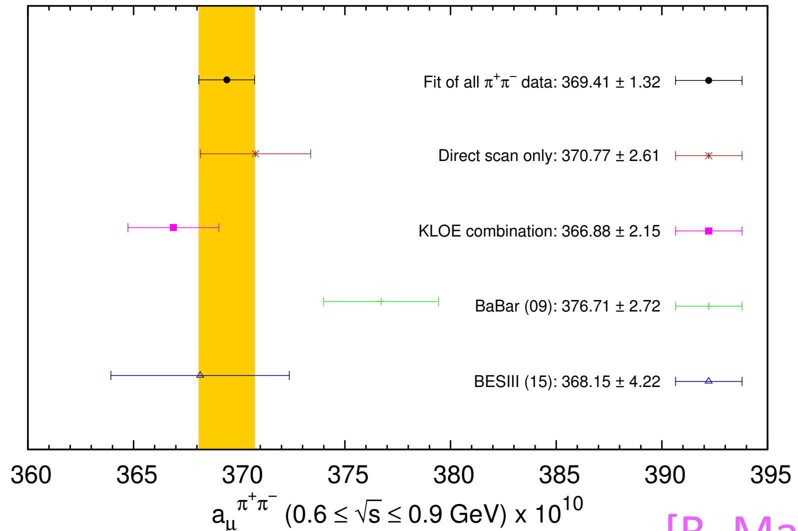
- KNT18 (PRD97,114025, arXiv:1802.02995)
- DHMZ17 (Eur. Phys. J. C77:827)

Channel	**This work (KNT18)**	**DHMZ17** [78]	**Difference**
Data based channels ($\sqrt{s} \le 1.8$ GeV)			
$\pi^0\gamma$ (data + ChPT)	4.58 ± 0.10	4.29 ± 0.10	0.29
$\pi^+\pi^-$ (data + ChPT)	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$ (data + ChPT)	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
. . .			
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

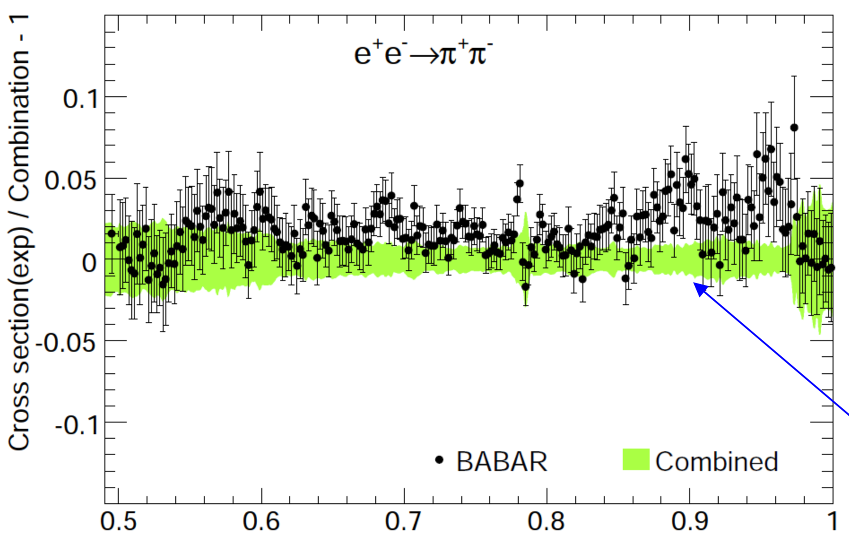
- Very small error,
  KNT18: 2.5 x10^{-10} [0.37%] and DHMZ17 3.4 x10^{-10} [0.49%]
- Good agreement for total, individual channels have a tention.
- Difference in how to combine experiments and energy bins, correlations among them

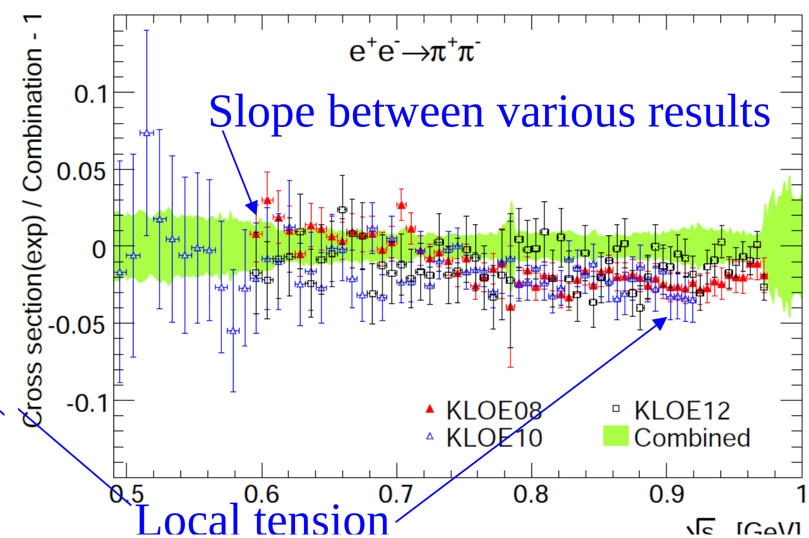
Dispersive method status

- BaBar and KLOE 2π contribution differ ~ 10(4) $\times 10^{-10}$ compared with quoted uncertainties, {2.5 or 3.4} $\times 10^{-10}$



[B. Malaescu's talk @Mainz g-2 2018]





[T. Blum PRL91 (2003) 052001]

HVP from Lattice

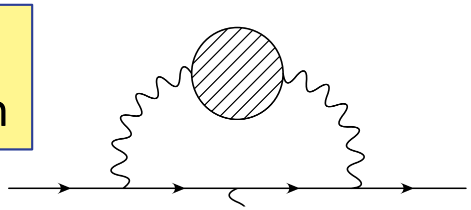
- Analytically continue to Euclidean/space-like momentum $K^2 = -q^2 > 0$
- Vector current 2pt function

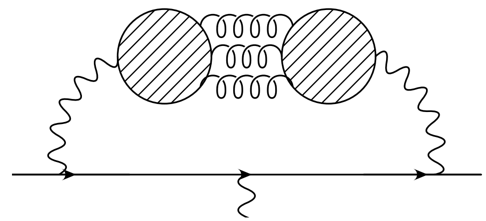
$$a_\mu = \frac{g-2}{2} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dK^2 f(K^2)\hat{\Pi}(K^2)$$

$$\Pi^{\mu\nu}(q) = \int d^4x e^{iqx} \langle J^\mu(x) J^\nu(0) \rangle$$

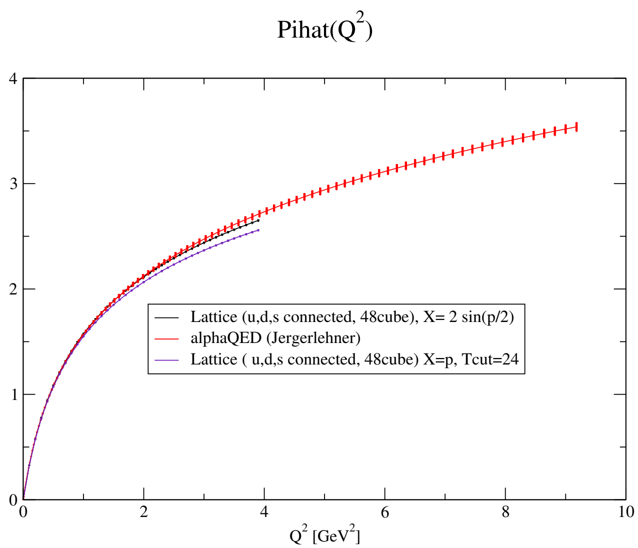
- Low Q2, or long distance, part of Π (Q2) is relevant for g-2

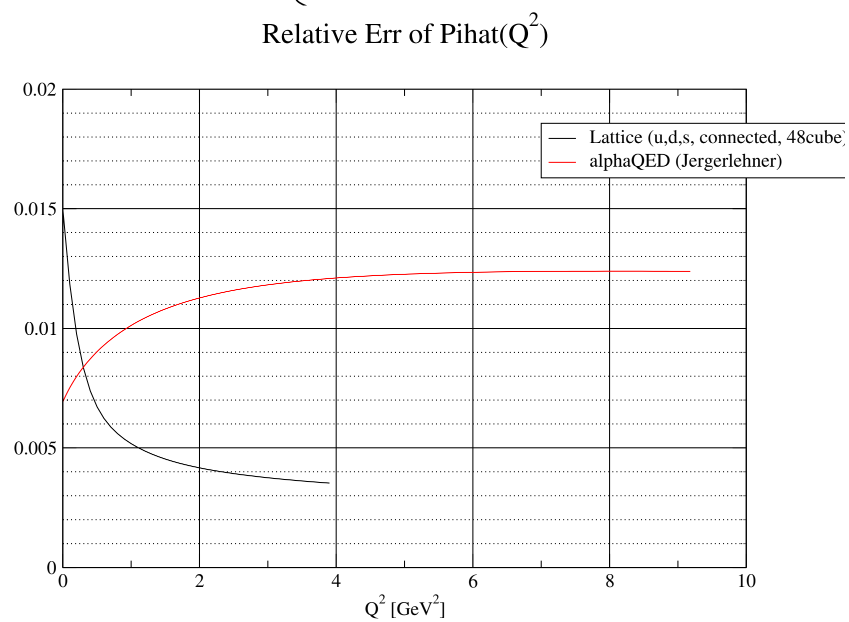
connected contribution





disconnected contribution





Euclidean Time Momentum Representation

[Bernecker Meyer 2011, Feng et al. 2013]

In Euclidean space-time, project verctor 2 pt to zero spacial momentum, $\vec{p} = 0$:

$$C(t) = \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle$$

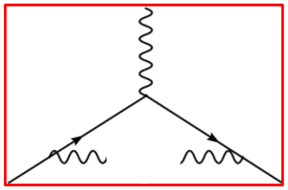
g-2 HVP contribution is

$$a_\mu^{HVP} = \sum_t w(t) C(t)$$

$$w(t) = 2 \int_0^\infty \frac{d\omega}{\omega} f_{\text{QED}}(\omega^2) \left[\frac{\cos \omega t - 1}{\omega^2} + \frac{t^2}{2} \right]$$

w(t) ~ t^4

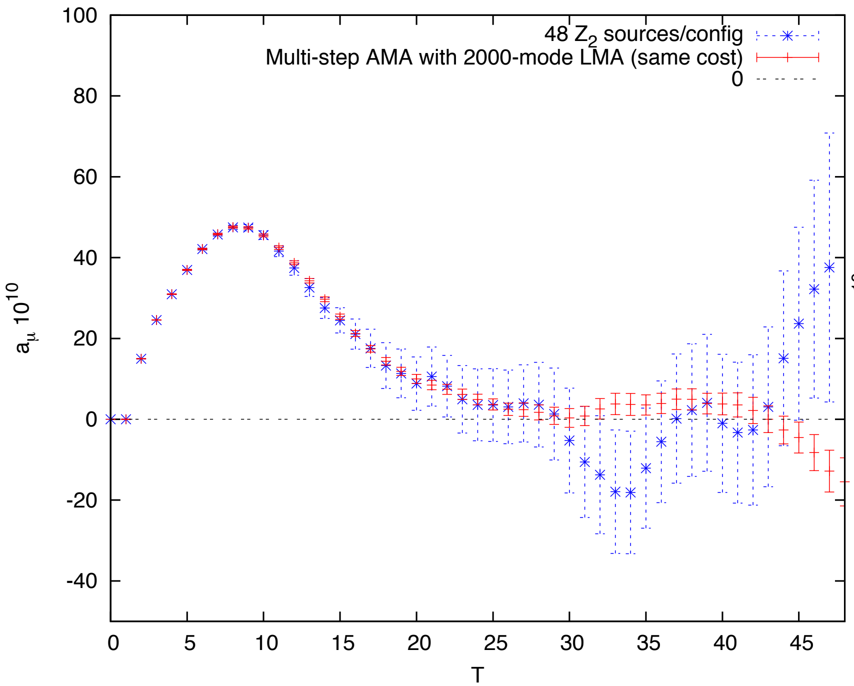
$f_{QED}(\omega^2)$

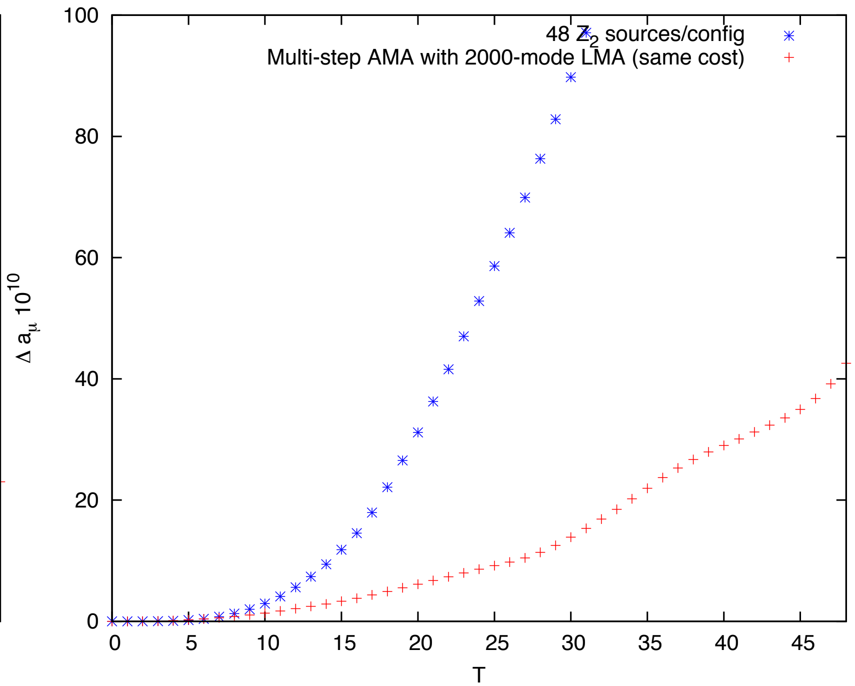


- Subtraction $\Pi(0)$ is performed.
  Noise/Signal $\sim e^{(E_{\pi\pi} - m_\pi)t}$, is improved [Lehner et al. 2015] .

- Corresponding $\hat{\Pi}(Q^2)$ has exponentially small volume error [Portelli et al. 2016] . $w(t)$ includes the continuum QED part of the diagram

DWF light HVP [2016 Christoph Lehner]

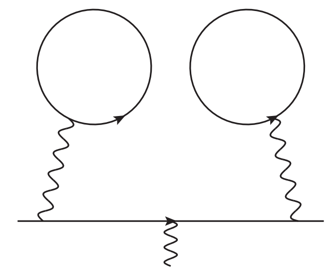




120 conf (a=0.11fm), 80 conf (a=0.086fm) physical point Nf=2+1 Mobius DWF
4D full volume LMA with 2,000 eigen vector (of e/o preconditioned zMobius $D^{+}D$)
EV compression (1/10 memory) using local coherence [C. Lehner Lat2017 Poster]
In addition, 50 sloppy / conf via multi-level AMA
more than x 1,000 speed up compared to simple CG

disconnected quark loop contribution

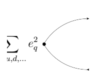
- [C. Lehner et al. (RBC/UKQCD 2015, arXiv:1512.09054, PRL)]
- Very challenging calculation due to statistical noise
- Small contribution, vanishes in SU(3) limit, Qu+Qd+Qs = 0
- Use low mode of quark propagator, treat it exactly (all-to-all propagator with sparse random source)
- First non-zero signal

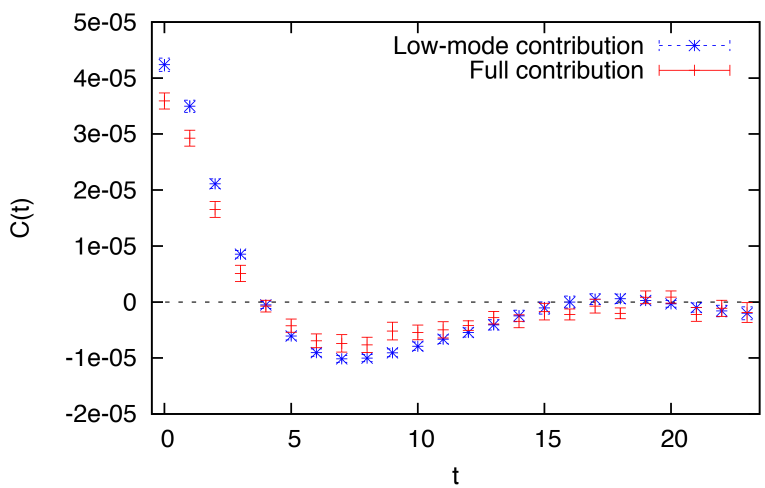


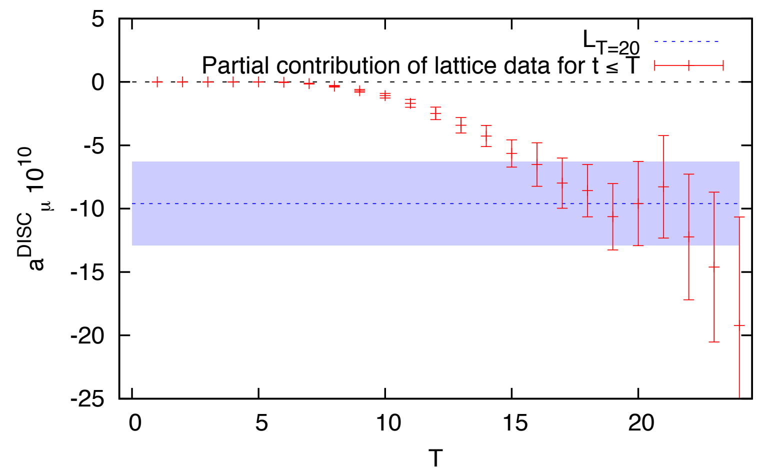
$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

Sensitive to m_π

crucial to compute at physical mass

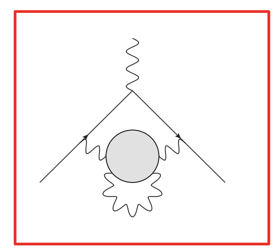


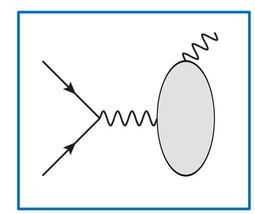


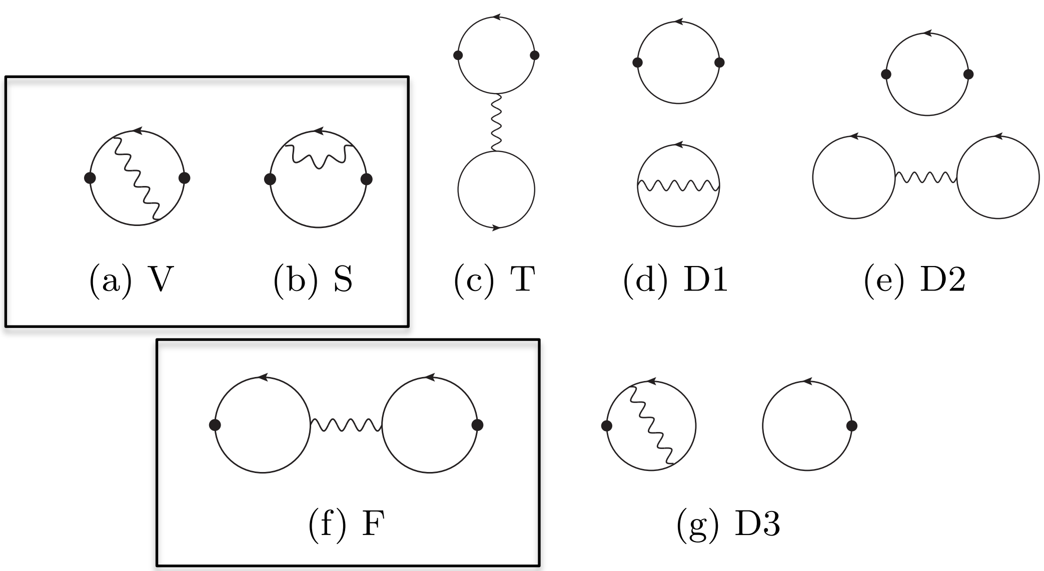


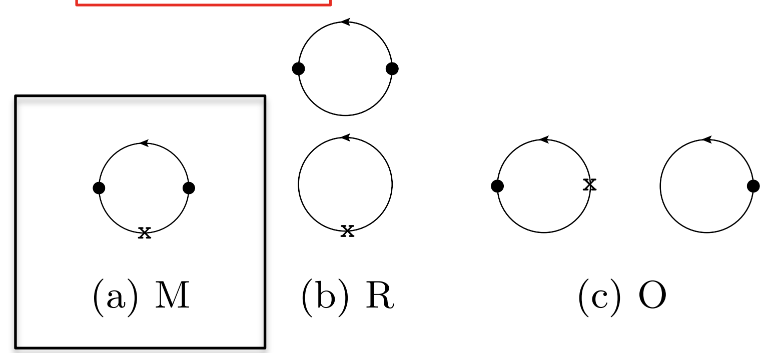
HVP QED+ strong IB corrections

- HVP is computed so far at Iso-symmetric quark mass, needs to compute isospin breaking corrections : Qu, Qd, mu-md ≠0
- u,d,s quark mass and lattice spacing are re-tuned using {charge,neutral} x{pion,kaon} and (Omega baryon masses)
- For now, V, S, F, M are computed : assumes EM and IB of sea quark and also shift to lattice spacing is small (correction to disconnected diagram)
- Point-source method : stochastically sample pair of 2 EM vertices a la important sampling with exact photon







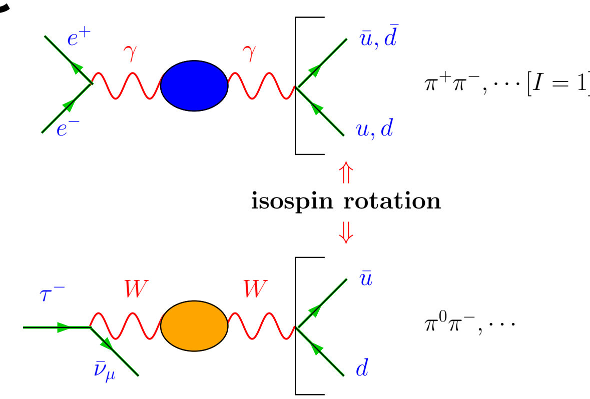


Tau input for HVP IB+QED corrections

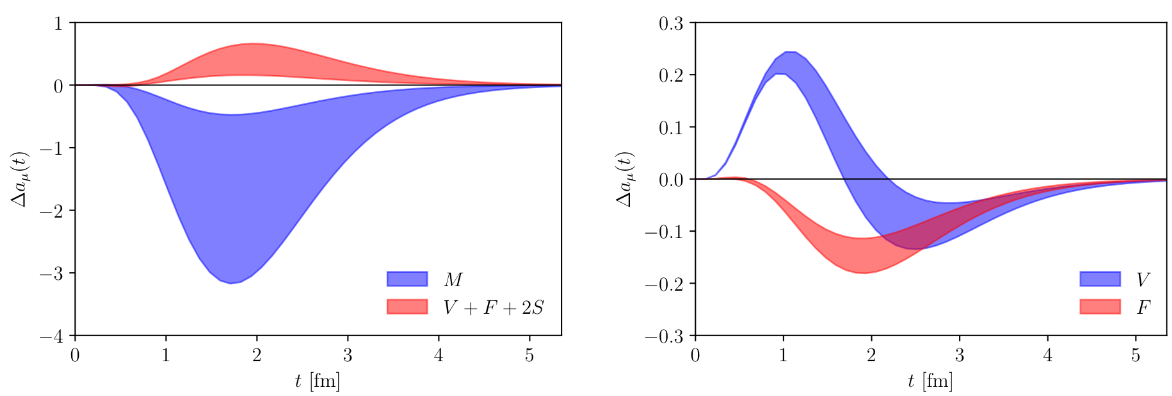
[T. Konno's talk]
[Y. Maekawa's talk]

- Could also compute the difference IB correction of
  $\Delta a_\mu = a_\mu(e+e-) - a_\mu(\tau)$

  [M. Bruno et al, arXiv:1811.00508]



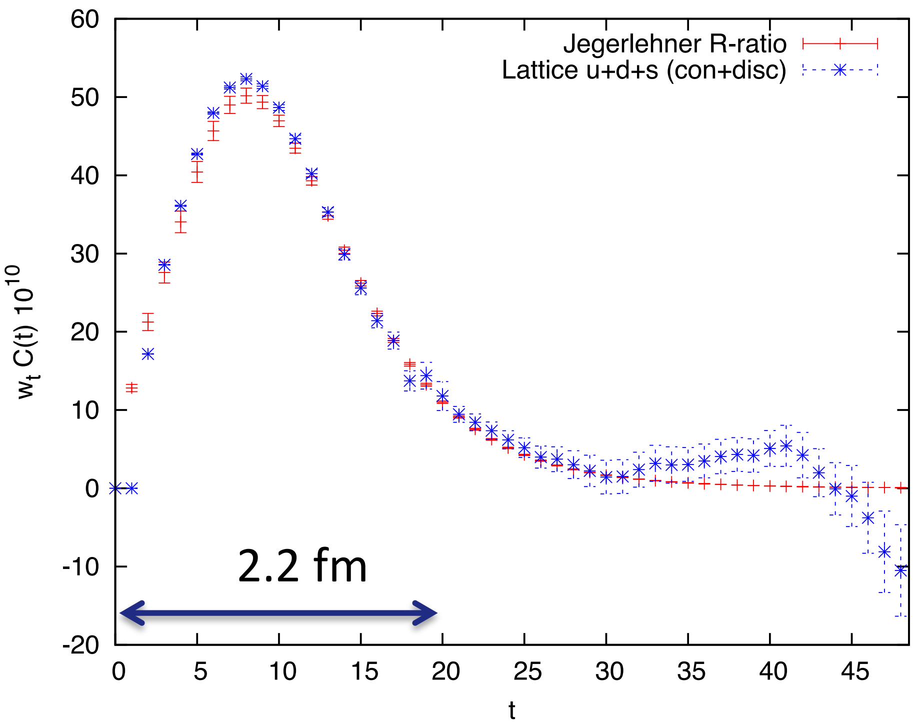
- I=0 to I=1 contribution from Strong IB+EM effect (left), I=1 contribution EM effects (right)

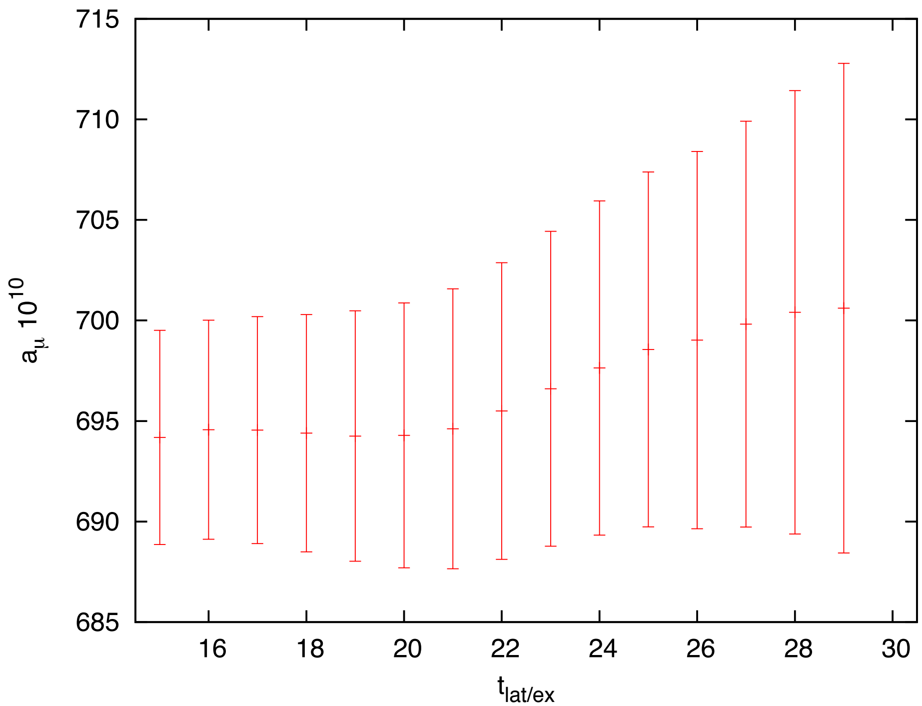


Interplay between Lattice and Experiment

- Check consistency between Lattice and R-ratio
- Short distance from Lattice, Long distance from R-ratio :
  error already 0.5 – 1.2% around $t_{lat/exp}$ = 2fm

$$a_\mu^{\mathrm{HVP}} = \left[\sum_{t=0}^{t_{\mathrm{lat/exp}}} w(t)C(t)\right]^{\mathrm{LAT}} + \left[\int_{t_{\mathrm{lat/exp}}}^{\infty} dt\, w(t)C(t)\right]^{\mathrm{EXP}}$$





Euclidean time correlation from e^+e^- $R(s)$ data

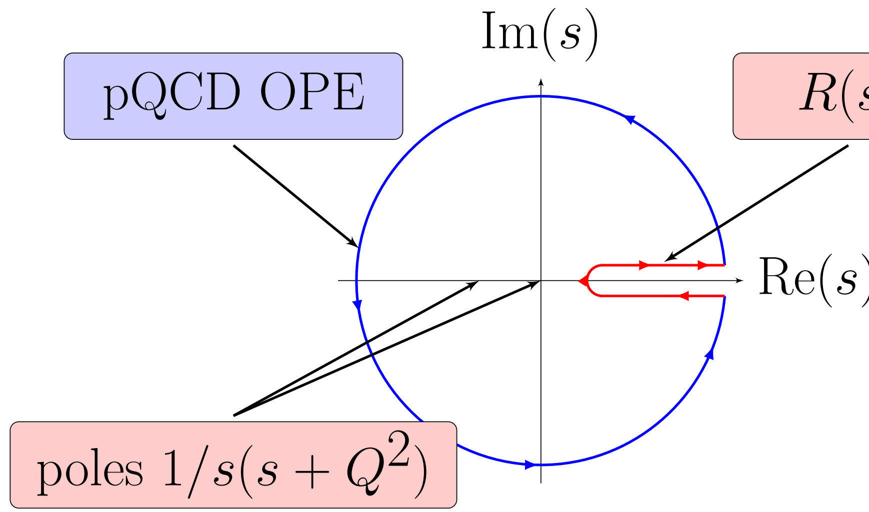
From e^+e^- $R(s)$ ratio, using disparsive relation, zero-spacial momentum projected Euclidean correlation function $C(t)$ is obtained

$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

Lattice can compute Integral of Inclusive cross sections accurately

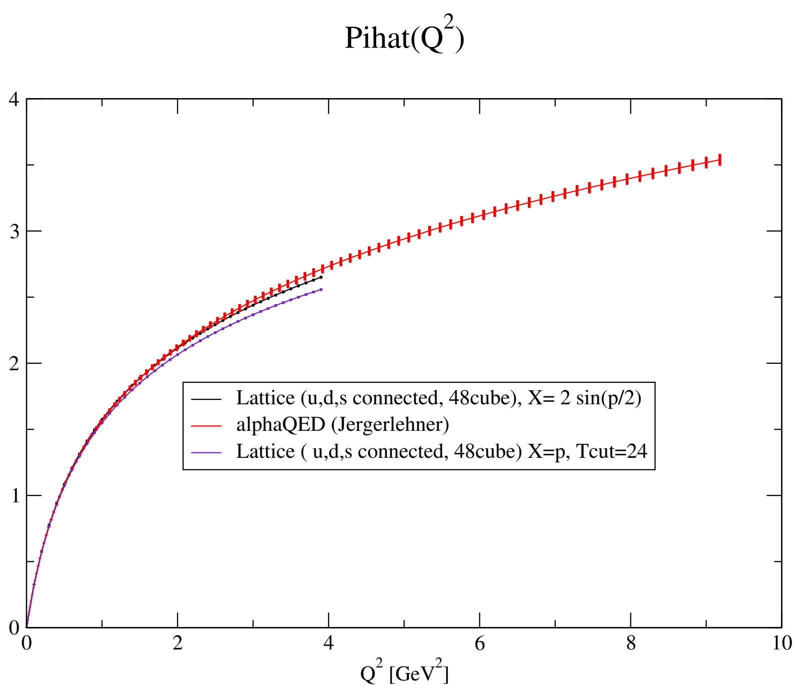
$$C^{\text{R-ratio}}(t) = \frac{1}{12\pi^2}\int_0^\infty \frac{d\omega}{2\pi}\hat{\Pi}(\omega^2)e^{i\omega t} = \frac{1}{12\pi^2}\int_0^\infty ds\,\sqrt{s}R(s)e^{-\sqrt{s}t}$$

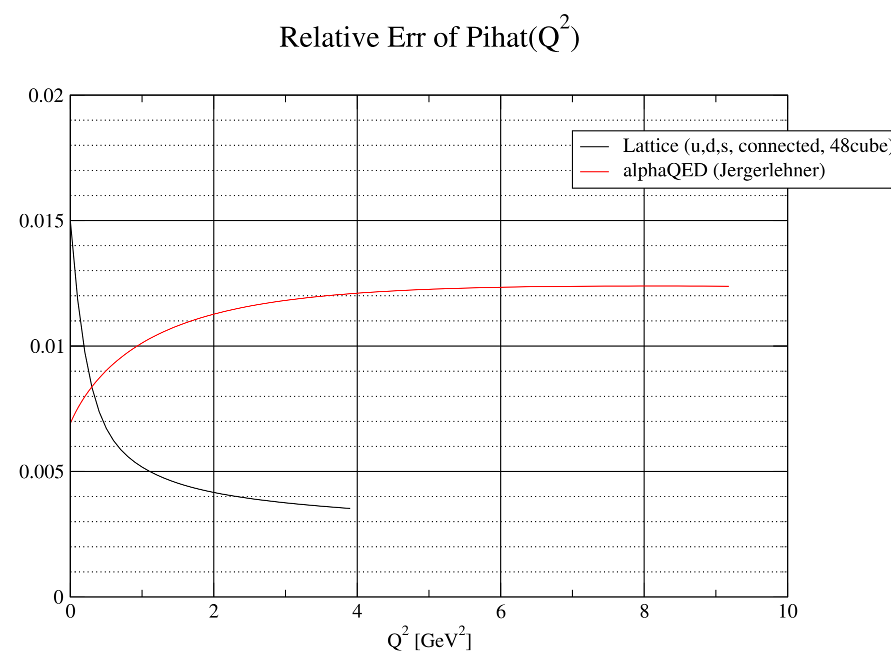
- $C(t)$ or $w(t)C(t)$ are directly comparable to Lattice results with the proper limits ($m_q \to m_q^{\text{phys}}, a \to 0, V \to \infty$, QED ...)
- Lattice: long distance has large statistical noise, (short distance: discretization error, removed by $a \to 0$ and/or pQCD)
- R-ratio : short distance has larger error



$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

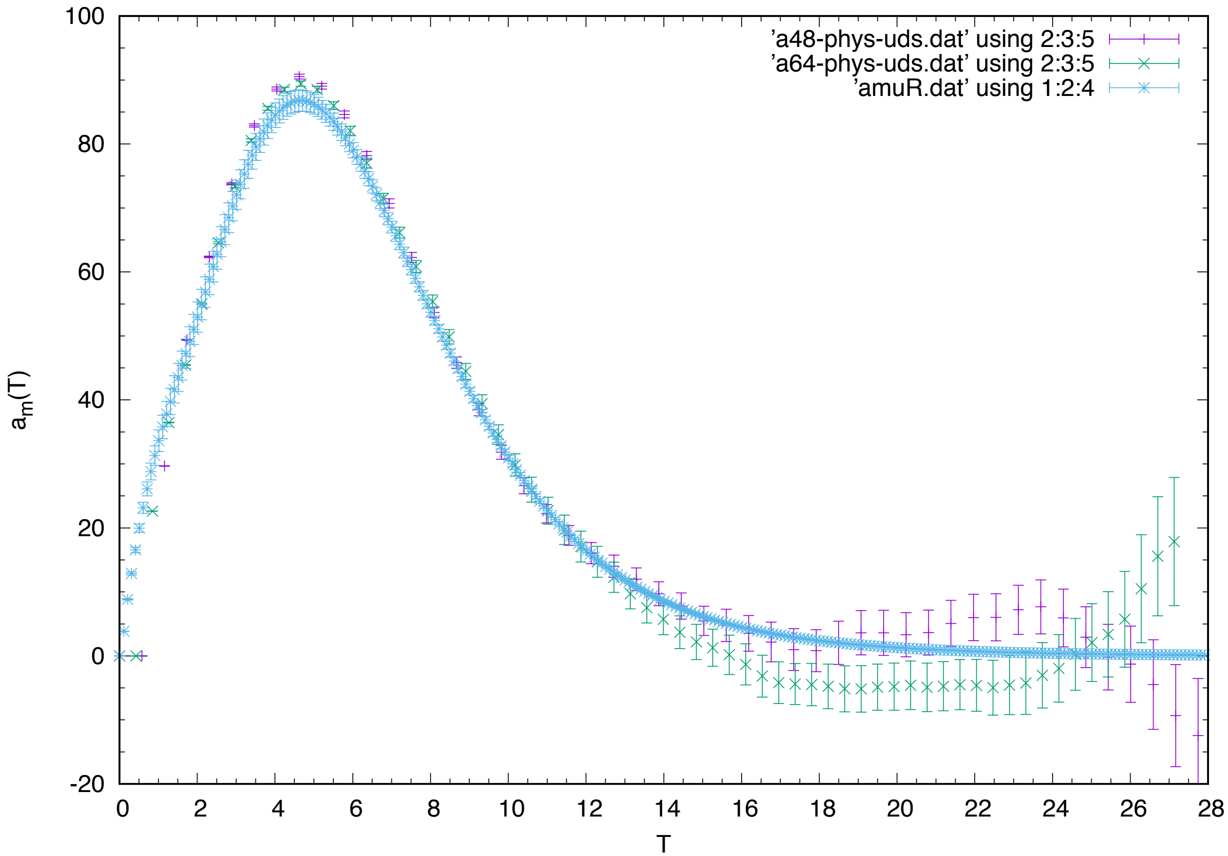
($1/a = 1.78$ **GeV,** **Relative statistical error)**





Comparison of R-ratio and Lattice
[F. Jegerlehner alphaQED 2016]

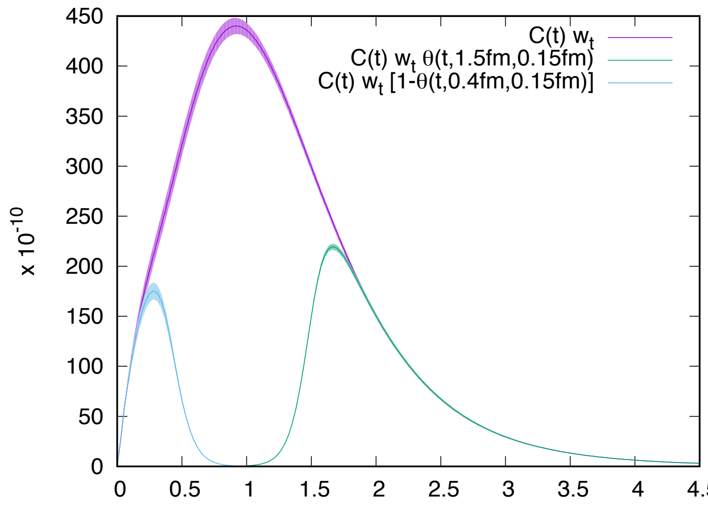
- Covariance matrix among energy bin in R-ratio is not available, assumes 100% correlated



Combine R-ratio and Lattice
[Christoph Lehner et al PRL18]

- Use short and long distance from R-ratio using smearing function, and mid-distance from lattice

$$\Theta(t, \mu, \sigma) \equiv [1 + \tanh[(t - \mu)/\sigma]]/2$$

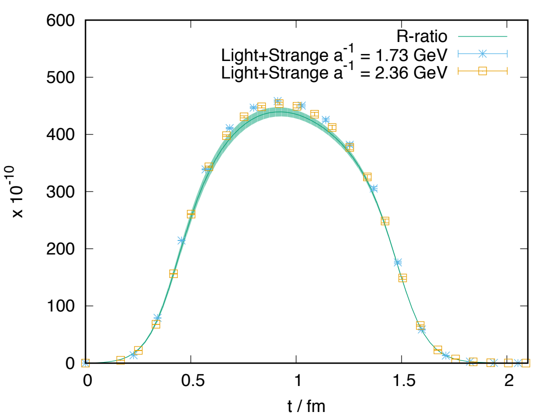


$$a_\mu = \sum_t w_t C(t) \equiv a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$

$$a_\mu^{\rm SD} = \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)] \,,$$

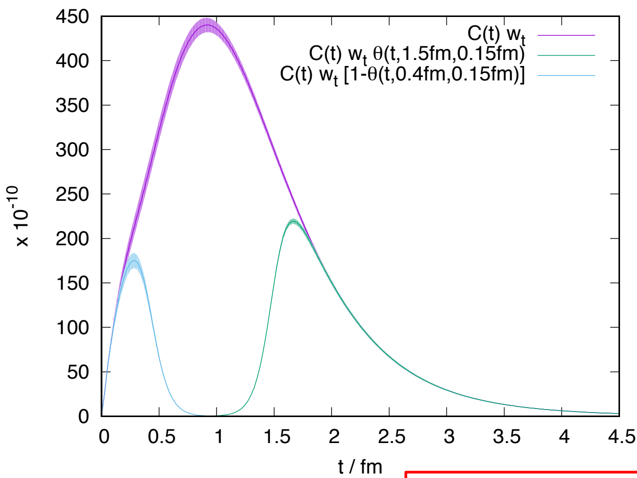
$$a_\mu^{\rm W} = \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)] \,,$$

$$a_\mu^{\rm LD} = \sum_t C(t) w_t \Theta(t, t_1, \Delta)$$

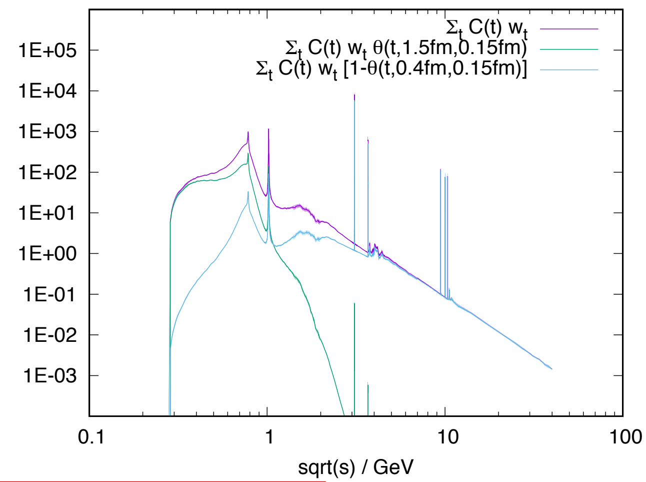


How does this translate to the time-like region?

Euclidean from Lattice

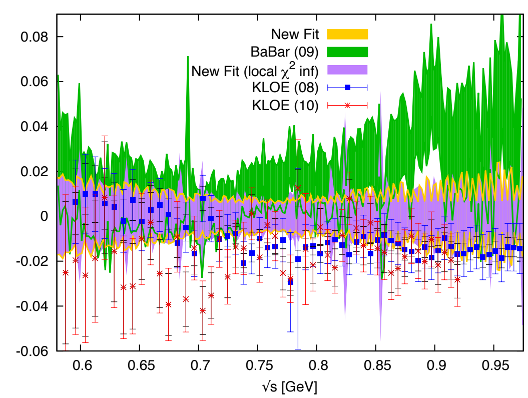


Time-like from e+e-



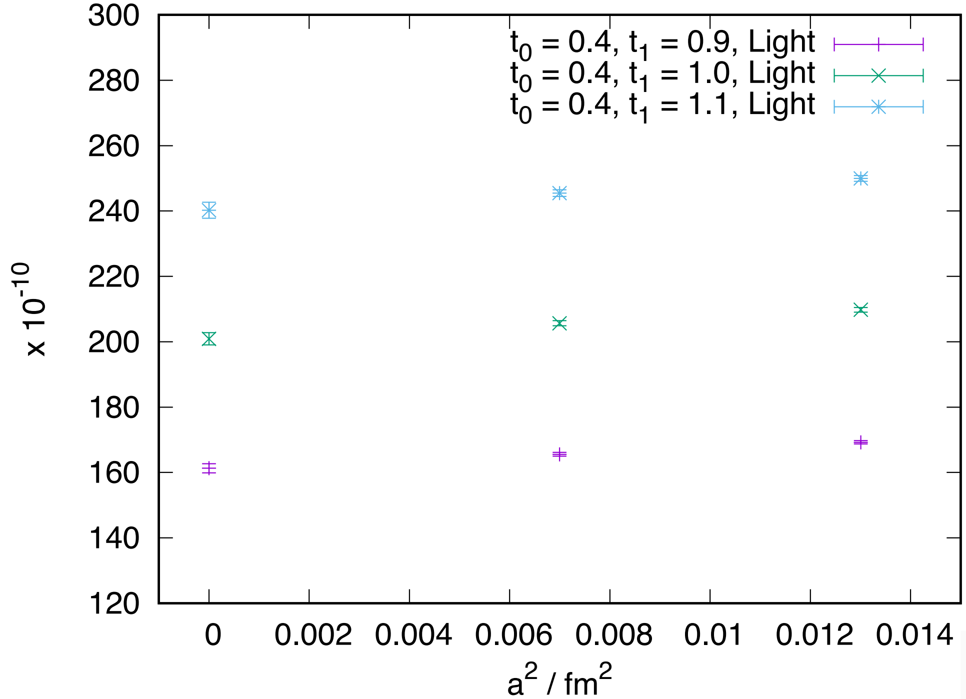
Blue : low-pass window
Green: high-pass window
Purple : total

Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.



Continuum limit of a^W

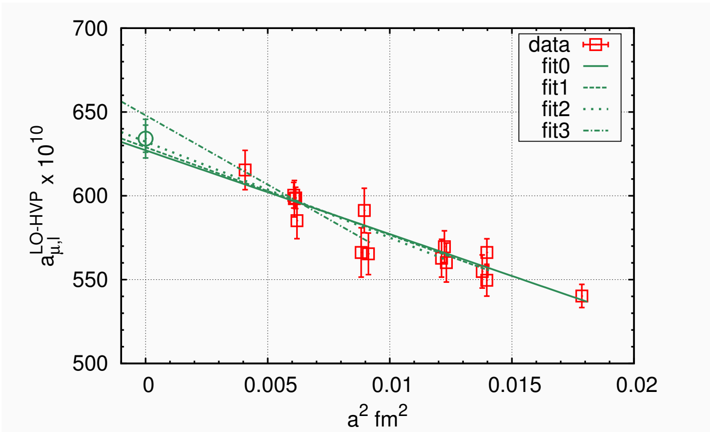
Continuum limit of a_μ^{W} from our lattice data; below $t_0 = 0.4$ fm and $\Delta = 0.15$ fm



RBC/UKQCD [C. Lehner Lat17]

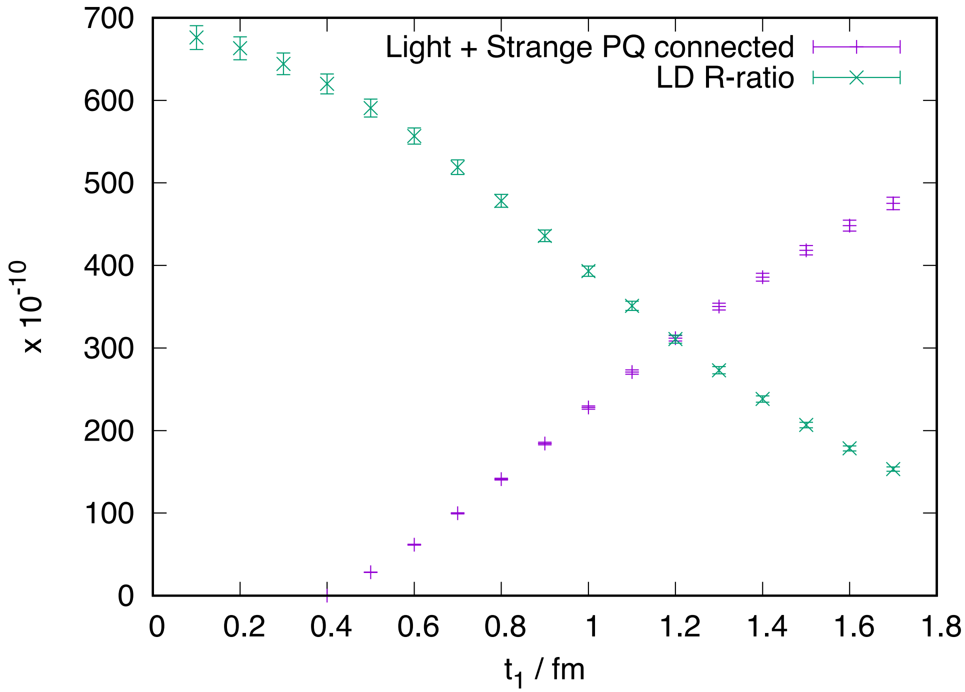
Continuum extrapolation is mild

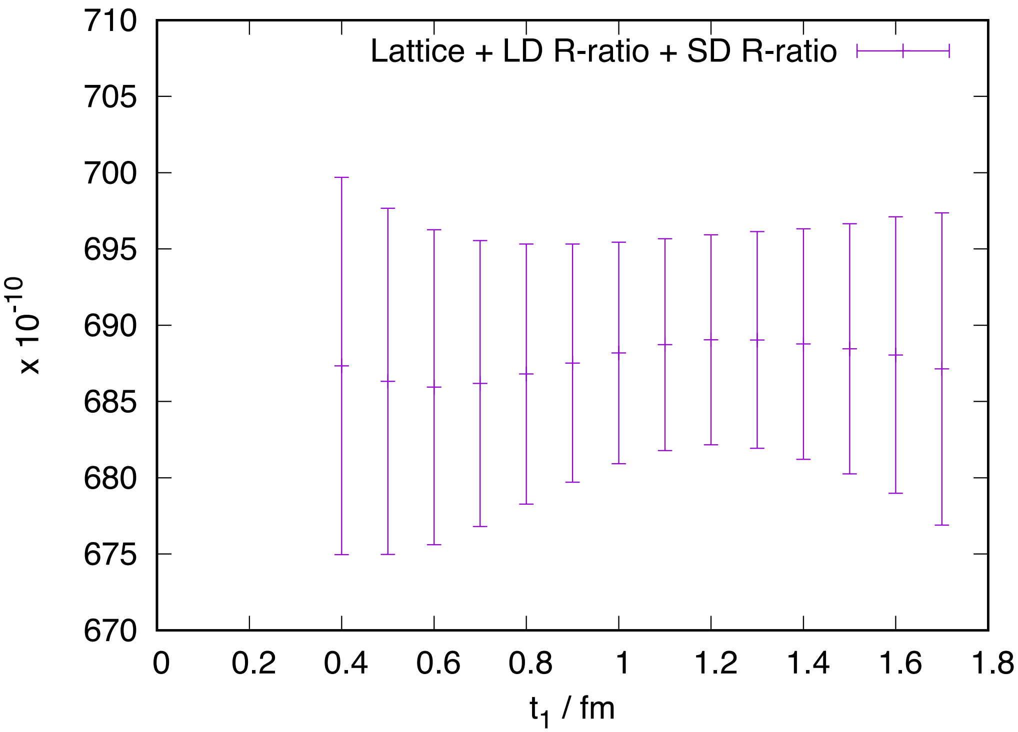
c.f BMWc [K. Miura Lat17]



R-ratio + Lattice

t0 = 0.4 fm



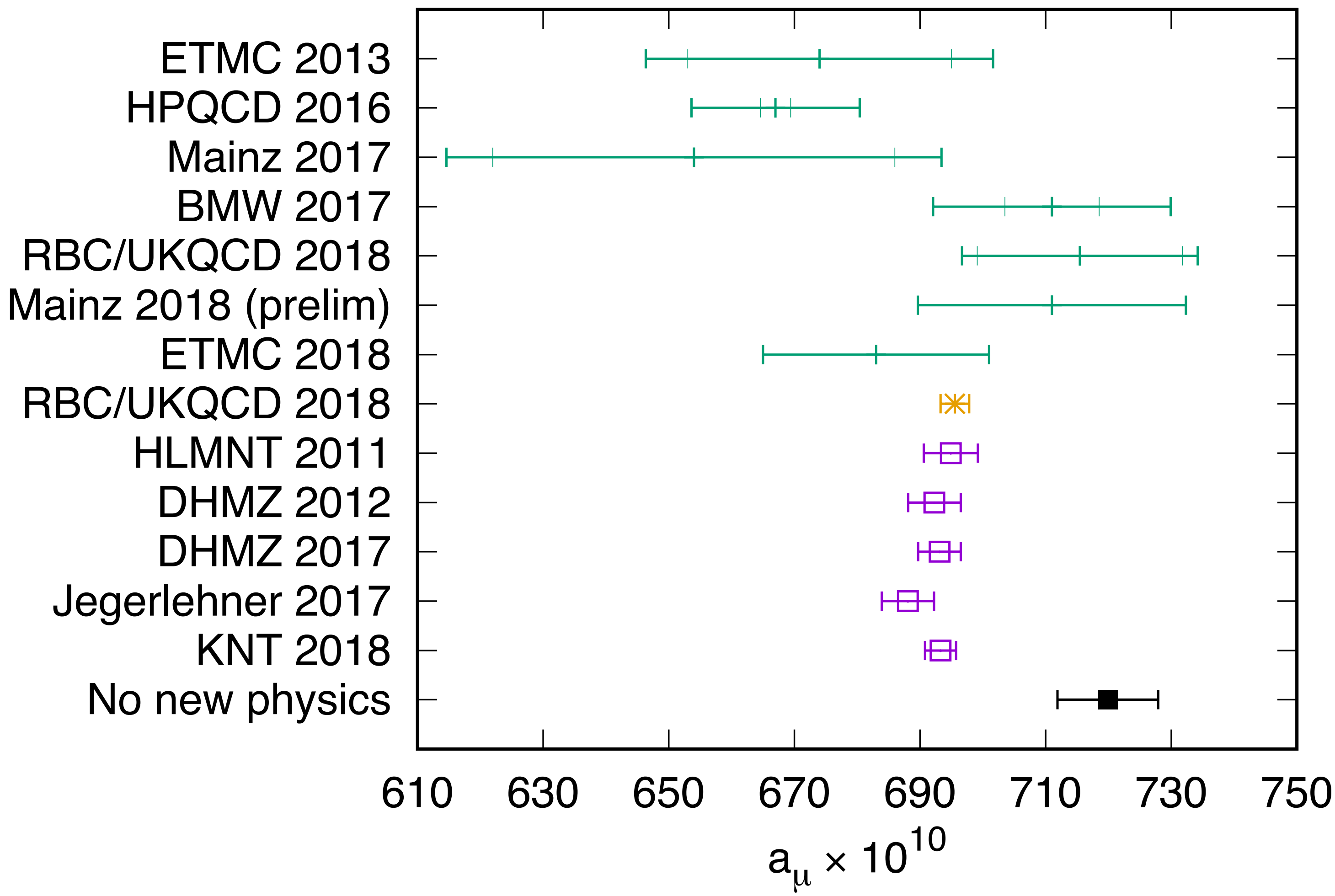


t1 dependence is flat => a consistency between R-ratio and Lattice

t1 = 1.2 fm, R-ratio : Lattice = 50:50

t1=1.2 fm current error (note 100% correlation in R-ratio) is minimum

HVP results



Pure Lattice (RBC/UKQCD)
715.4 (16.3)(9.2) [2.6%]

Lat+R-ratio Hybrid
692.5 (1.4)(2.3) [0.39%]

KNT18
693.3 (2.5) [0.36%]

- Significant improvements is in progress for statistical error using 2π and 4π (!) states in addition to EM current (GEVP, GS-parametrization)
- Checking finite volume and discretization error as well as Isospin V effects

Example error budget from RBC/UKQCD 2018 (**Fred's alphaQED17 results used for window result)**

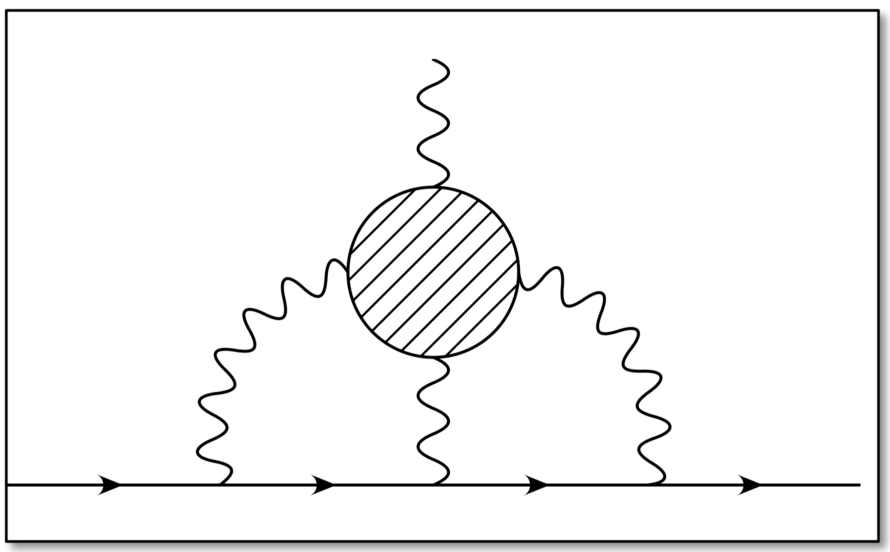
	Window t=[0.4, 1 fm]	Pure Lattice
$a_\mu^{\rm ud,\ conn,\ isospin}$	$202.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.2)_{\rm A}(0.2)_{\rm Z}$	$649.7(14.2)_{\rm S}(2.8)_{\rm C}(3.7)_{\rm V}(1.5)_{\rm A}(0.4)_{\rm Z}(0.1)_{\rm E48}(0.1)_{\rm E64}$
$a_\mu^{\rm s,\ conn,\ isospin}$	$27.0(0.2)_{\rm S}(0.0)_{\rm C}(0.1)_{\rm A}(0.0)_{\rm Z}$	$53.2(0.4)_{\rm S}(0.0)_{\rm C}(0.3)_{\rm A}(0.0)_{\rm Z}$
$a_\mu^{\rm c,\ conn,\ isospin}$	$3.0(0.0)_{\rm S}(0.1)_{\rm C}(0.0)_{\rm Z}(0.0)_{\rm M}$	$14.3(0.0)_{\rm S}(0.7)_{\rm C}(0.1)_{\rm Z}(0.0)_{\rm M}$
$a_\mu^{\rm uds,\ disc,\ isospin}$	$-1.0(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}$	$-11.2(3.3)_{\rm S}(0.4)_{\rm V}(2.3)_{\rm L}$
$a_\mu^{\rm QED,\ conn}$	$0.2(0.2)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$5.9(5.7)_{\rm S}(0.3)_{\rm C}(1.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.1)_{\rm E}$
$a_\mu^{\rm QED,\ disc}$	$-0.2(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$-6.9(2.1)_{\rm S}(0.4)_{\rm C}(1.4)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E}$
$a_\mu^{\rm SIB}$	$0.1(0.2)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E48}$	$10.6(4.3)_{\rm S}(0.6)_{\rm C}(6.6)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E48}$
$a_\mu^{\rm udsc,\ isospin}$	$231.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm M}$	$705.9(14.6)_{\rm S}(2.9)_{\rm C}(3.7)_{\rm V}(1.8)_{\rm A}(0.4)_{\rm Z}(2.3)_{\rm L}(0.1)_{\rm E48}(0.1)_{\rm E64}(0.0)_{\rm M}$
$a_\mu^{\rm QED,\ SIB}$	$0.1(0.3)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}$	$9.5(7.4)_{\rm S}(0.7)_{\rm C}(6.9)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.7)_{\rm E}(1.3)_{\rm E48}$
$a_\mu^{\rm R-ratio}$	$460.4(0.7)_{\rm RST}(2.1)_{\rm RSY}$	
a_μ	$692.5(1.4)_{\rm S}(0.2)_{\rm C}(0.2)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}(0.0)_{\rm b}(0.1)_{\rm c}(0.0)_{\overline{\rm S}}(0.0)_{\overline{\rm Q}}(0.0)_{\rm M}(0.7)_{\rm RST}(2.1)_{\rm RSY}$	$715.4(16.3)_{\rm S}(3.0)_{\rm C}(7.8)_{\rm V}(1.9)_{\rm A}(0.4)_{\rm Z}(1.7)_{\rm E}(2.3)_{\rm L}(1.5)_{\rm E48}(0.1)_{\rm E64}(0.3)_{\rm b}(0.2)_{\rm c}(1.1)_{\overline{\rm S}}(0.3)_{\overline{\rm Q}}(0.0)_{\rm M}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

For the pure lattice number the dominant errors are (S) statistics, (V) finite-volume errors, and (C) the continuum limit extrapolation uncertainty.

For the window method there are additional R-ratio systematic (RSY) and R-ratio statistical (RST) errors.

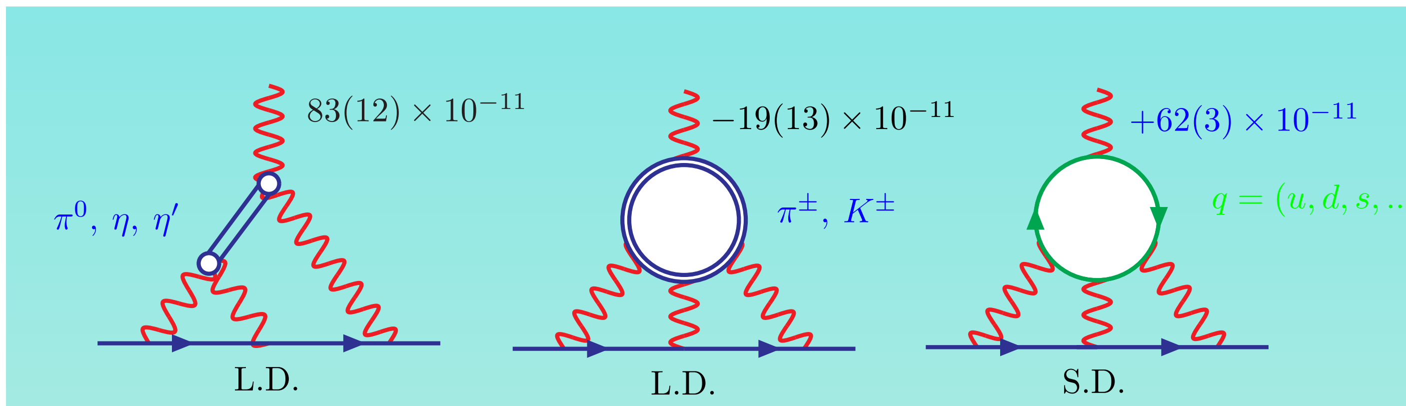
Hadronic Light-by-Light (HLbL) contributions



HLbL from Models

- Model estimate with non-perturbative constraints at the chiral / low energy limits using anomaly : (9—12) x 10^{-10} with 25-40% uncertainty

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 28.8(6.3)_{\rm exp}(4.9)_{\rm SM} \times 10^{-10} \quad [3.6\sigma]$$



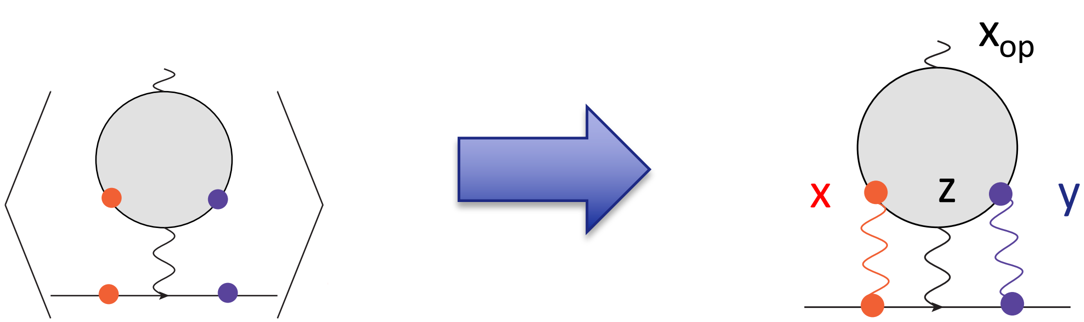
F. Jegerlehner , x 10^{11}

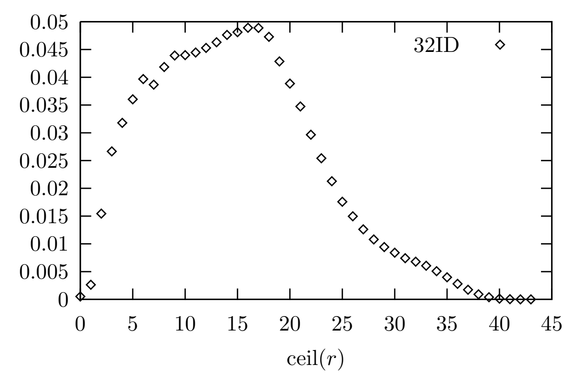
Contribution	BPP	HKS	KN	MV	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	0±10	−19±19	−19±13
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	15±10	22± 5
scalars	−6.8±2.0	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	105±26	116±39

Coordinate space Point photon method

[Luchang Jin et al. , PRD93, 014503 (2016)]

- Treat all 3 photon propagators exactly (3 analytical photons) , which makes the quark loop and the lepton line connected :
  disconnected problem in Lattice QED+QCD -> connected problem with analytic photon
- QED 2-loop in coordinate space. Stochastically sample, two of quark-photon vertex location x,y, z and x_{op} is summed over space-time exactly





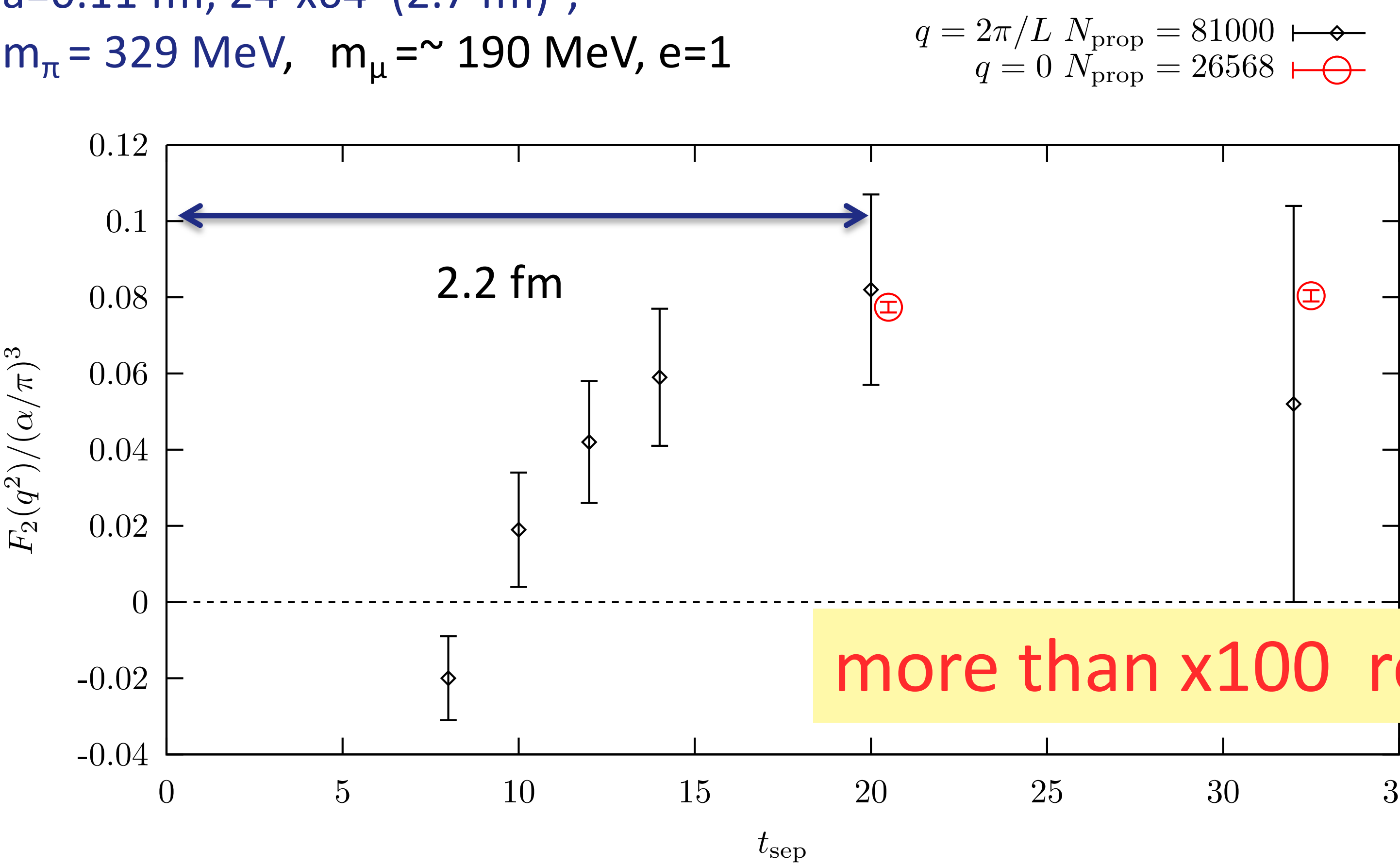
- Short separations, Min[|x-z|,|y-z|,|x-y|] < R ~ O(0.5) fm, which has a large contribution due to confinement, are summed for all pairs
- longer separations, Min[|x-z|,|y-z|,|x-y|] >= R, are done stochastically with a probability shown above (Adaptive Monte Carlo sampling)

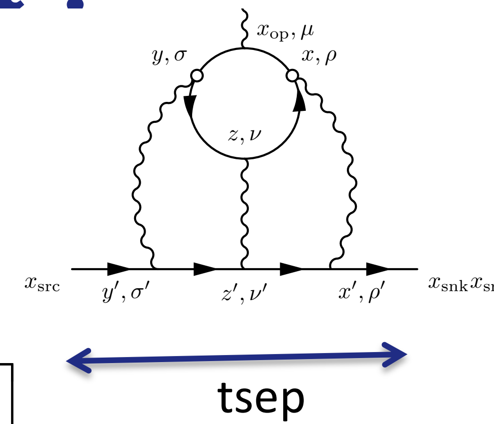
Dramatic Improvement !

Luchang Jin

a=0.11 fm, 24^3x64 (2.7 fm)^3,

m_π = 329 MeV, m_μ =~ 190 MeV, e=1



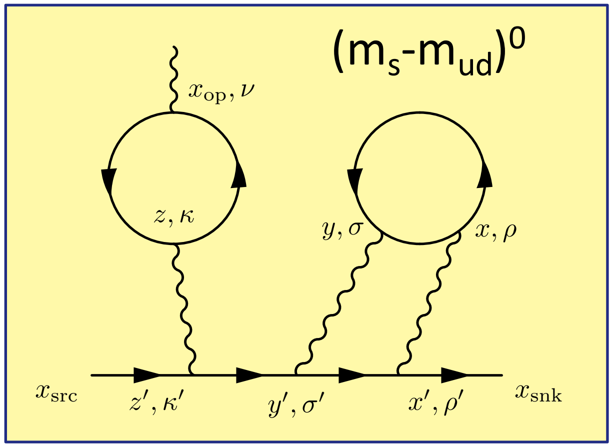


more than x100 reduced cost !

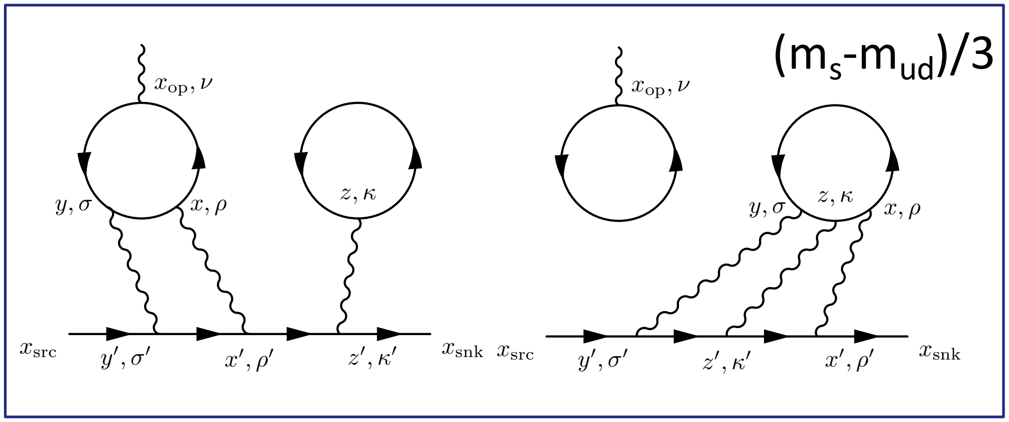
Method	$F_2/(\alpha/\pi)^3$	N_{conf}	N_{prop}	$\sqrt{\text{Var}}$
Conserved	0.0825(32)	12	$(118+128)\times 2\times 7$	0.65
Mom.	0.0804(15)	18	$(118+128)\times 2\times 3$	0.24

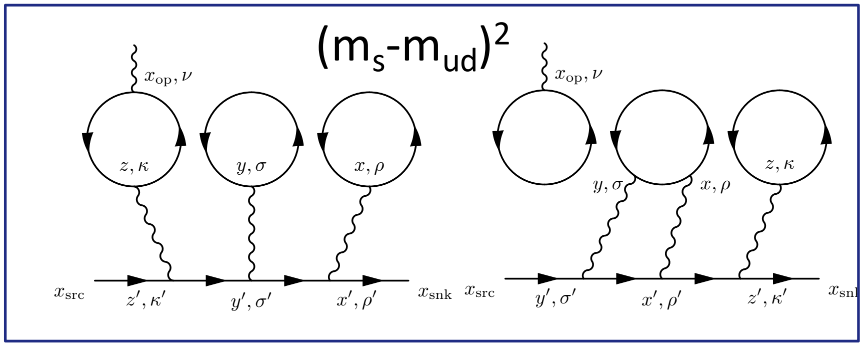
SU(3) hierarchies for d-HLbL

- At $m_s = m_{ud}$ limit, following type of disconnected HLbL diagrams survive $Q_u + Q_d + Q_s = 0$
- Physical point run using similar techniques to c-HLbL.



- other diagrams suppressed by
  $O(m_s - m_{ud}) / 3$ and $O((m_s - m_{ud})^2)$

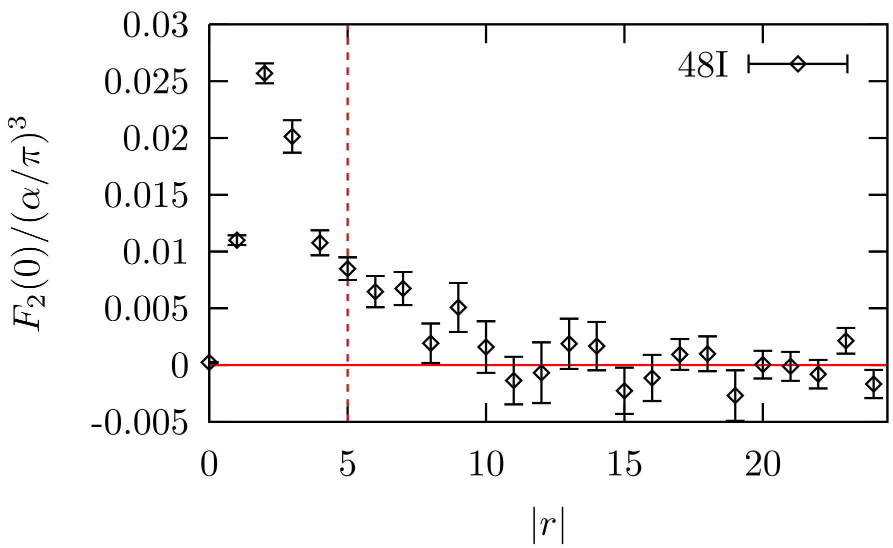


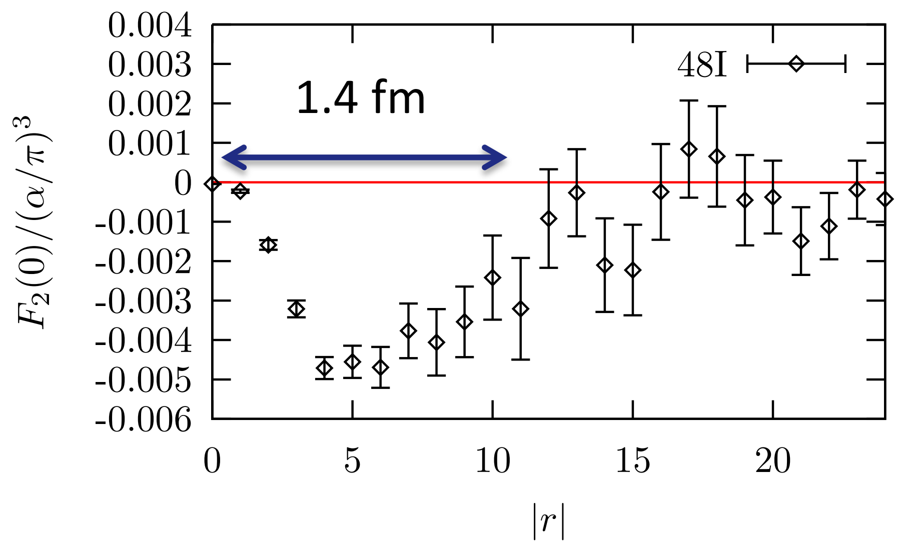


140 MeV Pion, connected and disconnected LbL results

[Luchang Jin et al. , Phys.Rev.Lett. 118 (2017) 022005]

- left: connected, right : leading disconnected

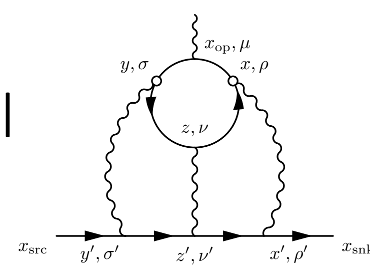




- Using AMA with 2,000 zMobius low modes, AMA

(statistical error only)

r = |x-y|



$$\left.\frac{g_\mu - 2}{2}\right|_{\mathrm{cHLbL}} = (0.0926 \pm 0.0077) \times \left(\frac{\alpha}{\pi}\right)^3 = (11.60 \pm 0.96) \times 10^{-10}$$

$$\left.\frac{g_\mu - 2}{2}\right|_{\mathrm{dHLbL}} = (-0.0498 \pm 0.0064) \times \left(\frac{\alpha}{\pi}\right)^3 = (-6.25 \pm 0.80) \times 10^{-10}$$

$$\left.\frac{g_\mu - 2}{2}\right|_{\mathrm{HLbL}} = (0.0427 \pm 0.0108) \times \left(\frac{\alpha}{\pi}\right)^3 = (5.35 \pm 1.35) \times 10^{-10}$$

Lattice 2017 Updates from PRL (2017)

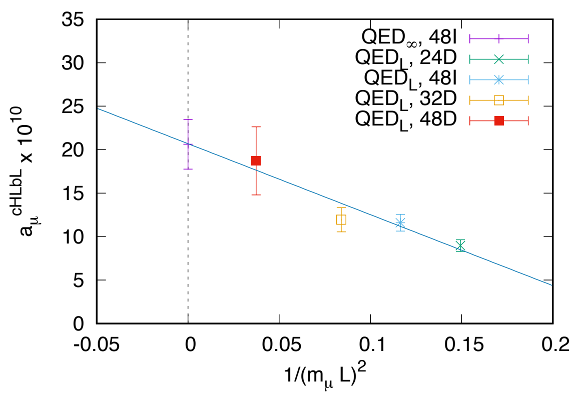
- Discretization error

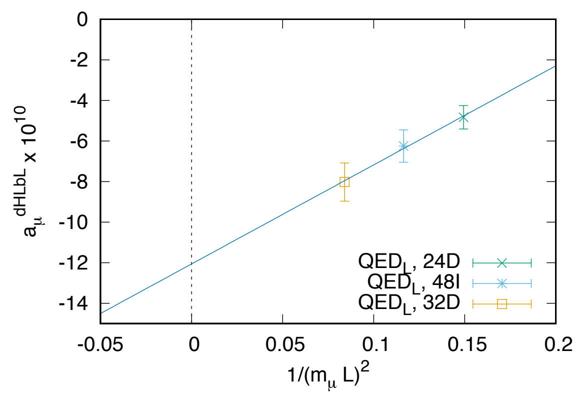
  → a scaling study for 1/a = 2.7, 1.4, 1.0 GeV at physical quark mass for both connected and disconnected is being finalized

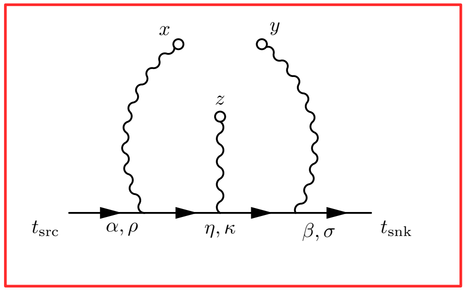
- Finite volume

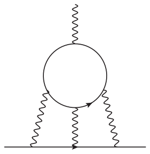
  QED_L (photon/lepton in a box) [08 Hayakawa Uno]

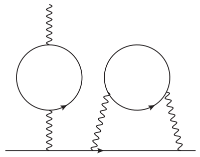
  Infinite Volume and continuum lepton + photon diagrams











Summary

- Lattice calculation for g-2 calculation is improved very rapidly
- **HVP**
  - New methods using low mode for connected at physical quark mass,
  - disconnected quark loop at physical quark mass, QED and IB studies are included
  - Combining with R-ratio experiment data for cross-check and improvement => 0.4 % error
  - Eventually the window will be enlarged for a pure LQCD prediction (currently 2.6 % error)

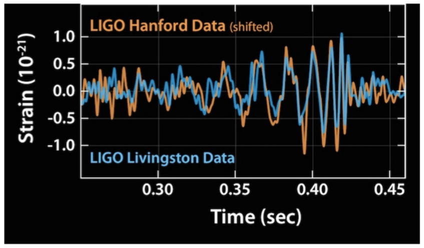
  - Significant improvements is in progress for statistical error using 2π and 4π (!) states in addition to EM current (GEVP, GS-parametrization)
  - Checking finite volume and discretization error as well as Isospin V effects

  - We could compute Inclusive hadron cross sections at Euclidean q^2 from the first principle Lattice QCD with Isospin breaking effects !
    e+e- -> hadron
    tau -> nu + hadrons
    tau inclusive decay and |Vus| arXiv:1803.07228 (to appear in PRL)

- **HLbL**
  - computing connected and leading disconnected diagrams :
    -> 8 % stat error in connected, 13 % stat error in leading disconnected
  - coordinate-space integral using analytic photon propagator with adaptive probability (point photon method), config-by-config conserved external current
  - Improving statistics right now.
  - Various size of Lattice ensemble / method for systematic error as well as higher disconnected diagram
    Comparing with Mainz group's results (for connected at heavy pion mass)

- Goal : HVP sub 1% (then 0.25%) , HLbL 10% error

Can we see the next physics Revolution (c.f GW) ?



Simulation details [RBC/UKQCD 2015]

two gauge field ensembles generated by RBC/UKQCD collaborations

Domain wall fermions: chiral symmetry at finite a

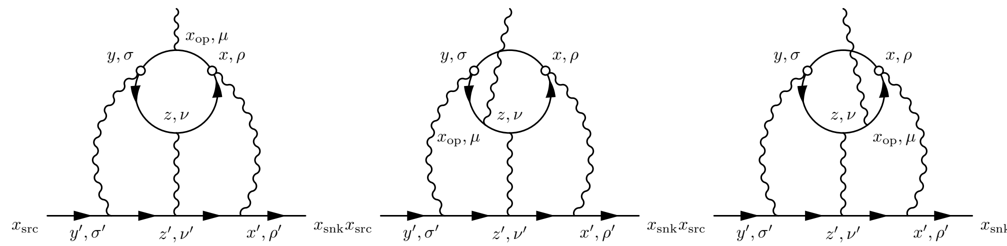
Iwasaki Gauge action (gluons)

- pion mass $m_\pi = 139.2(2)$ and 139.3(3) MeV ($m_\pi L \lesssim 4$)
- lattice spacings $a = 0.114$ and 0.086 fm
- lattice scale $a^{-1} = 1.730$ and 2.359 GeV
- lattice size $L/a = 48$ and 64
- lattice volume $(5.476)^3$ and $(5.354)^3$ fm^3

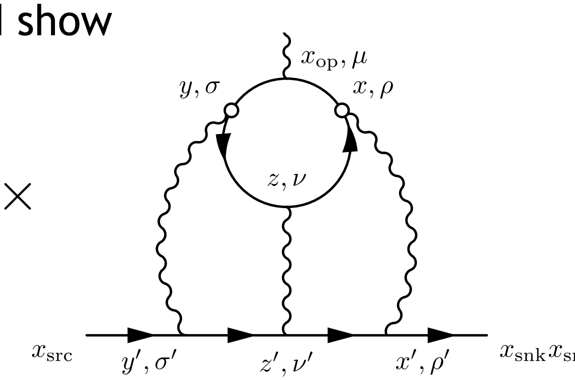
Use all-mode-average (AMA) [Blum et al 2012] and low-mode- averaging (LMA) [Giusti et al, 2004, Degrand et al 2005, Lehner 2016 for HVP] techniques for improved statistics by more than three orders of magnitudes compared to basic CG, and $\times 10$ smaller memory via multigrid-Lanczos [Lehner 2017] .

Conserved current & moment method

- [conserved current method at finite q2] To tame UV divergence, one of quark-photon vertex (external current) is set to be conserved current (other three are local currents). All possible insertion are made to realize conservation of external currents config-by-config.



- [moment method , q2→0] By exploiting the translational covariance for fixed external momentum of lepton and external EM field, q->0 limit value is directly computed via the first moment of the relative coordinate, xop – (x+y)/2, one could show



$$\frac{\partial}{\partial q_i}\mathcal{M}_\nu(\vec{q})|_{\vec{q}=0} = i \sum_{x,y,z,x_{\rm op}} (x_{\rm op} - (x+y)/2)_i \times$$

to directly get $F_2(0)$ without extrapolation.

$$\text{Form factor}: \ \Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\, m_l} F_2(q^2)$$

Current conservation & subtractions

- conservation => transverse tensor

$$\Pi^{\mu\nu}(q) = (\hat{q}^2\delta^{\mu\nu} - \hat{q}^\mu\hat{q}^\nu)\Pi(\hat{q}^2)$$

- In infinite volume, q=0, $\Pi_{\mu\nu}(q) = 0$
- For finite volume, $\Pi_{\mu\nu}(0)$ is exponentially small (L.Jin, use also in HLbL)

$$\int_V dx^4 \langle V_\mu(x)\mathcal{O}(0)\rangle = \int_V dx^4\, \partial_x\left(x\langle V_\mu(x)\mathcal{O}(0)\rangle\right)$$

$$= \int_{\partial V} dx^3\, x\langle V_\mu(x)\mathcal{O}(0)\rangle \propto L^4 \exp(-ML/2) \to 0$$

- e.g. DWF L=2, 3, 5 fm $\Pi_{\mu\nu}(0)$ = 8(3)e-4, 2(13)e-5, -1(5)e-8
- Subtract $\Pi_{\mu\nu}(0)$ alternates FVE, and reduce stat error
  "-1" subtraction trick :

$$\Pi^{\mu\nu}(q) - \Pi^{\mu\nu}(0) = \int d^4x(e^{iqx} - 1)\langle J^\mu(x)J^\nu(0)\rangle$$