Recent Experimental Results on Semi-Leptonic D Decays
& Extractions of $|V_{cd}|$ and $|V_{cs}|$

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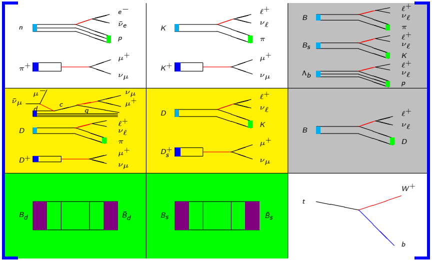
Outline

1. Introduction
2. Semileptonic $D \to V\ell^+\nu_\ell$ Decays [V=vector meson]
3. Semileptonic $D \to P\ell^+\nu_\ell$ Decays [P=pseudoscalar meson]
4. Extraction of $|V_{cd}|$ and $|V_{cs}|$
5. Checks of the CKM-matrix Unitarity and Others
6. Summary

- Semi-leptonic D decays provide a window to investigate both the strong and weak physics.
- $D \to V e^+ \nu_e$ can be used to determine $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$, ...
- $D \to \pi(K) e^+ \nu_e$ can be used
  1. to determine form factors $f_+^{\pi(K)}(q^2)$
  2. to extract $|V_{cd}|$ and $|V_{cs}|$



D, D_s — D, D_s+LQCD B, B_s+LQCD — D, D_s+LQCD B, B_s+DØ,CDF

Experiment / **Theory**

Decay Rate = Know × (Form Factor)^2 × |CKM Element|^2

Decay Rate ⇑

$D \to K(\pi)\ell^+\nu$ [also $D^+_{(s)} \to \ell^+\nu$]

(Form Factor)^2 ⇓

- Use unitarity for CKM elements to determine form factors $f_+^K(0)$ and $f_+^\pi(0)$, [also decay constant $f_{D^+_{(s)}}$], and test QCD

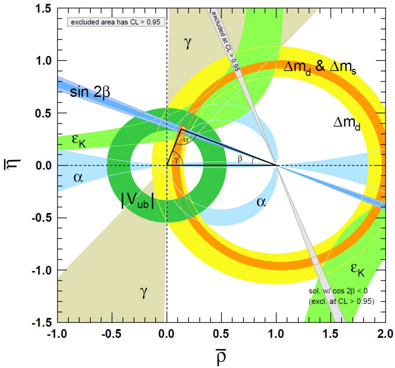
|CKM Element|^2 ⇓

- Use theory for form factors to extract $|V_{cd}|$ and $|V_{cs}|$

- Indirectly improve precision of the CKM element(s) of $|V_{ub}|$
- Check unitarity of the CKM matrix and search for New Physics

Charm's Supporting Role in Test of the SM

Constraints on Parameters of CKM unitarity triangle



Current Status

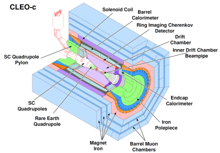
- The brown band dominated by uncertainty of $f_{B_{(s)}}$ calculated in LQCD
- The dark green band dominated by uncertainty of $f_+^{B\to\pi}(0)$ calculated in LQCD
- To reduce uncertainty of the apex of the triangle, $f_{B_{(s)}}$ and $f_+^{B\to\pi}(0)$ needs to be validated and precision of these calculated in LQCD need to be improved

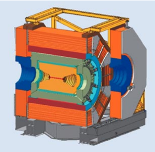
- With (semi-)leptonic D and D_s^+ decays, $f_{D^+_{(s)}}$ and $f_+^{\pi(K)}(q^2)$ can be precisely measured. These can be used to validate $f_{D^+_{(s)}}$ and $f_+^{\pi(K)}(q^2)$ calculated in LQCD, and then improve LQCD calculations of $f_{B_{(s)}}$ and $f_+^{B\to\pi}(0)$. These help improve the overall constraint of the CKM unitarity triangle.
- The (semi-)leptonic D and D_s^+ decays do help precisely test the SM and search for New Physics

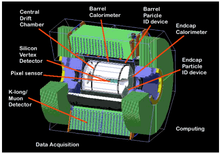
Status of e^+e^- Experiments

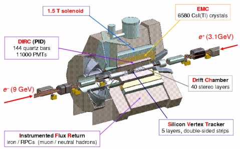
Charm Machines & B Factories

- Experiments near charm threshold
  1. CLEO-c/CESR: $D \to P\ell^+\nu_\ell$, $V\ell^+\nu_\ell$ [also $D^+ \to \mu^+\nu_\mu$, $D_s^+ \to \ell^+\nu_\ell$]
     (818 pb^{-1} data @ 3.773 GeV, 600 pb^{-1} data @ 4.170 GeV)
  2. BESIII/BEPCII: $D \to P\ell^+\nu_\ell$, $V\ell^+\nu_\ell$ [also $D^+ \to \mu^+\nu_\mu$]
     (2.92 fb^{-1} data @ 3.773 GeV)





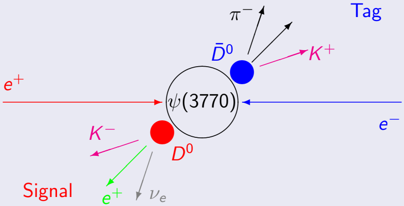




- Experiments at B factories
  1. Belle/KEK: $D^0 \to K^-\ell^+\nu_\ell$, $\pi^-\ell^+\nu_\ell$ [also $D_s^+ \to \ell^+\nu_\ell$]
     (913 fb^{-1} data @ $\Upsilon(4S)$, $\Upsilon(5S)$)
  2. BaBar/PEP-II: $D \to P\ell^+\nu_\ell$, $V\ell^+\nu_\ell$ [also $D_s^+ \to \ell^+\nu_\ell$]
     (521 fb^{-1} data @ 10.58 GeV)

Analysis Technique

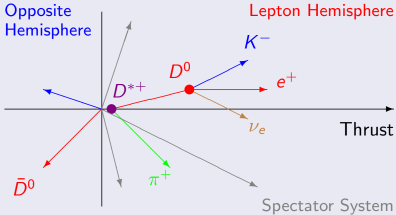
Threshold Experiments at 3.773 GeV



- $e^+e^- \to D\bar{D}$ production
- Event is very clean
- Double tag analysis
  - $M_{\rm b} = \sqrt{E^2_{\rm beam} - \sum_i^n |\vec{p}_i|^2}$
  - $U_{\rm miss} = E_{\rm miss} - |\vec{p}_{\rm miss}|$
  - $M^2_{\rm miss} = E^2_{\rm miss} - |\vec{p}_{\rm miss}|^2$

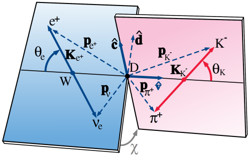
High Energy Experiments at $\sim$ 10.6 GeV

- $e^+e^- \to c\bar{c}$ fragmentation, $D^{*+} \to D^0\pi^+$, $D^0 \to K^- e^+ \nu_e$
- $e^+e^- \to D^{(*)}_{\rm tag} D^{*-}_{\rm sig} X$, $D^{*-}_{\rm sig} \to \bar{D}^0_{\rm sig}\pi^-$, $X = \pi^\pm$, π^0, or $K^\pm$ (it is also a double tag analysis)



Semileptonic $D \to V\ell^+\nu_\ell$ Decays

For example: $D^+ \to K^{*0}e^+\nu_e$, $K^{*0} \to K^-\pi^+$; For details, please see PRD83, 072001(2011)



- $m^2 = (p_{\pi^+} + p_{K^-})^2$
- $\cos(\theta_K) = \frac{\hat{\boldsymbol{\nu}}\cdot\boldsymbol{K}_{K^-}}{|\boldsymbol{K}_{K^-}|}$
- $\cos(\chi) = \hat{\boldsymbol{c}}\cdot\hat{\boldsymbol{d}}$
- $q^2 = (p_{e^+} + p_{\nu_e})^2$
- $\cos(\theta_e) = -\frac{\hat{\boldsymbol{\nu}}\cdot\boldsymbol{K}_{e^+}}{|\boldsymbol{K}_{e^+}|}$
- $\sin(\chi) = (\hat{\boldsymbol{c}}\times\hat{\boldsymbol{\nu}})\cdot\hat{\boldsymbol{d}}$

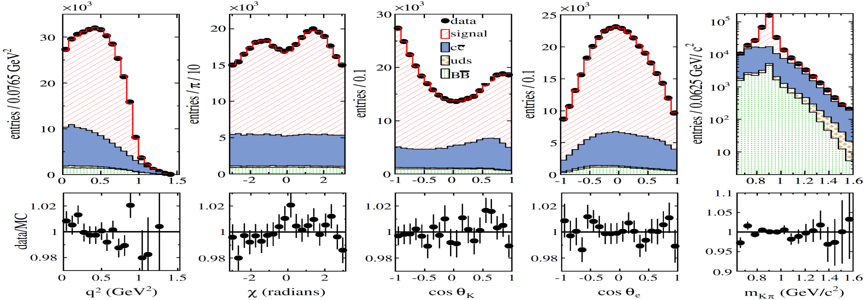
Decay rate depend on 5 variables and 3 form factors

$$d^5\Gamma = \frac{G_F^2|V_{cs}|^2}{(4\pi)^6 m_D^2} X\beta\mathcal{I}(m^2, q^2, \theta_K, \theta_e, \chi)dm^2dq^2d\cos(\theta_K)d\cos(\theta_e)d\chi$$

- $X = p_{K\pi}m_D$, $p_{K\pi}$ is the momentum of the $K\pi$ system in the D rest frame
- $\beta = 2p^*/m$, p^* is the breakup momentum of the $K\pi$ system in its rest frame
- $\mathcal{I}$ can be expressed in terms of helicity amplitudes $H_{0,\pm}$:

  $H_0(q^2) = \frac{1}{2mq}\left[(m_D^2 - m^2 - q^2)(m_D + m)A_1(q^2) - 4\frac{m_D^2 p_{K\pi}^2}{m_D+m}A_2(q^2)\right]$

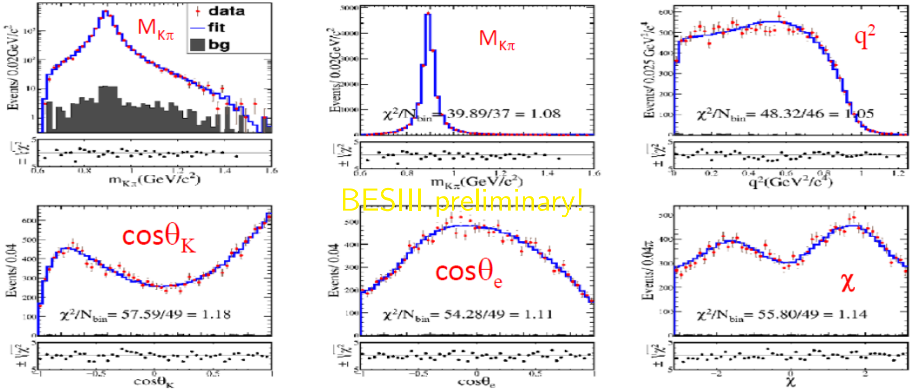
  $H_\pm(q^2) = (m_D + m)A_1(q^2) \mp \frac{2m_D p_{K\pi}}{m_D+m}V(q^2)$
- Vector form factor: $V(q^2) = \frac{V(0)}{1-q^2/m_V^2}$; or: FF ratio $r_V = V(0)/A_1(0)$
- Axial-vector form factor: $A_1(q^2) = \frac{A_1(0)}{1-q^2/m_A^2}$, $A_2(q^2) = \frac{A_2(0)}{1-q^2/m_A^2}$; or: FF ratio $r_2 = A_2(0)/A_1(0)$



- $K^{*0}(892)$ mass and width:
  $m_{K^{*0}(892)} = (895.4 \pm 0.2 \pm 0.2)\ \mathrm{MeV}/c^2$
  $\Gamma_{K^{*0}(892)} = (46.5 \pm 0.3 \pm 0.2)\ \mathrm{MeV}/c^2$
- Decay branching fractions
  $B(D^+ \to K^- \pi^+ e^+ \nu_e) = (4.00 \pm 0.03 \pm 0.04 \pm 0.09)\%$
  $B(D^+ \to K^- \pi^+ e^+ \nu_e)_{K^{*0}(892)} = (3.77 \pm 0.04 \pm 0.05 \pm 0.09)\%$
  $B(D^+ \to K^- \pi^+ e^+ \nu_e)_{\mathrm{S-wave}} = (0.232 \pm 0.007 \pm 0.007 \pm 0.005)\%$
- pole mass:
  $m_A = (2.63 \pm 0.10 \pm 0.13)\ \mathrm{GeV}/c$
- FF ratios:
  $r_V = 1.463 \pm 0.017 \pm 0.031$
  $r_2 = 0.801 \pm 0.020 \pm 0.020$

BESIII: $D^+ \to K^-\pi^+e^+\nu_e$

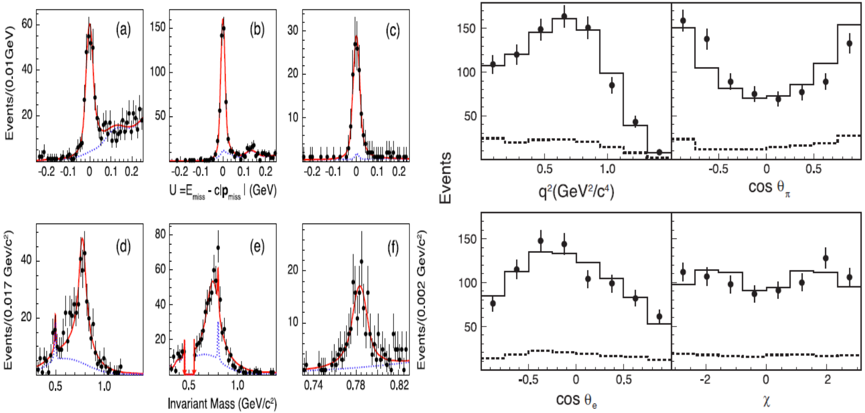
Partial Wave Analysis — Preliminary (2015) 2.92 fb^{-1} data @ 3.773 GeV



- $K^{*0}(892)$ mass and width:
  $m_{K^{*0}(892)} = (894.60 \pm 0.25 \pm 0.08)\ \mathrm{MeV}/c^2$
  $\Gamma_{K^{*0}(892)} = (46.42 \pm 0.56 \pm 0.15)\ \mathrm{MeV}/c^2$
- The fractions of the components
  $f(D^+ \to (K^-\pi^+)_{K^{*0}(892)}e^+\nu_e) = (93.93 \pm 0.22 \pm 0.18)\%$
  $f(D^+ \to (K^-\pi^+)_{S-\mathrm{wave}}e^+\nu_e) = (6.05 \pm 0.22 \pm 0.18)\%$
- pole mass:
  $m_V = (1.81^{+0.25}_{-0.17} \pm 0.02)\ \mathrm{GeV}/c^2$
  $m_A = (2.61^{+0.22}_{-0.17} \pm 0.03)\ \mathrm{GeV}/c^2$
- FF ratios:
  $r_V = 1.411 \pm 0.058 \pm 0.007$
  $r_2 = 0.788 \pm 0.042 \pm 0.008$

CLEO-c: $D^0 \to \rho^- \ell^+ \nu_\ell, D^+ \to \rho^0 \ell^+ \nu_\ell$ and $D^+ \to \omega \ell^+ \nu_\ell$

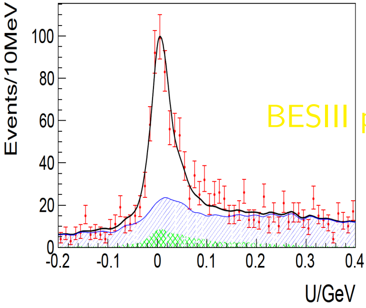
Physical Review Letters **110**, 131802 (2013); 818 pb^{-1} data @ 3.773 GeV

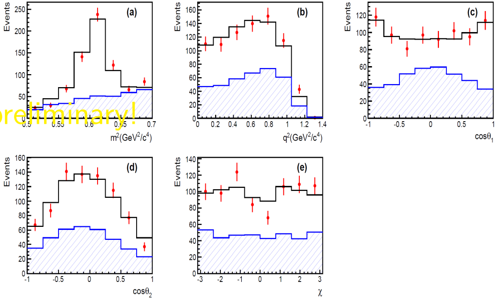


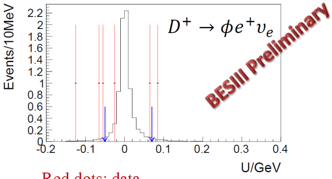
- $B(D^0 \to \rho^- \ell^+ \nu_\ell) = (1.77 \pm 0.12 \pm 0.10)\%$
- $B(D^+ \to \rho^0 \ell^+ \nu_\ell) = (2.17 \pm 0.12^{+0.12}_{-0.22})\%$; $\frac{\Gamma(D^0 \to \rho^- \ell^+ \nu_\ell)}{2\Gamma(D^+ \to \rho^0 \ell^+ \nu_\ell)} = 1.03 \pm 0.09^{+0.08}_{-0.02}$
- $B(D^+ \to \omega \ell^+ \nu_\ell) = 1.82 \pm 0.18 \pm 0.07)\%$
- $r_V = V(0)/A_1(0) = 1.48 \pm 0.15 \pm 0.05$; $r_2 = A_2(0)/A_1(0) = 0.83 \pm 0.11 \pm 0.04$

BESIII: $D^+ \to \omega e^+ \nu_e$ and $D^+ \to \phi e^+ \nu_e$

Preliminary (2015) 2.92 fb^{-1} data @ 3.773 GeV







Red dots: data

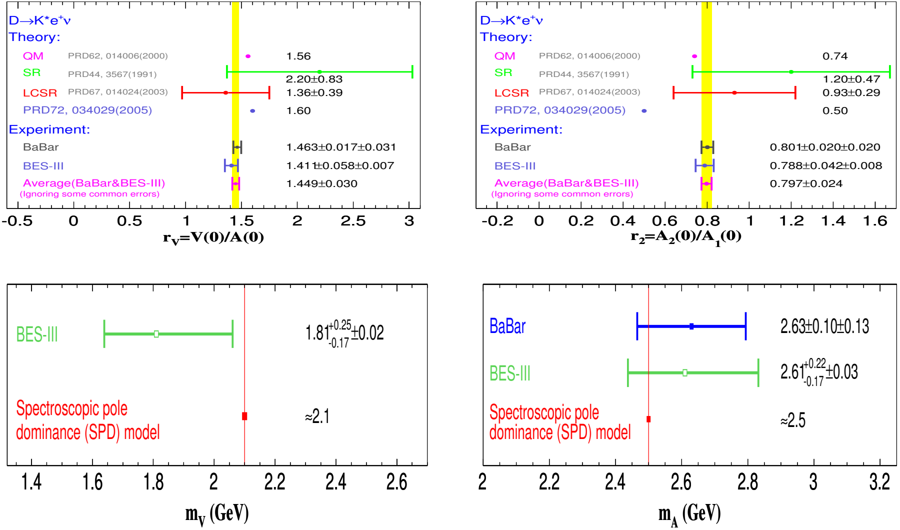
Black histogram: signal MC simulation

Arrows: signal region

- Decay branching fractions
  - $\mathcal{B}(D^+ \to \omega e^+ \nu_e) = (0.163 \pm 0.011 \pm 0.008)\%$
  - $\mathcal{B}(D^+ \to \phi e^+ \nu_e) < 1.3 \times 10^{-5}$ at 90% C.L.
- Form Factor ratios
  - $r_V \equiv \frac{V(0)}{A_1(0)} = 1.24 \pm 0.09 \pm 0.06$
  - $r_2 \equiv \frac{A_2(0)}{A_1(0)} = 1.06 \pm 0.15 \pm 0.05$

Comparison of $r_V = V(0)/A(0), r_2 = A_2(0)/A_1(0), m_V, m_A$

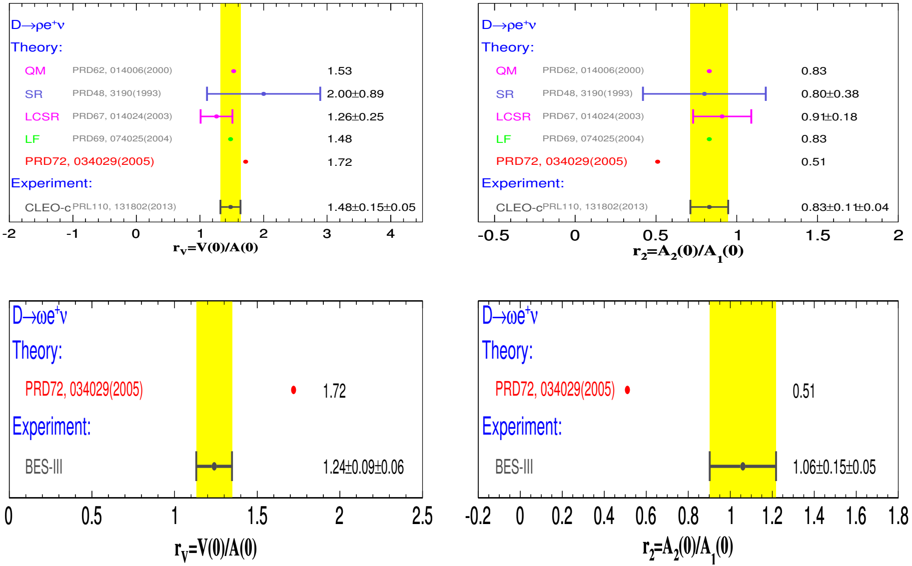
$D \to K^-\pi^+e^+\nu_e$, PRL101,131802(2013), PRD83, 072001(2011), BESIII preliminary results



The BaBar made higher precision measurements of these r_V, r_2 and m_A.

Comparison of $r_V = V(0)/A(0), r_2 = A_2(0)/A_1(0)$

$D \to \rho/\omega e^+ \nu_e$, Physical Review Letters **110**,131802(2013); BESIII preliminary results



Semileptonic $D \to P\ell^+\nu_\ell$ Decays

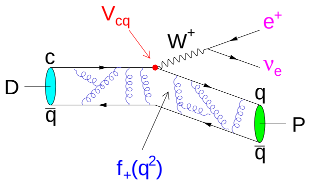
- Differential Rates of $D \to Pe^+\nu_e$ $(P = \pi, K)$ Decay:

$$\frac{d\Gamma}{dq^2} = X\frac{G_F^2 p^3}{24\pi^3}|f_+(q^2)|^2|V_{cd(s)}|^2$$

$$X = 1 \text{ for } K^-, \pi^-, \bar{K}^0; \quad X = 1/2 \text{ for } \pi^0$$

Form Factor Parameterizations

Simple Pole Model	$f_+(q^2) = \frac{f_+(0)}{(1-q^2/M^2_{\rm pole})}$
Modified Pole Model	$f_+(q^2) = \frac{f_+(0)}{(1-q^2/M^2_{\rm pole})/(1-\alpha q^2/M^2_{\rm pole})}$
ISGW2 Model	$f_+(q^2) = f_+(q^2_{\rm max})(1+\frac{r^2}{12}(q^2_{\rm max}-q^2))^{-2}$
$z-$Series Expansion	$f_+(q^2) = \frac{1}{P(q^2)\Phi(q^2,t_0)}\sum_{k=0}^{\infty} a_k(t_0)[z(q^2,t_0)]^k$

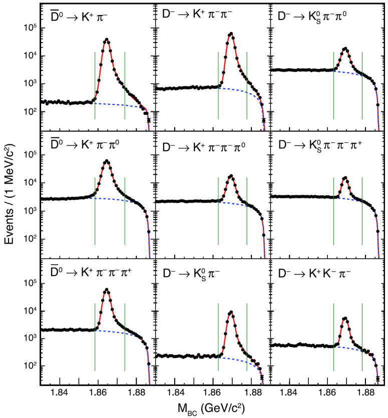


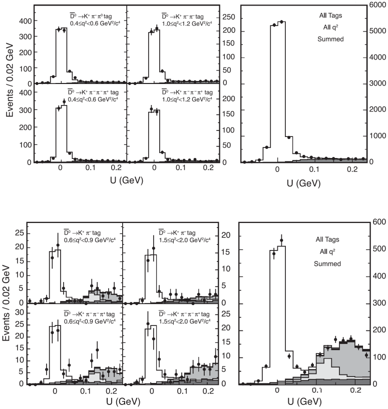
- Directly measured $f_+(q^2)|V_{cd(s)}|$
- Determine form factors $f_+^{\pi}(q^2)$ and $f_+^{K}(q^2)$ with inputs of $|V_{cd}|$ and $|V_{cs}|$
  - Validate $f_+^{\pi(K)}(q^2)$ calculated in LQCD, improve $f_+^{B\to\pi}(0)$ calculated in LQCD
  - Improved LQCD calculation of $f_+^{B\to\pi}(0)$ reduce error of $|V_{ub}|$
  - Improved measurements of $f_+^{\pi}(q^2)$ (z-expansion coefficients a_k) can help in reducing error of $|V_{ub}|$ as well
- Extract $|V_{cd}|$ and $|V_{cs}|$

CLEO-c: $D^0 \to K^- e^+ \nu_e$, $\pi^- e^+ \nu_e$, $D^+ \to \bar{K}^0 e^+ \nu_e$, $\pi^0 e^+ \nu_e$

PRD 80, 032005 (2009) (818 pb^{-1} data collected at 3.773 GeV)

Method: $e^+e^- \to \psi(3770) \to D^0\bar{D}^0, D^+D^-$

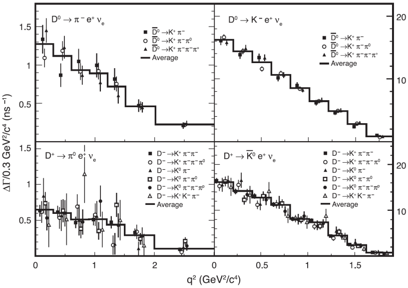


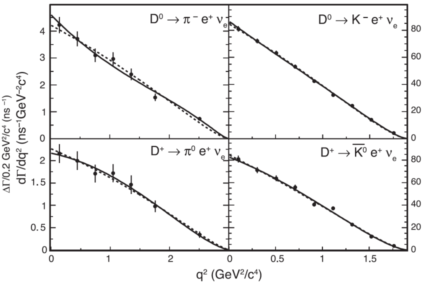


$N_{\bar{D}^0_{\rm tag}} = (0.662 \pm 0.001) \times 10^6$	$N_{D^0 \to K^- e^+ \nu_e} = 11836 \pm 121$	$N_{D^0 \to \pi^- e^+ \nu_e} = 1374 \pm 39$
$N_{D^-_{\rm tag}} = (0.263 \pm 0.001) \times 10^6$	$N_{D^+ \to \bar{K}^0 e^+ \nu_e} = 8467 \pm 91$	$N_{D^+ \to \pi^0 e^+ \nu_e} = 838 \pm 33$

CLEO-c: $D^0 \to K^- e^+ \nu_e$, $\pi^- e^+ \nu_e$, $D^+ \to \bar{K}^0 e^+ \nu_e$, $\pi^0 e^+ \nu_e$

PRD 80, 032005 (2009) (818 pb^{-1} data collected at 3.773 GeV)





Branching fraction results

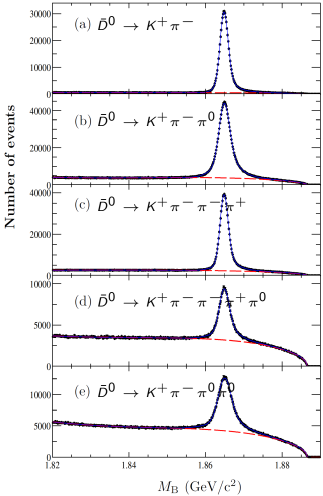
Mode	$\mathcal{B}$ (%)
$D^0 \to \pi^- e^+ \nu_e$	$0.288 \pm 0.008 \pm 0.003$
$D^0 \to K^- e^+ \nu_e$	$3.50 \pm 0.03 \pm 0.04$
$D^+ \to \pi^0 e^+ \nu_e$	$0.405 \pm 0.016 \pm 0.009$
$D^+ \to \bar{K}^0 e^+ \nu_e$	$8.83 \pm 0.10 \pm 0.20$

- Form factors
  - $f_+^K(0) = 0.739 \pm 0.007 \pm 0.005 \pm 0.000$
  - $f_+^\pi(0) = 0.666 \pm 0.019 \pm 0.004 \pm 0.003$
- CKM matrix element
  - $|V_{cd}| = 0.234 \pm 0.007 \pm 0.002 \pm 0.025$
  - $|V_{cs}| = 0.985 \pm 0.009 \pm 0.006 \pm 0.103$

BESIII: $D^0 \to K^- e^+ \nu_e$, $\pi^- e^+ \nu_e$

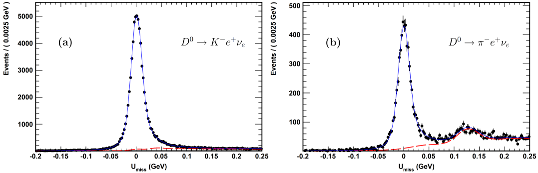
Preliminary (2.92 fb^{-1} data collected at 3.773 GeV)

New results based on 2.92 fb^{-1} data supersede those preliminary results presented at CHARM2012 which was based on $\sim 1/3$ data.



Method: $e^+e^- \to \psi(3770) \to D^0\bar{D}^0$

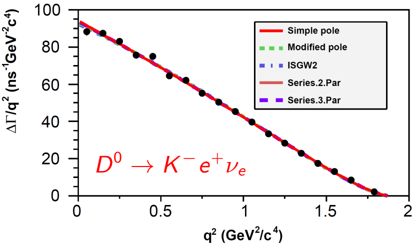
- $(2.793 \pm 0.004) \times 10^6$ $\bar{D}^0$ tags are reconstructed in 5 hadronic decay modes

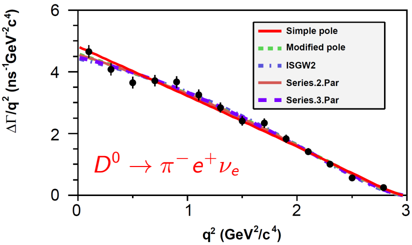


- $N(D^0 \to K^- e^+ \nu_e) = (70.7 \pm 0.3) \times 10^3$
  $\hookrightarrow \mathcal{B}_{D^0 \to K^- e^+ \nu_e} = (3.505 \pm 0.014 \pm 0.033)\%$
- $N(D^0 \to \pi^- e^+ \nu_e) = (6.3 \pm 0.1) \times 10^3$
  $\hookrightarrow \mathcal{B}_{D^0 \to \pi^- e^+ \nu_e} = (0.2950 \pm 0.0041 \pm 0.0026)\%$

BESIII: $D^0 \to K^- e^+ \nu_e$, $\pi^- e^+ \nu_e$

Preliminary (2.92 fb^{-1} data collected at 3.773 GeV)





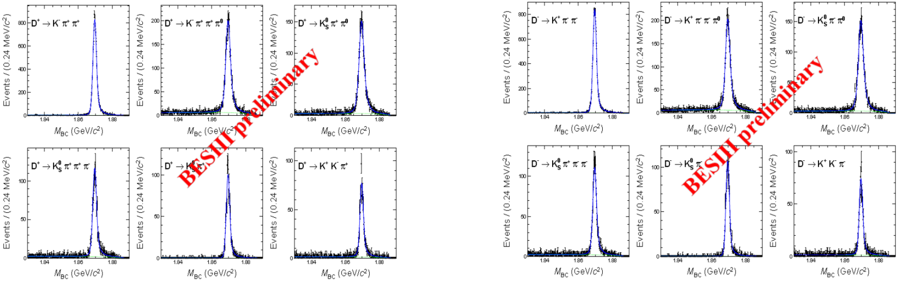
Model	$D^0 \to K^- e^+ \nu_e$		$D^0 \to \pi^- e^+ \nu_e$	
Simple pole	$f_+^K(0)\|V_{cs}\|$	$0.7209 \pm 0.0022 \pm 0.0033$	$f_+^\pi(0)\|V_{cd}\|$	$0.1475 \pm 0.0014 \pm 0.0005$
	$M_{\rm pole}$	$1.9207 \pm 0.0103 \pm 0.0069$	$M_{\rm pole}$	$1.9114 \pm 0.0118 \pm 0.0038$
Mod. pole	$f_+^K(0)\|V_{cs}\|$	$0.7163 \pm 0.0024 \pm 0.0034$	$f_+^\pi(0)\|V_{cd}\|$	$0.1437 \pm 0.0017 \pm 0.0008$
	α	$0.3088 \pm 0.0195 \pm 0.0129$	α	$0.2794 \pm 0.0345 \pm 0.0113$
ISGW2	$f_+^K(0)\|V_{cs}\|$	$0.7139 \pm 0.0023 \pm 0.0034$	$f_+^\pi(0)\|V_{cd}\|$	$0.1415 \pm 0.0016 \pm 0.0006$
	$r_{\rm ISGW2}$	$1.6000 \pm 0.0141 \pm 0.0091$	$r_{\rm ISGW2}$	$2.0688 \pm 0.0394 \pm 0.0124$
Series.2.Par	$f_+^K(0)\|V_{cs}\|$	$0.7172 \pm 0.0025 \pm 0.0035$	$f_+^\pi(0)\|V_{cd}\|$	$0.1435 \pm 0.0018 \pm 0.0009$
	r_1	$-2.2278 \pm 0.0864 \pm 0.0575$	r_1	$-2.0365 \pm 0.0807 \pm 0.0260$
Series.3.Par	$f_+^K(0)\|V_{cs}\|$	$0.7196 \pm 0.0035 \pm 0.0041$	$f_+^\pi(0)\|V_{cd}\|$	$0.1420 \pm 0.0024 \pm 0.0010$
	r_1	$-2.3331 \pm 0.1587 \pm 0.0804$	r_1	$-1.8434 \pm 0.2212 \pm 0.0690$
	r_2	$3.4223 \pm 3.9090 \pm 2.4092$	r_2	$-1.3871 \pm 1.4615 \pm 0.4677$

BESIII: $D^+ \to K_L^0 e^+ \nu_e$

Preliminary (2015) 2.92 fb^{-1} data @ 3.773 GeV

- K_L^0 reconstruction
  - Get position of the K_L^0 in EMC by finding a neutral cluster
  - Use the constraint of $U_{\mathrm{miss}} = 0$ to get momentum of the K_L^0

Candidates for double-tag events $D^+ \to K_L e^+ \nu_e$



- Branching Fraction: $\mathcal{B}(D^+ \to K_L^0 e^+ \nu_e) = (4.482 \pm 0.027_{\mathrm{stat}} \pm 0.103_{\mathrm{syst}})\%$
- CP asymmetry:

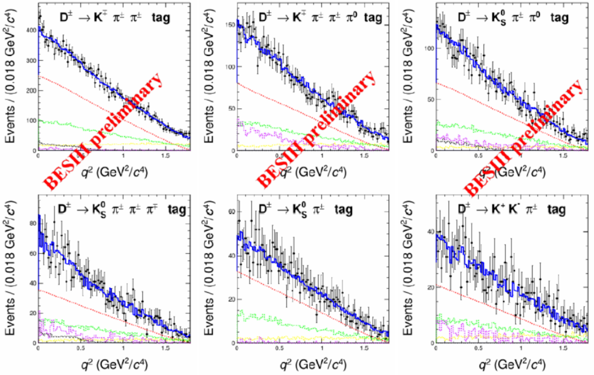
$$\mathcal{A}_{CP}^{D^+ \to K_L^0 e^+ \nu_e} = (-0.59 \pm 0.60_{\mathrm{stat}} \pm 1.48_{\mathrm{syst}})\%$$

No significant CP violation signal is observed from this decay.

BESIII: $D^+ \to K_L^0 e^+ \nu_e$

Preliminary (2015) 2.92 fb^{-1} data @ 3.773 GeV

- Number of doubly tagged events as a function of q^2 for six single tag modes



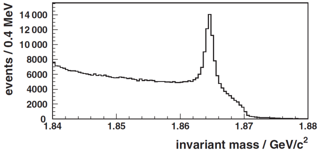
- Analysis these doubly tagged events yields: $f_+^K(0)|V_{cs}| = 0.728 \pm 0.006 \pm 0.011$
- $|V_{cs}| = 0.975 \pm 0.008 \pm 0.015 \pm 0.025$ [with $f_+^K(0) = 0.747 \pm 0.019$ (PRD82, 114506(2010))].

Belle: $D^0 \to K^-\ell^+\nu_\ell$, $\pi^-\ell^+\nu_\ell$

PRL 97, 061804 (2006) (282 fb^{-1} data collected at 10.58 GeV)

Method: $e^+e^- \to D^{(*)}_{\rm tag} D^{*-}_{\rm sig} X$, $D^{*-}_{\rm sig} \to \bar{D}^0_{\rm sig}\pi^-$, $X = \pi^\pm$, π^0, or $K^\pm$

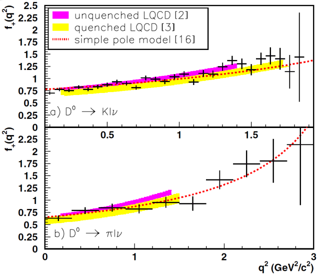
- From $D^{(*)}_{\rm tag} X \pi^-$, the $\bar{D}^0_{\rm sig}$ can be reconstructed
- $(56.5 \pm 0.3_{\rm stat} \pm 0.8_{\rm syst}) \times 10^3$ $\bar{D}^0_{\rm sig}$ were found
- $\bar{D}^0 \to K(\pi)^+\ell^-\bar{\nu}_\ell$ are reconstructed with $K(\pi)^+$ and ℓ^- candidates from the remaining tracks



Results

Channel	$K^-e^+\nu_e$	$K^-\mu^+\nu_\mu$	$\pi^-e^+\nu_e$	$\pi^-\mu^+\nu_\mu$
Yield	$1318 \pm 37 \pm 7$	$1249 \pm 37 \pm 25$	$126 \pm 12 \pm 3$	$106 \pm 12 \pm 6$

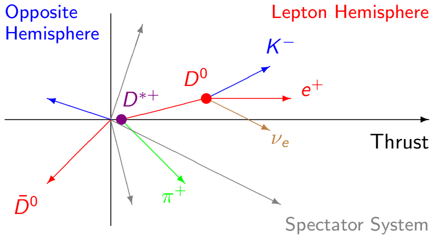
- Branching fractions
  - $\hookrightarrow \mathcal{B}(D^0 \to K^-\ell^+\nu_\ell) = (3.45 \pm 0.07 \pm 0.20)\%$
  - $\hookrightarrow \mathcal{B}(D^0 \to \pi^-\ell^+\nu_\ell) = (0.255 \pm 0.019 \pm 0.016)\%$
- Form factors
  - $\hookrightarrow f_+^K(0) = 0.695 \pm 0.007_{\rm stat} \pm 0.022_{\rm syst}$
  - $\hookrightarrow f_+^\pi(0) = 0.624 \pm 0.020_{\rm stat} \pm 0.030_{\rm syst}$

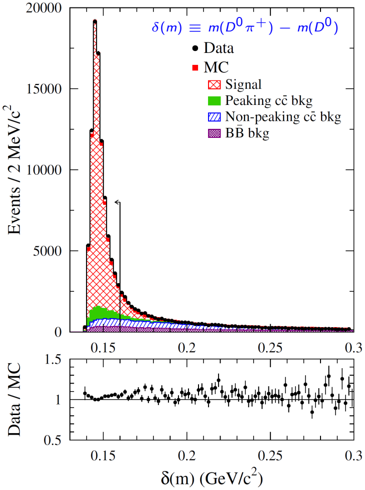


BaBar: $D^0 \to K^- e^+ \nu_e$

PRD 76, 052005 (2007) (75 fb^{-1} data collected at 10.58 GeV)

Method: $e^+e^- \to c\bar{c}$ fragmentation, $D^{*+} \to D^0\pi^+$, $D^0 \to K^- e^+ \nu_e$



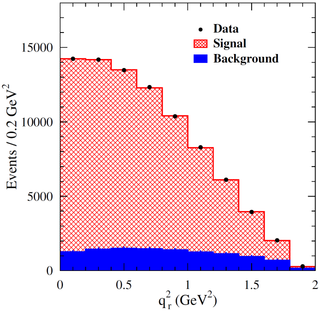


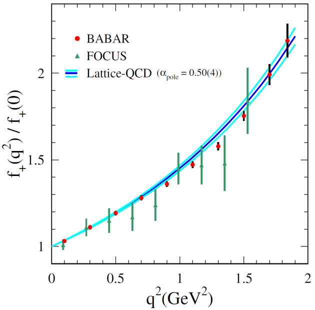
- Select K^-, e^+, π^+ in the same hemisphere
- Determine D^0 direction ($-\vec{p}_{\text{all tracks}\neq K^-, e^+}$)
- Estimate E_{ν_e} (missing energy in the lepton hemisphere)
- Constraint fit using D^0 and D^{*+} mass
- Fisher discriminant to reduce $B\bar{B}$, $c\bar{c}$ bkg

BaBar: $D^0 \to K^- e^+ \nu_e$

PRD 76, 052005 (2007) (75 fb^{-1} data collected at 10.58 GeV)

- 85.3×10^3 selected D^0 candidates containing 11.3×10^3 background events
- $(76.3 \pm 0.3) \times 10^3$ $D^0 \to K^- e^+ \nu_e$ events are observed

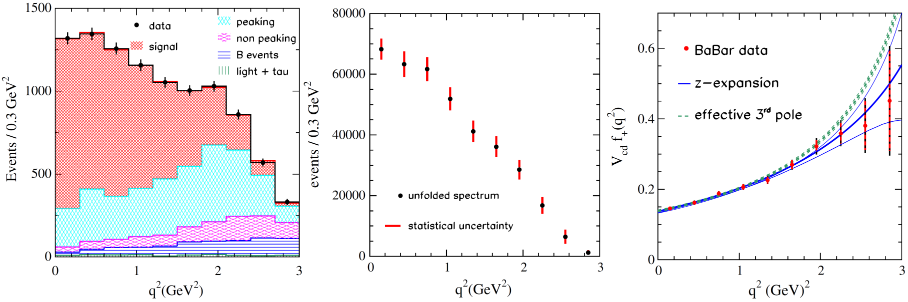




- $R_D = \frac{\mathcal{B}(D^0 \to K^- e^+ \nu_e)}{\mathcal{B}(D^0 \to K^- \pi^+)} = 0.927 \pm 0.007 \pm 0.012$
- Using $\mathcal{B}(D^0 \to K^- \pi^+)|_{\rm PDG06} = (3.80 \pm 0.07)\%$, BaBar determined $\mathcal{B}(D^0 \to K^- e^+ \nu_e) = (3.522 \pm 0.027 \pm 0.065)\%$
- $f_+^K(0) = 0.727 \pm 0.007 \pm 0.005 \pm 0.007$

BABAR: $D^0 \to \pi^- e^+ \nu_e$

Physical Review D **91**, 052022 (2015) 347.2 fb^{-1} data @ 10.58 GeV



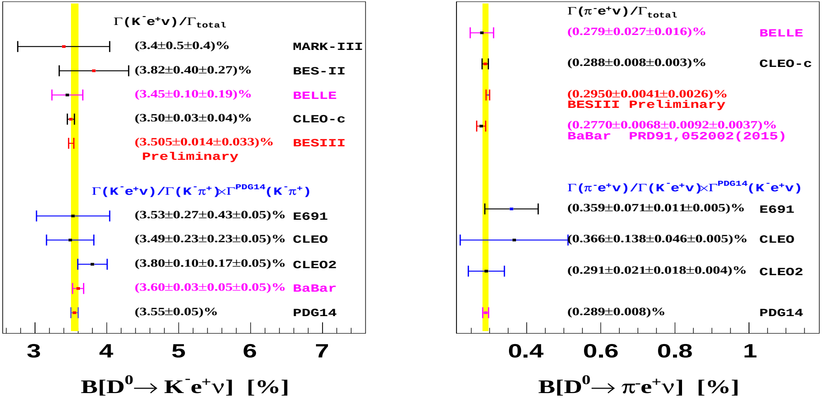
Results

1. $\mathcal{B}(D^0 \to \pi^- e^+ \nu_e) = (0.2770 \pm 0.0068_{\rm stat} \pm 0.0092_{\rm syst} \pm 0.0037_{\rm ext})\%$
2. $|V_{cd}| \times f^{\pi}_{+,D}(q^2 = 0) = 0.1374 \pm 0.0038_{\rm stat} \pm 0.0022_{\rm syst} \pm 0.0009_{\rm ext}$
   - Using the most recent LQCD prediction of $f^{\pi}_{+,D}(q^2 = 0) = 0.610 \pm 0.029$
     $\Rightarrow |V_{cd}| = 0.206 \pm 0.007_{\rm exp} \pm 0.009_{\rm LQCD}$
   - Assuming $|V_{cd}| = |V_{us}| = 0.2252 \pm 0.0009$
     $\Rightarrow f^{\pi}_{+,D}(q^2 = 0) = 0.610 \pm 0.020_{\rm exp} \pm 0.005_{\rm ext}$

Semileptonic $D \to P\ell^+\nu_\ell$ Decays

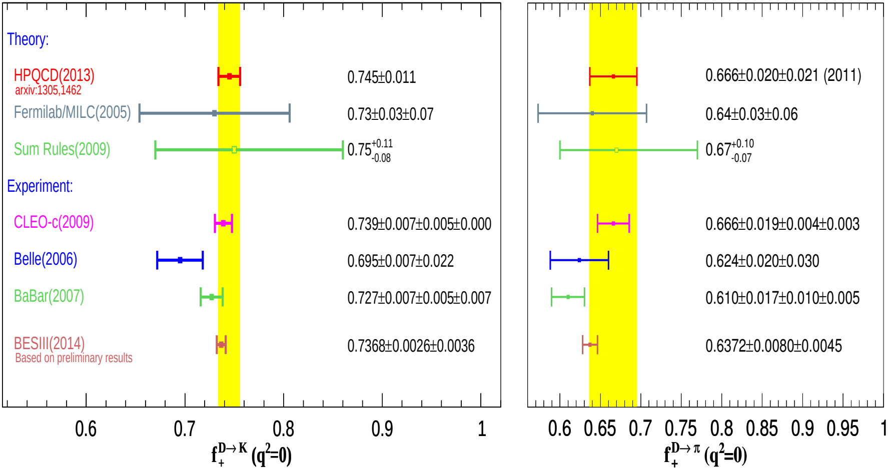
Comparison of $B(D \to Ke^+\nu_e)$ and $B(D \to \pi e^+\nu_e)$ from all available measurements

- Branching fractions measured at different experiments



- The most precise measurements of these decay branching fractions are from the BESIII experiment.

Comparison of Form Factors



The averages of these four determined form factors are

$$f_+^{D\to K}(0) = 0.735 \pm 0.004 \text{ and } f_+^{D\to\pi}(0) = 0.637 \pm 0.008.$$

Extraction of $|V_{cd}|$ and $|V_{cs}|$

Both semi-leptonic $D \to K(\pi)e^+\nu_e$ decays and leptonic $D^+_{(s)} \to l^+\nu_l$ decays are often used to extract $|V_{cd}|$ and $|V_{cs}|$

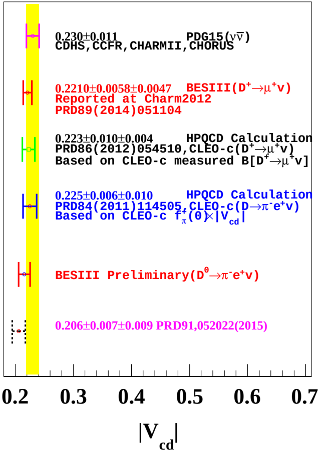
- Semileptonic D^0 and D^+ Decays

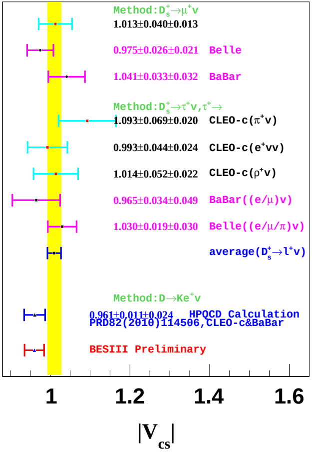
$$\frac{d\Gamma(D \to \pi(K)e^+\nu_e)}{dq^2} = X\frac{G_F^2}{24\pi^3}p^3|V_{cd(s)}|^2|f_+^{D\to\pi(K)}(q^2)|^2$$

  - $|V_{cd(s)}|^2|f_+^{D\to\pi(K)}(q^2)|^2$ is directly measured by analyzing differential decay rates
  - Extract $|V_{cd}|$ and $|V_{cs}|$ in conjunction with $f_+^{D\to\pi}(0)$ and $f_+^{D\to K}(0)$ calculated in LQCD

- Current status on extraction of $|V_{\rm cd}|$ and $|V_{\rm cs}|$
  There are many extractions of these quantities:
  - Several experiments and some authors extracted $|V_{cd(s)}|$ used different value of form factors
  - Some recent global analysis of all available measurements of semi-leptonic and leptonic D decays used updated values of the form factors and other quantities
    - Newly updated decay form factors calculated in LQCD
    - Masses and lifetimes of D mesons and leptons given in PDG2014

$|V_{cd}|$ and $|V_{cs}|$ extracted by original authors



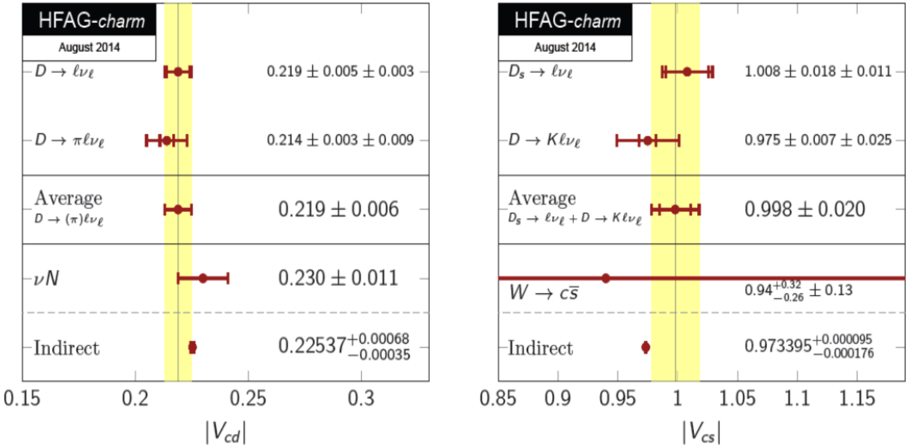


The BESIII used the leptonic $D^{+}\to\mu^{+}\nu_{\mu}$ decays to extract $|V_{cd}|$ for the first time, and precision of the $|V_{cd}|$ is higher than the PDG2014 average from $\nu\bar{\nu}$ interactions.

$|V_{cd}|$ and $|V_{cs}|$ extracted by HFAG-charm 2014

Presented at CKM2014, Sep., 2014

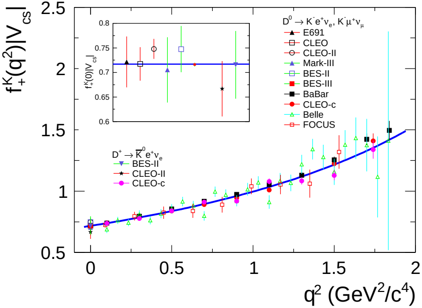
By averaging measurements of (semi-)leptonic D and D_s^+ decays, the HFAG extracted the values of $|V_{cd}|$ and $|V_{cs}|$, and presented these at the CKM2014.

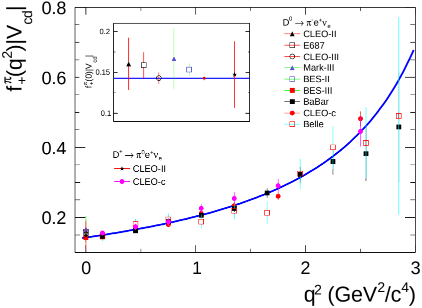


$|V_{cd}|$ and $|V_{cs}|$ extracted by analyzing all available data

Semi-leptonic D decays, Eur.Phys.J.C (2015)75:10; ; Phys.Lett.B743, 315 (2015)

- Global analysis of all available experimental results on semileptonic D decays

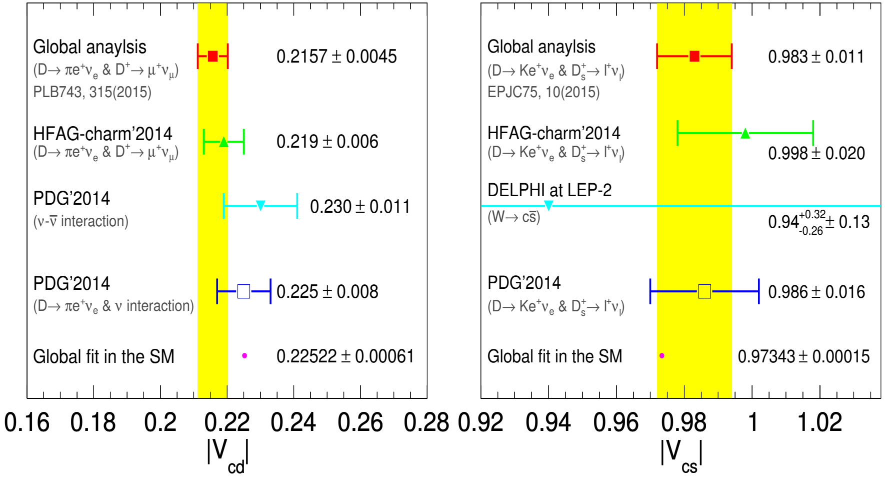




1. $f_+^K(0)|V_{cs}| = 0.717 \pm 0.004$, with $f_+^K(0) = 0.745 \pm 0.011$ (arXiv:1305.1462)
   $\Rightarrow |V_{cs}|^{D\to Ke^+\nu_e} = 0.962 \pm 0.005 \pm 0.014$
2. $f_+^\pi(0)|V_{cd}| = 0.1428 \pm 0.0019^{+0.0019}_{-0.0011}$, with $f_+^\pi(0) = 0.666 \pm 0.020 \pm 0.021$ (PRD84,114505(2011)
   $\Rightarrow |V_{cd}|^{D\to\pi e^+\nu_e} = 0.2144^{+0.0040}_{-0.0033} \pm 0.0093$

Comparisons of $|V_{cs}|$ and $|V_{cd}|$

PDG2014, HFAG-charm, Global analysis of all available measurements



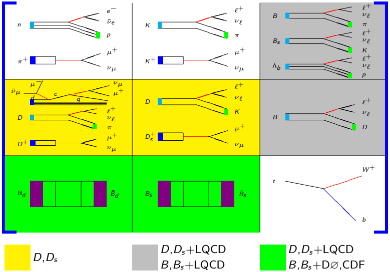
The $|V_{cd}| = 0.2157 \pm 0.0045$ from global analysis deviates from the $|V_{cd}| = 0.22522 \pm 0.00061$ obtained from the SM global fit by 2.1 standard deviation.

Test the consistency of the SM

Test the unitarity of the CKM matrix, and the lattice approach to the charm sector

Using the most up-to-date values for $|V_{cd}|$ and $|V_{cs}|$, one can test the consistency of the SM:

- $|V_{\rm cd(s)}|^{\rm Measured}/|V_{\rm cd(s)}|^{\rm SMFit} = 1$?
- Check unitarity of CKM matrix
  1. First column
     $|V_{\rm ud}|^2 + |V_{\rm cd}|^2 + |V_{\rm td}|^2 = 1 \quad ?$
  2. Second column
     $|V_{\rm us}|^2 + |V_{\rm cs}|^2 + |V_{\rm ts}|^2 = 1 \quad ?$
  3. Second row
     $|V_{\rm cd}|^2 + |V_{\rm cs}|^2 + |V_{\rm cb}|^2 = 1 \quad ?$



If any one significantly deviate from 1, it may indicate some NP involved in decays.

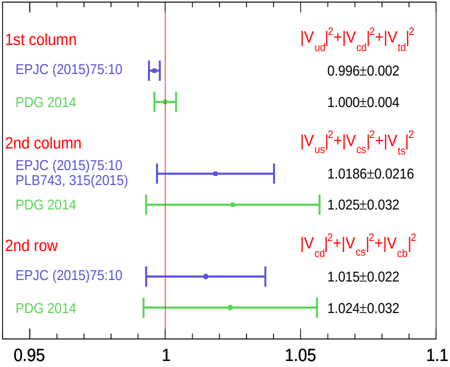
Another way to check the consistency of the SM:

- Comparing $[m_D f_+^{\pi(K)}(0)/f_{D^+_{(s)}}]^{\rm LQCD}$ and $[m_D f_+^{\pi(K)}(0)/f_{D^+_{(s)}}]^{\rm EXP}$ to check:
  1. the consistency of the SM
  2. the LQCD approach to charm sector

The most up-to-date test of the CKM-matrix Unitarity

- Using $|V_{cd}| = 0.2157 \pm 0.0045$, $|V_{cs}| = 0.983 \pm 0.011$ [EPJC(2015)75:10; PLB743,315(2015)] extracted from D decays

- Plus using PDG2014 values:
  1. $|V_{td}| = (8.4 \pm 0.6) \times 10^{-3}$
  2. $|V_{cb}| = (41.1 \pm 1.3) \times 10^{-3}$
  3. $|V_{us}| = 0.2253 \pm 0.0008$
  4. $|V_{ud}| = 0.97425 \pm 0.00022$
  5. $|V_{ts}| = (40.0 \pm 2.7) \times 10^{-3}$



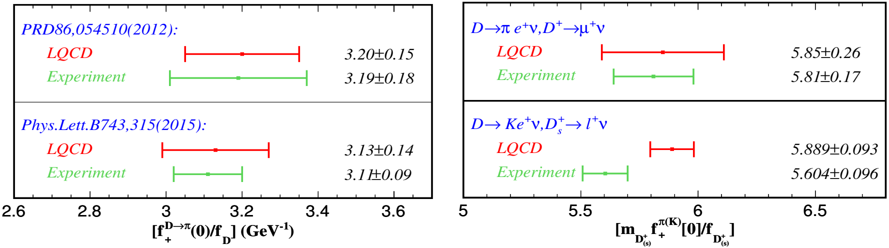
Results of the most up-to-date test

- The most up-to-date test of the first column unitarity from experiments becomes $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 - 1 = -0.004 \pm 0.002$, which differ from unity by 2σ, improving on PDG'2014 value 0.000 ± 0.004.
- Also giving more stringent test of the unitarity for the 2nd column and 2nd row than these given in PDG2014.

Theoretical and experimental ratio $f_+^{\pi(K)}(0)/f_{D^+_{(s)}}$

Test LQCD Approach to Charm Sector or Check the Consistency of Standard Model; PRD86,054510(2012); EPJC(2015)75:10; PLB743, 315(2015)

- Two ratios: $[f_+^{\pi}(0)/f_{D^+}]^{\rm LQCD}$ and $[f_+^{\pi}(0)/f_{D^+}]^{\rm EXP}$
  1. The two ratios from $D \to \pi e^+ \nu_e$ and $D^+ \to \ell^+ \nu_\ell$ are in excellent agreement
  2. $\Rightarrow$ LQCD approach to charm sector is excellent



- Two dimensionless ratios: $[m_{D_{(s)}} f_+^{\pi(K)}(0)/f_{D^+_{(s)}}]^{\rm LQCD}$ and $[m_{D_{(s)}} f_+^{\pi(K)}(0)/f_{D^+_{(s)}}]^{\rm EXP}$
  1. These two ratios are in excellent agreement for $D \to \pi e^+ \nu_e$ and $D^+ \to \ell^+ \nu_\ell$
  2. These two ratios from $D \to K e^+ \nu_e$ and $D_s^+ \to \ell^+ \nu_\ell$ decays differ by 2.1σ

Summary

- In the last few years, precise measurements of semi-leptonic $D \to \pi(K)\ell^+\nu_\ell$ decays made precision test of form factors calculated in LQCD
  1. $D \to K^*$ F.F. ratio: $r_V = 1.449 \pm 0.030$; $r_2 = 0.797 \pm 0.024$ (BaBar, BESIII)
  2. $\Delta r_V / r_V \sim 2.1\%$, $\Delta r_2 / r_2 \sim 3.0\%$ (Experiment)
  3. $f_+^{D\to K}(0) = 0.735 \pm 0.004$ (CLEO-c, BaBar, Belle, BESIII)
  4. $f_+^{D\to \pi}(0) = 0.637 \pm 0.008$ (CLEO-c, BaBar, Belle, BESIII)
  5. $[\Delta f_+^{D\to K}(0)/f_+^{D\to K}(0)]^{\rm EXP} \sim 0.5\%$; $[\Delta f_+^{D\to \pi}(0)/f_+^{D\to \pi}(0)]^{\rm EXP} \sim 1.3\%$
  6. $[\Delta f_+^{D\to K}(0)/f_+^{D\to K}(0)]^{\rm LQCD} \sim 1.5\%$; $[\Delta f_+^{D\to \pi}(0)/f_+^{D\to \pi}(0)]^{\rm LQCD} \sim 4.5\%$
- All available measurements of (semi-)leptonic D and D_s^+ decays together with recent LQCD calculations of FF and $f_{D^+_{(s)}}$ yields precise extractions of $|V_{\rm cd(s)}|$
  1. Globally analyzing all measurements of (semi-)leptonic D decays yields the most precise value:
     $$|V_{cd}| = 0.2157 \pm 0.0045 \qquad (\Delta|V_{cd}|/|V_{cd}| = 2.1\%)$$
  2. PDG2014 value: $|V_{cd}| = 0.225 \pm 0.008$
  3. Globally analyzing all measurements of (semi-)leptonic D and D_s^+ decays yields the most precise value:
     $$|V_{cs}| = 0.983 \pm 0.011 \qquad (\Delta|V_{cs}|/|V_{cs}| = 1.1\%)$$
  4. PDG2014 value: $|V_{cs}| = 0.986 \pm 0.016$

Summary

- Using the recently extracted $|V_{cd}|$ and $|V_{cs}|$ together with other CKM-matrix elements from PDG'2014, the most up-to-date test of the first column unitarity becomes $|V_{\rm ud}|^2 + |V_{\rm cd}|^2 + |V_{\rm td}|^2 - 1 = -0.004 \pm 0.002$ (2σ difference)
- $|V_{cd}|^{\rm Measured}/|V_{cd}|^{\rm SMFit} - 1 = -0.0423 \pm 0.0205$ (2.1σ deference)
- Ratio $[m_{D^+_{(s)}} f^{\pi(K)}(0)/f_{D^+_{(s)}}]^{\rm LQCD\ or\ EXP}$
  1. $[m_{D^+} f_+^{\pi}(0)/f_{D^+}]^{\rm LQCD} = 5.85 \pm 0.26$ (PRD84,114505(2011), PRD90,074509(2014))
  2. $[m_{D^+} f_+^{\pi}(0)/f_{D^+}]^{\rm EXP} = 5.81 \pm 0.17$ (Based on all available measurements,PLB715(2015))
  3. These two ratios from $D \to \pi e^+ \nu_e$ and $D^+ \to \ell^+ \nu_\ell$ decays are in excellent agreement, indicating that LQCD Approach to Charm Sector is excellent.
  4. $[m_{D_s^+} f_+^{K}(0)/f_{D_s^+}]^{\rm LQCD} = 5.889 \pm 0.093$
  5. $[m_{D_s^+} f_+^{K}(0)/f_{D_s^+}]^{\rm EXP} = 5.604 \pm 0.096$
  6. These two ratios from $D \to K e^+ \nu_e$ and $D_s^+ \to \ell^+ \nu_\ell$ decays are 2.1σ difference

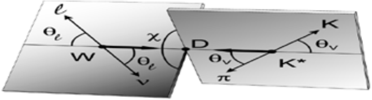
Need more charm data and higher-precision LQCD calculations of $f_+^{\pi(K)}(0)$ and $f_{D^+_{(s)}}$ to reduce these uncertainties, and to more precisely validate LQCD calculations of these quantities, as well as to more stringently test the unitarity.

Thank you very much for your attention!

Back Up

$D^+ \to K^{*0}\ell^+\nu_\ell$

Measurements of non-parametric Form Factors at the CLEO-c and BES-III



$$D^+ \to \bar{K}^{*0} e^+ \nu_e$$
$$D^+ \to \bar{K}^{*0} \mu^+ \nu_\mu$$

$$\frac{d^5\Gamma}{d\cos\theta_\ell\, d\cos\theta_V\, d\chi\, dq^2\, dm^2_{K\pi}} = \frac{|\mathcal{A}|^2 K P_\ell P^*}{256\pi^6 m_D^2 \sqrt{q^2}\, m_{K\pi}}$$

where $|\mathcal{A}|^2$ is the decay intensity, K is the $K^-\pi^+$ momentum in the D^+ rest frame, P^* is the momentum of the kaon in the $K^-\pi^+$ rest frame, and $|\vec{P}_\ell|$ is the momentum of the ℓ^+ in the $\ell^+\nu$ rest frame. Upon integration over χ, the differential decay width is proportional to:

$$\int |\mathcal{A}|^2 d\chi = \frac{q^2 - m_\ell^2}{8} \left\{ \begin{array}{c} ((1+\cos\theta_\ell)\sin\theta_V)^2 |H_+(q^2)|^2 |\beta|^2 \\ +((1-\cos\theta_\ell)\sin\theta_V)^2 |H_-(q^2)|^2 |\beta|^2 \\ +(2\sin\theta_\ell\cos\theta_V)^2 |H_0(q^2)|^2 |\beta|^2 \\ +8\sin^2\theta_\ell\cos\theta_V H_0(q^2) h_0(q^2) \mathrm{Re}\{A e^{-i\delta}\beta\} \end{array} \right\} + \frac{|\beta|^2}{8}(q^2 - m_\ell^2)\frac{m_\ell^2}{q^2}$$
$$\times \left\{ \begin{array}{c} (\sin\theta_\ell \sin\theta_V)^2 |H_+(q^2)|^2 + (\sin\theta_\ell\sin\theta_V)^2 |H_-(q^2)|^2 \\ +(2\cos\theta_\ell\cos\theta_V)^2 |H_0(q^2)|^2 \\ +(2\cos\theta_V)^2 |H_t(q^2)|^2 + 8\cos\theta_\ell\cos^2\theta_V H_0(q^2) H_t(q^2) \end{array} \right\}.$$

5 helicity basis form factors: $H_+(q^2)$, $H_-(q^2)$, $H_0(q^2)$, $H_t(q^2)$, $h_0(q^2)H_0(q^2)$

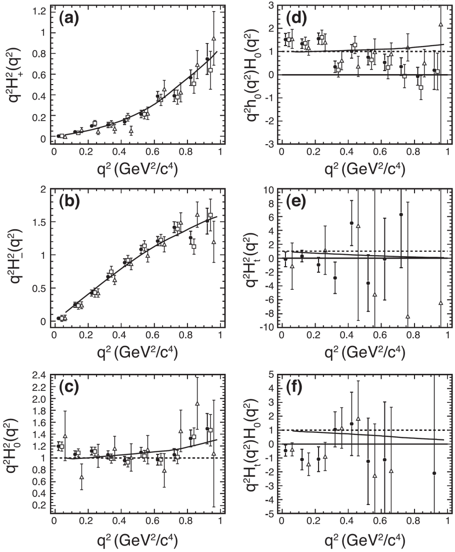
$H_t(q^2)$ is helicity suppressed by a factor of m_l^2/q^2, $H_t(q^2)=0$ for $D^+ \to K^{*0}e^+\nu_e$ decays, while the semimuonic decays are sensitive to the magnitude of the $H_t(q^2)$ form factor

$h_0(q^2)H_0(q^2)$ describing a non-resonance, s-wave $D^+ \to K^-\pi^+ e^+\nu_e$ contribution

CLEO-c: $D^+ \to K^-\pi^+\ell^+\nu_\ell$

Physical Review D **81**, 112001 (2010) 818 pb^{-1} data @ 3.773 GeV

Measurement of non-parametric Form Factor Products

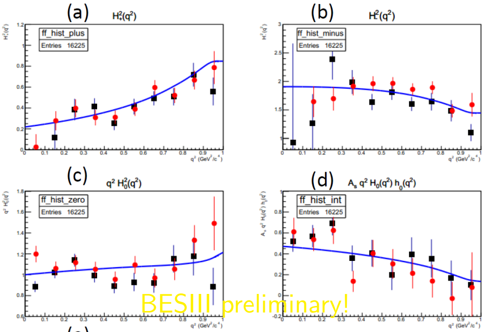


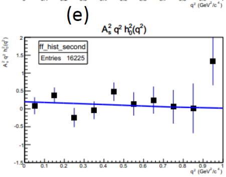
- Model-independent extraction of 6 helicity-basis form-factor products: $q^2H_+^2(q^2)$, $q^2H_-^2(q^2)$, $q^2H_0^2(q^2)$, $q^2h_0(q^2)H_0^2(q^2)$, $q^2H_t^2(q^2)$, $q^2H_0^2(q^2)H_t(q^2)$
- FOCUS projective weighting technique is used [PLB 633, 183(2006)]
- Using decay-angle distributions to separate contributions of FFs
- *solid curves:* SPD (spectroscopic pole dominance) model for dominant FFs.

BESIII: $D^+ \to K^-\pi^+e^+\nu_e$

non-parametric Form Factors — Preliminary (2015) 2.92 fb^{-1} data @ 3.773 GeV

non-parametric measurements of q^2 dependence of helicity-basis FFs



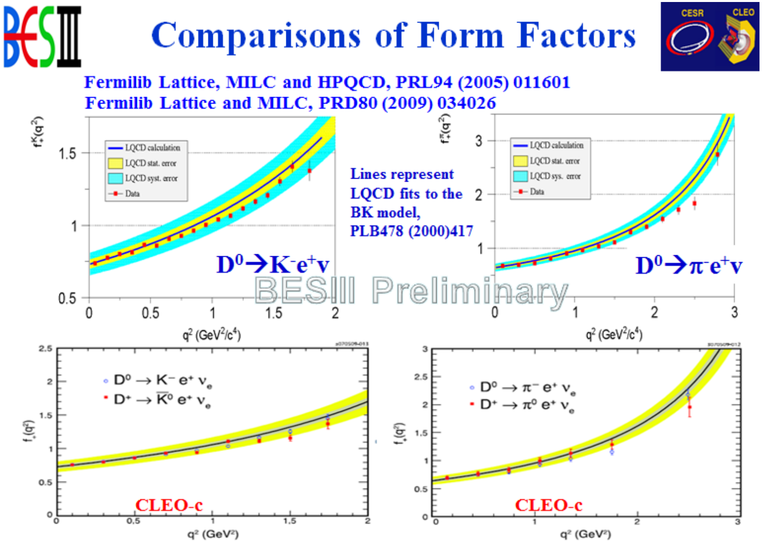


Normalization:
$q^2|H_0(q^2)|^2 \to 1$ as $q^2 \to 0$

(see plot (c))

- Model-independent extraction of 5 helicity-basis form factors
- Similar technique as the one [FOCUS, PLB633,183(2006)] used at CLEO-c
- Red dots: BESIII model-independent measurement
- Black dots: CLEO-c model-independent measurement
- Good agreement with SPD model (blue line) for dominant FFs.

Note: In (d) $A_s = Ae^{-i\delta}$ with $A = 0.33$ and $\delta = 39^o$



Comparisons of Form Factors
Fermilib Lattice, MILC and HPQCD, PRL94 (2005) 011601
Fermilib Lattice and MILC, PRD80 (2009) 034026
LQCD calculation
LQCD stat. error
LQCD syst. error
Data
Lines represent LQCD fits to the BK model, PLB478 (2000)417
D0→K-e+v
D0→π-e+v
BESIII Preliminary
q2 (GeV2/c4)
D0 → K− e+ νe
D+ → K̄0 e+ νe
D0 → π− e+ νe
D+ → π0 e+ νe
CLEO-c
q2 (GeV2)

$D \to \pi(K)e^+\nu_e$ FF parameters determined with all available measurements

Semi-leptonic D decays, Eur.Phys.J.C (2015)75:10; Phys.Lett.B743, 315 (2015)

$D \to \pi e^+\nu_e$

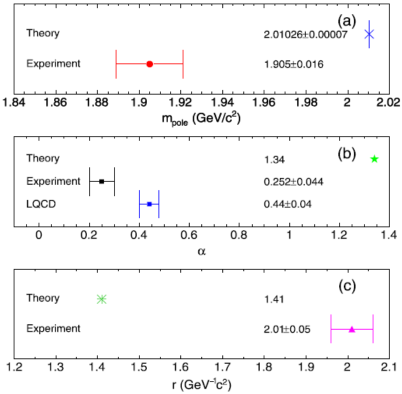


Fig. 4. Comparisons of the form factor parameters determined from experimental measurements and the theoretical expectations: (a) the pole mass m_{ploe} in single pole model, (b) α in the BK model, and (c) r in the ISGW2 model.

$D \to K e^+\nu_e$

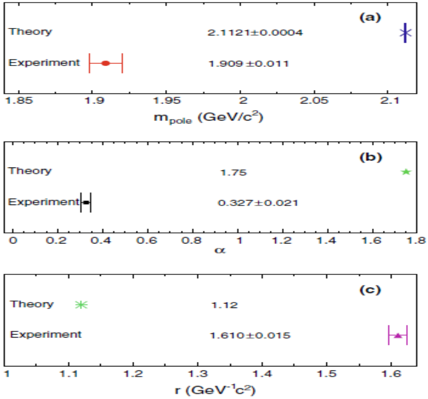
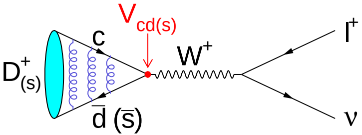


Fig. 4 Comparisons of the form factor parameters determined from experimental measurements and the theoretical expectations: **a** the pole mass m_{pole} in single pole model, **b** α in the BK model, and **c** r in the ISGW2 model

Extractions of $|V_{cd}|$ and $|V_{cs}|$ from Leptonic $D^+_{(s)}$ decays

- Leptonic D^+ and D^+_s Decays



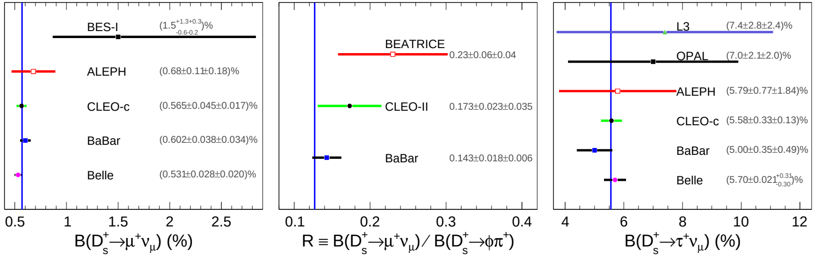
$$\Gamma(D^+_{(s)} \to \ell^+\nu_\ell) = \frac{G_F^2}{8\pi} m_\ell^2 m_{D_{(s)}} \left(1 - \frac{m_\ell^2}{m^2_{D_{(s)}}}\right)^2 f^2_{D^+_{(s)}} |V_{cd(s)}|^2$$

  - Directly determined $f^2_{D^+_{(s)}} |V_{cd(s)}|^2$ with $B(D^+_{(s)} \to \ell^+\nu_\ell)$ which were measured at different experiments
  - $G_{\rm F}$, $m_{D_{(s)}}$, m_ℓ, and lifetime of the meson are taken from PDG2014
  - Input $f_{D^+} = (212.6 \pm 0.4^{+1.0}_{-1.2})$ MeV [PRD90,074509(2014)] and $f_{D_s^+} = (249.0 \pm 0.3^{+1.1}_{-1.5})$ MeV [PDR90,074509(2014)]
  - Extract $|V_{cd}|$ and $|V_{cs}|$
- The BESIII measured $|V_{cd}| = 0.2210 \pm 0.0058 \pm 0.0047$ with $D^+ \to \mu^+\nu_\mu$ decays for the first time in 2014 (First reported at Charm2012)
- Several extractions of $|V_{cs}|$ from $D^+_{(s)} \to \ell^+\nu_\ell$ decays are avariable

$|V_{cd}|$ and $|V_{cs}|$ extracted by analyzing all available data

Leptonic $D^+_{(s)}$ decays, Eur.Phys.J.C (2015)75:10; Phys.Lett.B743, 315 (2015)

- Global fit to all experimental data of leptonic D_s^+ decays



- $f_{D_s^+}|V_{cs}| = (252.0 \pm 3.7 \pm 1.8)$ MeV, with $f_{D_s^+} = (249.0 \pm 0.3^{+1.1}_{-1.5})$ MeV (PRD90, 074509(2014))
  $\Rightarrow |V_{cs}|^{D_s^+ \to \ell^+ \nu_\ell} = 1.012 \pm 0.015 \pm 0.009$
- Use $\mathcal{B}(D^+ \to \mu^+ \nu_\mu) = (3.74 \pm 0.17) \times 10^{-4}$ measured at CLEO-c and BESIII
  $f_{D+}|V_{cd}| = (45.92 \pm 1.04 \pm 0.15)$ MeV, with $f_{D^+} = (212.6 \pm 0.4^{+1.0}_{-1.2})$ MeV (PRD90, 074509(2014))
  $\Rightarrow |V_{cd}|^{D^+ \to \mu^+ \nu_\mu} = 0.2160 \pm 0.0049 \pm 0.0014$

$B(D^+_{(s)} \to \ell^+\nu_\ell)$ and $f^2_{D^+_{(s)}}|V_{cd(s)}|^2$

See original publications and arXiv:1411.3868

- $B(D^+_{(s)} \to \ell^+\nu_\ell)$ and $f^2_{D^+_{(s)}}|V_{cd(s)}|^2$

Table 2: Summary of decay branching fractions $B(D_s^+ \to \mu^+\nu_\mu)$, $B(D_s^+ \to \tau^+\nu_\tau)$ and branching ratio $R_B = B(D_s^+ \to \mu^+\nu_\mu)/B(D_s^+ \to \phi\pi^+)$ measured at different experiments.

Experiment		$B(D_s^+ \to \mu^+\nu_\mu)$ (%)	Note
BES-I [8]		$1.5^{+1.3+0.3}_{-0.6-0.2}$	D
ALEPH [9]		$0.68 \pm 0.11 \pm 0.18$	D
CLEO-c [10]		$0.565 \pm 0.045 \pm 0.017$	P
BaBar [11]		$0.602 \pm 0.038 \pm 0.034$	P
Belle [3]		$0.531 \pm 0.028 \pm 0.020$	P
Experiment	R_B	$B(D_s^+ \to \mu^+\nu_\mu)$ (%)	R
BEATRICE [12]	$0.23 \pm 0.06 \pm 0.04$	$1.04 \pm 0.27 \pm 0.18 \pm 0.09$	R
CLEO-II [13]	$0.173 \pm 0.023 \pm 0.035$	$0.779 \pm 0.104 \pm 0.158 \pm 0.069$	R
BaBar [14]	$0.143 \pm 0.018 \pm 0.006$	$0.644 \pm 0.081 \pm 0.027 \pm 0.057$	R
Experiment		$B(D_s^+ \to \tau^+\nu_\tau)$ (%)	
L3 [15]		$7.4 \pm 2.8 \pm 2.4$	D
OPAL [16]		$7.0 \pm 2.1 \pm 2.0$	D
ALEPH [9]		$5.79 \pm 0.77 \pm 1.84$	D
CLEO-c [17]		$5.58 \pm 0.33 \pm 0.13$	P
BaBar [11]		$5.00 \pm 0.35 \pm 0.49$	P
Belle [3]		$5.70 \pm 0.21^{+0.31}_{-0.30}$	P

Table 3: $f_{D_s^+}|V_{cs}|$ and $f_{D_s^+}$ determined by fitting different decay branching fractions in groups of DPR, DP and P shown in Table 2.

Group of $B(D_s^+ \to \ell^+\nu_{ell})$	DPR	DP	P
$f_{D_s^+}\|V_{cs}\|$	$252.0 \pm 3.7 \pm 1.8$ [7]	$250.7 \pm 3.8 \pm 1.8$	$250.2 \pm 3.8 \pm 1.8$
$f_{D_s^+}$	258.9 ± 4.2 [7]	257.5 ± 4.3	257.0 ± 4.3

- $f_{D_s^+}|V_{cs}| = (250.7 \pm 3.8 \pm 1.8)$ MeV from "DP" fit