Status of the lattice QCD calculations of the hadronic vacuum polarization contribution to the muon $g-2$ by the BMW collaboration

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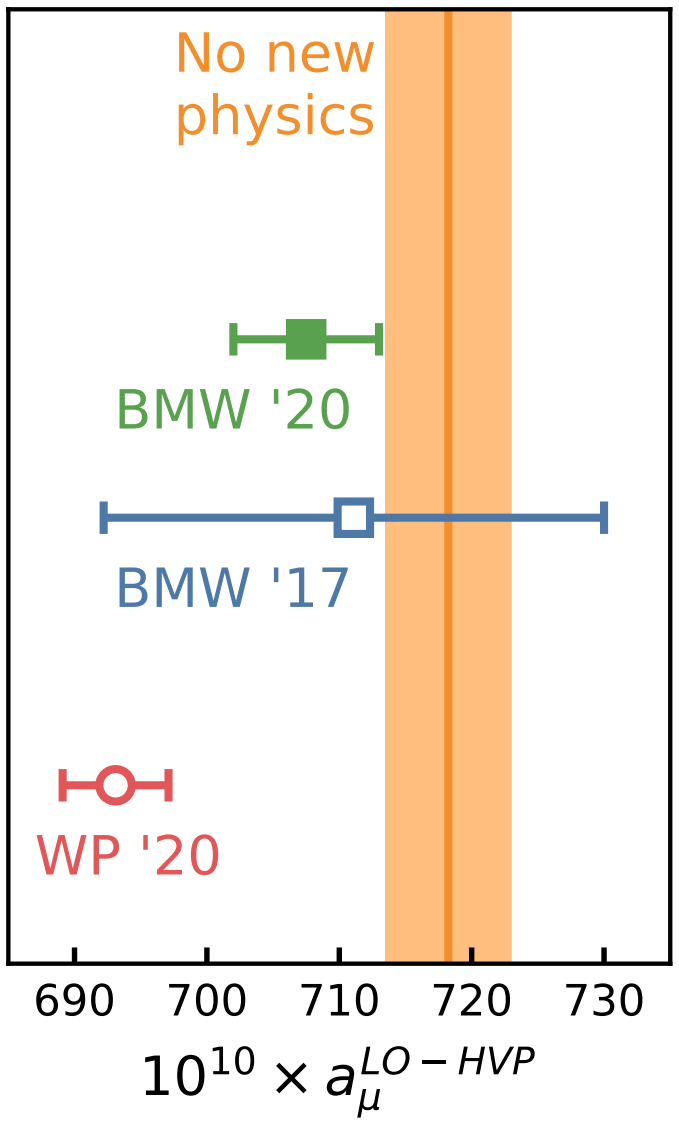
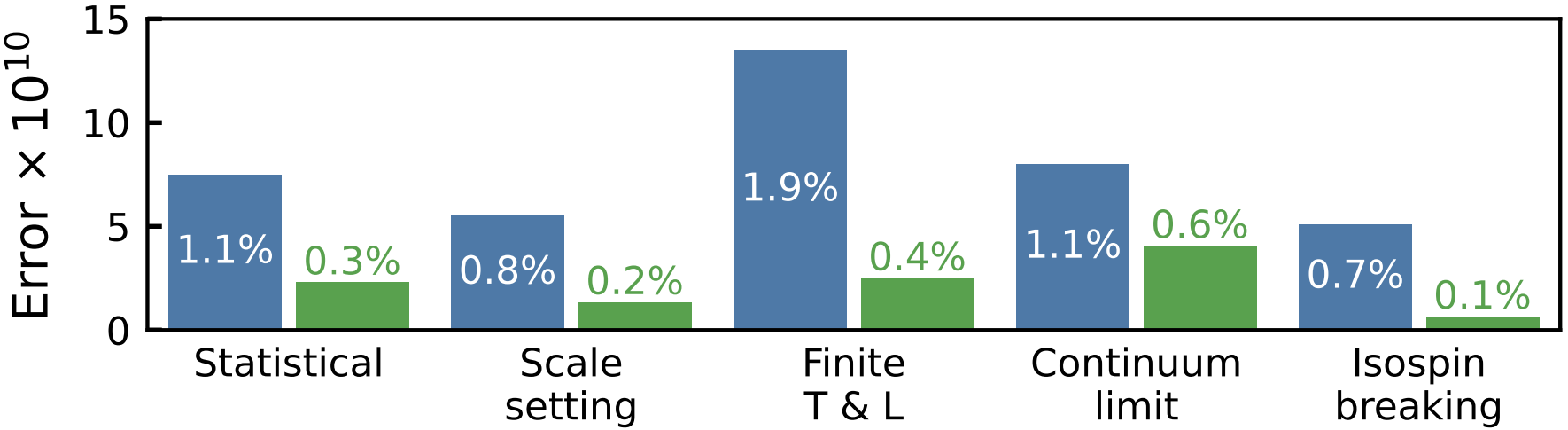
on behalf of the Budapest-Marseille-Wuppertal collaboration

Three years of progress

- Recently published sub-percent determination of HVP contribution to $g_\mu - 2$ [BMWc '20]
- First lattice calculation with errors comparable to data-driven determinations
- 3.4× increase in precision over our earlier work [BMWc '17]
- Many improvements needed to attain this precision, thanks to the work of many groups around the world



Key improvements



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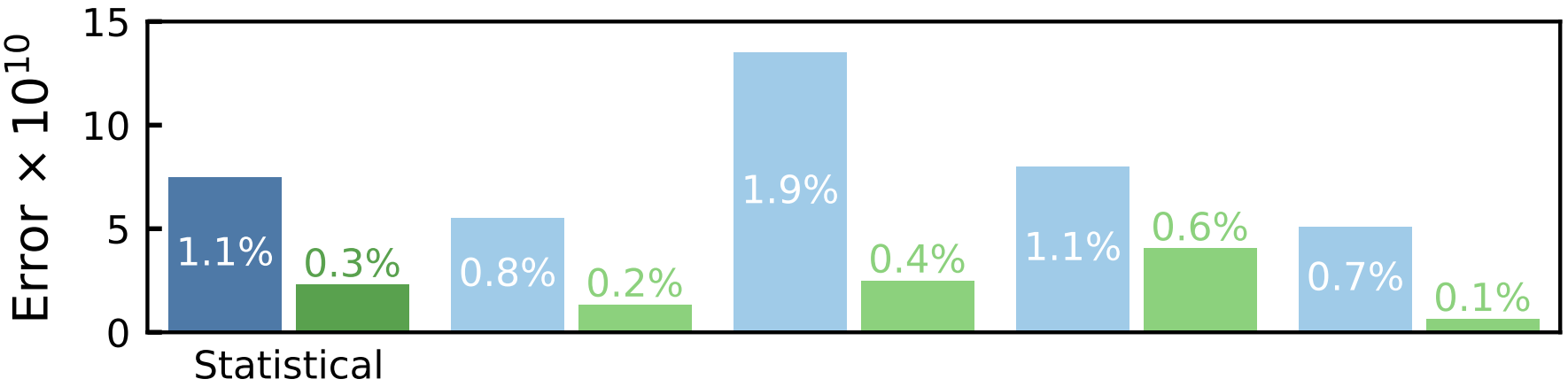

- Statistical noise in u/d contributions grows exponentially at large t
- Algorithmic improvements (EigCG, solver truncation [Bali et al '09], all mode averaging [Blum et al '13]) to generate more statistics
- Exact treatment of IR modes to reduce long-distance noise (low mode averaging [Neff et al '01, Giusti et al '04, ...])
- Rigorous upper/lower bounds on long-distance contribution [Lehner '16, BMWc '17]

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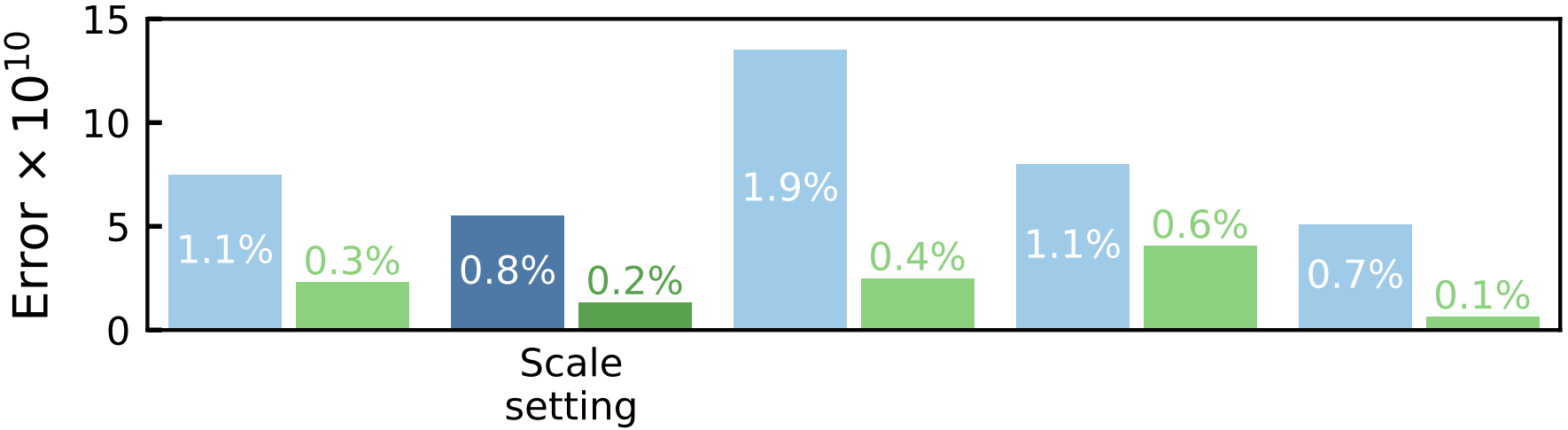

- Naïvely, relative errors in lattice spacing are doubled
- Requires permille determination of scale
- Use Ω^- baryon mass computed with 0.2% error
  - Partially subsumed into statistical error
- Wilson-flow scale [Lüscher '10, BMWc '12] for isospin decomposition

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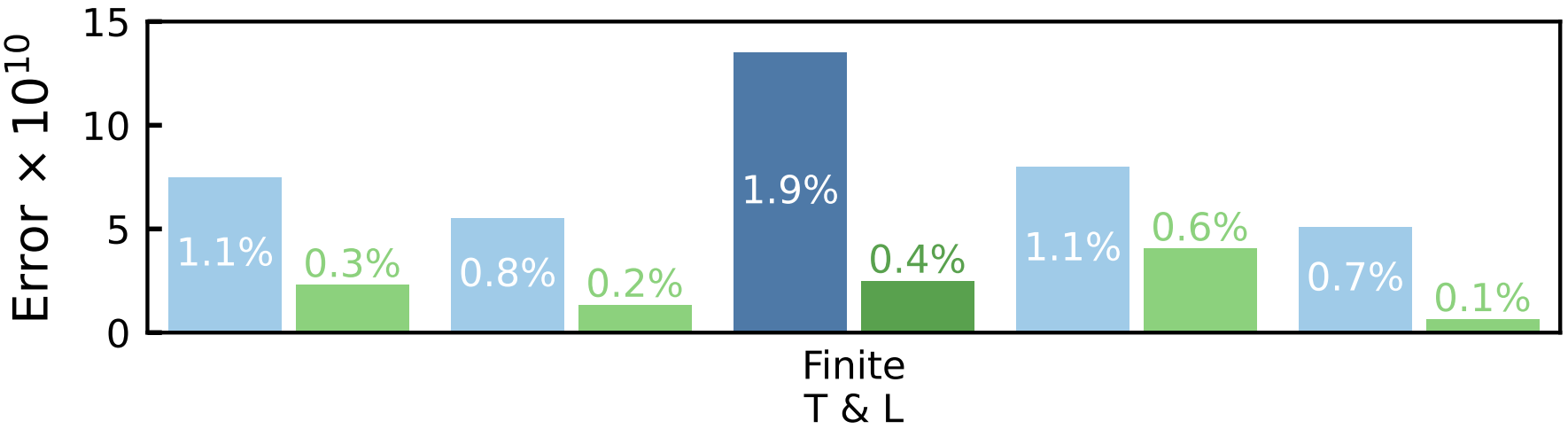
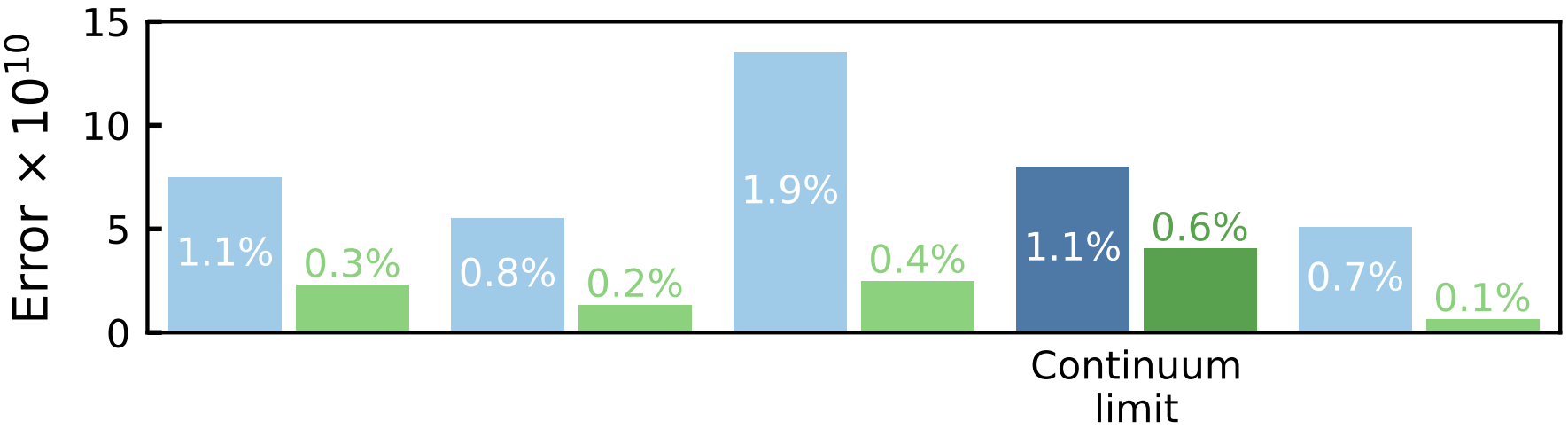


- Even in our large volumes ($L \gtrsim 6.1fm$, $T \geq 8.7fm$), exponentially suppressed FV effects are significant
- One-loop SU(2) χPT [Aubin et al '16] suggests $\sim 2\%$ effect
- Perform dedicated FV study with even larger volumes: $(\sim 11fm)^4$
- χPT & other models validated by comparing to lattice data
- Use two-loop χPT [Aubin et al '20] for tiny, residual correction

Key improvements



- Need controlled continuum ($a \to 0$) limit
- Perform all calculations at 6 lattice spacings: $0.134fm$–$0.064fm$
- Statistical error at finest a reduced from 1.9% to 0.3%!
- Improve continuum limit w/ EFTs and phenomenological models (SRHO) [Sakurai '60, Jegerlehner et al '11, Chakraborty et al '17, BMWc '20]
  - 2-loop SU(2) SχPT for systematic error [Bijnens et al '99, BMWc '20]
  - Models validated with lattice data

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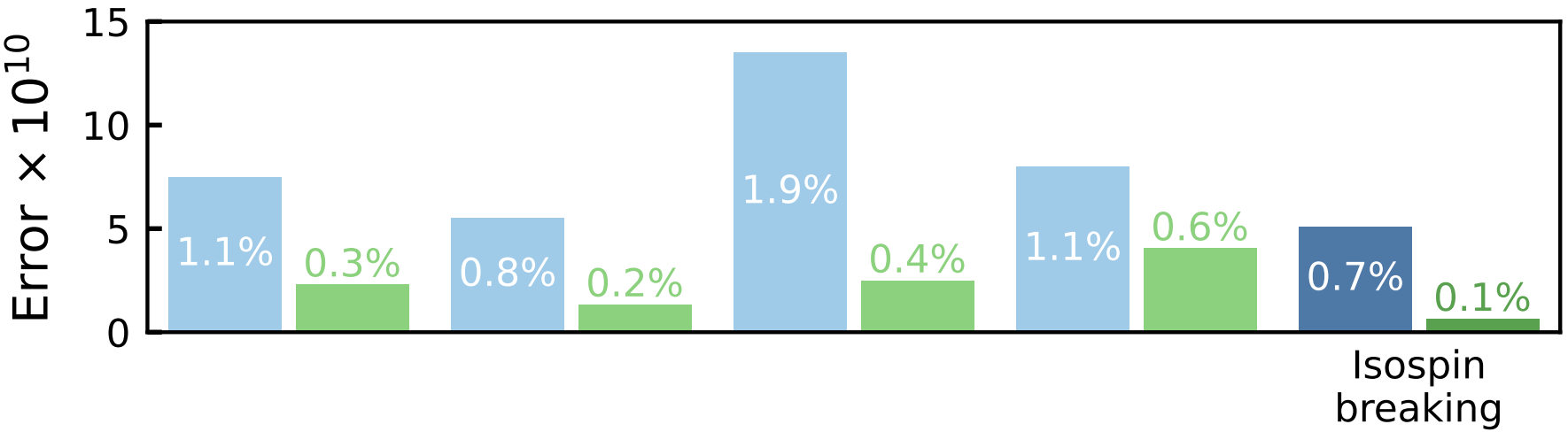

- Include all relevant isospin-breaking effects
- Compute *all* $O(\alpha)$ and $O(\delta m = m_d - m_u)$ effects on *all* quantities needed

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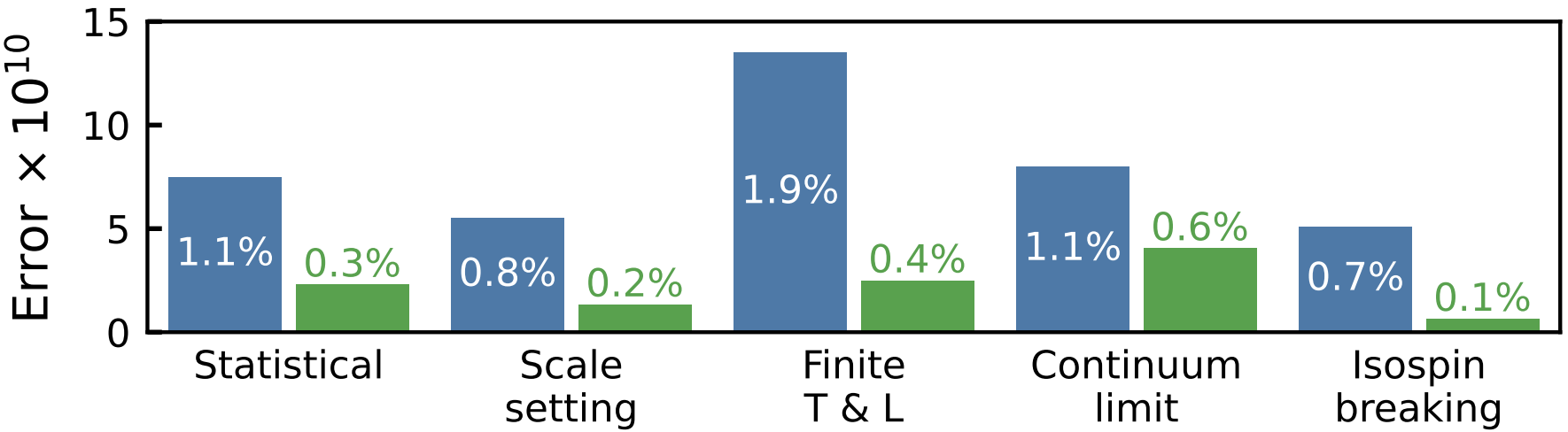

- Thorough & robust determination of statistical & systematic errors
- Statistical error: resampling methods
- Systematic error: extended frequentist approach [BMWc '08, '14]
  - Hundreds of thousands of different analyses of correlation functions
  - Weighted by AIC weight
  - Use median of distribution for central values & 68% confidence interval for total error

Recent improvements

- Result was surprising: put it on the arXiv (v1) and waited for six months for feedback from the community
- Incorporated suggestions (v2), and only then submitted for publication
  - increased our statistics
  - added a small neglected correction
  - changed continuum limit procedure
- Underwent thorough refereeing process (v3)
  - improved taste breaking corrections (SχPT & SMLLGS $\rightarrow$ SRHO)
  - included $a^2\alpha_s^3$ polynomials
- Despite many improvements, result changed by approximately one sigma

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Taste-breaking: SRHO model	-1.2

Conclusion

- Significant improvement in
  - Statistical noise
  - Scale setting
  - Finite size effects
  - Continuum limit
  - Isospin breaking
- Reduction in total error from 2.7% to 0.8%
- Shows surprising agreement with no-new-physics scenario
- Important to have lattice cross-checks
  - Particularly of $a_{\mu,win}$
- Important to understand disagreement with R-ratio