

“Quantum Sensing”*)

— Sec. I & II —

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*) C.L. Degen, F. Reinhard and P. Cappellaro, Rev. Mod. Phys. 89, 035002 (2017)

Sec. I Introduction (1)

- *Promising real-world applications of quantum mechanics?*
 - *Quantum computers,*
 - *Deutsch, D., 1985, Proc. R. Soc. A 400, 97.*
 - *DiVincenzo, D. P., 2000, Fortschr. Phys. 48, 771.*
 - *Quantum cryptography,*
 - *Gisin, N., G. Ribordy, W. Tittel, and H. Zbinden, 2002, Rev. Mod. Phys. 74, 145.*
 - *Quantum sensors.*
 - *For magnetic and electric fields, time and frequency, rotations, temperature and pressure.*
 - *Capitalize on the central weakness of quantum systems, their strong sensitivity to external disturbances.*
 - *This trend in quantum technology is curiously reminiscent of the history of semi-conductors.*
 - *Sensors have found commercial applications decades before computers.*
 - *e.g. light meters based on selenium photocells: Weston, E., 1931, "Exposure Meter," U.S. Patent 2016469.*

Sec. I Introduction (2)

- *Many concepts of quantum sensing have resulted from **decades of development in high-resolution spectroscopy**.*
 - *In **atomic physics and magnetic resonance**, especially.*
 - *e.g. atomic clocks, atomic vapor magnetometers and superconducting quantum interference devices.*
- *What's "new" in quantum sensing?*
 - *Quantum systems are increasingly **investigated at the single-atom level**,*
 - ***Entanglement** is used as a resource **for increasing the sensitivity**,*
 - *Quantum systems and quantum manipulations are **specifically designed and engineered for sensing purpose**.*
- *This review focuses on key concepts and methods of quantum sensing.*
 - *With particular attention to **practical aspects that emerge from nonideal experiments**.*
 - *Will consider mostly **"qubits" — two-level quantum systems**.*
 - *Will overview actual implementations, but not in specific detail.*
 - *Will not cover related fields, e.g. atomic clocks or photon-based sensors.*
 - *Will consider theory only up to the point necessary to introduce the key concepts.*
- *This review intends to ...*
 - *Offer an introduction to whom new to the field,*
 - *Provide a basic reference for whom already active in the field.*

Sec. I.A Content (1)

- *Some basic definitions for “quantum sensing” (Sec. II.A).*
- *The elementary criteria for a quantum system to be useful as a quantum sensor (Sec. II.B).*

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- *An overview of the most important physical implementations (Sec. III).*

- *The core concepts of quantum sensing:*
 - *the basic measurement protocol (Sec. IV),*
 - *the sensitivity of a quantum sensor (Sec. V).*

Sec. I.A Content (2)

- *Covering the important area of*
 - *time-dependent signals (Sec. VI),*
 - *quantum spectroscopy (Sec. VII).*

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- *Some advanced quantum sensing techniques:*
 - *adaptive methods developed to greatly enhance the dynamic range of the sensor (Sec. VIII),*
 - *techniques that involve multiple qubits (Secs. IX and X), particularly the entanglement-enhanced sensing, quantum storage, and quantum error correction (QEC) schemes.*

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- *A brief outlook on possible future developments (Sec. XI).*

Sec. I.A Content (3)

- *Several reviews that covered different aspects of quantum sensing and excellent introductions into the field.*
 - *A review:*
 - *Budker, D., and M. Romalis, 2007, Nat. Phys. 3, 227.*
 - *A book on atomic vapor magnetometry:*
 - *Budker, D., and D. F. J. Kimball, 2013, Optical Magnetometry (Cambridge University Press, Cambridge, UK).*
 - *A paper on magnetometry with nitrogen-vacancy centers in diamond:*
 - *Taylor, J. M., P. Cappellaro, L. Childress, L. Jiang, D. Budker, P. R. Hemmer, A. Yacoby, R. Walsworth, and M. D. Lukin, 2008, Nat. Phys. 4, 810.*
 - *Papers on entanglement-assisted sensing, sometimes referred to as “quantum metrology,” “quantum-enhanced sensing,” or “second generation quantum sensors”:*
 - *Bollinger, J. J., W. M. Itano, D. J. Wineland, and D. J. Heinzen, 1996, Phys. Rev. A 54, R4649.*
 - *Giovannetti, V., S. Lloyd, and L. Maccone, 2004, Science 306, 1330.*
 - *Giovannetti, V., S. Lloyd, and L. Maccone, 2006, Phys. Rev. Lett. 96, 010401.*
 - *Giovannetti, V., S. Lloyd, and L. Maccone, 2011, Nat. Photonics 5, 222.*

Sec. II.A Quantum sensing

- Quantum sensing is *one of the following*:
 - (I) Use of a *quantum object to measure a physical quantity* (classical or quantum).
 - “Quantum object”: *characterized by quantized energy levels*,
 - Such as *electronic, magnetic or vibrational states of ...*
 - *Superconducting or spin qubits, neutral atoms or trapped ions.*
 - (II) Use of *quantum coherence to measure ...*
 - “Quantum coherence”: *i.e. wavelike spatial or temporal superposition states.*
 - (III) Use of *quantum entanglement to improve the sensitivity or precision* of a measurement.
 - *Sensitivity or precision beyond what is possible classically.*
- *Type I&II*
 - *Broadly cover many physical systems, including not-strictly-“quantum” ones.*
 - *e.g. classical wave interference in optical or mechanical systems.*
 - *Novotny, L., 2010, Am. J. Phys. 78, 1199.*
 - *Faust, T., J. Rieger, M. J. Seitner, J. P. Kotthaus, and E. M. Weig, 2013, Nat. Phys. 9, 485.*
 - *Often close to applications*, → *extensively discussed in this review.*
 - *May not exploit the full power of quantum mechanics, but already provide advantages:*
 - *e.g. operation at nanoscales that are not accessible to classical sensors.*
- *Type III*
 - *A truly quantum definition.*
 - *Relies on entanglement and, hence, requires >1 sensing qubits.*
 - *e.g. the use of maximally entangled states to reach a Heisenberg-limited measurement.*

Sec. II.B Quantum sensors (1)

From the criteria for quantum computation:

▸ DiVincenzo, D. P., 2000, Fortschr. Phys. 48, 771.

- Four necessary attributes for a quantum system to be a quantum sensor:

(1) Has *discrete, resolvable energy levels*.

- Will *focus on a two-level system “qubit”* (Fig. 1) (or an ensemble of two-level systems),
- Through which many properties of more complex quantum systems can be modeled.
- Goldstein, G., M. D. Lukin, and P. Cappellaro, 2010, arXiv:1001.4804.

(2) Must be possible *to initialize into a well-known state and to read out its state*.

(3) Can be *coherently manipulated, typically by time-dependent fields*. (Not strictly required for all protocols.)

- Exception ex.: CW spectroscopy or relaxation rate measurements.

(4) *Interacts with a relevant physical quantity $V(t)$* , such as an electric or magnetic field.

- *Quantified by a coupling or transduction parameter $\gamma = \partial^q E / \partial V^q$* which relates changes in the transition energy E to changes in the external parameter V .
- Coupling is *mostly either linear ($q=1$) or quadratic ($q=2$)*.
- Leading to *a shift of the quantum system’s energy levels or to transitions between energy levels* (Fig. 1).

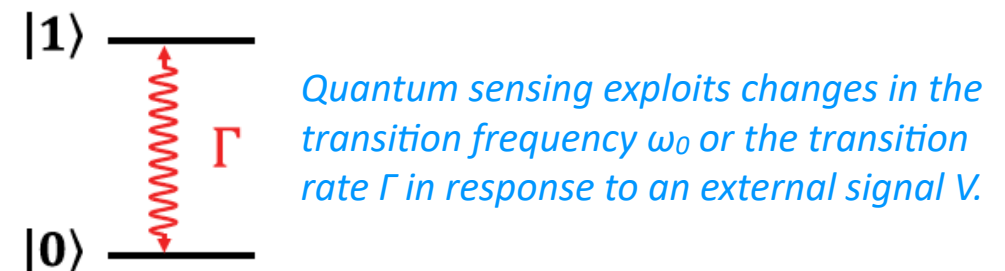
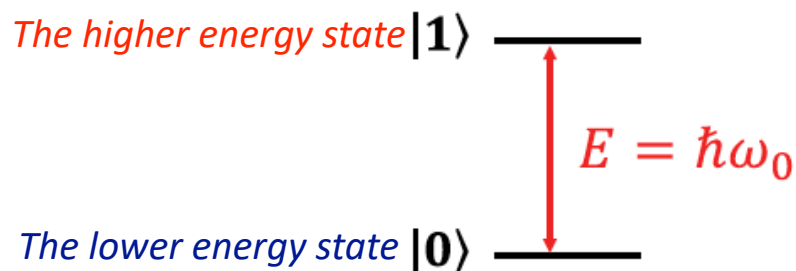


FIG. 1. Basic features of a two-state quantum system.

This review uses $\hbar = 1$ and expresses all energies in units of angular frequency.

Sec. II.B Quantum sensors (2)

- *Key physical characteristics to compare experimental realizations.*
 - *External parameter(s) they respond to.*
 - *Charged systems, e.g. trapped ions: electric fields,*
 - *Spin-based systems: magnetic fields,*
 - *Some systems: several physical parameters.*
 - *“Intrinsic sensitivity” (Eq. 1) → will see in Sec. V.*
 - *Strong response to wanted signals, while minimal influence of unwanted noise.*
 - *γ should be large, e.g. by the choice of an appropriate physical realization.*
 - *T_χ should be made as long as possible (to be discussed in the later sections).*

$$\text{sensitivity} \propto \frac{1}{\gamma \sqrt{T_\chi}}, \quad (1)$$

γ ($= \partial^q E / \partial V^q$): transduction parameter
 T_χ : decoherence or relaxation time