Theoretical status of semi-leptonic and rare *B* decays

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FPCP 2015
Nagoya

Outline

Motivation

Exclusive decays

Inclusive decays

Leptonic decays

Physics case

Semi-leptonic & Rare
B decays

Flavour changes in the Standard Model (SM)

$U_i = \{u, c, t\}$:

$Q_U = +2/3$

$D_j = \{d, s, b\}$:

$Q_D = -1/3$

$$\mathcal{L}_{\rm CC} = \frac{g_2}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma^\mu P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_\mu$$

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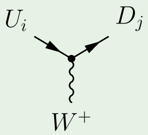

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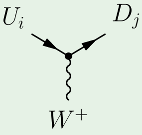

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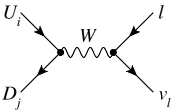

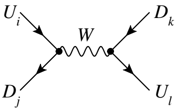

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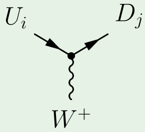

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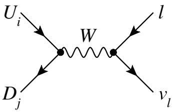

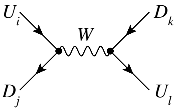

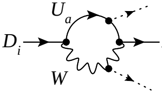

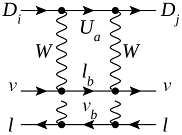

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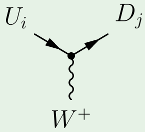

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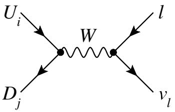

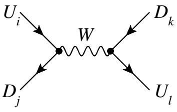

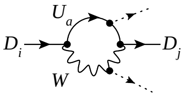

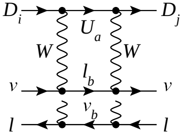

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$\mathcal{A} \sim G_F V_{ij}$

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$\sim G_F g \sum_a V_{ai} V^*_{aj} f(m_a)$

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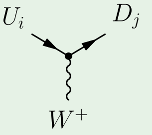

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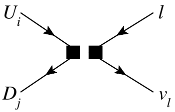

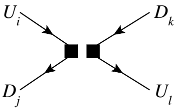

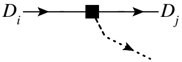

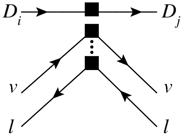

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Decoupling for $m_M \ll m_W$ ⇒ effective theory à la Fermi

$\mathcal{A} \sim G_F V_{ij}$ $\sim G_F V_{ij} V_{lk}^*$ $\sim G_F\, C(V_{ij}, m_a)$ $\sim G_F\, C(V_{ij}, m_a, m_b)$

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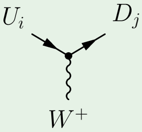


In SM: FCNC- w.r.t. CC-decays are . . .

quantum fluctuations = loop-suppressed

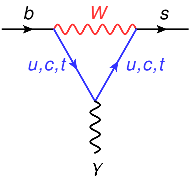
- no suppression of contributions beyond SM (BSM) wrt SM itself
- **indirect search for BSM signals**

  ⇒ additional contribution to

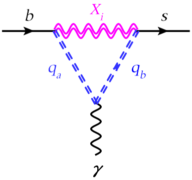
  effective coupling = $C^{\rm SM} + C^{\rm NP}$

BUT requires high precision, experimentally and theoretically !!!

SM



BSM



$C^{\rm SM}(V_{ij}, m_a) \quad + \quad C^{\rm NP}(W_{ij}, m_X, m_q)$

Fit of CKM matrix: Tree-level + $\Delta B = 2$ decays

$\Rightarrow$ fit of CKM-Parameters ...

4 Wolfenstein parameters

$$\boldsymbol{\lambda \sim 0.22,\ A,\ \rho,\ \eta}$$

$$V_{ij} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$\Rightarrow$ nowadays sophisticated fit: "combine and overconstrain" [CKMfitter, arXiv:1106.4041]

CKM	Process	Observables				Theoretical inputs
$\lvert V_{ud}\rvert$	$0^+ \to 0^+$ transitions	$\lvert V_{ud}\rvert_{\rm nucl}$	$=$	0.97425 ± 0.00022	[6]	Nuclear matrix elements
$\lvert V_{us}\rvert$	$K \to \pi\ell\nu$	$\lvert V_{us}\rvert_{\rm semi} f_+(0)$	$=$	0.2163 ± 0.0005	[7]	$f_+(0) = 0.9632 \pm 0.0028 \pm 0.0051$
	$K \to e\nu_e$	$\mathcal{B}(K \to e\nu_e)$	$=$	$(1.584 \pm 0.0020) \cdot 10^{-5}$	[8]	$f_K = 156.3 \pm 0.3 \pm 1.9$ MeV
	$K \to \mu\nu_\mu$	$\mathcal{B}(K \to \mu\nu_\mu)$	$=$	0.6347 ± 0.0018	[7]	
	$\tau \to K\nu_\tau$	$\mathcal{B}(\tau \to K\nu_\tau)$	$=$	0.00696 ± 0.00023	[8]	
$\lvert V_{us}\rvert / \lvert V_{ud}\rvert$	$K \to \mu\nu / \pi \to \mu\nu$	$\frac{\mathcal{B}(K \to \mu\nu_\mu)}{\mathcal{B}(\pi \to \mu\nu_\mu)}$	$=$	$(1.3344 \pm 0.0041) \cdot 10^{-2}$	[7]	$f_K/f_\pi = 1.205 \pm 0.001 \pm 0.010$
	$\tau \to K\nu / \tau \to \pi\nu$	$\frac{\mathcal{B}(\tau \to K\nu_\tau)}{\mathcal{B}(\tau \to \pi\nu_\tau)}$	$=$	$(6.33 \pm 0.092) \cdot 10^{-2}$	[9]	
$\lvert V_{cd}\rvert$	$D \to \mu\nu$	$\mathcal{B}(D \to \mu\nu)$	$=$	$(3.82 \pm 0.32 \pm 0.09) \cdot 10^{-4}$	[10]	$f_{D_s}/f_D = 1.186 \pm 0.005 \pm 0.010$
$\lvert V_{cs}\rvert$	$D_s \to \tau\nu$	$\mathcal{B}(D_s \to \tau\nu)$	$=$	$(5.29 \pm 0.28) \cdot 10^{-2}$	[11]	$f_{D_s} = 251.3 \pm 1.2 \pm 4.5$ MeV
	$D_s \to \mu\nu$	$\mathcal{B}(D_s \to \mu\nu_\mu)$	$=$	$(5.90 \pm 0.33) \cdot 10^{-3}$	[11]	
$\lvert V_{ub}\rvert$	semileptonic decays	$\lvert V_{ub}\rvert_{\rm semi}$	$=$	$(3.92 \pm 0.09 \pm 0.45) \cdot 10^{-3}$	[11]	form factors, shape functions
	$B \to \tau\nu$	$\mathcal{B}(B \to \tau\nu)$	$=$	$(1.68 \pm 0.31) \cdot 10^{-4}$	[4]	$f_{B_s} = 231 \pm 3 \pm 15$ MeV
						$f_{B_s}/f_B = 1.209 \pm 0.007 \pm 0.023$
$\lvert V_{cb}\rvert$	semileptonic decays	$\lvert V_{cb}\rvert_{\rm semi}$	$=$	$(40.89 \pm 0.38 \pm 0.59) \cdot 10^{-3}$	[11]	form factors, OPE matrix elts
α	$B \to \pi\pi,\ \rho\pi,\ \rho\rho$	branching ratios, CP asymmetries			[11]	isospin symmetry
β	$B \to (c\bar{c})K$	$\sin(2\beta)_{[c\bar{c}]}$	$=$	0.678 ± 0.020	[11]	
γ	$B \to D^{(*)}K^{(*)}$	inputs for the 3 methods			[11]	GGSZ, GLW, ADS methods
$V_{tq}^* V_{tq'}$	Δm_d	Δm_d	$=$	0.507 ± 0.005 ps^{-1}	[11]	$\hat{B}_{B_s}/\hat{B}_{B_d} = 1.01 \pm 0.01 \pm 0.03$
	Δm_s	Δm_s	$=$	17.77 ± 0.12 ps^{-1}	[12]	$\hat{B}_{B_s} = 1.28 \pm 0.02 \pm 0.03$
$V_{tq}^* V_{tq'},\ V_{cq}^* V_{cq'}$	ϵ_K	$\lvert\epsilon_K\rvert$	$=$	$(2.229 \pm 0.010) \cdot 10^{-3}$	[8]	$\hat{B}_K = 0.730 \pm 0.004 \pm 0.036$
						$\kappa_\epsilon = 0.940 \pm 0.013 \pm 0.023$

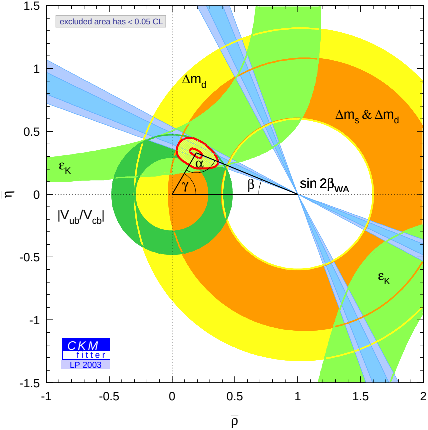
Fit of CKM matrix: Tree-level + $\Delta B = 2$ decays

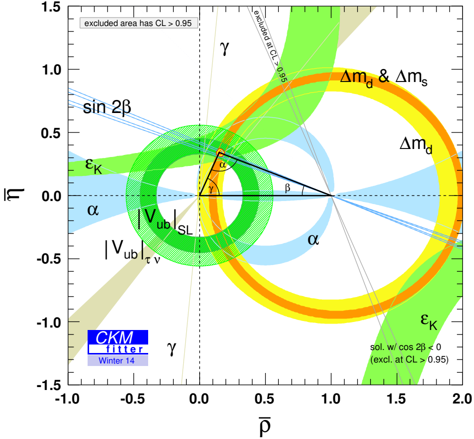
$\Rightarrow$ fit of CKM-Parameters ... 2003 $\rightarrow$ 2014

`http://ckmfitter.in2p3.fr/` :

improved by *B*-factories, Tevatron, LHC

Unitarity: $V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0$





See also “UTfit collaboration” **`http://www.utfit.org/UTfit/`**

See also “SCAN Method” [Eigen et al. arXiv:1301.5867 + 1503.02289]

Rich phenomenology in FCNC's – example $b \to s$

$b \to s + \gamma$

$B \to K^*\gamma \quad (B_s \to \phi\gamma)$

- Br
- time-dependent CP asy's: S, C, H
- iso-spin asymmetry Δ_{0-}

$B \to X_s\gamma$

- Br, dBr/dE_γ
- $A_{\rm CP}$ in $B \to X_s\gamma$ and $B \to X_{s+d}\gamma$

$B_s \to \gamma\gamma$

- Br
- $A_{\rm CP}$

$b \to s + \bar{\ell}\ell$

$B_s \to \bar{\ell}\ell$

- Br

$B \to K + \bar{\ell}\ell$

- $d^2Br/dq^2\,d\cos\theta_\ell \to dBr/dq^2$, $A_{\rm FB}$, F_H

$B \to K^*(\to K\pi) + \bar{\ell}\ell \quad (B_s \to \phi(\to \bar{K}K) + \bar{\ell}\ell)$

- $d^4Br/dq^2\,d\cos\theta_\ell\,d\cos\theta_{K^*}\,d\phi$

12 angular observables $J^{(s,c)}_{1,\ldots,9}(q^2)$ + CP-conj.

$\to dBr/dq^2$, $A_{\rm FB}$, F_L, $A_T^{(2,3,4,{\rm re},{\rm im})}$, $H_T^{(1,2,3,4,5)}$, …

$B \to X_s + \bar{\ell}\ell$

- $d^2Br/dq^2\,d\cos\theta_\ell$, $A_{\rm FB}$, H_T (or H_L)

… in $b \to s + \{\gamma, \gamma\gamma, \bar{\ell}\ell\}$ FCNC's to test short-distance **effective couplings**:

C_i for $i = 7, (7')$

C_i for $i = 7, 9, 10, (7', 9', 10', \ldots)$

BUT need **non-perturbative hadronic quantities**: (complementarity of exclusive and inclusive)

Decay constants and LCDA's for $B_{d,s}$, K, K^*, ϕ, …

Form factors: $(B \to K) \to f_{+,T,0}$ and $(B \to K^*, B_s \to \phi) \to V, A_{0,1,2}, T_{1,2,3}$

B-Hadron decays are a Multi-scale problem . . .

. . . with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in *B* restframe	$\gg$	QCD-bound state effects
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		$\Lambda_{\rm QCD} \approx 0.5$ GeV

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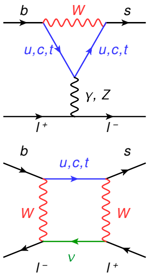
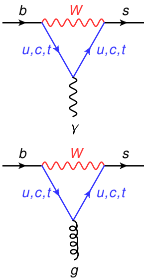
electroweak IA	$\gg$	ext. mom'a in *B* restframe	$\Rightarrow$	decoupling heavy particles
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		*W*, *Z*-boson, top-quark

$$\mathcal{L}_{\rm eff} \sim G_F\, V_{\rm CKM} \times \left[\sum_{9,10} C_i^{\ell\bar\ell}\, \mathcal{O}_i^{\ell\bar\ell} + \sum_{7\gamma,\,8g} C_i\, \mathcal{O}_i + \text{CC} + (\text{QCD \& QED-peng}) \right]$$

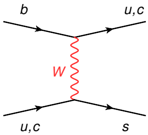
semi-leptonic



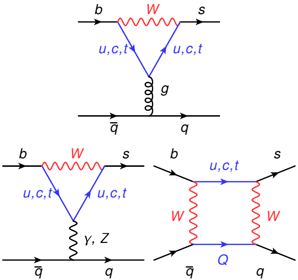
electro- & chromo-mgn



charged current



QCD & QED -penguin



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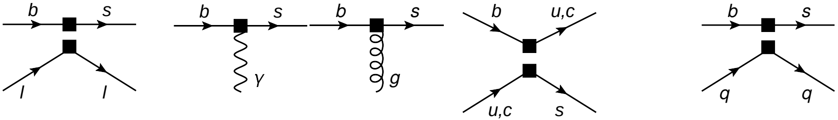
electroweak IA	$\gg$	ext. mom'a in *B* restframe	$\Rightarrow$	effective theory
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		at scales below m_B

$$\mathcal{L}_{\rm eff} \sim G_F\, V_{\rm CKM} \times \Big[\sum_{9,10} C_i^{\ell\bar\ell}\, \mathcal{O}_i^{\ell\bar\ell} + \sum_{7\gamma, 8g} C_i\, \mathcal{O}_i + \text{CC} + (\text{QCD \& QED-peng}) \Big]$$

semi-leptonic | electro- & chromo-mgn | charged current | QCD & QED -penguin



C_i = Wilson coefficients: contains short-dist. pmr's (heavy masses $M_t, \ldots$ – CKM factored out) and leading logarithmic QCD-corrections to all orders in α_s

$\Rightarrow$ in SM known up to NNLO QCD and NLO EW/QED

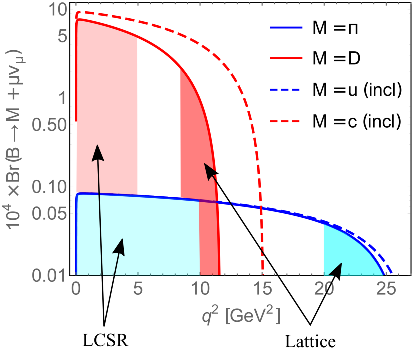
$\mathcal{O}_i$ = higher-dim. operators: flavour-changing coupling of light quarks

Exclusive decays

$$B \to (P, V) + \ell\bar{\nu}_\ell$$

$$B \to (K, K^*) + \bar{\ell}\ell$$

Overview $B \to (P, V) + \ell\bar{\nu}_\ell$

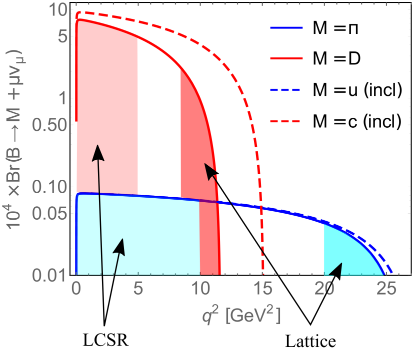


exclusive $B \to (P, V) + \ell\bar{\nu}_\ell$ decays require "only" hadronic form factors (FF) as input

$B \to (P, V)$ FF's commonly calculated via

- at low q^2 = large recoil:
  light-cone sum rules (LCSR)
- at high q^2 = low recoil:
  lattice QCD (LQCD)

Overview $B \to (P, V) + \ell\bar{\nu}_\ell$



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Example $B \to P$ vector FF's $f_{+,0}(q^2)$

$$\langle P(p-q)|\bar{q}\,\gamma_\mu\, b|B(p)\rangle = f_+(2p-q)_\mu + [f_0 - f_+]\frac{m_B^2 - m_P^2}{q^2}q_\mu$$

differential branching fraction — only f_+ relevant for $m_\ell \ll q^2$ $(\ell = e, \mu)$

$$\frac{d\mathcal{B}[B \to P\ell\bar{\nu}_\ell]}{dq^2} \propto \tau_B|V_{qb}|^2\left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\vec{p}|\left[m_B^2|\vec{p}|^2\left(1 - \frac{m_\ell^2}{2\,q^2}\right)^2 f_+^2 + \frac{3m_\ell^2}{8\,q^2}(m_B^2 + m_P^2)^2 f_0^2\right]$$

$B \to (P, V)$ FF uncertainties

Compilation of (some) latest LCSR and LQCD (lattice) results

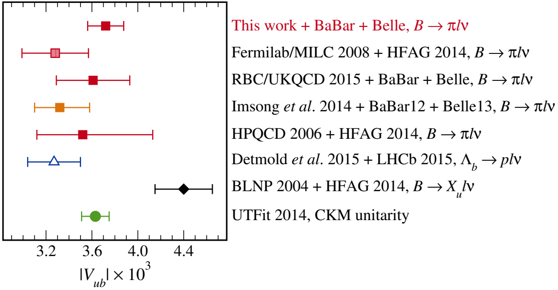
FF	method	q^2-region	uncertainty	Ref.	
		$B \to \pi$			$\lvert V_{ub}\rvert \times 10^3$
f_+	LCSR	$q^2 < 10$ GeV^2	$\approx 7\,\%$	Imsong et al. 1409.7816	$3.32^{+0.26}_{-0.22}$
$f_{+,0}$	LQCD	19 GeV$^2 < q^2$	$8 - 14\,\%$	RBC & UKQCD 1501.05373	3.61 ± 0.32
$f_{+,0}$	LQCD	20 GeV$^2 < q^2$	$\approx 4\,\%$	FNAL/MILC 1503.07839	3.72 ± 0.16
		$B \to \rho, \omega$			$\lvert V_{ub}\rvert \times 10^3$
V, A_i, T_j	LCSR	$q^2 < 14$ GeV^2	≈ 10 & $14\,\%$	Bharucha et al. 1503.05534	6 & 9 % th. err.
		$B \to D$			$\lvert V_{cb}\rvert \times 10^3$
$f_{+,0}$	LCSR	$q^2 < 6$ GeV^2	$\approx 27\,\%$	Faller et al. 0809.0222	–
$f_{+,0}$	LQCD	8.5 GeV$^2 < q^2$	$\approx 1.5\,\%$	FNAL/MILC 1503.07237	39.6 ± 1.7
$f_{+,0}$	LQCD	9.5 GeV$^2 < q^2$	$\approx 5\,\%$	HPQCD 1505.03925	40.2 ± 2.1
		$B \to D^*$			$\lvert V_{cb}\rvert \times 10^3$
V, A_i	LCSR	$q^2 < 6$ GeV^2	$\approx 27\,\%$	Faller et al. 0809.0222	–
$\mathcal{F}(1)$	LQCD	$q^2 = q^2_{\rm max}$	$1.4\,\%$	FNAL/MILC 1403.0635	39.04 ± 0.75

Future measurements of other channels welcome:

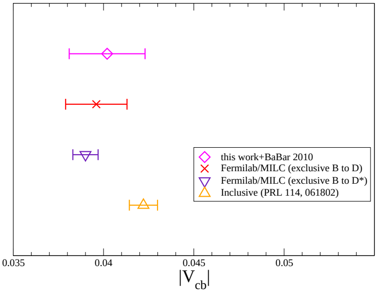
$B_s \to K$ LQCD HPQCD 1406.279, RBC & UKQCD 1501.05373, $B_s \to D_s$ LQCD Atoui et al. 1310.5238,

$\Lambda_b \to p$, $\Lambda_b \to \Lambda_c$ LQCD Detmold et al. 1503.01421

V_{ub} V_{cb}

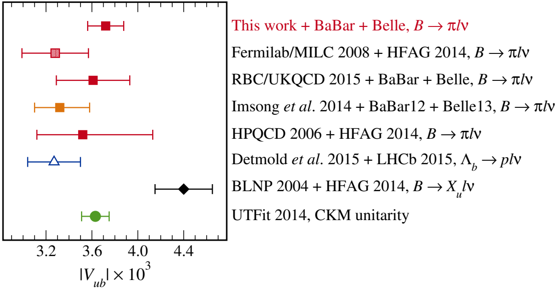


[FNAL/MILC arXiv:1503.07839]

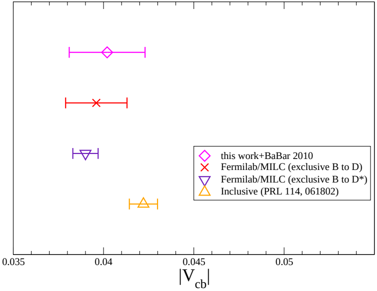


[HPQCD arXiv:1505.03925]

V_{ub} and V_{cb}



[FNAL/MILC arXiv:1503.07839]



[HPQCD arXiv:1505.03925]

$R(D)$ and $R(D^*)$

$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu}_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu}_\ell)}$$

$R(D)|_{\text{Babar}} = 0.440 \pm 0.072$ [Babar 1205.5442]

$R(D)|_{\text{Belle}} = 0.390 \pm 0.100$ [Belle$^{*)}$]

$R(D)|_{\text{SM}} = 0.299 \pm 0.011$ [FNAL/MILC 1503.07237]

$R(D)|_{\text{SM}} = 0.300 \pm 0.008$ [HPQCD 1505.03925]

- $R(D)$ No heavy quark limit

Belle Updates this conference !!!

$R(D^*)|_{\text{Babar}} = 0.332 \pm 0.030$ [Babar 1303.0571]

$R(D^*)|_{\text{Belle}} = 0.347 \pm 0.050$ [Belle$^{*)}$]

$R(D^*)|_{\text{SM}} = 0.252 \pm 0.003$

[Fajfer/Kamenik/Nišandžić 1203.2654]

- $R(D^*)$ with heavy quark limit

[$^{*)}$ Belle arXiv:0706.4429, 0910.4301, 1005.2302]

Angular analysis of $\overline{B} \to \overline{K}^* [\to \overline{K}\pi] + \bar{\ell}\ell$

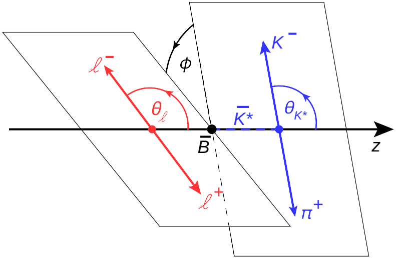
4-body decay with on-shell $\overline{K}^*$ (vector)

1) $q^2 = m^2_{\bar{\ell}\ell} = (p_\ell + p_{\bar{\ell}})^2 = (p_{\bar{B}} - p_{\bar{K}^*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle (\vec{p}_{\bar{B}}, \vec{p}_\ell)$ in $(\bar{\ell}\ell)$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle (\vec{p}_{\bar{B}}, \vec{p}_{\bar{K}})$ in $(\bar{K}\pi)$ – c.m. system

4) $\phi \angle (\vec{p}_{\bar{K}} \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



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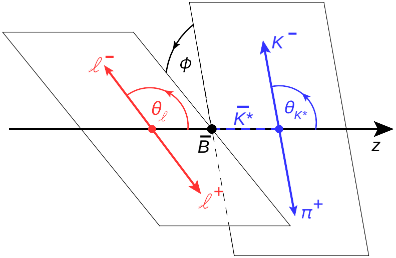

$J_i(q^2)$ = "Angular Observables"

$$\frac{32\pi}{9}\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_\ell$$
$$+ J_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + J_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_\ell + J_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ J_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + J_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi$$

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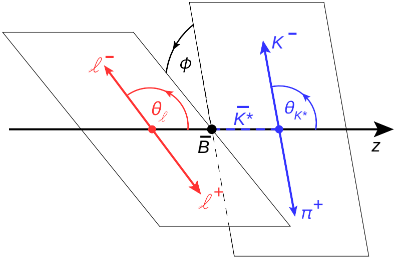

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$$+ J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$

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$$+ J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$

$\Rightarrow$ "$2 \times (12 + 12) = 48$" if measured separately: A) decay + CP-conj and B) for $\ell = e, \mu$

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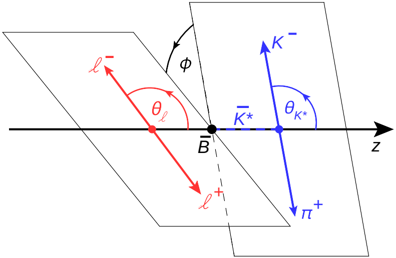


$\Rightarrow$ CP-averaged and CP-asymmetric angular observables

$$S_i = \frac{J_i + \bar{J}_i}{\Gamma + \bar{\Gamma}}, \qquad A_i = \frac{J_i - \bar{J}_i}{\Gamma + \bar{\Gamma}},$$

[Krüger/Sehgal/Sinha/Sinha hep-ph/9907386]
[Altmannshofer et al. arXiv:0811.1214]

CP-conj. decay $B^0 \to K^{*0}(\to K^+\pi^-)\ell^+\ell^-$: $d^4\bar{\Gamma}$ from $d^4\Gamma$ by replacing

$$\text{CP-even} : \quad J_{1,2,3,4,7} \longrightarrow +\bar{J}_{1,2,3,4,7}[\delta_W \to -\delta_W]$$

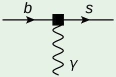
$$\text{CP-odd} : \quad J_{5,6,8,9} \longrightarrow -\bar{J}_{5,6,8,9}[\delta_W \to -\delta_W]$$

with weak phases δ_W conjugated

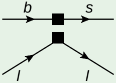
Exclusive $B \to K^* (\to K\pi)\, \bar{\ell}\ell$ . . . using narrow width appr. & intermediate K^* on-shell

Hadronic amplitude $B \to K^* (\to K\pi)\, \bar{\ell}\ell$ — neglecting 4-quark operators

$\mathcal{A}_\lambda = \langle \bar{\ell}\ell\, K^*_\lambda | \quad C_7 \times$



$+ \quad C_{9,10} \times$



$|B\rangle$

$\mathcal{A}_\lambda$ = transversity amplitudes of K^* $(\lambda = \perp, \parallel, 0)$

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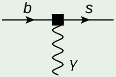

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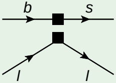


$|B\rangle$

$\mathcal{A}_\lambda$ = transversity amplitudes of K^* $(\lambda = \perp, \parallel, 0)$

- "Naive factorisation" of leptonic and quark currents: $\mathcal{A}_\lambda \sim C_i\ [\bar{\ell}\,\Gamma'_i\,\ell] \otimes \langle K^* | \bar{s}\,\Gamma_i\, b | B \rangle$
- "just" requires $B \to K^*$ form factors (=FF): V, $A_{1,2}$, $T_{1,2,3}$ $\qquad$ (A_0 contribution $\sim 2m_\ell/\sqrt{q^2}$)

$$A_\perp^{L,R} \simeq \sqrt{2\,\lambda}\left[(C_9 \mp C_{10})\frac{V}{m_B + m_{K^*}} + \frac{2\,m_b}{q^2} C_7\, T_1\right]$$

$$A_\parallel^{L,R} \simeq -\sqrt{2}\,(m_B^2 - m_{K^*}^2)\left[(C_9 \mp C_{10})\frac{A_1}{m_B - m_{K^*}} + \frac{2\,m_b}{q^2} C_7\, T_2\right]$$

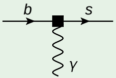
$$A_0^{L,R} \simeq -\frac{1}{2\,m_{K^*}\sqrt{q^2}}\left\{(C_9 \mp C_{10})\left[\ldots A_1 + \ldots A_2\right] + 2\,m_b C_7\left[\ldots T_2 + \ldots T_3\right]\right\}$$

- FF's @ low q^2: light-cone sum rules $\qquad$ [Ball/Zwicky hep-ph/0412079, Khodjamirian et al. arXiv:1006.4945]
- FF's @ high q^2: lattice calculations $\qquad$ [Horgan/Liu/Meinel/Wingate arXiv:1310.3722, 1310.3887]

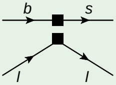
Exclusive $B \to K^*(\to K\pi)\,\bar{\ell}\ell$ … using narrow width appr. & intermediate K^* on-shell

Hadronic amplitude $B \to K^*(\to K\pi)\,\bar{\ell}\ell$ — including 4-quark operators

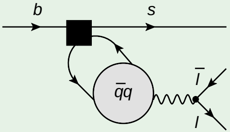
$\mathcal{A}_\lambda = \langle \bar{\ell}\ell\, K^*_\lambda |\quad C_7 \times$



$+\quad C_{9,10} \times$



$+ \sum_i C_i \times$



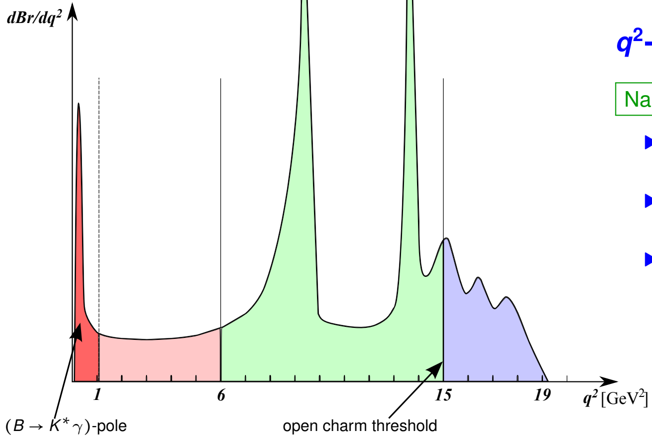
$|B\rangle$

… but 4-Quark operators and $\mathcal{O}_{8g}$ have to be included ⇒ no “naive factorisation” !!!

- current-current $b \to s + (\bar{u}u,\, \bar{c}c)$ ($b \to s\,\bar{u}u$ suppressed by $V_{ub}V^*_{us}$)
- QCD-penguin operators $b \to s + \bar{q}q$ ($q = u, d, s, c, b$) (small Wilson coefficients)

⇒ large peaking background around certain $q^2 = (m_{J/\psi})^2,\ (m_{\psi'})^2$:

$B \to K^{(*)}(\bar{q}q) \to K^{(*)}\bar{\ell}\ell$



q^2-Regions in $B \to K^* \bar{\ell}\ell$

Narrow resonances

- dominated by charged-cur. (tree-level) op's
- not sensitive to new physics in $b \to s\bar{\ell}\ell$
- nonperturbative predictions via: dispersion relations + $B \to K^*(\bar{c}c)$ data

Large Recoil (low-q^2)

- very low-q^2 ($\lesssim 1$ GeV^2) dominated by $\mathcal{O}_7$
  low-q^2 ($[1, 6]$ GeV^2) dominated by $\mathcal{O}_{9,10}$
- 1) QCD factorization or SCET
  2) LCSR
  3) non-local OPE of $\bar{c}c$-tails

[Beneke/Feldmann/Seidel hep-ph/0106067 + 0412400;
Lyon/Zwicky et al. 1212.2242 +1305.4797; Khodjamirian et al. 1006.4945 + 1211.0234]

Low Recoil (high-q^2)

- dominated by $\mathcal{O}_{9,10}$
- local OPE (+ HQET) ⇒ theory only for sufficiently large q^2-integrated obs's
  [Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann 1101.5118]

"Optimized observables" in $B \to K^* \bar{\ell}\ell$

Idea: reduce form factor (=FF) sensitivity by combination (usually ratios) of angular obs's J_i

⇒ guided by large energy limit @ low-q^2 and Isgur-Wise @ high-q^2 FF-relations

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@ low q^2 = large recoil

$$A_T^{(2)} = P_1 = \frac{J_3}{2\,J_{2s}}, \qquad A_T^{(\mathrm{re})} = 2\,P_2 = \frac{J_{6s}}{4\,J_{2s}}, \qquad A_T^{(\mathrm{im})} = -2\,P_3 = \frac{J_9}{2\,J_{2s}},$$

$$P_4' = \frac{J_4}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_5' = \frac{J_5/2}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_6' = \frac{-J_7/2}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_8' = \frac{-J_8}{\sqrt{-J_{2c}J_{2s}}},$$

$$A_T^{(3)} = \sqrt{\frac{(2\,J_4)^2 + J_7^2}{-2\,J_{2c}\,(2\,J_{2s} + J_3)}}, \qquad A_T^{(4)} = \sqrt{\frac{J_5^2 + (2\,J_8)^2}{(2\,J_4)^2 + J_7^2}}$$

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571]

[Becirevic/Schneider arXiv:1106.3283]

[Matias/Mescia/Ramon/Virto arXiv:1202.4266]

[Descotes-Genon/Matias/Ramon/Virto arXiv:1207.2753]

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⇒ guided by large energy limit @ low-q^2 and Isgur-Wise @ high-q^2 FF-relations

@ high q^2 = low recoil

$$H_T^{(1)} = P_4 = \frac{\sqrt{2} J_4}{\sqrt{-J_{2c}(2J_{2s} - J_3)}},$$

$$H_T^{(2)} = P_5 = \frac{J_5/\sqrt{2}}{\sqrt{-J_{2c}(2J_{2s} + J_3)}}, \qquad H_T^{(3)} = \frac{J_{6s}/2}{\sqrt{(2J_{2s})^2 - (J_3)^2}},$$

$$H_T^{(4)} = Q = \frac{\sqrt{2} J_8}{\sqrt{-J_{2c}(2J_{2s} + J_3)}}, \qquad H_T^{(5)} = \frac{-J_9}{\sqrt{(2J_{2s})^2 - (J_3)^2}},$$

$$\frac{A_9}{A_{\rm FB}} = \frac{J_9}{J_{6s}}, \qquad \text{and} \qquad \frac{J_8}{J_5}$$

[CB/Hiller/van Dyk arXiv:1006.5013]
[Matias/Mescia/Ramon/Virto arXiv:1202.4266]
[CB/Hiller/van Dyk arXiv:1212.2321]

Theory uncertainties in $B \to K^* \bar{\ell}\ell$

Form factors available from LCSR and LQCD

- @ low-q^2 LCSR results of $(B \to K, K^*)$, $(B_s \to \phi)$ FF's: 8 – 10 % uncertainty
  [Bharucha/Straub/Zwicky 1503.05534]
- @ high-q^2 LQCD results of $(B \to K, K^*)$, $(B_s \to \phi)$ FF's: 6 – 9 % uncertainty
  [Bhouchard et al. 1306.2384, Horgan et al. 1310.3722, 1501.00367]

$\Rightarrow$ alone 15 – 20 % FF-uncertainty in "non-optimised" observables

??? What about "optimised" observables — example P_5'

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Ref.	$q^2 \in [2.5, 4]$ GeV^2	$q^2 \in [4, 6]$ GeV^2	$q^2 \in [1, 6]$ GeV^2
LHCb (3/fb)	$-0.07^{+0.34}_{-0.36}$	-0.30 ± 0.16	$-0.05^{+0.11}_{-0.10}$
ABSZ (qua)	-0.50 ± 0.10	-0.77 ± 0.07	-0.44 ± 0.08
ABSZ (lin)	-0.50 ± 0.16	-0.77 ± 0.11	—
DHMV (qua)	$-0.47^{+0.16}_{-0.17}$	$-0.82^{+0.10}_{-0.12}$	—
DHMV (lin)	$-0.47^{+0.28}_{-0.32}$	$-0.82^{+0.18}_{-0.23}$	—
JMC 1 (lin)	$-0.25^{+0.31}_{-0.27}$	—	$-0.28^{+0.30}_{-0.26}$
JMC 2 (lin)	—	—	$-0.28^{+0.38}_{-0.36}$

errors added : lin = linearly, qua = in quadrature

LHCb = LHCb-CONF-2015-002, ABSZ = 1503.05534 + 1503.06199, DHMV = 1503.03328, JMC 1 / 2 = 1212.2263 / 1412.3183

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??? What about "optimised" observables — example P'_5

- ABSZ (contrary to DHMV and JMC) uses full QCD FF's from LCSR
  ⇒ do not employ FF relations @ LO in QCDF
  ⇒ do not consider subleading corrections to FF's, only to $B \to K^* \bar{\ell}\ell$ amplitudes
- DHMV try to implement error estimates as closely to JMC 1 as possible
  ⇒ same parameterisation of FF-relation breaking corrections
- for linearly added errors: uncertainties comparable for DHMV and JMC 1
- central values between DHMV/ABSZ and JMC very different,
  due to choice of central values of FF-relation breaking corrections

LHCb = LHCb-CONF-2015-002, ABSZ = 1503.05534 + 1503.06199, DHMV = 1503.03328, JMC 1 / 2 = 1212.2263 / 1412.3183

Inclusive decays

$$B \to X_c + \ell\bar{\nu}_\ell$$

$$B \to X_{d,s} + (\gamma, \bar{\ell}\ell)$$

Inclusive decays = Heavy Quark Expansion (HQE)

1) $B(p_B) \to X(p_X) + L(q)$ via optical theorem $\Rightarrow$ absorptive part of $B \to B$

$X = X_{u,d,s,c}$ and $L = (\gamma,\, \ell\bar{\nu}_\ell,\, \bar{\ell}\ell)$

$$d\Gamma = \frac{(2\pi)^4}{2M_B} \sum_X d[PS]\delta^{(4)}(p_B - p_X - q)\langle B|i\mathcal{L}^\dagger_{\text{eff}}|X + L\rangle\langle L + X|i\mathcal{L}_{\text{eff}}|B\rangle$$

$$\sim \frac{(2\pi)^4}{2M_B} d[PS]\delta^{(4)}(p_B - p_X - q)\ \text{Im}\langle B|\mathrm{T}\{\mathcal{L}^\dagger_{\text{eff}}\mathcal{L}_{\text{eff}}\}|B\rangle$$

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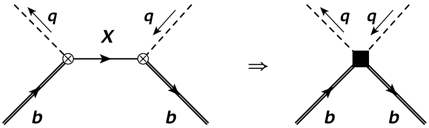
2) local OPE + (HQET-) matrix elements – z_i = Wilson coefficients $(\mu \sim m_b \gg \Lambda_{\rm QCD})$

$$\Rightarrow \quad \mathrm{T}\{\mathcal{L}^\dagger_{\rm eff}\mathcal{L}_{\rm eff}\} \overset{!}{=} z_1(\bar{b}b) + \frac{z_2}{m_b^2}(\bar{b}g\sigma\cdot Gb) + \sum \frac{z_{qi}}{m_b^3}(\bar{b}\Gamma_i q)(\bar{q}\Gamma_i b) + \ldots$$



$$p_X^2 = (p_B - q)^2 < (m_b - \sqrt{q^2})$$

OPE = expansion in $\Lambda_{\rm QCD}/(m_b - \sqrt{q^2})$

$\Rightarrow$ breaks down for $\sqrt{q^2} \to m_b^2$

(q^2-integrated quantities still supposed to be reliable)

$$\Rightarrow \quad \langle B|\bar{b}b|B\rangle \overset{\rm (HQET)}{=} 1 + \frac{1}{2m_b^2}\langle B|\bar{h}_b(iD)^2 h_b|B\rangle + \frac{1}{4m_b^2}\langle B|\bar{h}_b(g\sigma\cdot G)h_b|B\rangle + \mathcal{O}\left[\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3\right]$$

$$d\Gamma \sim \text{parton result} + \mathcal{O}\left[\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2\right]$$

Phase space cuts ...

... usually required by experiment to suppress backgrounds (bkg):

- $B \to X_u \ell \bar{\nu}_\ell$ complicated due to huge charm-bkg, shape functions from $B \to X_s \gamma$ moments
- $B \to X_c \ell \bar{\nu}_\ell$ E_ℓ lepton energy
- $B \to X_s \gamma$ $E_\gamma \in [1.7, 2.0]$ GeV (photon energy in B-meson restframe)
- $B \to X_s \bar{\ell}\ell$ $M_{X_s} < [1.8, 2.0]$ GeV to remove double semileptonic bkg
  practically irrelevant at high q^2, but not at low q^2

!!! extrapolations beyond cuts introduce model-dependent uncertainties in measurements

OR ...

... introduce new scales in theory

$\Rightarrow$ rate less inclusive $\Rightarrow$ additional non-perturbative effects (shape functions etc.)

... so far extrapolation beyond cuts mostly left to experimentalists

Inclusive $B \to X_c \ell \bar{\nu}_\ell$

Decay width and also moments (in lepton energy, hadronic mass)

$\Rightarrow$ double expansion in $a_s \equiv \alpha_s(\mu)/\pi$ and Λ_{QCD}/m_b

$$M_i = M_i^{(0)} + a_s M_i^{(1)} + a_s^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + a_s M_i^{(\pi,0)}\right) \frac{\mu_\pi^2}{m_b^2} + \left(M_i^{(G,0)} + a_s M_i^{(G,0)}\right) \frac{\mu_G^2}{m_b^2}$$
$$+ M_i^{(D)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS)} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}\left(a_s^3, \frac{a_s^2}{m_b^2}, \frac{a_s}{m_b^3}, \frac{1}{m_b^4}, \frac{1}{m_b^3 m_c^2}\right)$$

[see refs. in review Gambino arXiv:1501.00314]

- $M_i^{(j)}$ depend on m_c, m_b, E_{cut} and μ (renormalization schemes: "kinetic" or "1S")
- dim-5: $\mu_\pi^2 \propto \langle B|\bar{b}_v(\vec{D})^2 b_v|B\rangle$, $\mu_G^2(\mu) \propto \langle B|\bar{b}_v \sigma_{\mu\nu} G^{\mu\nu} b_v|B\rangle$ and dim-6: $\rho^3_{D,\,LS}$
- $\mathcal{O}(1/m_Q^{4,5})$ contributions estimated 1.3% effect to Γ, less on moments

[Mannel/Turzcyk/Uraltsev 1009.4622, Heinonen/Mannel 1407.4384]

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Combined fit of V_{cb} with $m_{b,c}$, $\mu^2_{\pi,G}$ and $\rho^3_{D,LS}$ from

- data (Babar, Belle, CLEO, DELPHI, CDF): (total + partial) width + moments
- external input on quark masses (m_c), hyperfine splitting ($M_{B^*} - M_B$) for μ_G, ...

Final precision of the fit of $|V_{cb}|_{\rm incl} = (42.21 \pm 0.78) \times 10^{-3}$

only 2% from combination of exp. and th. uncertainties

[Alberti/Gambino/Healey/Nandi 1411.6560]

Inclusive $B \to X_s\gamma$

$$\Gamma(B \to X_q\gamma) = \Gamma(b \to q\gamma)_{\rm p} + \delta\Gamma_{\rm np}$$

$$\propto (|C_7|^2 + |C_7'|^2)$$

- $\Gamma(b \to q\gamma)_{\rm p}$ = perturbatively calculable part @ NNLO
- $\delta\Gamma_{\rm np}$ = non-perturbative part
  around 5% uncertainty @ $E_\gamma \geq 1.6$ GeV
  [Benzke/Lee/Neubert/Paz arXiv:1003.5012]
- $b \to du\bar{u}\gamma$ sizeable in $b \to d\gamma$
  [Asatrian/Greub et al. arXiv:1305.6464]

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  [Asatrian/Greub et al. arXiv:1305.6464]

Latest SM updates @ NNLO QCD for $E_\gamma \geq 1.6$ GeV [Misiak et al. arXiv:1503.01789]

$$\mathcal{B}(B \to X_s\gamma)|_{\rm SM} = (3.36 \pm 0.23) \times 10^{-4}$$

$$\mathcal{B}(B \to X_d\gamma)|_{\rm SM} = (1.73^{+0.12}_{-0.22}) \times 10^{-5}$$

uncertainty budget due to:

- 5% non-perturbative
- 3% higher order
- 3% interpolation of m_c-dep. in NNLO corr.
- 2% parametric

Better adopted for actual measurement without strange tagging $\Rightarrow X_{s+d}$:

$$R_\gamma \equiv \frac{\mathcal{B}(B \to X_s\gamma) + \mathcal{B}(B \to X_d\gamma)}{\mathcal{B}(B \to X_x\ell\bar{\nu}_\ell)} = (3.31 \pm 0.22) \times 10^{-3}$$

Current world averages

$$\mathcal{B}(B \to X_s\gamma)|_{\rm Exp} = (3.43 \pm 0.22) \times 10^{-4} \qquad \mathcal{B}(B \to X_d\gamma)|_{\rm Exp} = (1.41 \pm 0.57) \times 10^{-5}$$

Inclusive $B \to X_s \bar{\ell}\ell$

3 observables in angular analysis $\qquad Br \propto (H_L + H_T)$ and $A_{\rm FB} \propto H_A$

$$\frac{8}{3}\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = \left(1+\cos^2\theta_\ell\right) H_T(q^2) + 2\left(1-\cos^2\theta_\ell\right) H_L(q^2) + 2\cos\theta_\ell H_A(q^2)$$

with different dependence on $C_{7,9,10}$ $\qquad \hat{s} = q^2/m_b^2$

$$H_T \propto \hat{s}\,(1-\hat{s})^2 \left[|C_9 + \frac{2}{\hat{s}} C_7|^2 + |C_{10}|^2\right] \qquad H_L \propto (1-\hat{s})^2 \left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$

$$H_A \propto -4\,\hat{s}(1-\hat{s})^2 \mathrm{Re}\left[(C_9 + \frac{2}{\hat{s}} C_7) C_{10}^*\right]$$

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SM predictions @ NNLO QCD and NLO QED [Huber/Hurth/Lunghi arXiv:1503.04849]

- theory unc. for $\mathcal{B}$ & $H_{L,T}$: 6 – 9 % in $q^2 \in [1, 3.5], [3.5, 6], [1, 6]$ GeV^2
- theory unc. for H_A: from 5 – 70 %, depend strongly on q^2-binning around zero-crossing
- zero-crossing of H_A predicted with $\lesssim 4\,\%$
- QED corrections lead to pronounced differences for $\ell = e$ and $\ell = \mu$
- at high-q^2 uncertainties larger: $\mathcal{B}$ about 30 %

Effects of cuts in M_{X_s} have been analysed in SCET at level of sub-leading shape functions
$\Rightarrow$ require combination of $B \to X_s\gamma$, $B \to X_s\bar{\ell}\ell$ and $B \to X_u\ell\bar{\nu}_\ell$

[Lee et al. hep-ph/0511334, 0512191, 0812.0001, Bernlocher et al.1101.3310, Bell et al. 1007.3758]

Leptonic decays

$$B \to \ell \bar{\nu}_\ell \quad \& \quad B_q \to \bar{\ell}\ell$$

Leptonic decays . . .

- . . . are helicity-suppressed in the SM $\propto m_\ell^2$

  $\Rightarrow$ largest Br's for $\ell = \tau$, but experimentally challenging

  $\Rightarrow$ enhanced sensitivity to scalar couplings

- . . . depend only on one hadronic input (@ LO in QED)

  B-meson decay constant: $ip_\mu f_B \equiv \langle 0|\bar{q}\gamma_\mu\gamma_5 b|B(p)\rangle$

  $\Rightarrow$ first lattice calculations with $\mathcal{O}(2\%)$ uncertainty [see details in FLAG 1310.8555 + website]

  @ NLO QED $\Rightarrow$ first conceptual studies for lattice calculations [Carrasco et al. arXiv:1502.00257]

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$B \to \ell\bar{\nu}_\ell$

$$\mathcal{B}(B^- \to \ell\bar{\nu}_\ell) = \frac{G_F^2 m_B m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

SM estimate f_B uncertainty on $\mathcal{B} \approx 4\%$

$\Rightarrow$ no CKM uncertainty if used to extract $|V_{ub}|$

Measurement

[Belle hep-ex/0611045, Babar arXiv:0903.1220, Belle arXiv:1503.05613, Babar arXiv:1207.0698]

$$\mathcal{B}(B^- \to e\bar{\nu}_e)|_{\rm SM} \sim 8 \times 10^{-12} \qquad \mathcal{B}(B^- \to e\bar{\nu}_e)|_{\rm Exp} < 9.8 \times 10^{-7} \quad @\,90\%\,\text{CL}$$

$$\mathcal{B}(B^- \to \mu\bar{\nu}_\mu)|_{\rm SM} \sim 3 \times 10^{-7} \qquad \mathcal{B}(B^- \to \mu\bar{\nu}_\mu)|_{\rm Exp} < 1.0 \times 10^{-6} \quad @\,90\%\,\text{CL}$$

$$\mathcal{B}(B^- \to \tau\bar{\nu}_\tau)|_{\rm SM} \sim 8 \times 10^{-5} \qquad \mathcal{B}(B^- \to \tau\bar{\nu}_\tau)|_{\rm Exp} = \begin{cases} (9.1 \pm 2.2) \times 10^{-5} & (4.6\,\sigma) \\ (17.9 \pm 4.8) \times 10^{-5} & (3.8\,\sigma) \end{cases}$$

$B_{d,s} \to \bar{\ell}\ell$

- NNLO QCD crrs. reduce μ_0-dep. from 1.8 % at NLO → 0.2 % at NNLO
  [Hermann/Misiak/Steinhauser arXiv:1311.1347]
- NLO EW crrs. reduce scheme-dependence from 7 % at LO → 0.3 % at NLO
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SM predictions @ NNLO QCD & NLO EW

$$\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9}$$

$$\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu)_{\rm SM} = (1.06 \pm 0.09) \times 10^{-10}$$

[CB/Gorbahn/Hermann/Misiak/Stamou/Steinhauser 1311.0903]

Measurement

$$\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)_{\rm Exp} = (2.8^{+0.7}_{-0.6}) \times 10^{-9} \qquad (6.2\sigma)$$

$$\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu)_{\rm Exp} = (3.9^{+1.6}_{-1.4}) \times 10^{-10} \qquad (3.2\sigma)$$

[LHCb + CMS 1411.4413]

⇒ future experimental uncertainties: 5 % @ LHCb (50/fb) and 15 % @ CMS (100/fb)

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[LHCb + CMS 1411.4413]

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Error budget	f_{B_q}	CKM	τ_H^q	m_t	α_s	other param.	non-param.	$\sum$
$\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)$	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%
$\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu)$	4.5%	6.9%	0.5%	1.6%	0.1%	$< 0.1\%$	1.5%	8.5%

Non-parametric uncertainties: !!! used $V_{cb}|_{\rm incl}$

- 0.3% from $\mathcal{O}(\alpha_{em})$ corrections from $\mu_b \in [m_b/2, 2m_b]$
- $2 \times 0.2\%$ from $\mathcal{O}(\alpha_s^3, \alpha_{em}^2, \alpha_s\alpha_{em})$ matching corrections from $\mu_0 \in [m_t/2, 2m_t]$
- 0.3% from top-mass conversion from on-shell to $\overline{\rm MS}$ scheme
- 0.5% further uncertainties (power corrections $\mathcal{O}(m_b^2/M_W^2)$, ...)

Summary

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- short-distance (SD) under control for $b \to (u, c)\ell\bar{\nu}_\ell$ and $b \to (s, d) + (\gamma, \bar{\ell}\ell)$
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- lattice QCD matured $\Rightarrow$ huge progress on B-decay constant and $B \to M$ form factors

Leptonic $B \to \ell\bar{\nu}_\ell$ & $B_q \to \bar{\ell}\ell$

✔ decay constant known nowadays (2 %) & SD under control (NLO EW, NNLO QCD)

$\Rightarrow$ ideal to extract V_{ub} and test NP – waiting for exp. to catch up (Belle II, LHCb, CMS, ATLAS)

Exclusive $B \to (P, V)\ell\bar{\nu}_\ell$

✔ at high-q^2 some form factors with $\lesssim 5\,\%$ accuracy & SD under control

✔ experimental uncertainties in $B \to D, D^*$ small, waiting for improvements in $B \to \pi$

✗ non-zero recoil lattice prediction for $B \to D^*$ desired

$\Rightarrow$ good prospects to extract V_{ub} & V_{cb}

Exclusive $B \to (K, K^*)\bar{\ell}\ell$

✔ form factors known 6 – 10 % accuracy & SD under control

✗ at low-q^2 theory needs better control of subleading corrections and $\bar{c}c$-contributions

$\Rightarrow$ theory proposes also data-driven checks of some issues (e.g. duality violation at high-q^2)

Summary

- short-distance (SD) under control for $b \to (u, c)\ell\bar{\nu}_\ell$ and $b \to (s, d) + (\gamma, \bar{\ell}\ell)$
- lattice QCD matured $\Rightarrow$ huge progress on B-decay constant and $B \to M$ form factors

Inclusive $B \to X_c \ell\bar{\nu}_\ell$

✔ theory under control, HQE seems to work

$\Rightarrow$ precision on V_{cb} is 2% (combined exp. and theory uncertainty)

Inclusive $B \to X_s(\gamma, \bar{\ell}\ell)$

✔ SD and pertubative part of HQE under control (NNLO QCD, NLO QED)

✗ non-reducible non-perturbative corrections of 5% in $\mathcal{B}(B \to X_s\gamma)$

$\Rightarrow$ $B \to X_s\gamma$ puts strong constraints on NP in C_7,
waiting for preciser updates of $B \to X_s\bar{\ell}\ell$

Backup Slides

Experimental number of events: $b \to s(d)\,\bar{\ell}\ell$

# of evts	BaBar	Belle	CDF	LHCb	CMS	ATLAS
	2012	2009	2011	2011 (+2012)	2011 (+2012)	2011
	471 M $\bar{B}B$	605 fb^{-1}	9.6 fb^{-1}	1 (+2) fb^{-1}	5 (+20) fb^{-1}	5 fb^{-1}
$B^0 \to K^{*0}\,\bar{\ell}\ell$	$137 \pm 44^\dagger$	$247 \pm 54^\dagger$	288 ± 20	2361 ± 56	415 ± 70	426 ± 94
$B^+ \to K^{*+}\bar{\ell}\ell$			24 ± 6	162 ± 16		
$B^+ \to K^+\bar{\ell}\ell$	$153 \pm 41^\dagger$	$162 \pm 38^\dagger$	319 ± 23	4746 ± 81	not yet	not yet
$B^0 \to K^0_S\,\bar{\ell}\ell$			32 ± 8	176 ± 17		
$B_s \to \phi\,\bar{\ell}\ell$			62 ± 9	174 ± 15		
$B_s \to \bar{\mu}\mu$				emerging	emerging	limit
$\Lambda_b \to \Lambda\,\bar{\ell}\ell$			51 ± 7	78 ± 12		
$B^+ \to \pi^+\,\bar{\ell}\ell$		limit		25 ± 7		
$B_d \to \bar{\mu}\mu$			limit	limit	limit	limit

- CP-averaged results
- J/ψ and ψ' q^2-regions vetoed
- † unknown mixture of B^0 and $B^\pm$
- $\ell = \mu$ for CDF, LHCb, CMS, ATLAS

Babar arXiv:1204.3933 + 1205.2201
Belle arXiv:0904.0770
CDF arXiv:1107.3753 + 1108.0695 + Public Note 10894
LHCb arXiv:1205.3422 + 1209.4284 + 1210.2645 + 1210.4492 + 1304.6325 + 1305.2168 + 1306.2577 + 1307.5024 + 1307.7595 + 1308.1340 + 1308.1707 + 1403.8044 + 1403.8045 + 1406.6482
CMS arXiv:1307.5025 + 1308.3409
ATLAS ATLAS-CONF-2013-038

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Outlook / Prospects

- **Belle** reprocessed all data 711 fb^{-1} $\to$ no final analysis yet!
- **LHCb** $\sim$ 2 fb^{-1} from 2012 to be analysed and $\gtrsim$ 8 fb^{-1} by the end of 2018
- **ATLAS / CMS** $\sim$ 20 fb^{-1} from 2012 to be analysed
- **Belle II** expects about (10-15) K events $B \to K^*\bar{\ell}\ell$ ($\gtrsim$ 2020) [Bevan arXiv:1110.3901]

Angular observables & form factor (=FF) relations

$$J_i(q^2) \sim \{\text{Re, Im}\}\left[A_m^{L,R}\left(A_n^{L,R}\right)^*\right]$$

$$\sim \sum_a (C_a F_a) \sum_b (C_b F_b)^*$$

$A_m^{L,R} \ldots K^*$-transversity amplitudes $m = \perp, \parallel, 0$

$C_a \ldots$ short-distance coefficients

$F_a \ldots$ FF's

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simplify when using FF relations:

low K^* recoil limit: $E_{K^*} \sim M_{K^*} \sim \Lambda_{\rm QCD}$ [Isgur/Wise PLB232 (1989) 113, PLB237 (1990) 527]

$$T_1 \approx V, \qquad T_2 \approx A_1, \qquad T_3 \approx A_2 \frac{M_B^2}{q^2}$$

large K^* recoil limit: $E_{K^*} \sim M_B$ [Charles et al. hep-ph/9812358, Beneke/Feldmann hep-ph/0008255]

$$\xi_\perp \equiv \frac{M_B}{M_B + M_{K^*}} V \approx \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 \approx T_1 \approx \frac{M_B}{2E_{K^*}} T_2$$

$$\xi_\parallel \equiv \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 - \frac{M_B - M_{K^*}}{M_{K^*}} A_2 \approx \frac{M_B}{2E_{K^*}} T_2 - T_3$$

Low-q^2 = Large Recoil: $E_{K^*} \sim m_b$

$\Rightarrow$ energetic "light" K^*, allows to calculate hard spectator scattering (HS) and weak annihilation (WA) in expansion in $\Lambda_{\rm QCD}/E_{K^*}$ and perturbatively in α_s

QCD Factorisation (QCDF)

[Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]

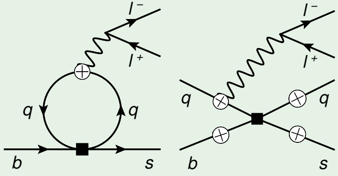
= (large recoil + heavy quark) limit (also Soft-Collinear Effective Theory = SCET)

$$\left\langle \bar\ell\ell\, K_a^* \,\middle|\, H_{\rm eff}^{(i)} \,\middle|\, B \right\rangle \sim$$

$$C_a^{(i)} \times \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + \mathcal{O}(\Lambda_{\rm QCD}/m_b)$$

$C_a^{(i)}$, $T_a^{(i)}$: perturbative kernels in α_s ($a = \perp, \parallel$, $i = u, t$)

ϕ_B, ϕ_{a,K^*} : B– and K_a^*–distribution amplitudes



- $C_a^{(i)}$ corrections $\sim$ universal form factors ξ_a
- $T_a^{(i)}$ HS and WA contributions - numerically small in most observables
- breaks down at subleading order in $1/m_b \to$ endpoint divergences

  [Feldmann/Matias hep-ph/0212158]

  $\Rightarrow$ may be large for some observables, especially optimised observables

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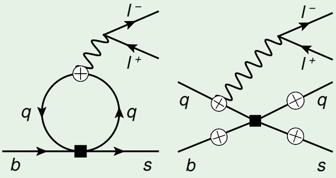

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  $\Rightarrow$ may be large for some observables, especially optimised observables

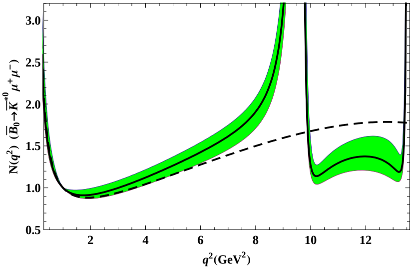
$\Rightarrow$ sub-leading soft gluon effects beyond QCDF from LCSR's

[Ball/Jones/Zwicky hep-ph/0612081, Dimou/Lyon/Zwicky arXiv:1212.2242, Lyon/Zwicky arXiv:1305.4797]

$\bar{c}c$-Resonances

@ low q^2 $\Rightarrow$ in general non-perturbative, $B \to K^* J/\psi(\to K^* \bar{\ell}\ell)$ colour-suppressed

[Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]



- $-4m_c^2 \le q^2 \le 2$ GeV^2 $\ll 4m_c^2$: non-local OPE near light-cone including soft-gluon emission
  - $\Rightarrow$ matrix elmnt. via LCSR with B-meson DA's and light-meson interpolating current
  [Khodjamirian/Mannel/Offen hep-ph/0504091 & 0611193]
- $B \to K^{(*)}$ form factors also via same LCSR
- $q^2 \gtrsim 4$ GeV^2: hadronic dispersion relation using measured $B \to K^{(*)} + (J/\psi, \psi')$ $\to$ some modelling of spectral density
- matching both regions: destructive interference between J/ψ and ψ' contributions
- affects rate up to (15-20) % for $1 \lesssim q^2 \lesssim 6$ GeV^2

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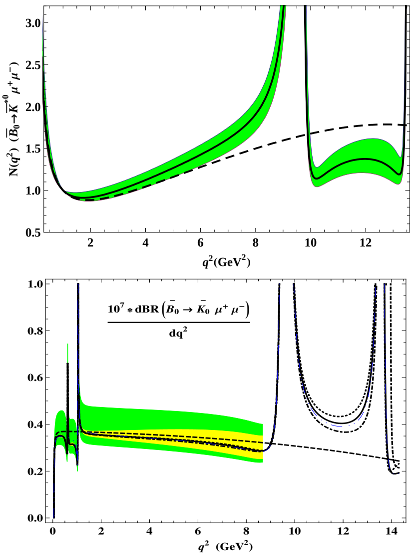
Extended to include light resonances $q = u, d, s$

for $B \to K\bar{\ell}\ell$ [Khodjamirian/Mannel/Wang arXiv:1211.0234]

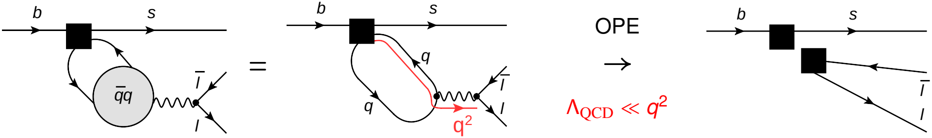
- non-local OPE done completely below hadronic threshold $q^2 < 0$



$\bar{c}c$-Resonances

@high q^2 [Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118]

Hard momentum transfer $(q^2 \sim M_B^2)$ through $(\bar{q}q) \to \bar{\ell}\ell$ allows local OPE



$$\mathcal{A}[B \to K^* \bar{\ell}\ell] \sim \frac{8\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x} \langle K^* | T\{\mathcal{L}^{\rm eff}(0), j_\mu^{\rm em}(x)\} | B\rangle\, [\bar{\ell}\gamma^\mu \ell]$$

$$= \left(\sum_a \mathcal{C}_{3a} \mathcal{Q}_{3a}^\mu + \frac{m_s}{m_b} \times \text{dim-4} + \sum_b \mathcal{C}_{5b} \mathcal{Q}_{5b}^\mu + \mathcal{O}(dim > 5) \right) [\bar{\ell}\gamma_\mu \ell]$$

$dim = 3$ usual $B \to K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$, also α_s matching corrections known

$dim = 5$ suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2$ %, explicite estimate @ $q^2 = 15$ GeV^2: $< 1\%$

beyond OPE duality violating effects [Beylich/Buchalla/Feldmann arXiv:1101.5118]

- based on Shifman model for c-quark correlator + fit to recent BES data
- ± 2 % for integrated rate $q^2 > 15$ GeV^2

Subleading corrections to TransAmp's

$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil

$$A_i^{L,R} \sim C^{L,R} \times f_i \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$)

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K^*}} V, \qquad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K^*})\,A_1, \qquad f_0 = \frac{(1-\hat{s}-\hat{M}_{K^*}^2)(1+\hat{M}_{K^*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K^*}(1+\hat{M}_{K^*})\sqrt{\hat{s}}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

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FF symmetry breaking

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s)$$

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Low hadronic recoil

$\Rightarrow$ small, apart from possible duality violations

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Large hadronic recoil

$$A_{\perp,\parallel}^{L,R} \sim \pm C_\perp^{L,R} \times \xi_\perp + \mathcal{O}(\alpha_s, \lambda), \qquad A_0^{L,R} \sim C_\parallel^{L,R} \times \xi_\parallel + \mathcal{O}(\alpha_s, \lambda)$$

2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

$$C_\perp^{L,R} = (C_9 \mp C_{10}) + \frac{2m_b M_B}{q^2} C_7, \qquad C_\parallel^{L,R} = (C_9 \mp C_{10}) + \frac{2\,m_b}{M_B} C_7,$$

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Large hadronic recoil ⇒ limited, end-point-divergences at $\mathcal{O}(\lambda)$

$$A_{\perp,\parallel}^{L,R} \sim \pm C_\perp^{L,R} \times \xi_\perp + \mathcal{O}(\alpha_s, \lambda), \qquad A_0^{L,R} \sim C_\parallel^{L,R} \times \xi_\parallel + \mathcal{O}(\alpha_s, \lambda)$$

2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

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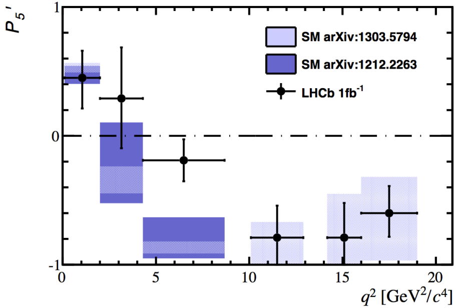
P'_5 & subleading corrections

tension in P'_5: 3.7σ for $q^2 \in [4.3, 8.7]$ GeV^2

2.5σ for $q^2 \in [1.0, 6.0]$ GeV^2

comparing experiment [LHCb arXiv:1308.1707]
with theory [Descotes-Genon/Hurth/Matias/Virto 1303.5794]

$\Rightarrow$ 2 "recipes" used to estimate subleading crr's @ low q^2 (mainly for FF's)



P_5' & subleading corrections

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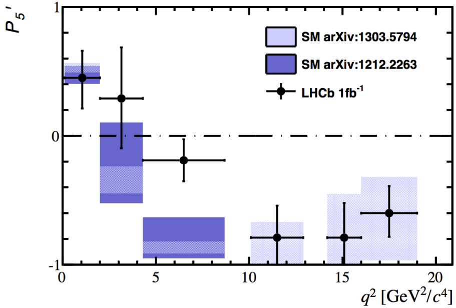

I) Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589

Introduce "rescaling factor ζ" for each K^*-transversity amplitude

$$A^{L/R}_{0,\perp,\parallel} \longrightarrow \zeta^{L/R}_{0,\perp,\parallel} \times A_{0,\perp,\parallel} \qquad 1 - \frac{\Lambda_{\rm QCD}}{m_b} \lesssim \zeta \lesssim 1 + \frac{\Lambda_{\rm QCD}}{m_b}$$

- mimic subleading crr's from A) FF relations and B) $1/m_b$ contr. to ampl.
- can account for q^2-dep.: introduce ζ for each q^2-bin
- used in most analysis/fits

P_5' & subleading corrections

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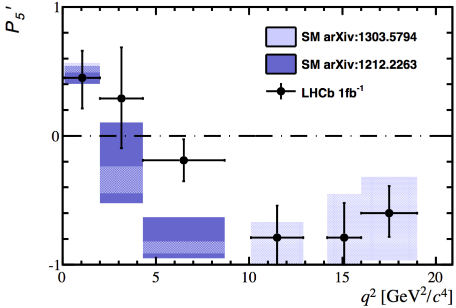

II) Jäger/Martin-Camalich arXiv:1212.2263 (updates in arXiv:1412.3183)

Keep track of subleadig crr.'s to FF-relations (ξ_j = universal FF)

$$FF_i \propto \xi_j + \alpha_s \Delta FF_i + a_i + b_i \frac{q^2}{m_B^2} + \ldots$$

with a_i, b_i from spread of nonperturbative FF-calculations (LCSR, quark models ...)

a_i, b_i are $\sim \Lambda_{\rm QCD}/m_b$ and ΔFF_i QCD crr's [Beneke/Feldmann hep-ph/0008255]

"Scheme-dependence" for definition of ξ_j in terms of QCD FF's

Scheme 1 $\quad \xi_\perp^{(1)} \equiv \frac{m_B}{m_B + m_{K^*}} V \quad \xi_\parallel^{(1)} \equiv \frac{m_B + m_{K^*}}{2E} A_1 - \frac{m_B - m_{K^*}}{m_B} A_2$

Scheme 2 $\quad \xi_\perp^{(2)} \equiv T_1 \quad \xi_\parallel^{(2)} \equiv \frac{m_{K^*}}{E} A_0$

P_5' & subleading corrections

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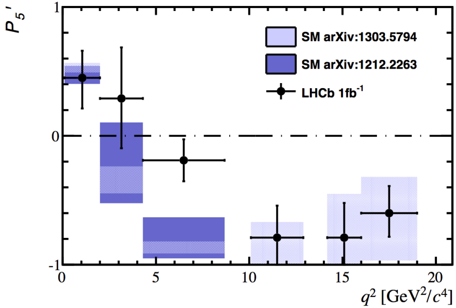
with theory [Descotes-Genon/Hurth/Matias/Virto 1303.5794]

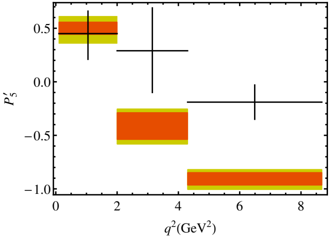
$\Rightarrow$ 2 "recipes" used to estimate subleading crr's @ low q^2 (mainly for FF's)



III) Descotes-Genon/Hofer/Matias/Virto arXiv:1407.8526

Update of method II) $\Rightarrow$ find smaller subleading FF corrections, contrary to II)

parametric + subleading $1/m_b$
$\bar{c}c$ estimate
LHCb data



- use LCSR results of FF's to estimate subleading $1/m_b$ contributions $\Rightarrow$ typically $\lesssim 10\%$
- contrary to II), do not fix central values of subleading contributions to zero, obtain them from fit
- contrary to II), use q^2-dep. of $\xi_{\perp,\parallel}$ as given by LCSR result of QCD FF's, do not use q^2-dep. as predicted by power count. in $m_b \to \infty$ limit
- Scheme 1 better for observables sensitive to $C_{9,10}$, Scheme 2 for observables $\sim C_7$

Angular analysis of $B \to K \bar{\ell}\ell$

Besides $d\Gamma/dq^2$, two more obs's measured LHCb 3/fb arXiv:1403.8045

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{F_H}{2} + A_{\rm FB}\cos\theta_\ell + \frac{3}{4}\left[1 - F_H\right]\sin^2\theta_\ell$$

In the SM:

- $F_H \sim m_\ell^2/q^2$ tiny for $\ell = e, \mu$ and reduced FF uncertainties @ low- & high-q^2
  CB/Hiller/Piranishvili arXiv:0709.4174, CB/Hiller/van Dyk/Wacker arXiv:1111.2558
- $A_{\rm FB} \simeq 0 + \mathcal{O}(\alpha_e) + \mathcal{O}(\text{dim} - 8)$ up to "QED-background" & higher dim. m_b^2/m_W^2

Beyond SM: test scalar & tensor operators CB/Hiller/Piranishvili arXiv:0709.4174

- $F_H \sim |C_T|^2 + |C_{T5}|^2 + \mathcal{O}(m_\ell)$
- $A_{\rm FB} \sim (C_S + C_{S'})C_T + (C_P + C_{P'})C_{T5} + \mathcal{O}(m_\ell)$

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Lepton-flavour violating (LFV) effects: generalise $C_i \to C_i^\ell$!!!

Take ratios of observables for $\ell = \mu$ over $\ell = e$ (or $\ell = \tau$) Krüger/Hiller hep-ph/0310219

$\Rightarrow$ FF's cancel in SM up to $\mathcal{O}(m_\ell^4/q^4)$ @ low-q^2 CB/Hiller/Piranishvili arXiv:0709.4174

$$R_M^{[q^2_{\rm min},\, q^2_{\rm max}]} = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma[B \to M\, \bar{\mu}\mu]}{dq^2}}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma[B \to M\, \bar{e}e]}{dq^2}}$$

for $M = K,\ K^*,\ X_s$

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for $M = K, K^*, X_s$

Recent measurement of

$R_K^{[1,6]} = 0.745^{+0.090}_{-0.074} \pm 0.036$ LHCb 3/fb arXiv:1406.6482

deviates by 2.6σ from SM

$R_{K,\rm SM}^{[1,6]} = 1.0008 \pm 0.0004$ Bouchard et al. arxiv:1303.0434

High-q^2 $\bar{c}c$-resonances in $B^+ \to K^+ \bar{\mu}\mu$

[Lyon/Zwicky arXiv:1406.0566]

factorization assumption for $B \to K + \Psi(nS)(\to \bar{\ell}\ell)$:

$$\langle \Psi(nS)\, K | (\bar{c}\Gamma c)(\bar{s}\Gamma' b) | B\rangle \approx \langle \Psi(nS) | \bar{c}\Gamma c | 0\rangle \otimes \langle K | \bar{s}\Gamma' b | B\rangle + \ldots \text{ nonfactorisable}$$

+ dispersion relations with BES II $\bar{e}e \to \bar{q}q$ data

+ comparison with LHCb 3 fb^{-1} of $B^+ \to K^+ \bar{\mu}\mu$ @ high-q^2

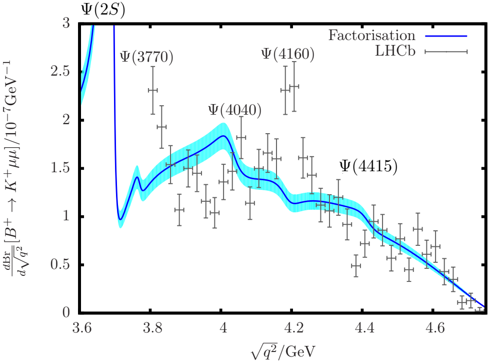
- factorization “badly fails” differentially in q^2
  - $\Rightarrow$ not unexpected, well-known from $B \to K\Psi(nS)$
  - $\Rightarrow$ “fudge factor” $\neq 1$
- does it invalidate the OPE ???

  this requires q^2-integration !!!
- investigate other $B \to M \bar{\ell}\ell$

  $M = K^*$ at LHCb

  $M = X_s$ (inclusive) at Belle II

  + including J/ψ and ψ'



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- a) no "fudge factor": $p = 0\%$

  various "generalisations of factorisable contributions"

  b) fit "fudge factor" = −2.6: $p = 1.5\%$

  c), d) fit rel. factors of $\Psi(nS)$: $p = 12\%$ and $p = 20\%$

  $\Rightarrow$ improve the combined fit of BES II and LHCb considerably

  (BES II data alone: $p = 44\%$)

- BUT can these parametrisations capture all features of non fact. contr.: Wilson coeffs. & q^2 ???
- can't be explained with NP in C_9

  $\Rightarrow$ can ease tension in P_5'
  $\Rightarrow$ NP in $b \to s\,\bar{c}c$?!