

新学術領域「新ハドロン」  
多彩なフレーバーで探る新しいハドロン存在形態の包括的研究  
第一回 評価委員会@名古屋大学, 8月4日(木)

# Composite and elementary natures of hadrons ～分子共鳴とクォーク核の状態混合～



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H. Nagahiro *et al.*, Phys. Rev. D83,111504(R) (2011)

# 本研究の目的

Exotic hadrons : *not* simple  $q\bar{q}$  (or  $qqq$ ) state

›  $\sigma$ ,  $f_0(980)$ ,  $a_0(980)$ , ... ,  $a_1(1260)$ ,  $K_1(1270)$ , ... ,  $N^*(1535)$ ,  $\Lambda(1405)$ , ... etc...

→ multi-quark system, ハドロン分子共鳴状態, ...

$a_1(1260) \rightarrow \pi\gamma$  decay as  $\pi\rho$  composite

H. N., L. Roca, A. Hosaka and E. Oset, PRD79(09)014015.

$a_1^+ \rightarrow \pi^+\gamma$  ...  $\Gamma_{\pi\gamma} \sim 130$  keV [実験値 :  $640 \pm 246$  keV]

$b_1^+ \rightarrow \pi^+\gamma$  ...  $\Gamma_{\pi\gamma} \sim 210$  keV [ " :  $230 + 60$  keV]

$$\left| a_1 \right\rangle = C_1 \left| \begin{array}{c} \rho \\ \pi \end{array} \right\rangle + C_2 \left| \begin{array}{c} a_1 \end{array} \right\rangle + \dots$$

physical hadron                      hadronic composite                       $qq^{\text{bar}}$ -core

$a_1$ に限らず全てのハドロン共鳴は、  
多かれ少なかれ、複数の要素の**混合状態**

✓ どのように混ざるか  
✓ どの程度混ざっているか

# $a_1(1260)$ axial vector meson ... a good example

$a_1(1260)$   $I^G(J^{PC}) = 1^-(1^{++})$  [Particle Data Group, JPG **37**, 075021 (2010)]

	<u>VALUE (MeV)</u>	<u>EVTS</u>		<u>VALUE (MeV)</u>	<u>EVTS</u>
<b>MASS</b>	<b><math>1230 \pm 40</math></b>	<b>OUR ESTIMATE</b>	<b>WIDTH</b>	<b>250 to 600</b>	<b>OUR ESTIMATE</b>

as an elementary field (or  $q\bar{q}$ ) : candidate for chiral partner of  $\rho$

[ $q\bar{q}$ -NJL] M. Wakamatsu *et al.*, ZPA311(88)173, A.Hosaka, PLB244(90)363-367, ...

[Lattice QCD] M. Wingate *et al.*, PRL74(95)4596, ...

[Hidden local sym.] Bando-Kugo-Yamawaki; PR164(88)217; Harada-Yamawaki, PR381(03)1, Kaiser-Meissner, NPA519(90)671, ...

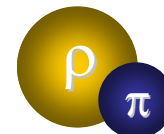
[Holographic QCD] T. Sakai, S. Sugimoto, PTP113 (05) 843; *ibid.*114(05)1083, ...



as a dynamically generated resonance **[as  $\pi\rho$  composite particle]**

[coupled-channel BS] Lutz-Kolomeitsev, NPA730(04)392, ...

[Chiral Unitary model] Roca-Oset-Singh, PRD72(05)014002, ...



## applications

[ $\tau$ -decay spectrum] M. Wagner and S. Leupold, PRD78(08)053001, ...

[radiative decay width] H.N, L. Roca, A. Hosaka, E. Oset, PRD79(09)014015, ...

# A good model for $a_1$ , $\pi$ and $\rho$ mesons

hidden local sym. or holographic model

Bando-Kugo-Yamawaki, PR164(88)217

Sakai-Sugimoto, PTP113(05)843

$$\mathcal{L}_{\text{WT}} = -\frac{g_4}{4f_\pi^2} \text{tr}([\rho^\mu, \partial^\nu \rho_\mu][\pi, \partial_\nu \pi]) \Rightarrow \begin{array}{cc} \pi & \text{---} & \pi \\ & \times & \\ \rho & \text{---} & \rho \end{array} \Rightarrow \text{composite } a_1$$

$$\mathcal{L}_{a_1\pi\rho} = -g_{a_1\pi\rho} \frac{i\sqrt{2}}{f_\pi} \left\{ \text{tr}[(\partial_\mu a_{1\nu} - \partial_\nu a_{1\mu})[\partial^\mu \pi, \rho^\nu]] + \text{tr}[(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)[\partial^\mu \pi, a_1^\nu]] \right\} \Rightarrow \begin{array}{c} \text{elementary } a_1 \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \pi \\ \text{---} \\ \rho \end{array}$$

A good model for **composite  $a_1$**  and **elementary  $a_1$**

elementary  $a_1$  meson の質量 : in holographic model

$$m_{a_1} = 1189 \text{ MeV}$$

Sakai-Sugimoto, PTP113(05)843; PTP114(05)1083]

$$f_\pi = 92.4 \text{ MeV}, m_\rho = 776 \text{ MeV}$$

input parameters

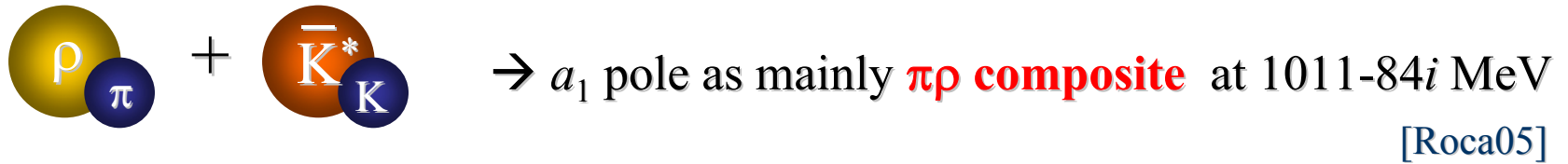
✓ この  $a_1$  meson を  $q\bar{q}$  状態 (elementary  $a_1$ ) と考える。

[hQCD is constructed in the large  $N_c$  limit]

# Dynamically generated resonances : **composite $a_1$ meson**

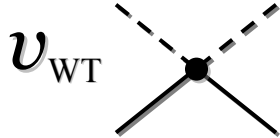
$\pi\rho$ -composite としての  $a_1$  meson : chiral unitary model

L.Roca, E.Oset and J.Singh, PRD72(05)014002



$\pi\rho \rightarrow \pi\rho$  散乱振幅 : coupled-channel 計算

$$T_{\pi\rho \rightarrow \pi\rho} = \frac{v_{WT}}{1 - v_{WT}G} \vec{\epsilon} \cdot \vec{\epsilon}'$$

$v_{WT}$  

$$= \text{[Series of diagrams: a cross, a loop, a double loop, etc.]} = \text{[Diagram with a wavy line between two green circles]} = g_R(s) \frac{1}{s - s_p} g_R(s)$$

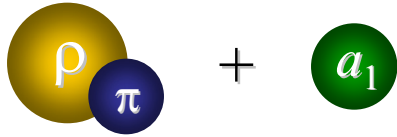
regularization constant

$a(\mu) = -1.85$  ( $\mu=900\text{MeV}$ ) [Roca *et al.*]  $\rightarrow a(\mu) = -0.2$  (natural)  
 to avoid the double counting.

[T. Hyodo, D.Jido, A.Hosaka, PRC78(08)025203]

# Formalism : elementary $a_1$ field through additional interaction

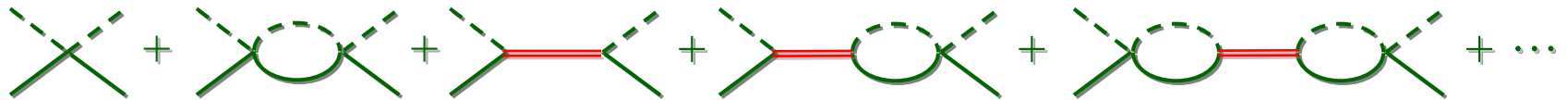
mixed states of  $\pi\rho$  composite  $a_1$  and elementary  $a_1$  mesons



elementary  $a_1$  meson は、additional interaction  $v_{a1}$  を通して散乱に寄与

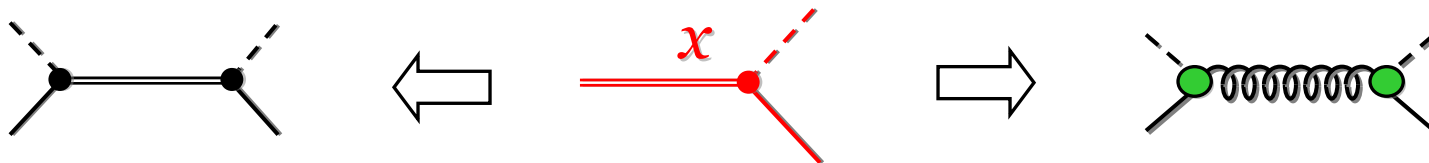
full scattering amplitude

$$T_{\pi\rho \rightarrow \pi\rho} = \frac{v_{WT} + v_{a_1}}{1 - (v_{WT} + v_{a_1})G} \vec{\epsilon} \cdot \vec{\epsilon}'$$

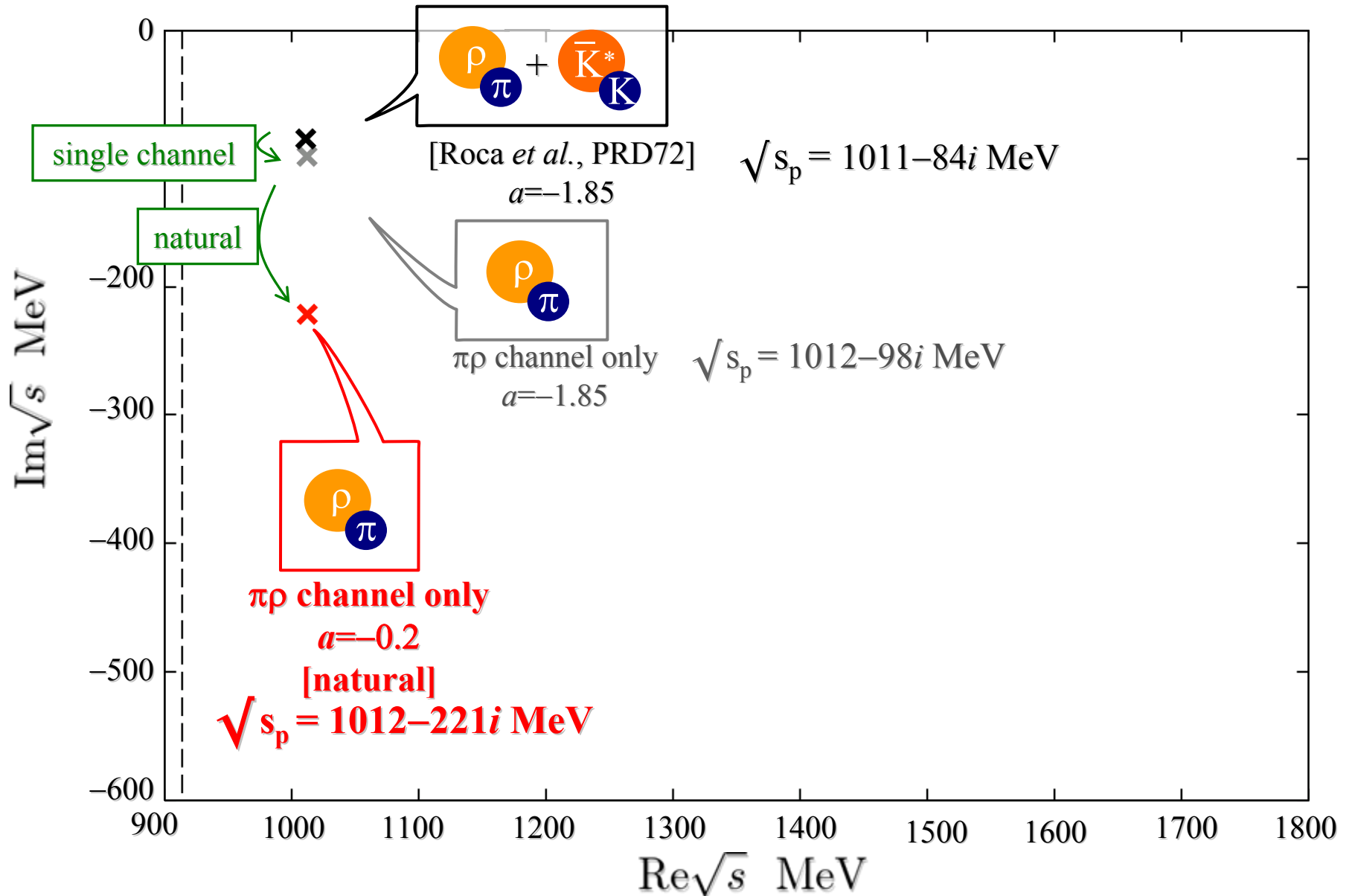


= physical resonant  $a_1$  states

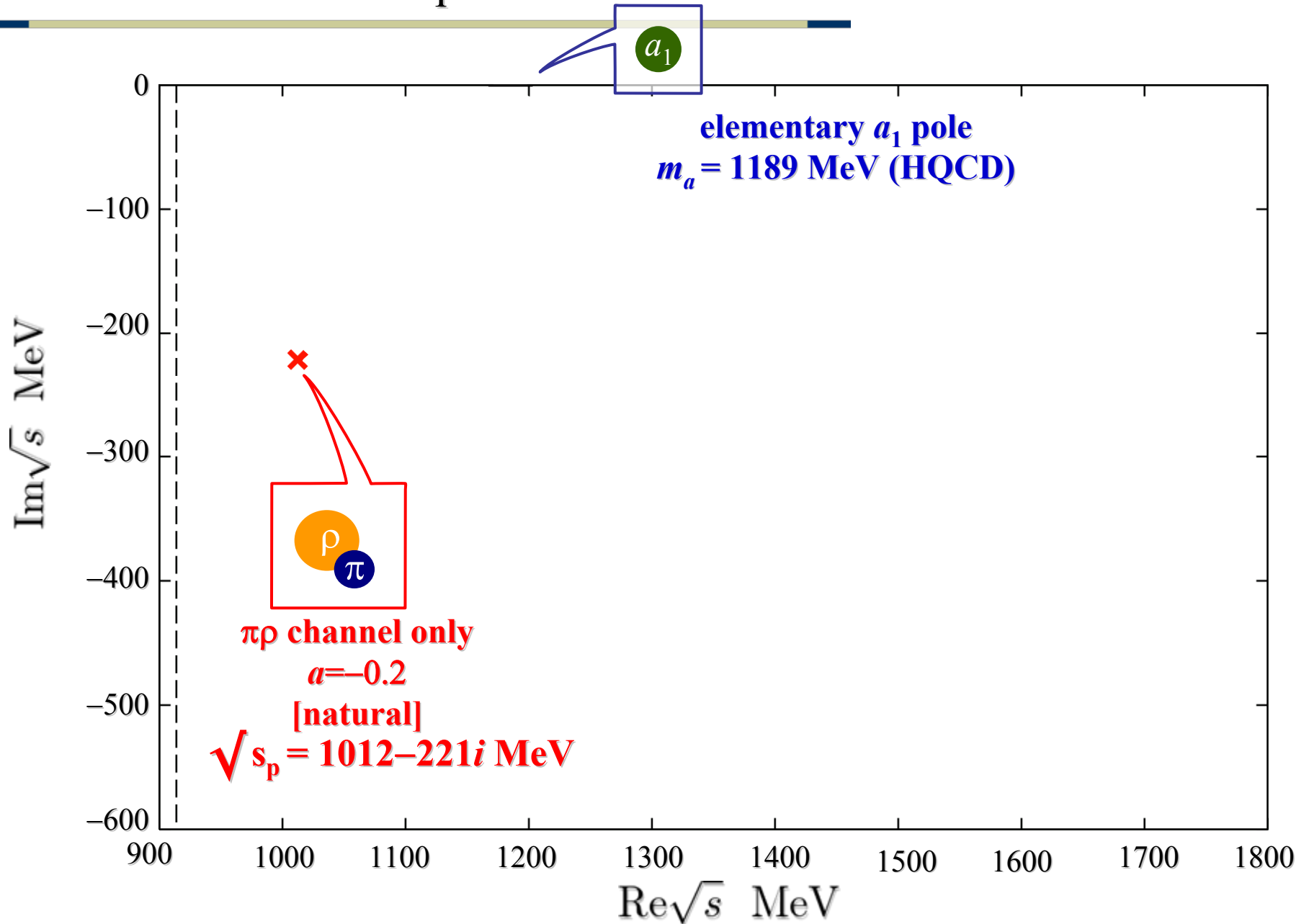
causes the mixing



# Numerical result 1 : pole-flow of T-matrix

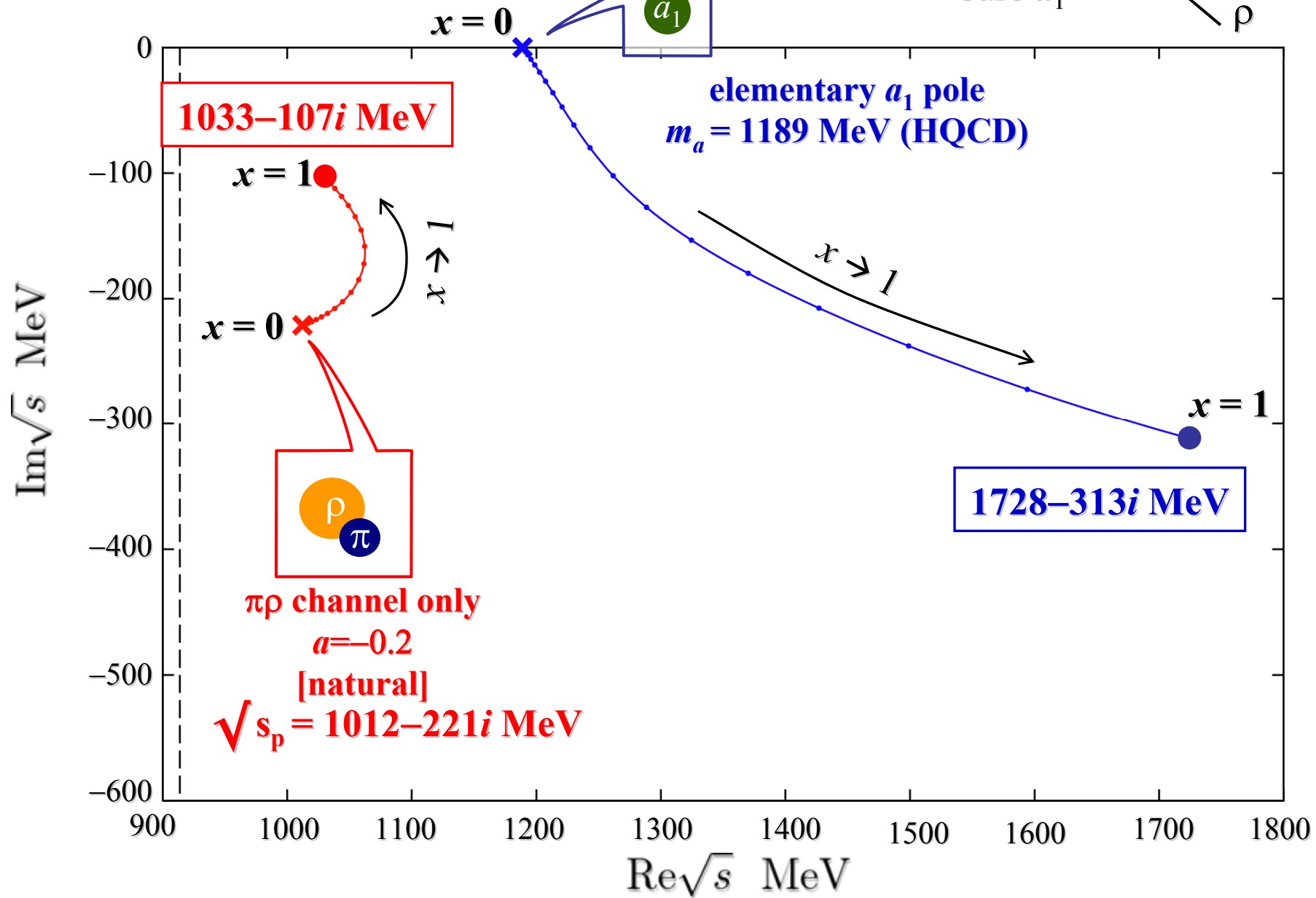


# Numerical result 1 : pole-flow of T-matrix



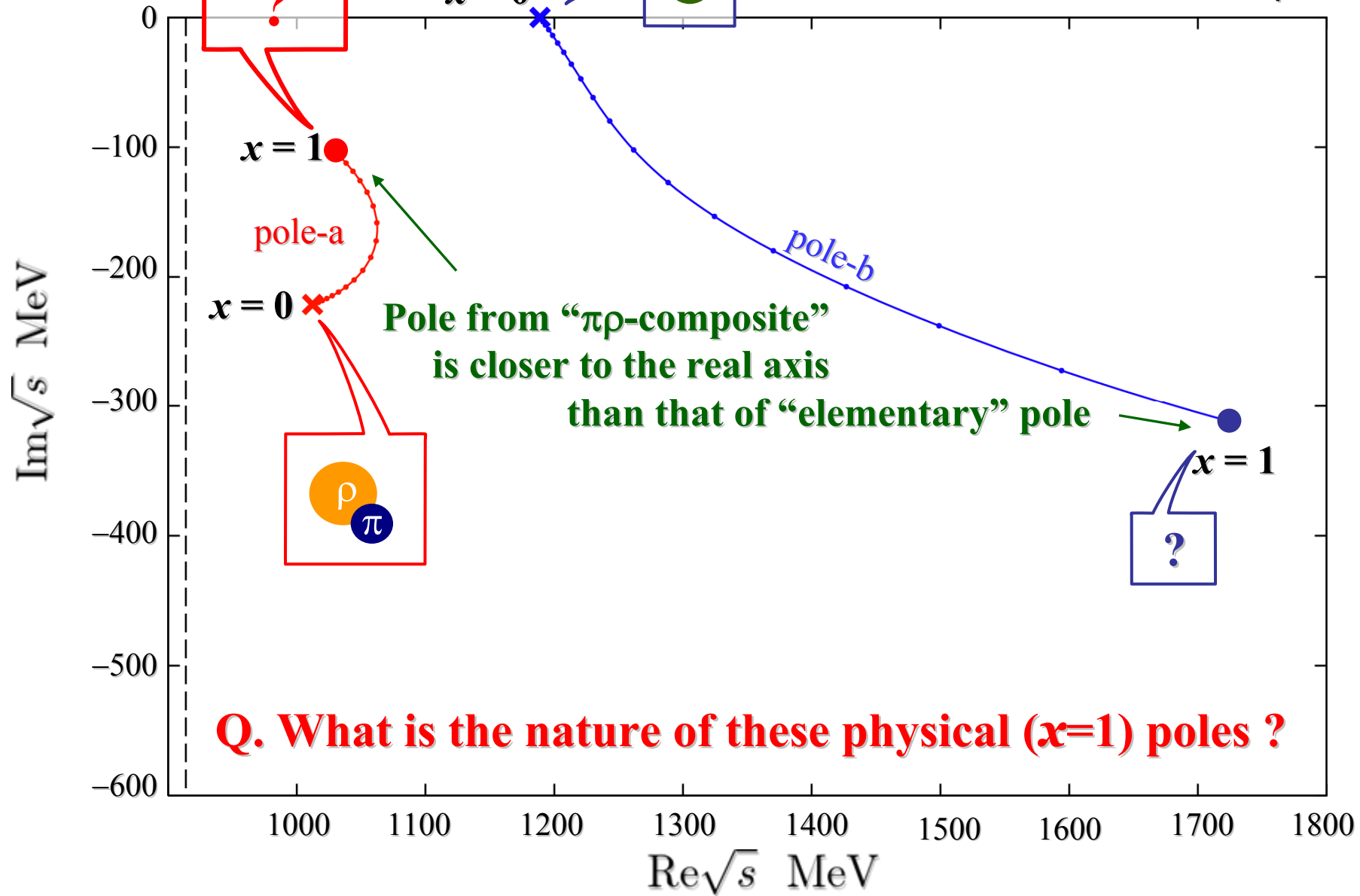


# Numerical result 1 : pole-flow of T-matrix





# Numerical result 1 : pole-flow of T-matrix





# Alternative expression for the *full* $\pi\rho$ scattering amplitude $T$

$$T = \frac{v_{WT} + v_{a_1}}{1 - (v_{WT} + v_{a_1})G} = (g_R, g) \left\{ \begin{pmatrix} s - s_p & \\ & s - m_{a_1}^2 \end{pmatrix} - \begin{pmatrix} g_R G g & \\ g G g_R & g G g \end{pmatrix} \right\}^{-1} \begin{pmatrix} g_R \\ g \end{pmatrix}$$

$$= \left( \begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} \right) \left\{ \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix}^{-1} - \begin{pmatrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{pmatrix} \right\}^{-1} \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix}$$

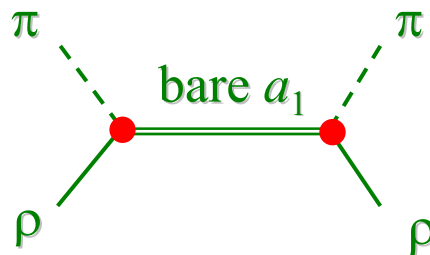
## $\pi\rho$ -composite $a_1$ pole

$$T_{WT} = \frac{v_{WT}}{1 - v_{WT}G}$$

$$\text{---} + \text{---} + \text{---} + \dots \equiv \text{---} = g_R(s) \frac{1}{s - s_p} g_R(s)$$

## $a_1$ pole term



$$V_{a_1} = g(s) \frac{\vec{\epsilon} \cdot \vec{\epsilon}'}{s - m_a^2} g(s)$$



# Alternative expression for the *full* $\pi\rho$ scattering amplitude $T$

$$T = \frac{v_{WT} + v_{a_1}}{1 - (v_{WT} + v_{a_1})G} = (g_R, g) \left\{ \begin{pmatrix} s-s_p & \\ & s-m_{a_1}^2 \end{pmatrix} - \begin{pmatrix} g_R G g & \\ g G g_R & g G g \end{pmatrix} \right\}^{-1} \begin{pmatrix} g_R \\ g \end{pmatrix}$$

$$= \left( \begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} \right) \left\{ \begin{pmatrix} \text{---} & \\ & \text{---} \end{pmatrix}^{-1} - \begin{pmatrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{pmatrix} \right\}^{-1} \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix} \quad \hat{D}$$

In this form, we can analyze the mixing nature of the physical  $a_1$  in terms of the **original two bases**:  and 

composite  $a_1$

elementary  $a_1$

$$= \begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} \boxed{\text{---} \text{---}} \text{---} \text{---} + \begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} \boxed{\text{---} \text{---}} \text{---} \text{---} + \begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} \boxed{\text{---} \text{---}} \text{---} \text{---} + \begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} \boxed{\text{---} \text{---}} \text{---} \text{---}$$


$D^{11} \quad D^{21} \quad D^{12} \quad D^{22}$

# full propagator D の性質

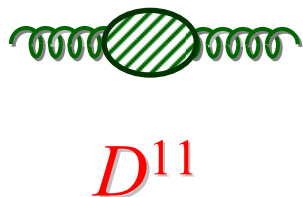
- ✓ それぞれの propagator は  $T_{\text{full}}$  と同じ所に pole を持つ。
- ✓ その residues  $z_a^{ii}$  は、mixing rate の意味を持つ。

$$z_a^{11} = |\langle 1|a\rangle|^2 = |\langle \text{pole-a} \rangle|^2$$

→ pole-a に, composite  $a_1$  を見いだす確率

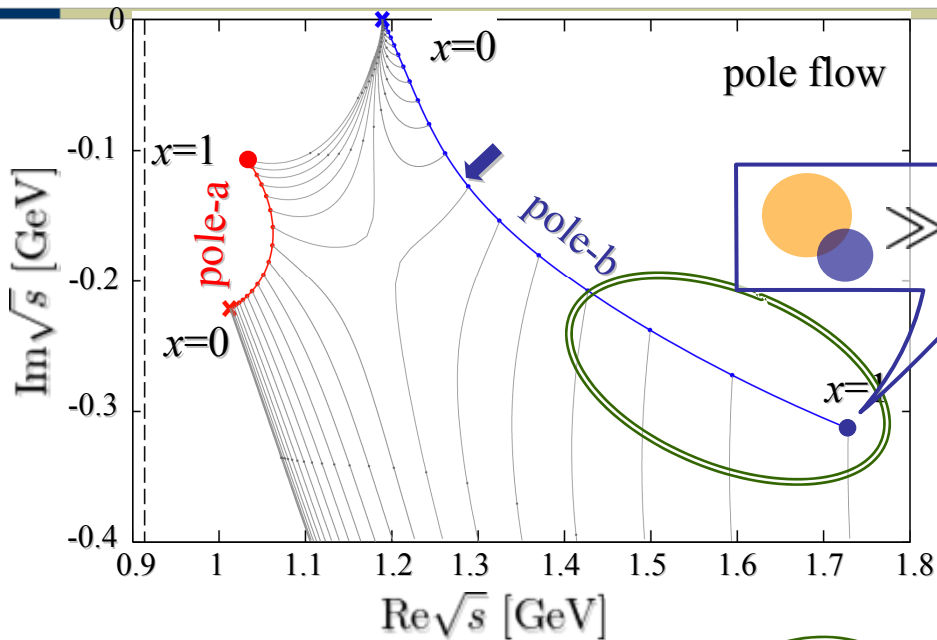
In this form, we can analyze the mixing nature of the physical  $a_1$  in terms of the **original two bases**:  and 

composite  $a_1$       elementary  $a_1$



$$D^{11} = \frac{z_a^{11}}{s - M_a^2} + \frac{z_b^{11}}{s - M_b^2} + \text{regular term}$$

# Residues : probabilities of finding two $a_1$ 's in pole-a and -b

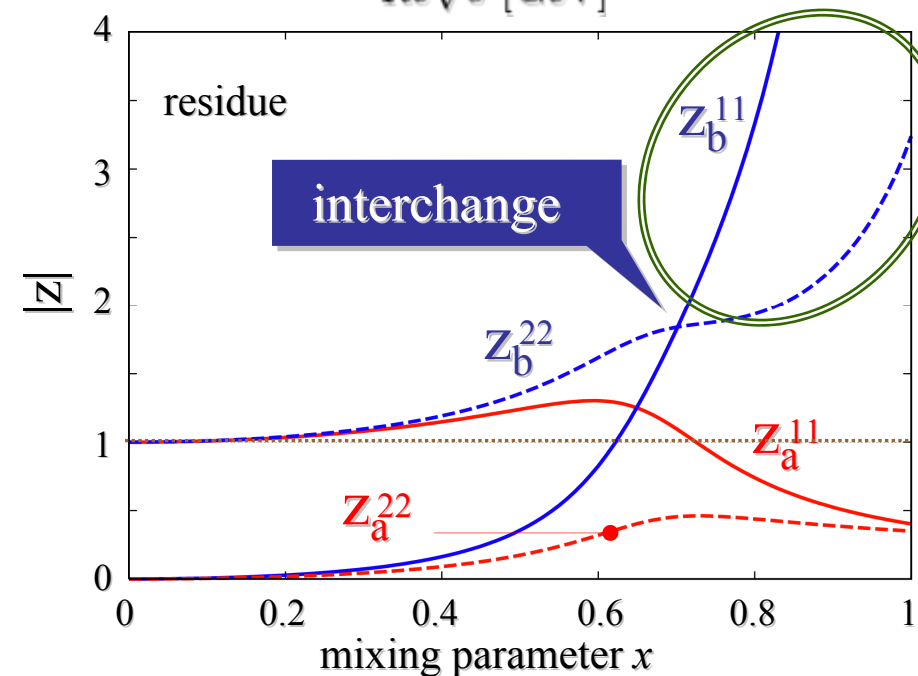


$$[\hat{D}_{\text{full}}]^{11} = \frac{z_a^{11}}{E - E_a} + \frac{z_b^{11}}{E - E_b} + (\text{regular})$$

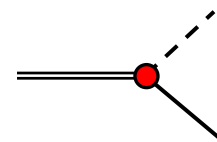
$$[\hat{D}_{\text{full}}]^{22} = \frac{z_a^{22}}{E - E_a} + \frac{z_b^{22}}{E - E_b} + (\text{regular})$$

$$|a\rangle = \sqrt{z_a^{11}} |\text{orange-blue}\rangle + \sqrt{z_a^{22}} |\text{green}\rangle$$

$$|b\rangle = \sqrt{z_b^{11}} |\text{orange-blue}\rangle + \sqrt{z_b^{22}} |\text{green}\rangle$$



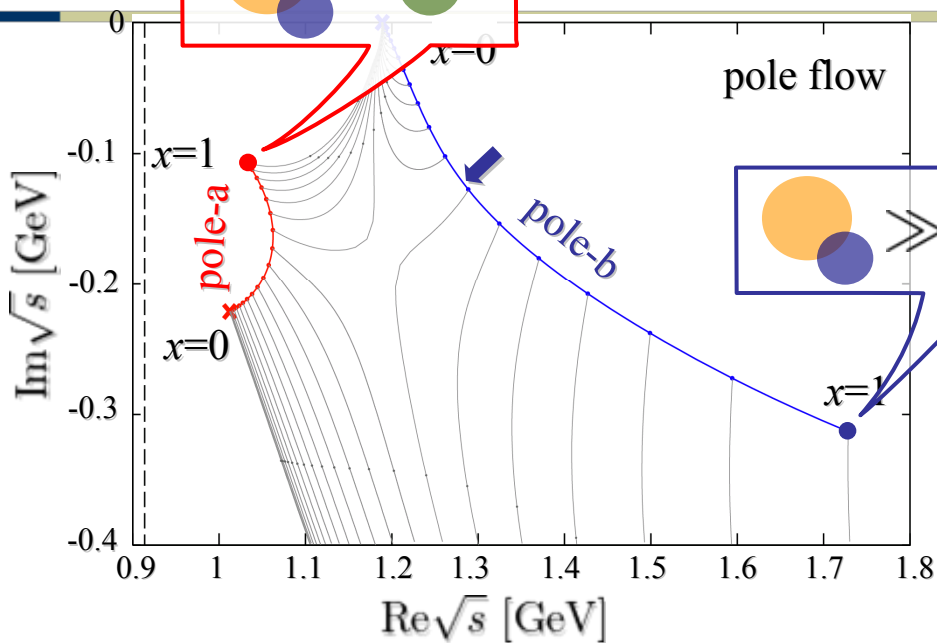
Large residue  
→ due to the ene-dep.



$$g(s) = \frac{2\sqrt{2}}{f_\pi} g_{a_1\pi\rho}(s - M_\rho^2)$$



# Residues: probabilities of finding two $a_1$ 's in pole-a and -b

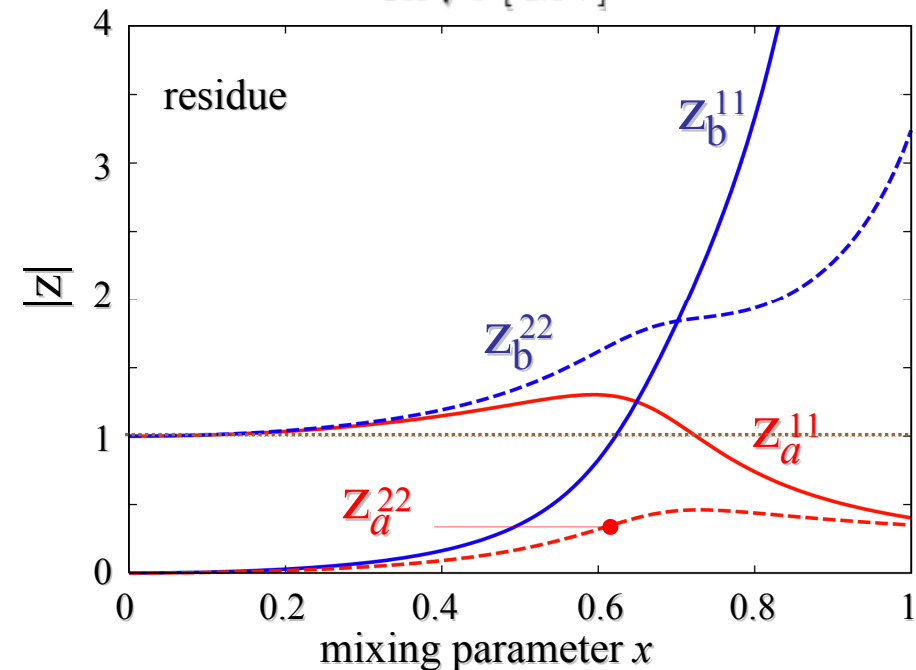


$$[\hat{D}_{\text{full}}]^{11} = \frac{z_a^{11}}{E - E_a} + \frac{z_b^{11}}{E - E_b} + (\text{regular})$$

$$[\hat{D}_{\text{full}}]^{22} = \frac{z_a^{22}}{E - E_a} + \frac{z_b^{22}}{E - E_b} + (\text{regular})$$

$$|a\rangle = \sqrt{z_a^{11}} |\text{orange-blue}\rangle + \sqrt{z_a^{22}} |\text{green}\rangle$$

$$|b\rangle = \sqrt{z_b^{11}} |\text{orange-blue}\rangle + \sqrt{z_b^{22}} |\text{green}\rangle$$



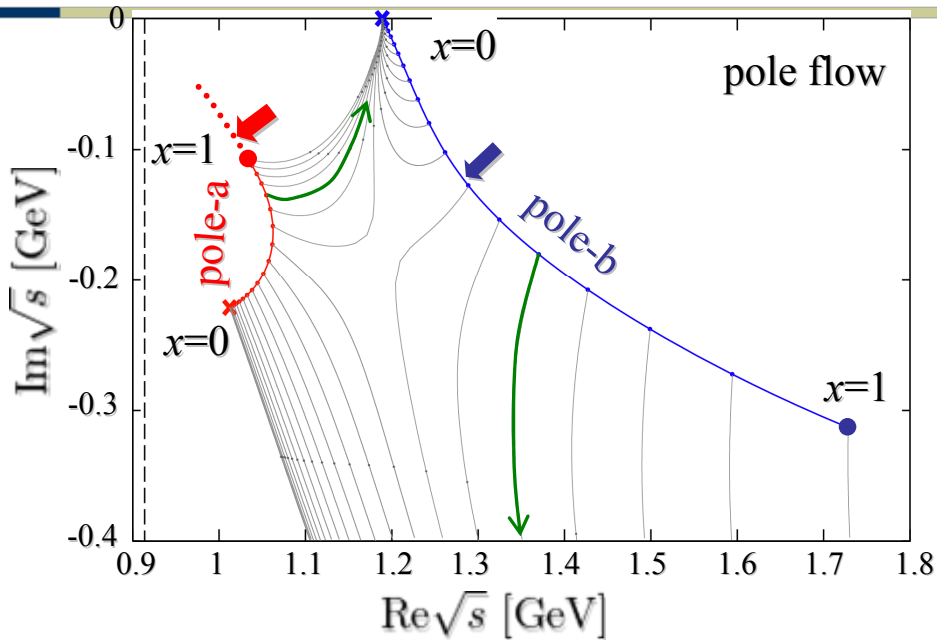
## at physical point ( $x=1$ )

- **pole-a** has a component of the elementary  $a_1$  meson *comparable to* that of composite  $a_1$ .  
(pole-a at  $x=1$  is possibly observed one)



non-zero comp. of

# Last question : large $N_C$ limit vs. nature of poles

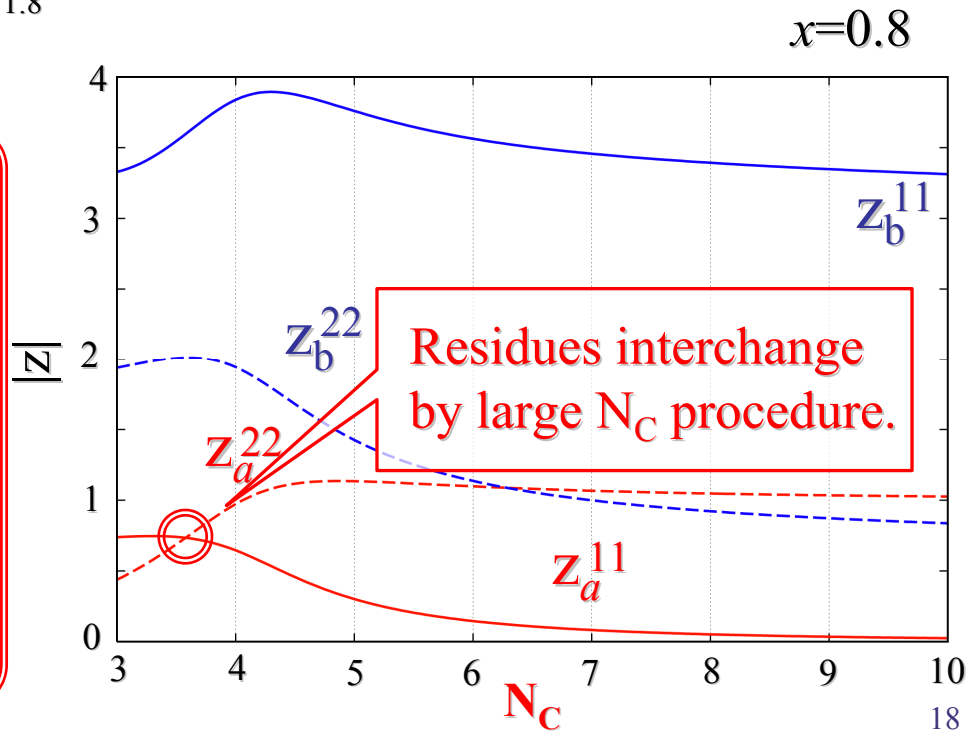


- for pole-b, large  $N_C$  flip point  $\sim$  residue-interchange
- for pole-a, large  $N_C$  flip point  $\ll$  residue-interchange

$\propto 1/N_C$                        $\propto 1/\sqrt{N_C}$

mixing nature *changes* as  $N_C$  is increased

**Large  $N_C$  limit doesn't always reflect the world at  $N_C=3$ .**



# Conclusions

- » We analyzed the nature of the hadronic resonance by residues
  - »  $a_1(1260)$  meson :  $\pi\rho$ 分子共鳴 + elementary  $a_1$ 
    - › bare  $a_1$  ... doesn't have molecule nature
    - ›  $\pi\rho$  molecule ... “natural” regularization
- Important to avoid the double-counting
- ✓ the pole expected to be observed is *pole-a*: having finite ● comp.
  - ✓ Non-trivial  $N_C$  dependence pole-nature  $\leftarrow ? \rightarrow$  large  $N_C$  limit

## Future works

### phenomenological interests

- »  $\tau$ -decay spectrum with our model parameter
- » radiative decay width, etc...

to see **how the nature of poles affects *observables***

### theoretical interests

- » application to other systems,  $\sigma$ ,  $N^*(1535)$ , etc...