

B decays with lattice QCD: status and prospects



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(University of Illinois)

Mini workshop on $D^{(*)}\tau\nu$ and related decays
KMI, 27-28 March 2017

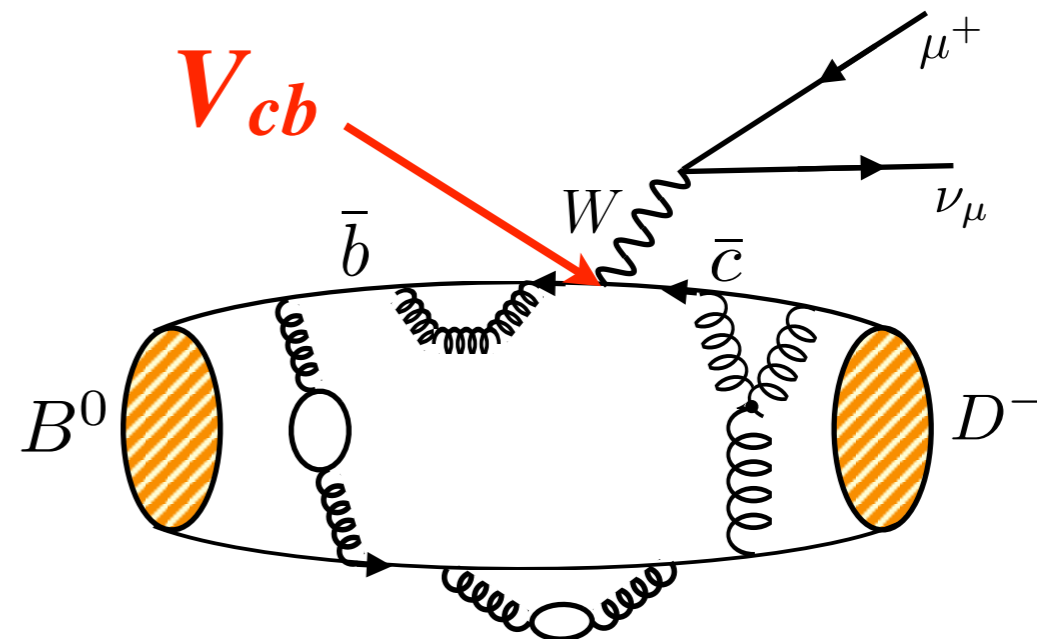


Outline

- Motivation and Introduction
- lattice QCD
- $B \rightarrow D^*$ at zero recoil
- $B \rightarrow D$ at all recoil
 - ◆ combined exp-lattice fit
- $|V_{cb}|$ determinations
- LFU τ/ℓ
- Summary and Outlook

Introduction

example: $B^0 \rightarrow D^- \mu^+ \nu_\mu$



Experiment vs. SM theory:

(experiment) = (known) x **CKM factor** x (had. matrix element)



$$\frac{d\Gamma(B \rightarrow \pi l \nu)}{dq^2}, \frac{d\Gamma(B \rightarrow K l^+ l^-)}{dq^2}, \dots$$

$$\frac{d\Gamma(B \rightarrow D l \nu)}{d\omega}, \frac{d\Gamma(B \rightarrow D \tau \nu)}{d\omega}, \dots$$

$$\Delta m_{d(s)}$$

⋮



Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

Introduction

- For $|V_{cb}|$ determinations use
 - ◆ $B_{(s)} \rightarrow D_{(s)} \ell \nu$, ($\ell = e, \mu$)
 - ◆ $B_{(s)} \rightarrow D_{(s)}^* \ell \nu$, ($\ell = e, \mu$)
- For tests of lepton flavor universality use
 - ◆ $B_{(s)} \rightarrow D_{(s)} \tau \nu_\tau / B_{(s)} \rightarrow D_{(s)} \ell \nu_\ell$
 - ◆ $B_{(s)} \rightarrow D_{(s)}^* \tau \nu_\tau / B_{(s)} \rightarrow D_{(s)}^* \ell \nu_\ell$
- We need form factors at nonzero recoil for both.

Introduction

$$B_{(s)} \rightarrow D_{(s)} \ell \nu$$

$$\frac{\langle D | V^\mu | B \rangle}{\sqrt{M_B M_B}} = h_+(\omega)(v_B + v_D)^\mu + h_-(\omega)(v_B - v_D)^\mu$$

$$\mathcal{G}(\omega) = h_+(\omega) + \frac{M_B - M_D}{M_B + M_D} h_-(\omega) \sim f_+(q^2)$$

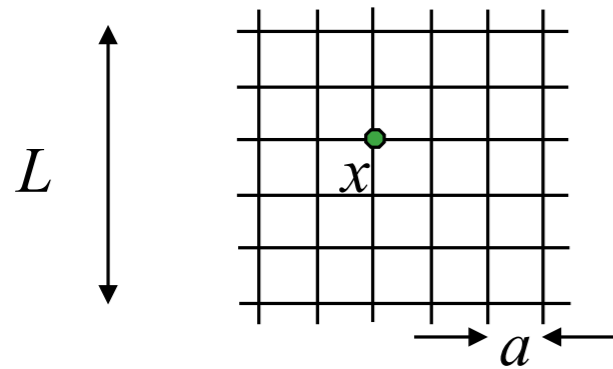
$$B_{(s)} \rightarrow D_{(s)}^* \ell \nu$$

$$\frac{\langle D^*(p_{D^*}, \epsilon^{(\alpha)}) | A^\mu | B(p_B) \rangle}{\sqrt{M_B M_{D^*}}} = \frac{i}{2} \epsilon_\nu^{(\alpha)*} [g^{\mu\nu} (1 + \omega) h_{A_1}(\omega) - v_B^\nu (v_B^\mu h_{A_2}(\omega) + v_{D^*}^\mu h_{A_3}(\omega))]$$

$$\frac{\langle D^*(p_{D^*}, \epsilon^{(\alpha)}) | V^\mu | B(p_B) \rangle}{\sqrt{M_B M_{D^*}}} = \frac{1}{2} \epsilon^{\mu\nu}{}_{\rho\sigma} \epsilon_\nu^{(\alpha)*} v_B^\rho v_{D^*}^\sigma h_V(\omega)$$

Lattice QCD Introduction

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$



- ◆ discrete Euclidean space-time (spacing a)
derivatives \rightarrow difference operators, etc...
- ◆ finite spatial volume (L)
- ◆ finite time extent (T)

adjustable parameters

❖ lattice spacing:

$$a \rightarrow 0$$



❖ finite volume, time:

$$L \rightarrow \infty, T > L$$



❖ quark masses (m_f):

$$M_{H,\text{lat}} = M_{H,\text{exp}}$$



tune using hadron masses
extrapolations/interpolations

$$m_f \rightarrow m_{f,\text{phys}}$$

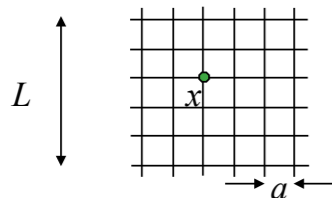
m_{ud}

m_s

m_c

m_b

❖ also: n_f = number of sea quarks: 3 (2+1), 4 (2+1+1)



Lattice QCD Introduction

systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on **EFT (Effective Field Theory)** descriptions of QCD

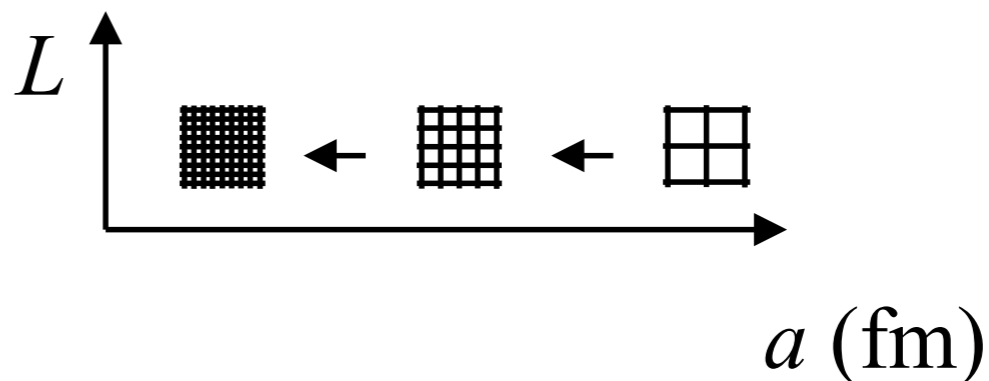
→ **ab initio**

The **EFT** description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...



Lattice guide

- Need to have several (≥ 2) lattice spacings.
Comparing lattice results with different actions provides good cross checks of methods used.
- “physical mass ensemble” means pion mass is at (or near) its physical value.
If larger than in Nature, there must be a range of light quark masses, where the pions shouldn't be too heavy at low end of the range.
- box size should have $m_\pi L \geq 4$.
- sea quark flavors: 2+1, 2+1+1, 1+1+1+1
- complete systematic error analysis and budget
- FLAG: compare/combine results from different lattice groups for specific quantities.

form factor for $B \rightarrow D^{(*)} \ell \nu$ & V_{cb}

$$\frac{d\Gamma(B \rightarrow D^* \ell \nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{1/2} |\mathcal{F}(\omega)|^2$$

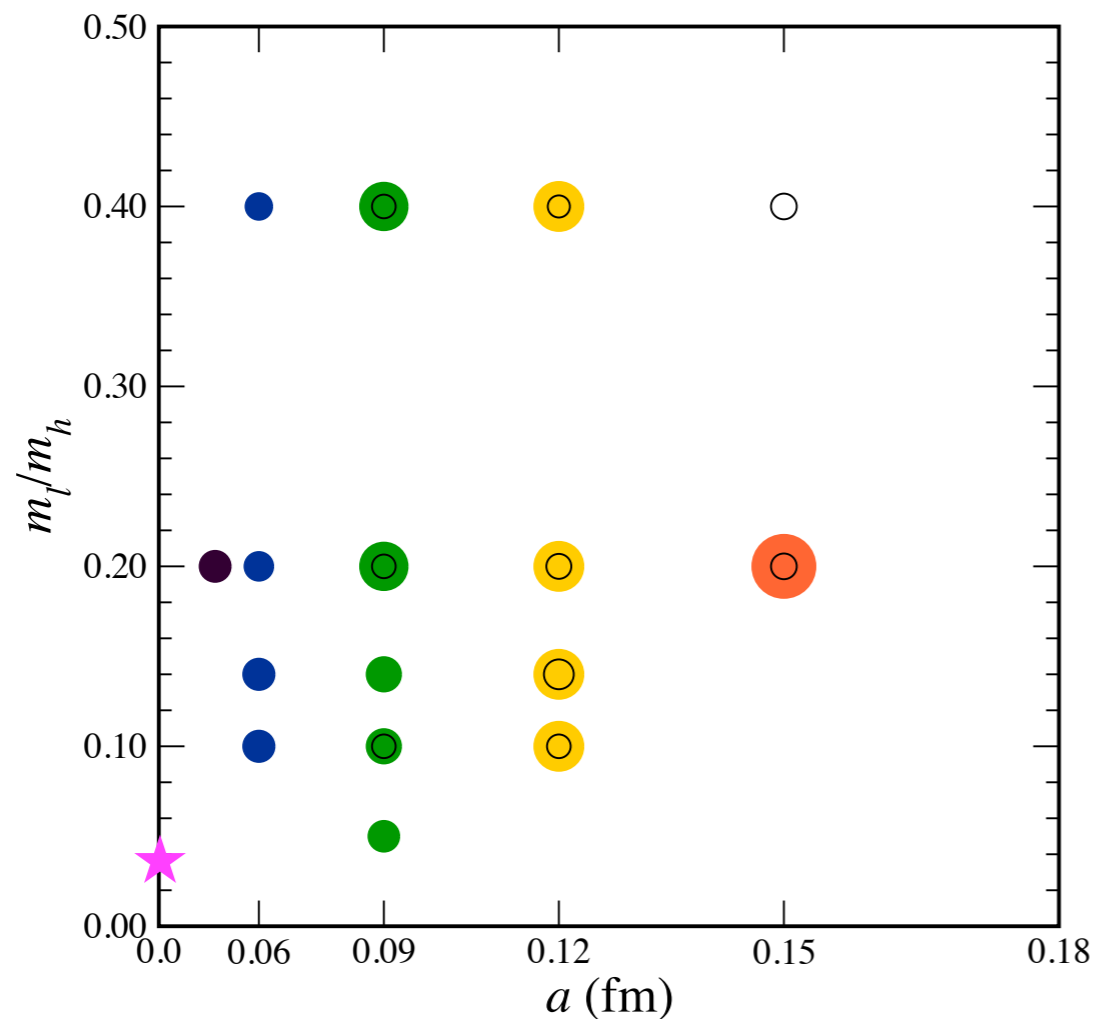
Using CLN to extrapolate to $\omega=1$ (HFAG 2016):

$$B \rightarrow D^* \ell \nu : \eta_{EW} |V_{cb}| \mathcal{F}(1) = (35.61 \pm 0.11 \pm 0.41) \times 10^{-3}$$

- ❖ [FNAL/MILC 2014](#) (J. Bailey et al, arXiv:1403.0635, 2014 PRD): $\mathcal{F}(1) = 0.906(4)(12)$
- ❖ [new: HPQCD](#) (J. Harrison @ Lattice 2016, preliminary)

Form factor for $B \rightarrow D^*$ at zero recoil

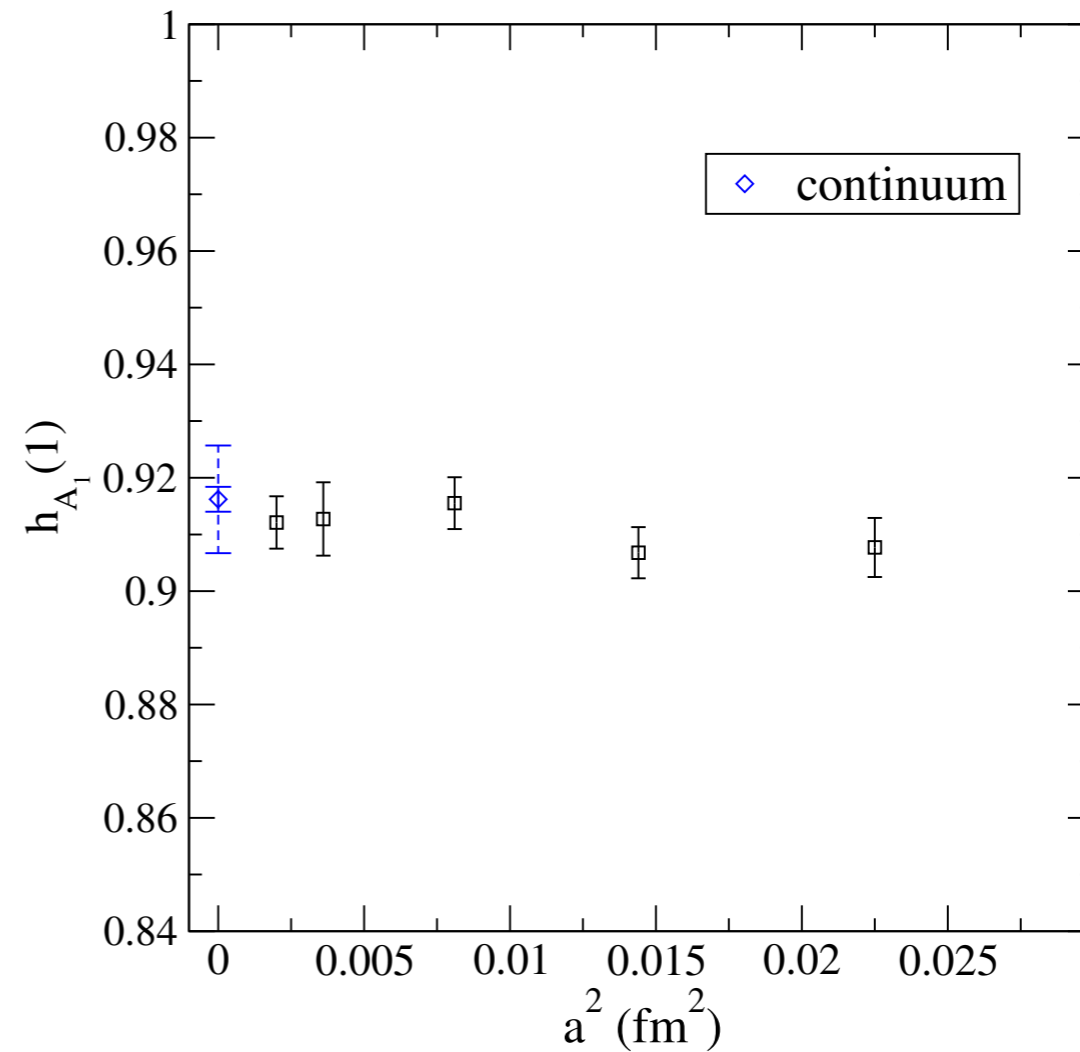
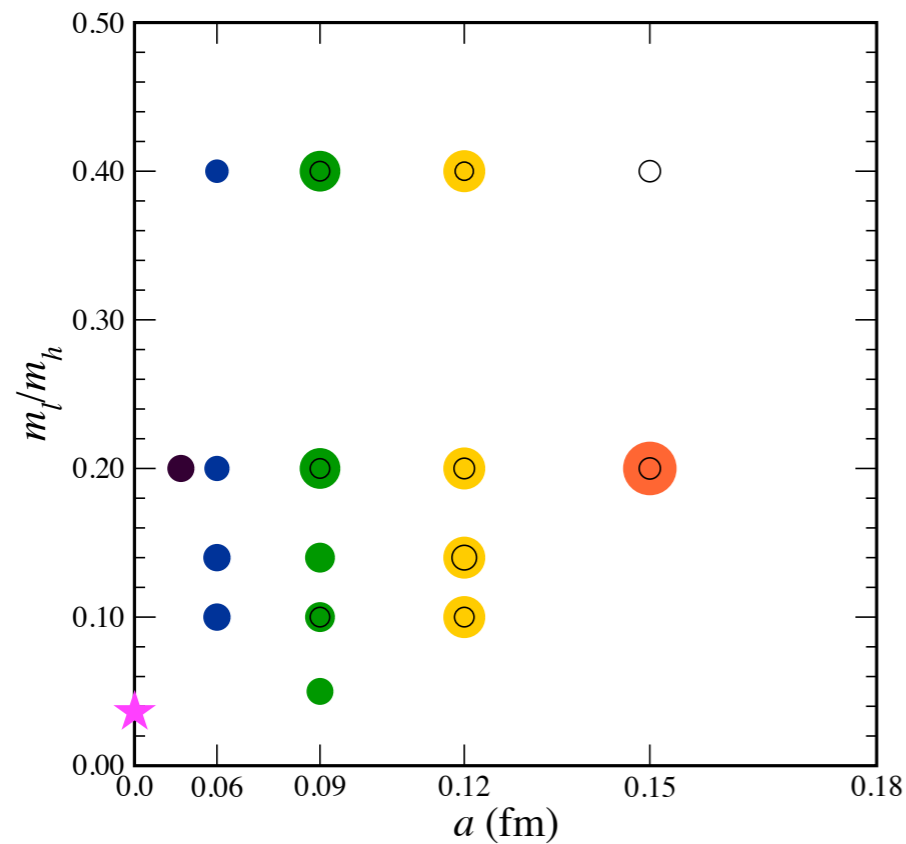
❖ FNAL/MILC 2014 (J. Bailey et al, arXiv:1403.0635, 2014 PRD):



- 15 MILC asqtad ensembles
5 lattice spacings
~ 4 sea quark masses per lattice spacing
~ 600 - 2000 configurations
× 4 time-sources per ensemble
- asqtad light valence quarks
- Fermilab b quarks
- $O(a)$ improved current
- mostly nonperturbative renormalization (mNPR)

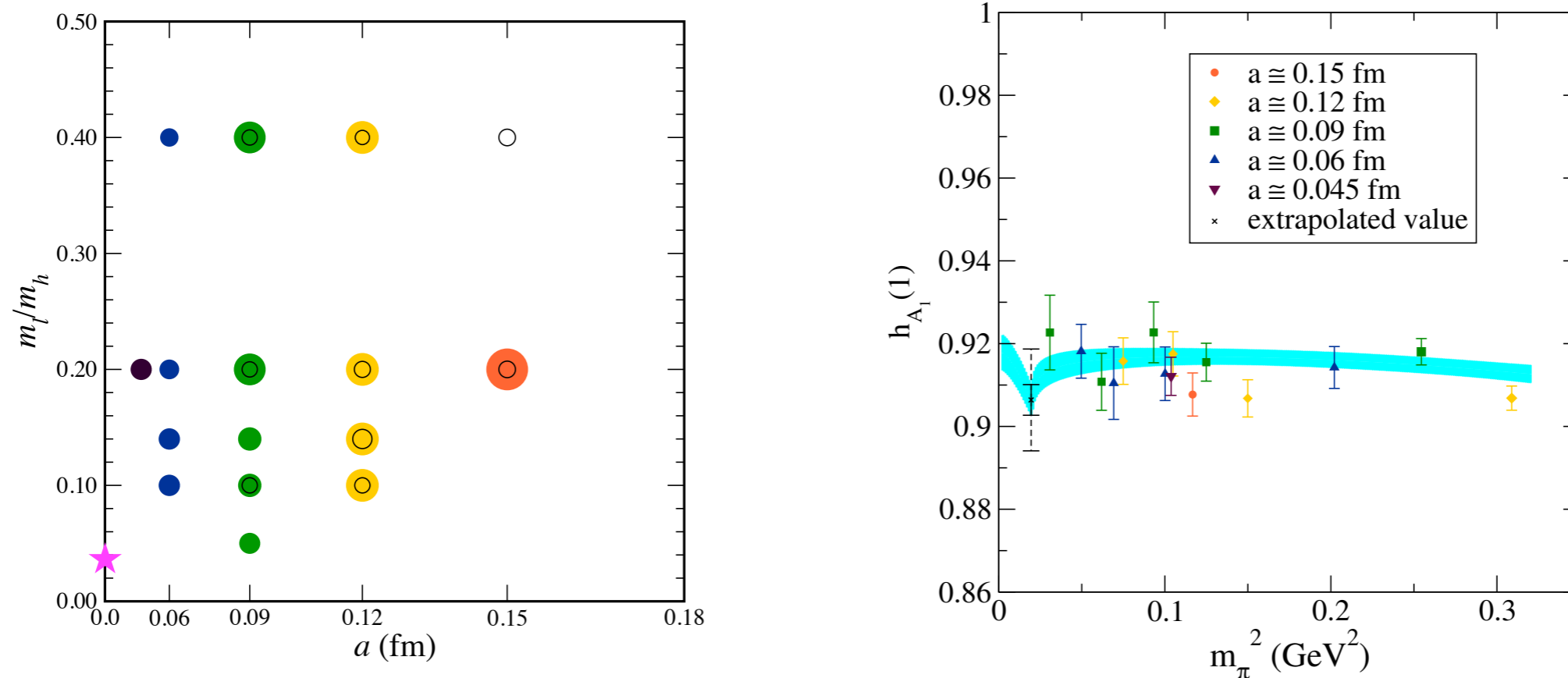
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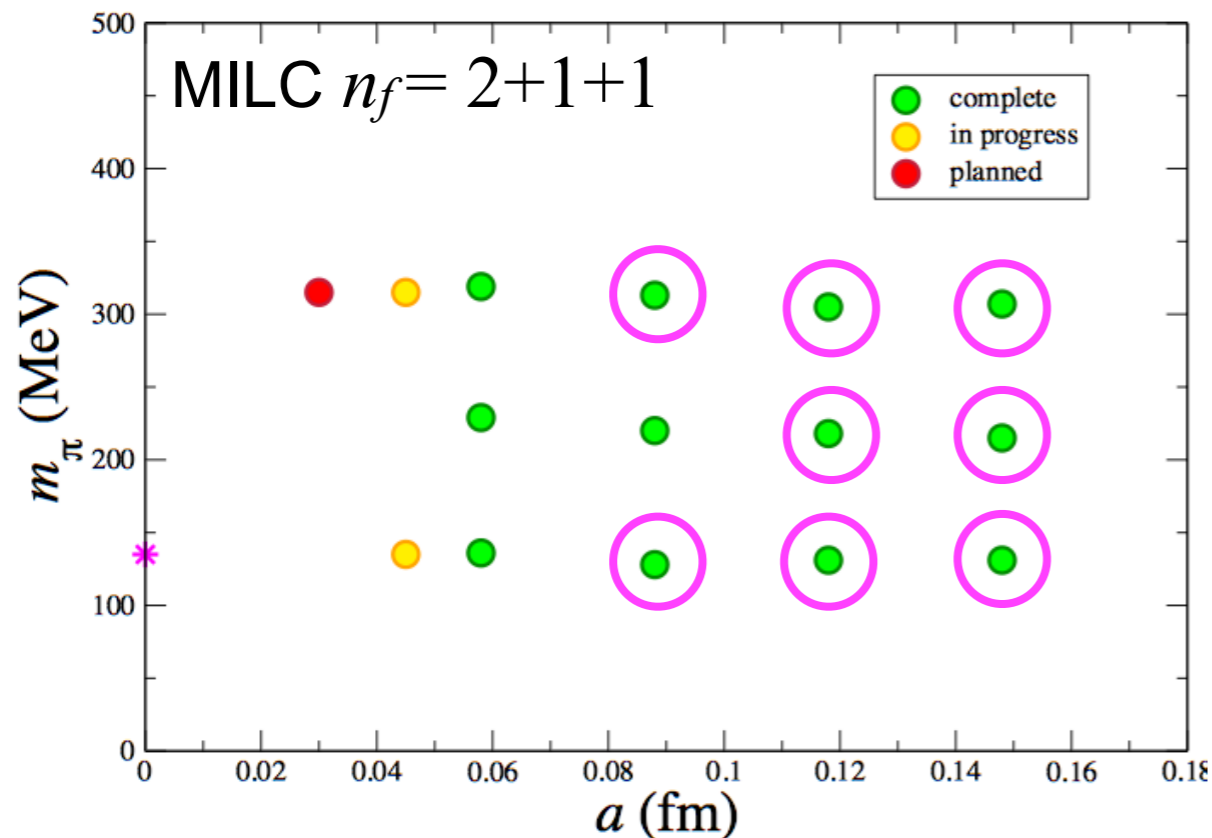
❖ FNAL/MILC 2014 (J. Bailey et al, arXiv:1403.0635, 2014 PRD):



- ❖ combined chiral-continuum extrapolation
- ❖ cusp due to $D^* \rightarrow D\pi$ and $m_{D^*} - m_D \sim m_\pi$
- ❖ included using ChPT with $D^*D\pi$ coupling as input.

Form factor for $B \rightarrow D^*$ at zero recoil

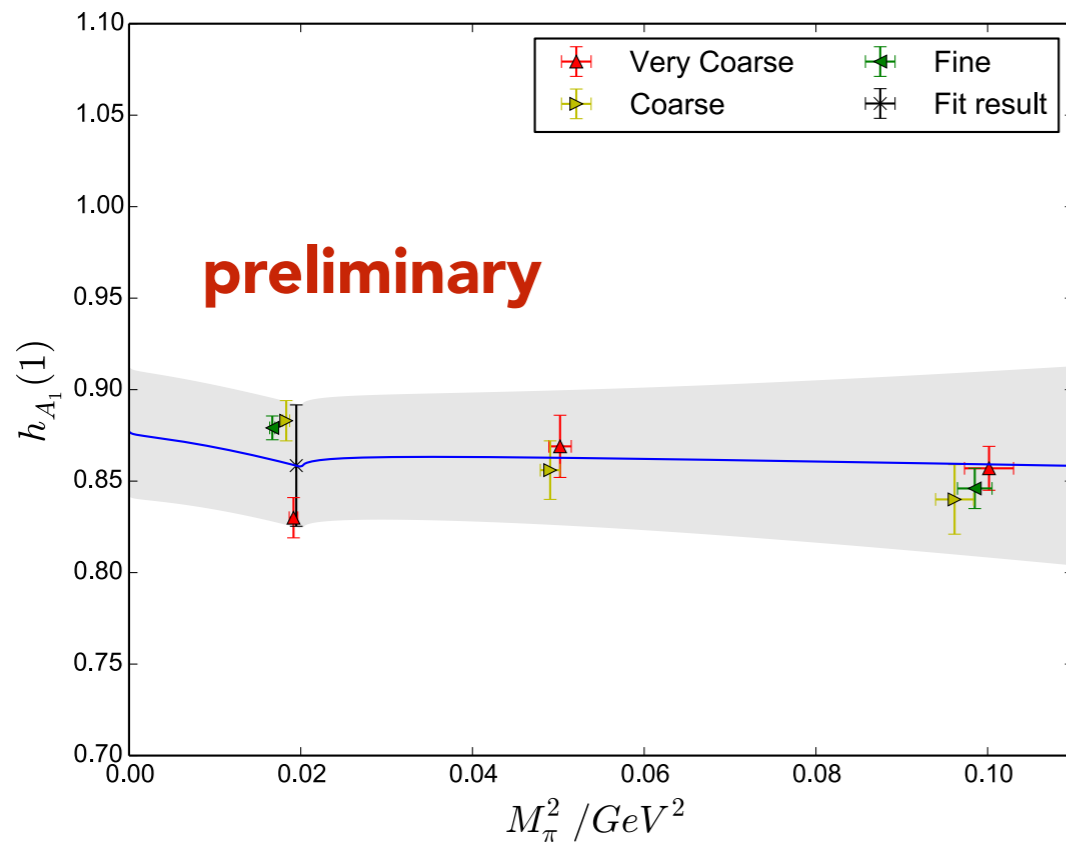
❖ HPQCD (J. Harrison @ Lattice 2016, **preliminary**):



- 8 MILC HISQ ensembles
3 lattice spacings
~ 3 sea quark masses per lattice spacing including one each at physical mass
- HISQ light valence quarks
- NRQCD b quarks
- $O(a)$ improved current
- 1-loop perturbative renormalization is the dominant source of error
- systematic error analysis in progress

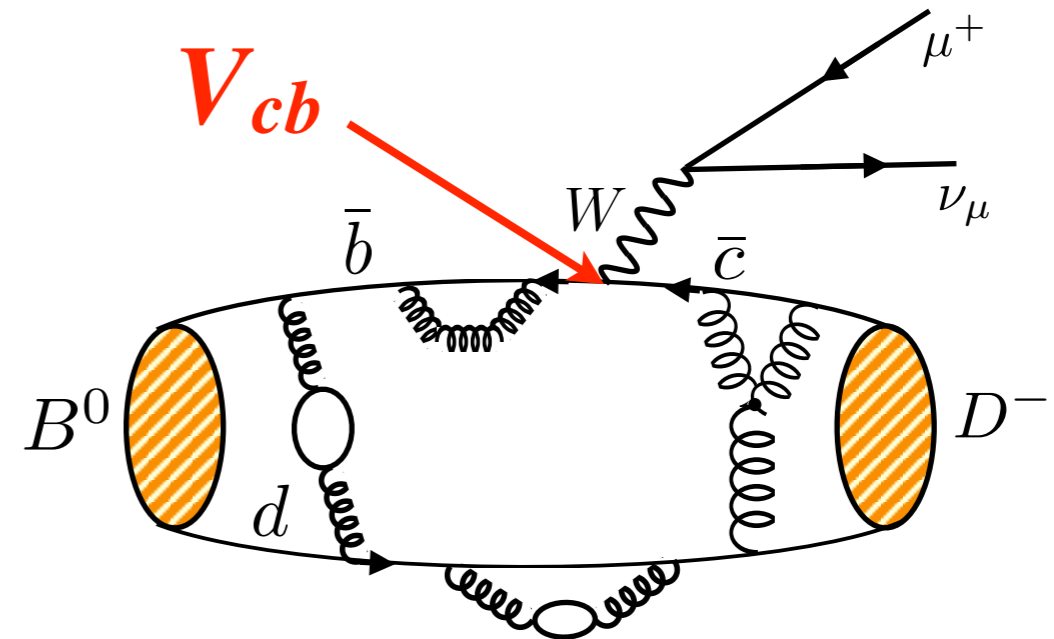
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Form factors for $B \rightarrow D \ell \nu$, ($\ell = e, \mu, \tau$)

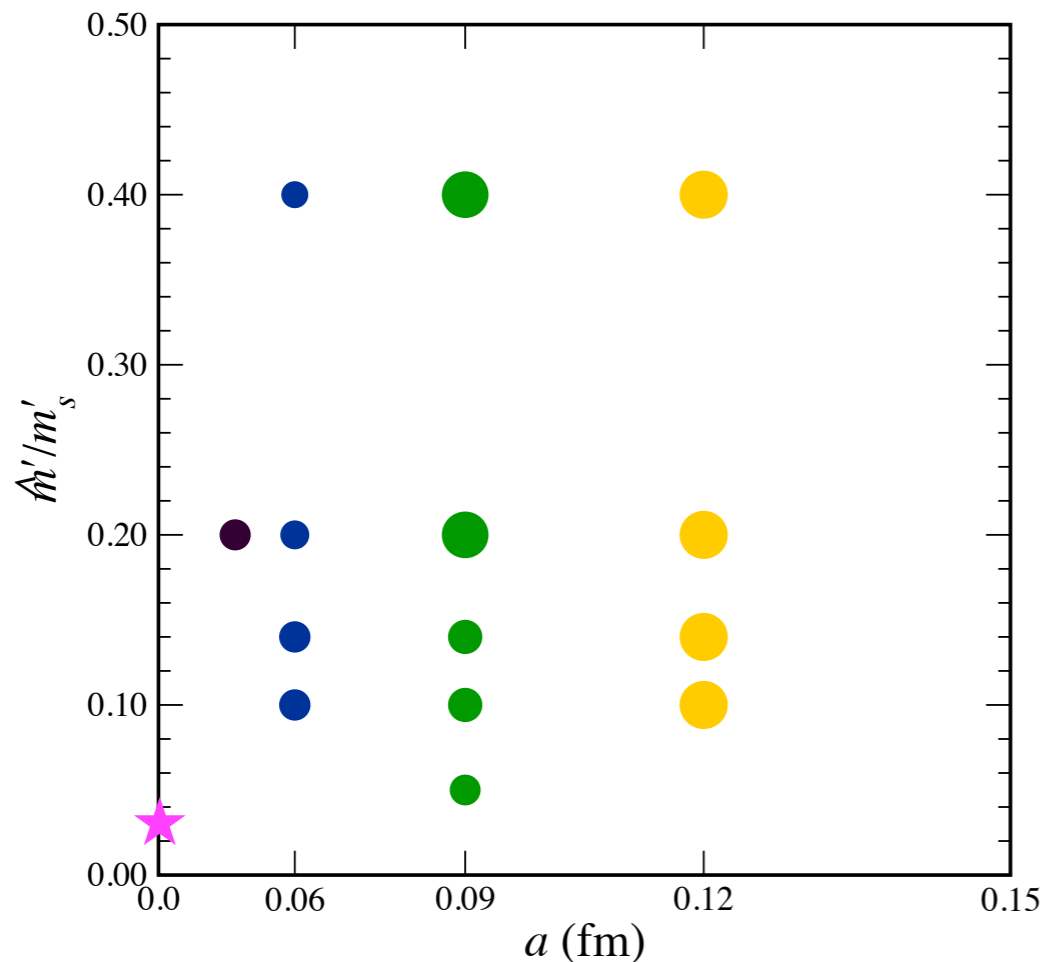


$$\frac{d\Gamma(B \rightarrow D \mu \nu)}{dq^2} = (\text{known}) \times |V_{cb}|^2 \times f_+^2(q^2)$$

- ★ calculate the form factors in the low recoil energy (high q^2) range.
- ★ use **z-expansion** for model-independent parameterization of q^2 dependence.
- ★ calculate the complete set of form factors, $f_+(q^2)$, $f_0(q^2)$.
- ★ for $f_+(q^2)$ compare shape between experiment and lattice.

Form factors for $B \rightarrow D \ell \nu$, ($\ell = e, \mu, \tau$)

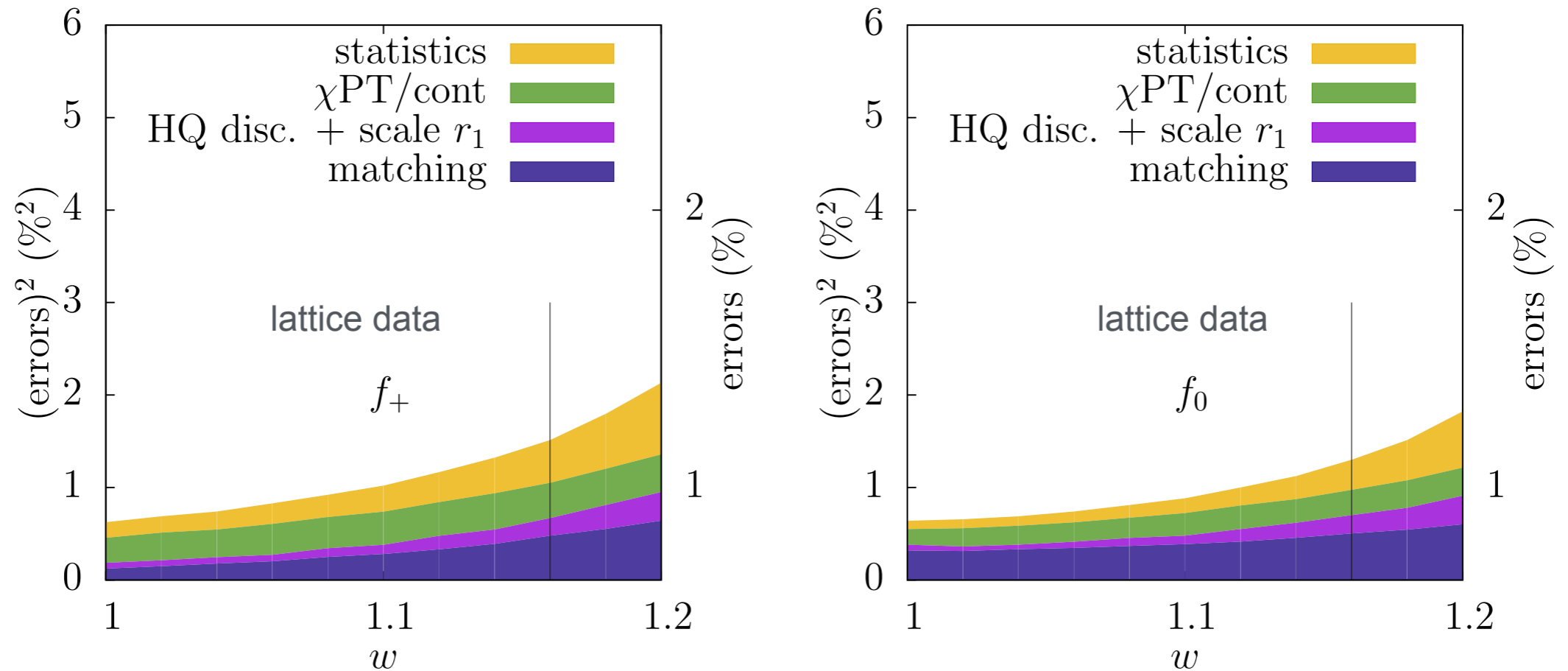
FNAL/MILC (arXiv:1503.07237, PRD 2015)



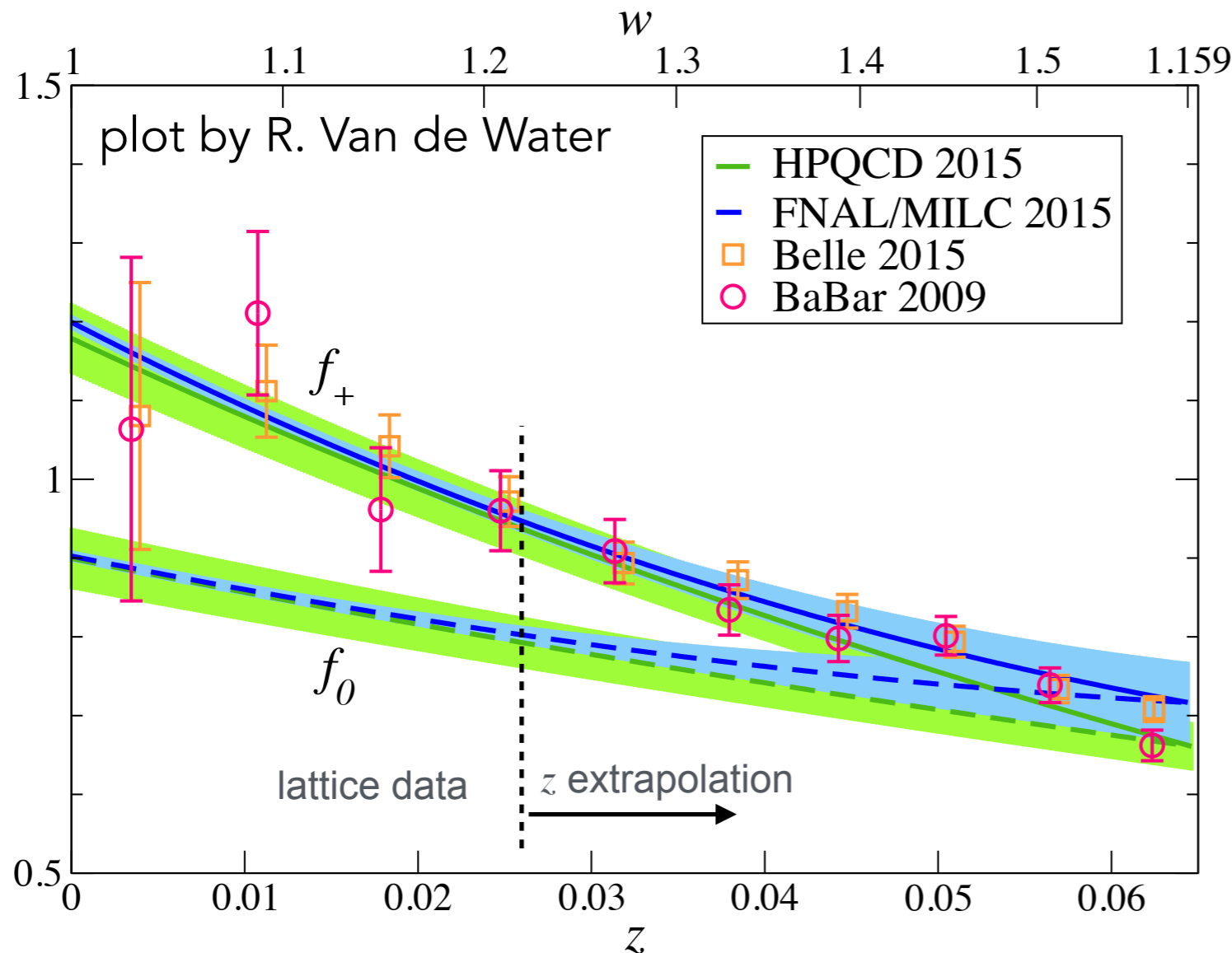
- 14 MILC asqtad ensembles
4 lattice spacings
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Form factors for $B \rightarrow D \ell \nu$, ($\ell = e, \mu, \tau$)

FNAL/MILC (arXiv:1503.07237, PRD 2015)



Form factors for $B \rightarrow D \ell \nu$, ($\ell = e, \mu, \tau$)



HPQCD (arXiv:1505.03925, PRD 2015)

FNAL/MILC (arXiv:1503.07237, PRD 2015)

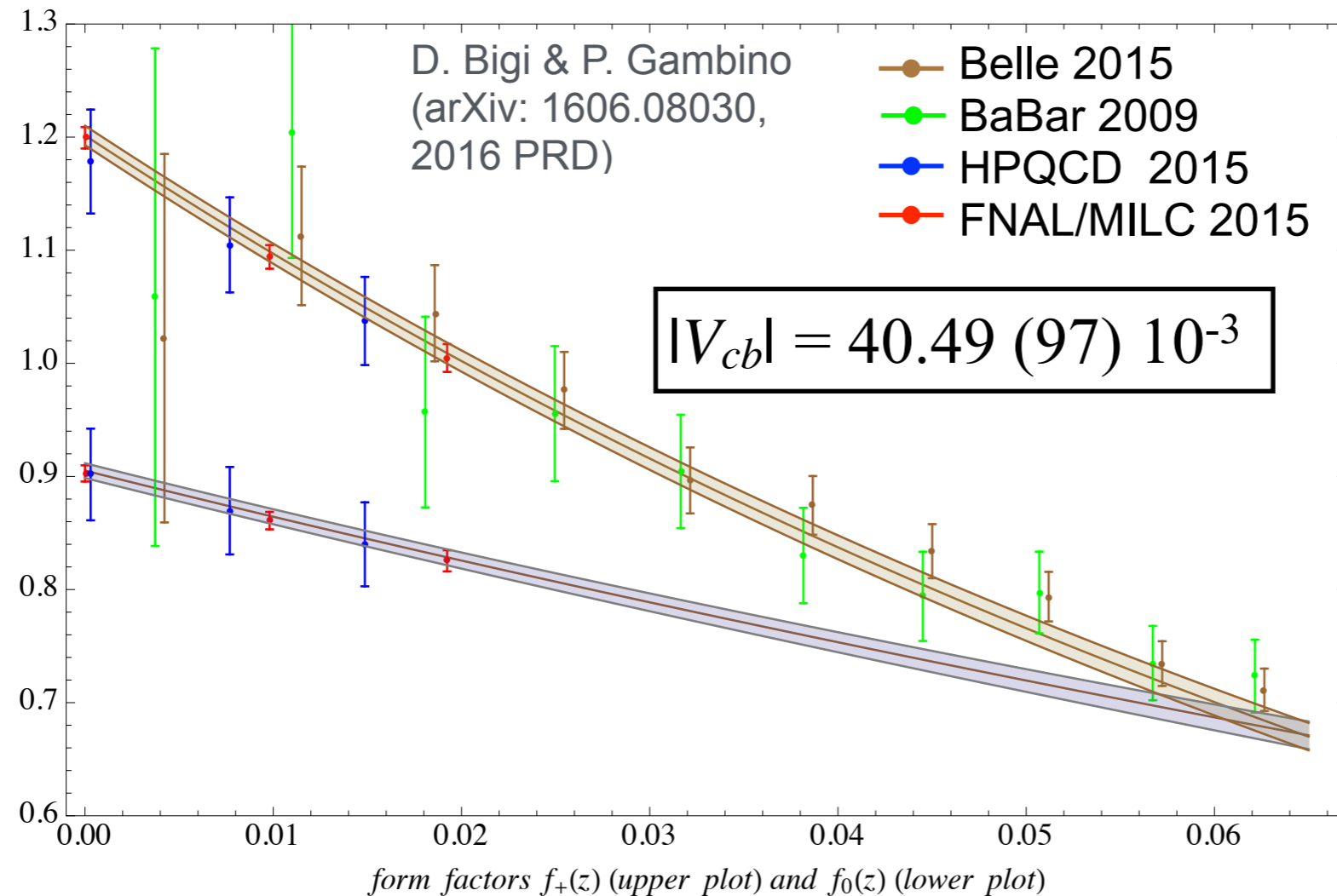
- ★ Two LQCD calculations (FNAL/MILC, HPQCD)
- ★ HPQCD uses 5 MILC ensembles and different valence quark actions
- ★ LQCD form factor uncertainties ($\sim 1.2\%$) smaller than experiment.

★ LQCD form factors can be used to calculate the CKM free ratio:

$$R(D) \equiv \frac{\mathcal{B}(B \rightarrow D\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu)}$$

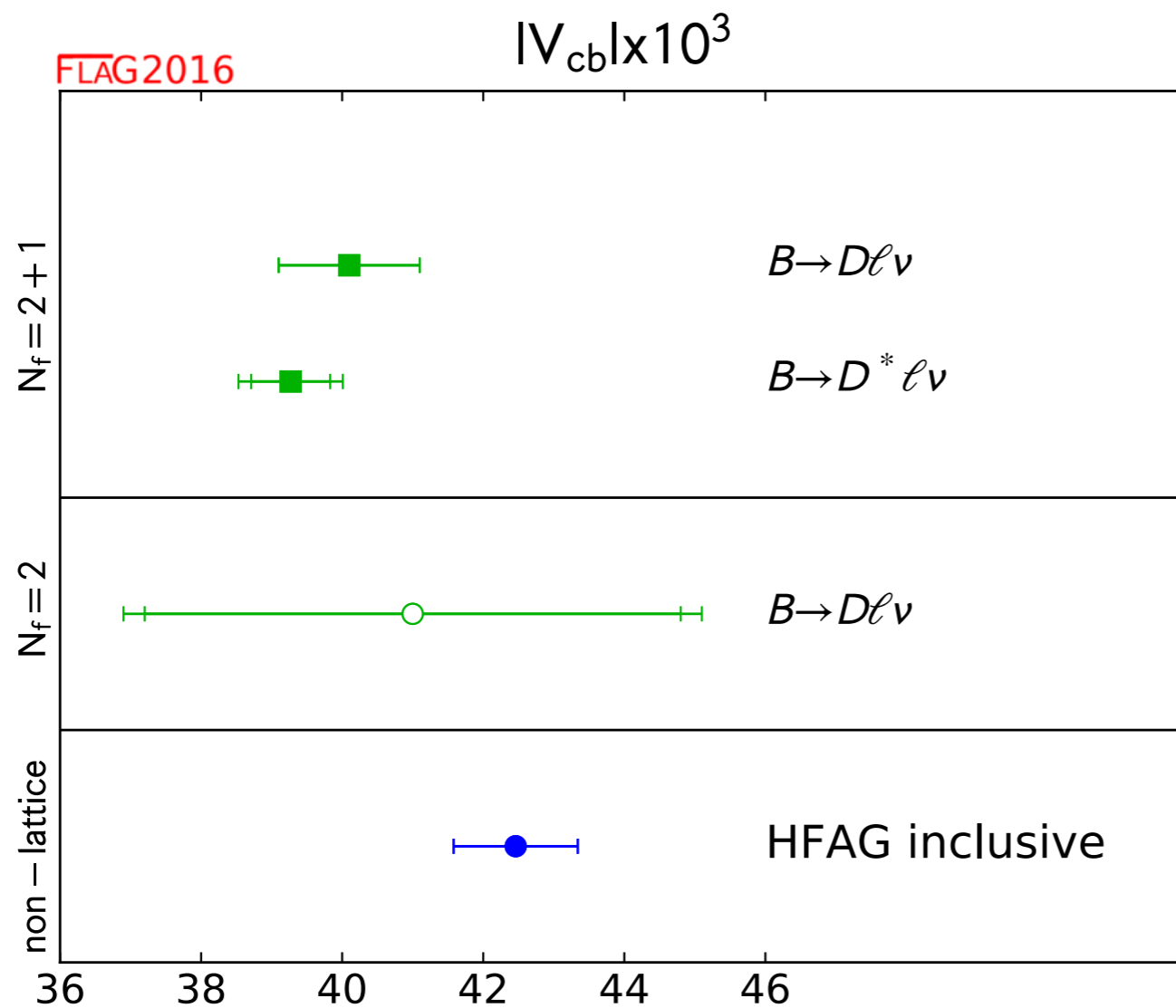
$B \rightarrow D \ell \nu$ & $|V_{cb}|$

- ★ combine LQCD form factors with experiment, using the BGL (Boyd, Grinstein, Lebed, hep-ph/9508211, 1996 NPB) parameterization:

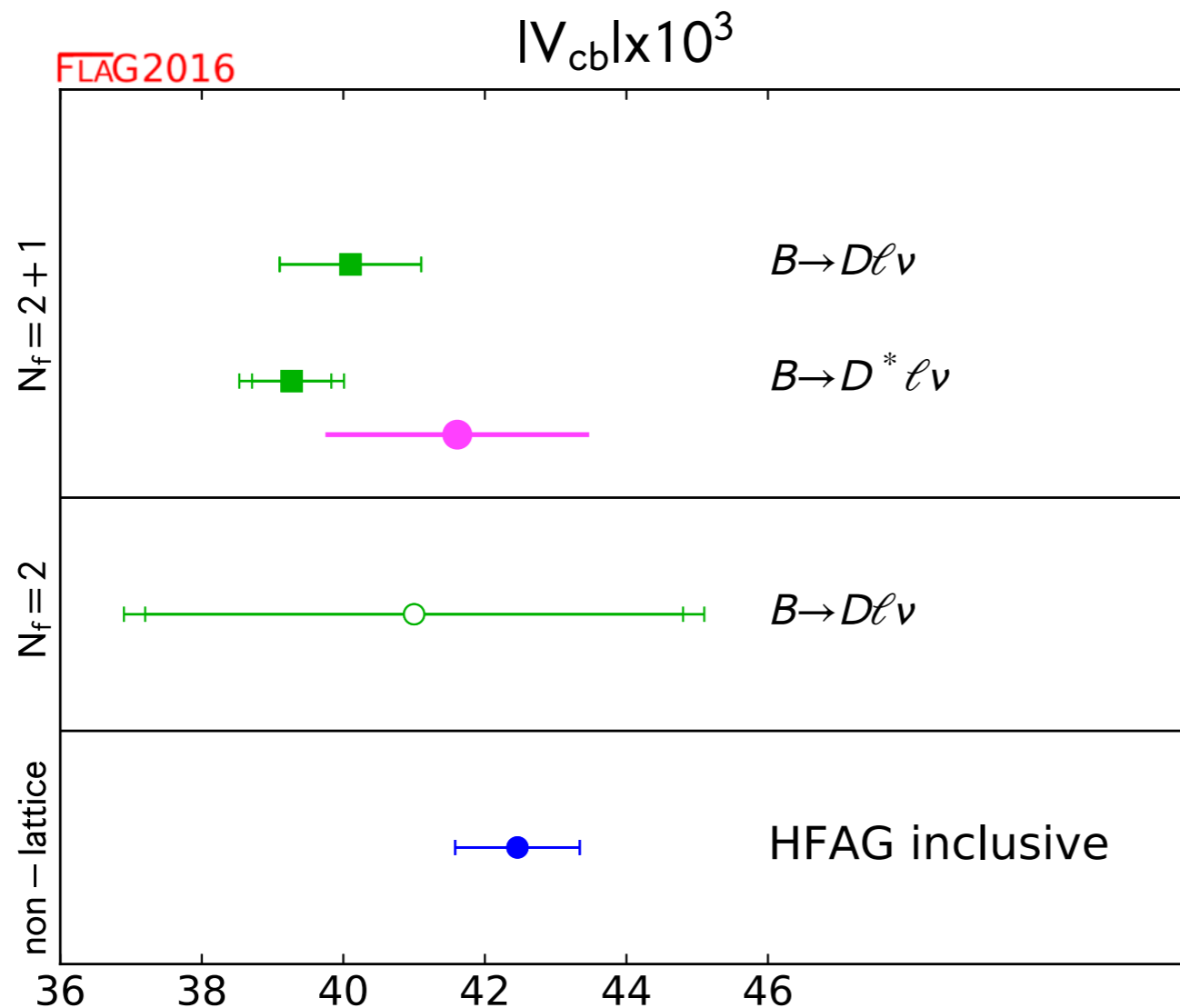


- ★ FLAG-3 (S. Aoki et al, arXiv:1607.00299, EPJC 2017) performs a similar combined fit using the BCL parameterization.

Implications for $|V_{cb}|$



Implications for $|V_{cb}|$



Two new analyses:

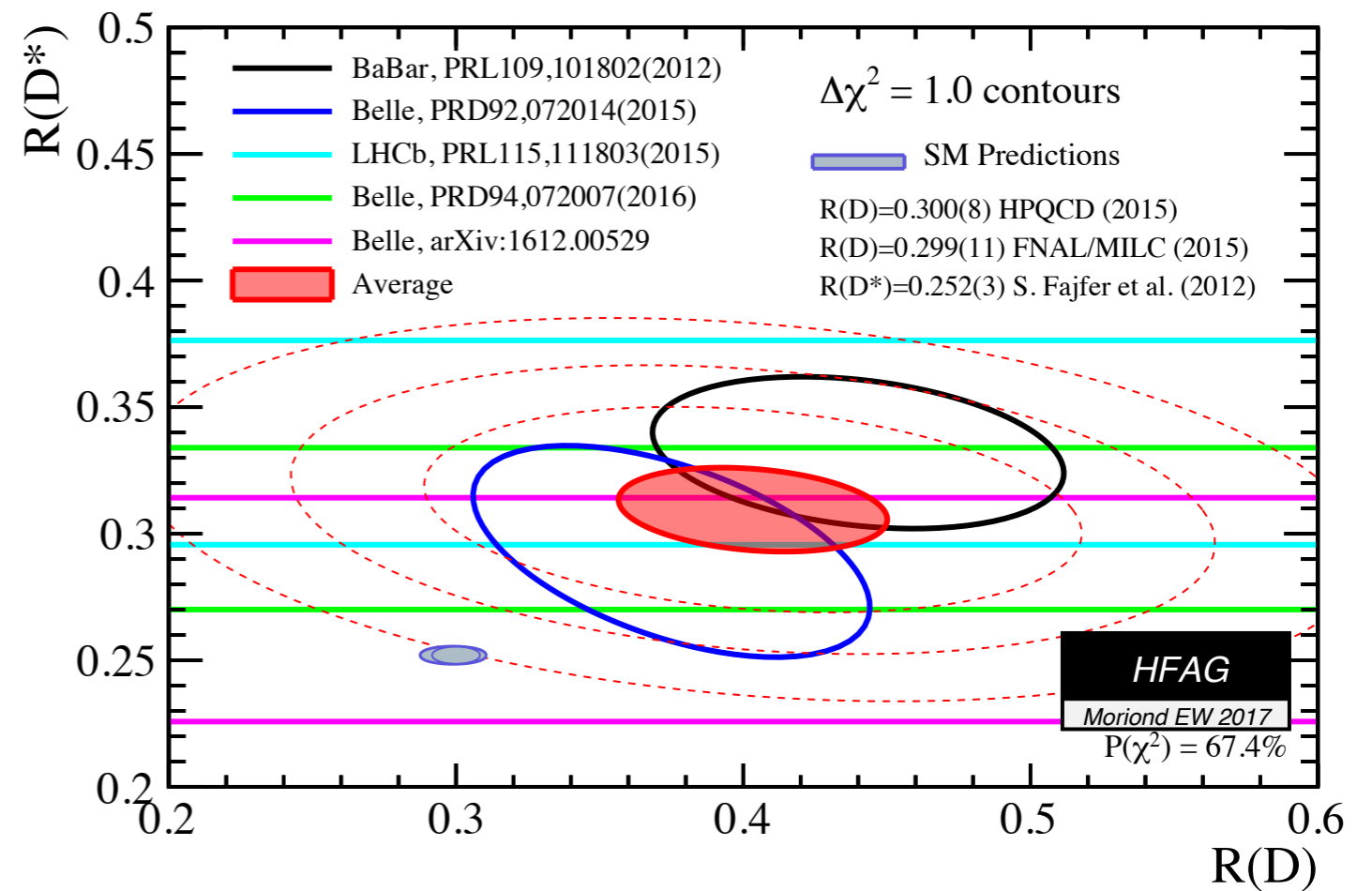
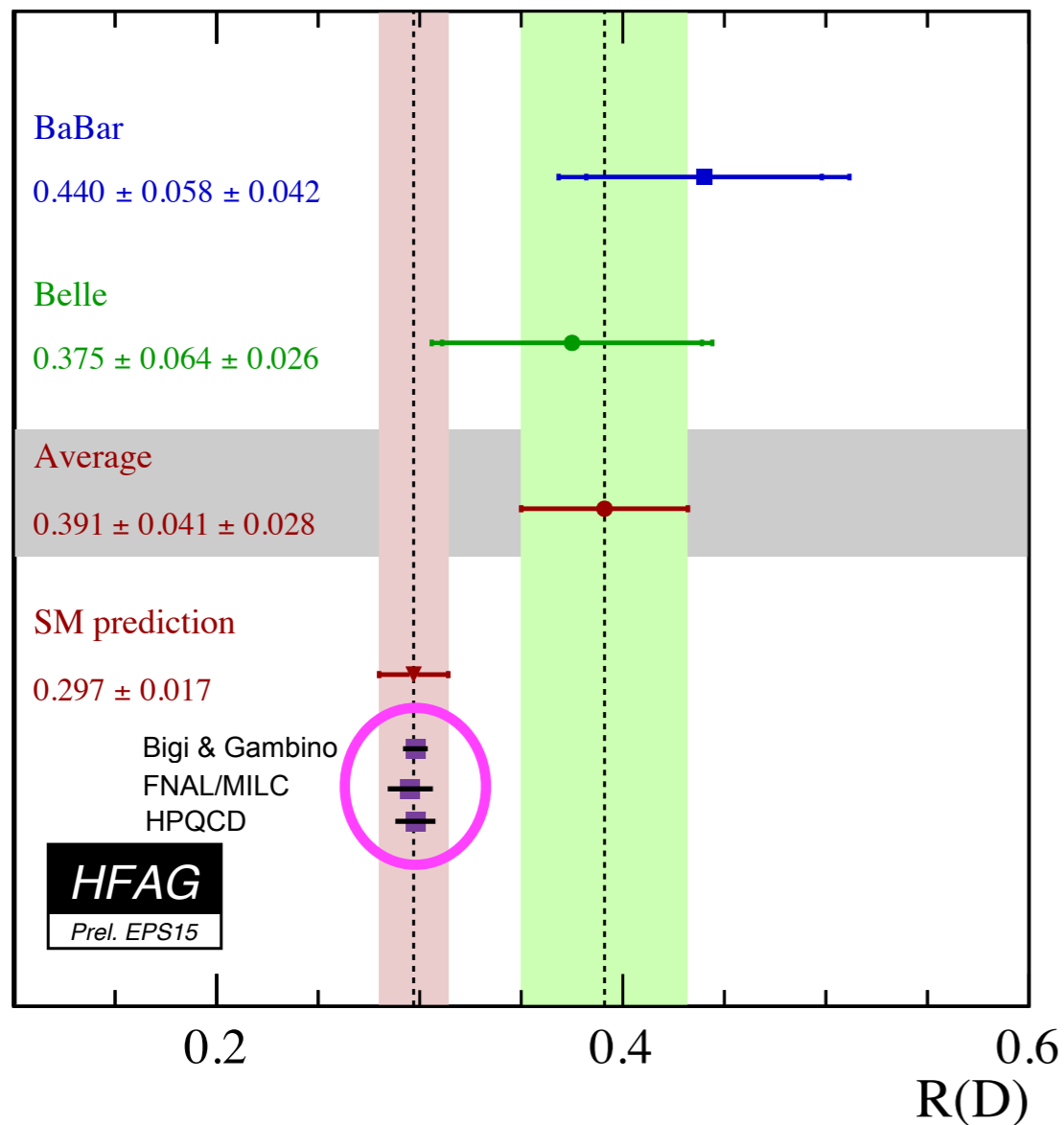
- Bigi, Gambino, Schacht (arXiv:1703.06124)
- Grinstein, Kobach (arXiv:1703.08170)

Both use new Belle data (arXiv:1702.01521) and BGL together with lattice $\mathcal{F}(1)$.

BSM phenomenology: LFU τ/ℓ

$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu)}$$

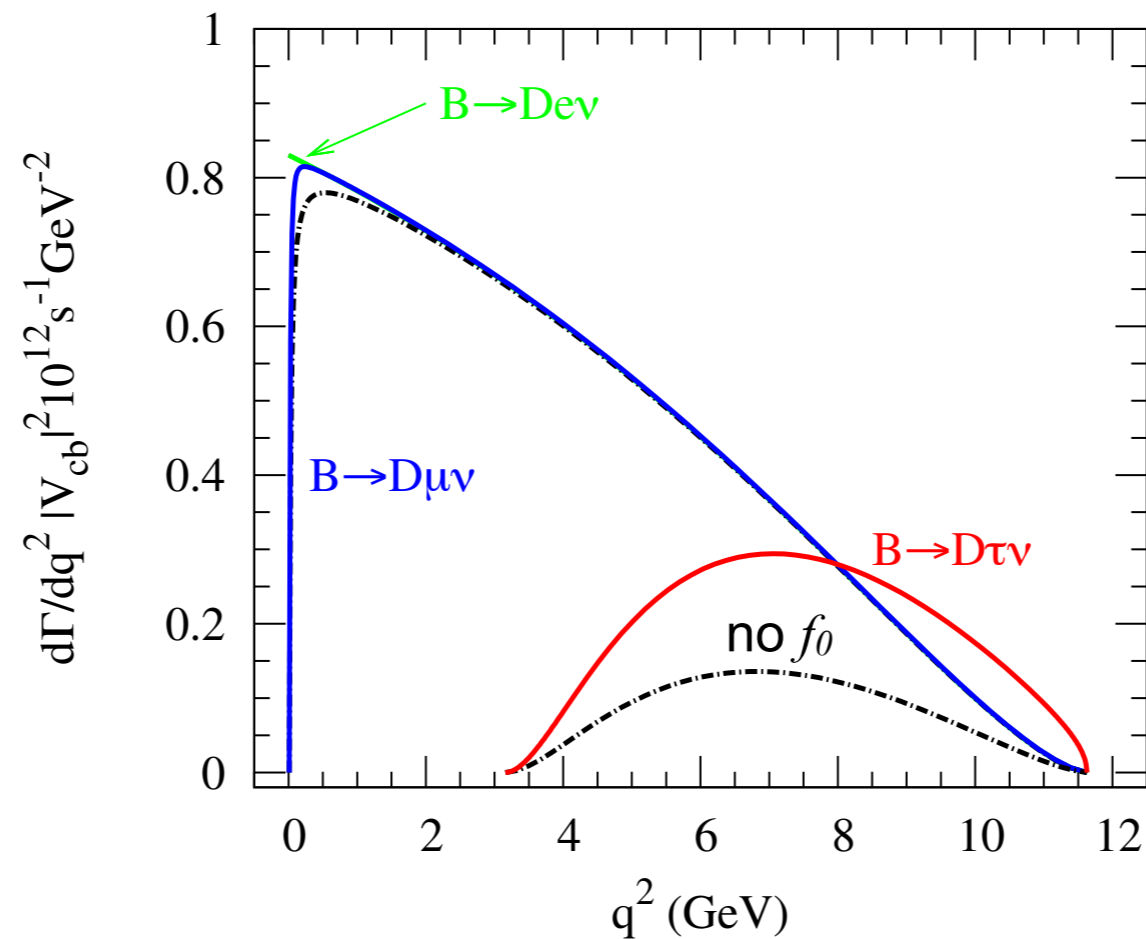
HFAG average for EPS 2015



HFAG 2017 average: combined 3.9σ excess

BSM phenomenology: LFU τ/ℓ

D. Du et al (arXiv:1510.02349, PRD 2016)

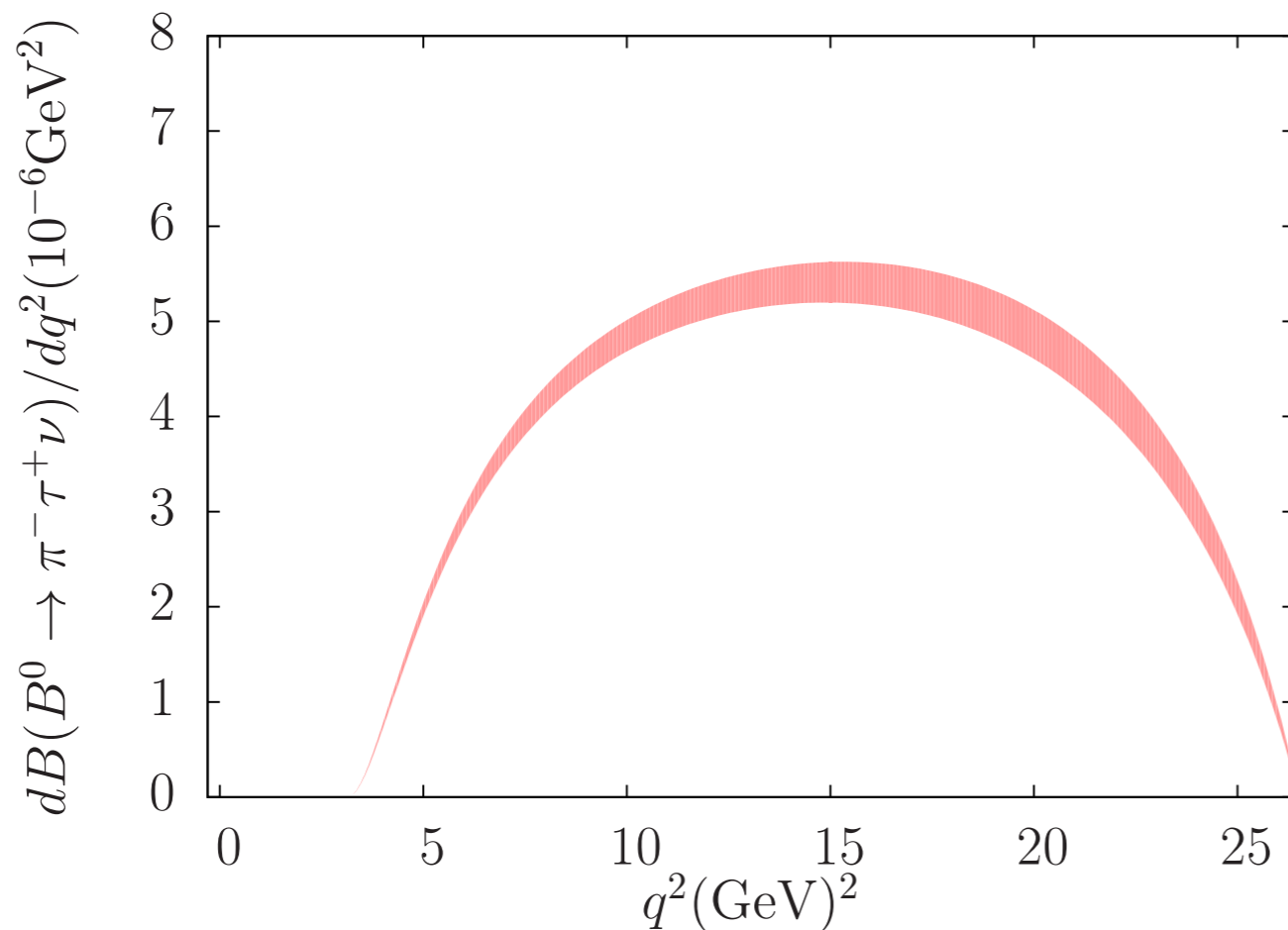


- The shape of the $B \rightarrow D^{(*)} \tau\nu$ rate is sensitive to f_0 contribution.
- Shape comparison: use (ratios of) differential or binned decay rates to compare theory and experiment.

BSM phenomenology: LFU τ/ℓ

D. Du et al (arXiv:1510.02349, PRD 2016)

$$\text{SM prediction for } R(\pi) = \frac{\mathcal{B}(B \rightarrow \pi \tau \nu_\tau)}{\mathcal{B}(B \rightarrow \pi \ell \nu)} = 0.641(17)$$



Uses the form factors from the combined LQCD + exp. fit to $d\mathcal{B}(B \rightarrow \pi \ell \nu) / dq^2$

Prospects for $B_{(s)} \rightarrow D_{(s)}$ form factors at all recoil

ongoing/planned LQCD calculations:

★ **RBC/UKQCD:**

$B \rightarrow D$ and $B_s \rightarrow D_s$ form factors

RHQ action for b quark, DWF charm on DWF (2+1) ensembles

preliminary results presented at Lattice 2016 (Witzel)

★ **HQCD:**

$B_s \rightarrow D_s$ form factors

NRQCD b quarks, HISQ charm on MILC asqtad (2+1) ensembles

preliminary results presented at Lattice 2016 (Monahan)

★ **FNAL/MILC:**

$B \rightarrow D$ and $B_s \rightarrow D_s$ form factors

Fermilab b, c quarks on MILC HISQ (2+1+1) ensembles (full set)

★ **LANL/SNU:**

$B \rightarrow D$ form factors

Oktay-Kronfeld b, c quarks on MILC HISQ (2+1+1) ensembles

First tests of discretization errors with OK action at Lattice 2016

Prospects for $B_{(s)} \rightarrow D_{(s)}^*$ form factors at all recoil

ongoing/planned LQCD calculations:

★ **HPQCD:**

NRQCD b quarks, HISQ charm on MILC HISQ (2+1+1) ensembles

★ **FNAL/MILC:**

1) Fermilab b, c quarks on MILC asqtad (2+1) ensembles (full set)

2) Fermilab b, c quarks on MILC HISQ (2+1+1) ensembles (full set)

★ **LANL/SNU:**

Oktay-Kronfeld b, c quarks on MILC HISQ (2+1+1) ensembles

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RHQ action for b quark, DWF charm on DWF (2+1) ensembles

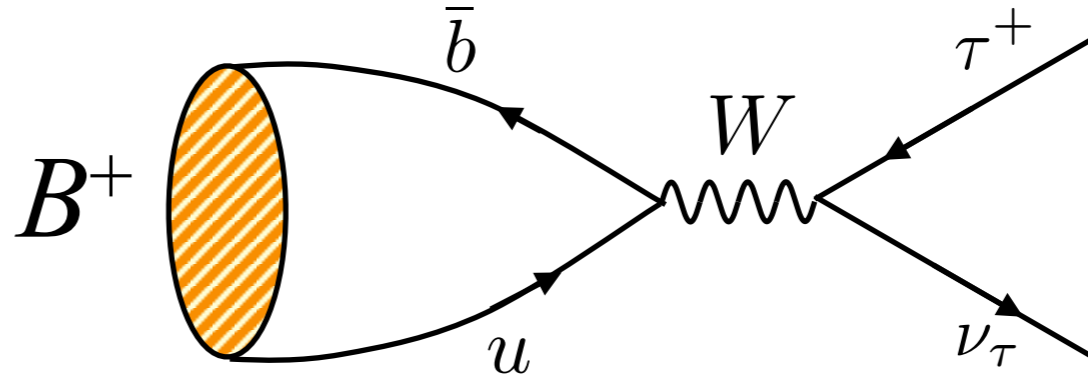
Combine binned experimental decay distributions (Belle, arXiv: 1702.01521) with LQCD form factors to extract $|V_{cb}|$ and obtain improved form factors to be used for SM predictions of $R(D^{(*)})$.

Electroweak corrections, η_{EW}

- ❖ It includes a log from $W/Z/\gamma$ boxes (Sirlin, 1982): $\eta_{EW} = 1 + \frac{\alpha}{\pi} \ln \frac{m_Z}{\mu}$
- ❖ In B^0 decay, there is a long-distance (universal) radiative correction due to Coulomb attraction between the final states: $\pi\alpha/2$
- ❖ Structure-dependent radiative corrections have not yet been calculated.

Leptonic B -meson decay

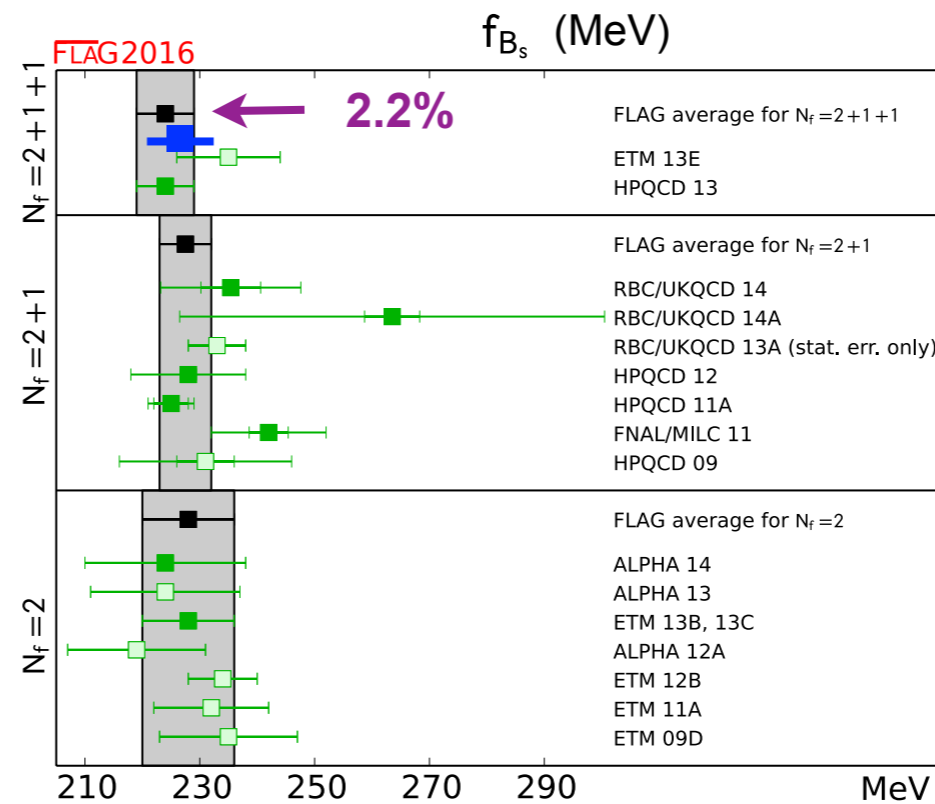
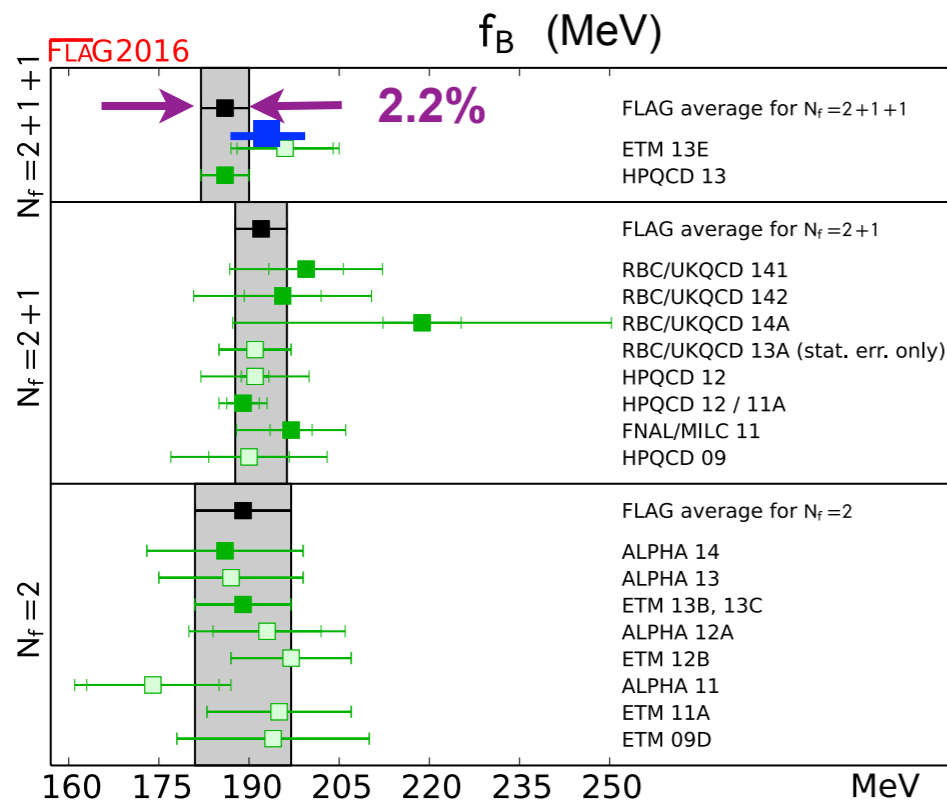
Example: $B^+ \rightarrow \tau^+ \nu_\tau$



$$\Gamma(B^+ \rightarrow \tau^+ \nu_\tau) = (\text{known}) \times |V_{ub}|^2 f_B^2$$

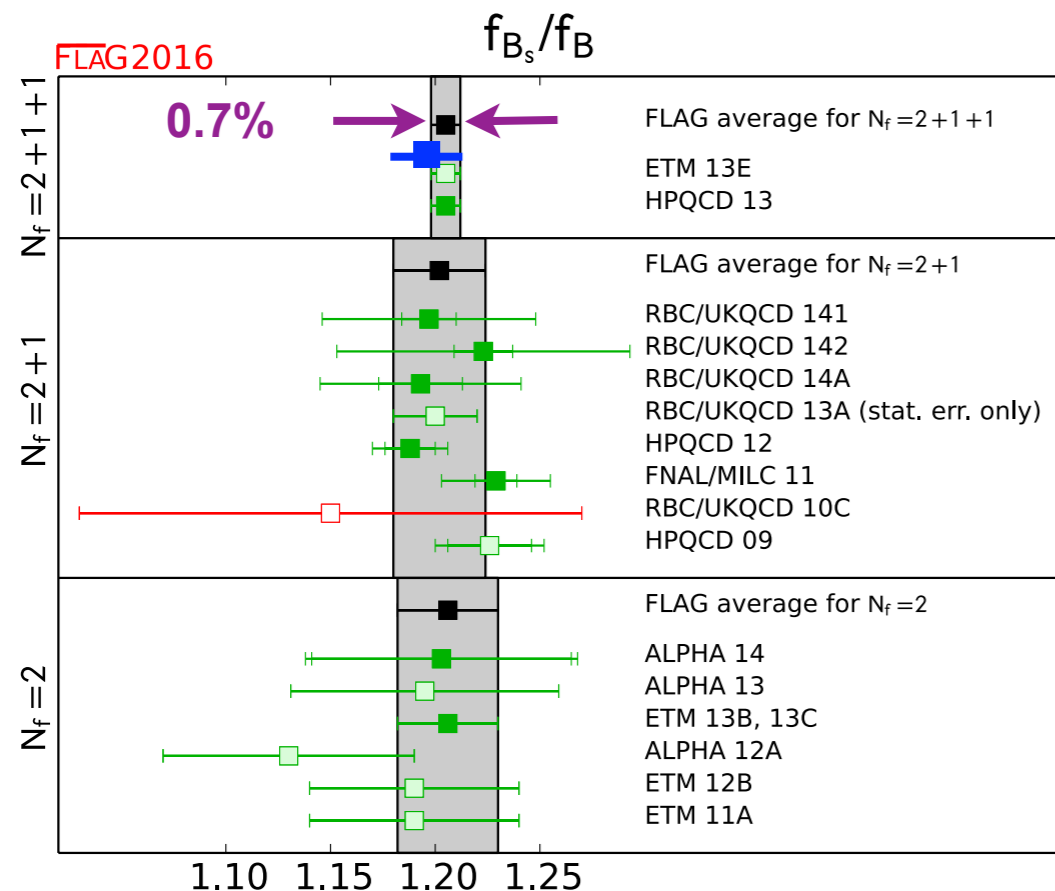
- use experiment + LQCD input for determination of CKM element or to search for new physics.
- SU(3) **ratio** f_{B_s}/f_{B_d} : statistical and systematic errors tend to cancel.
- Decay constants are also needed for rare leptonic decay, $B_{s(d)} \rightarrow \mu\mu$.

B decay constant summary



S. Aoki et al
(FLAG-3 review, arXiv:
1607.00299, EPJC 2017)

status
end 2015



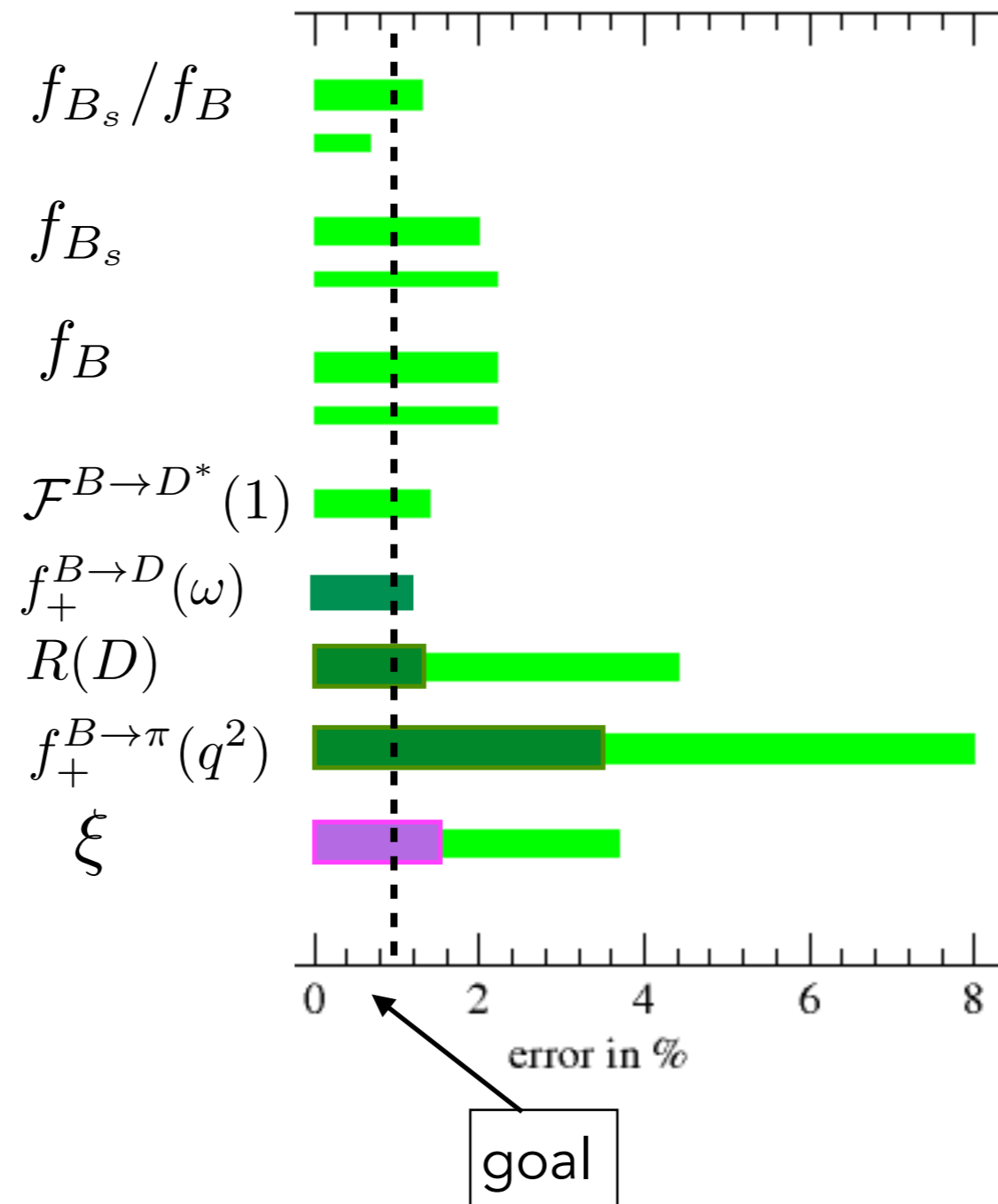
◆ new results by ETM (arXiv:1603.04306, 2016 PRD)

◆ ongoing work by
FNAL/MILC (Komijani @ Lattice 2016),
RBC/UKQCD, ...

➡ expect to reduce errors on f_B, f_{B_s} to $\approx 1\%$

B-meson summary

errors (in %) FLAG-2/3 averages + new results



Summary

- ★ LQCD results exist for $B \rightarrow D$ form factors at all recoil and $B \rightarrow D^*$ form factor at zero recoil with errors that are commensurate with experimental uncertainties.
- ★ Expect to see new LQCD results for $B_{(s)} \rightarrow D_{(s)}$ and $B_{(s)} \rightarrow D_{(s)}^*$ form factors at all recoil at Lattice 2017.
 - may affect the tension between exclusive and inclusive determinations of IV_{cb} .
 - will enable an improved SM estimate of $R(D^*)$.
- ★ For B decays to $D^{(*)}\tau\nu$ final states, shape comparison between theory and experiment would be useful.
- ★ LQCD (or combined lattice +exp) form factors can also be used to obtain the predictions for $R(D^{(*)})$ and other observables from BSM theories.
- ★ expect LQCD results for B -meson decay constants at 1% level soon.

Outlook



Amala Willenbrock

Outlook

Further improvements

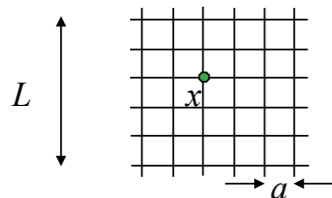
- ★ Gauge field ensembles with light sea quarks at their **physical masses** are being used in a growing number of LQCD calculations.
- ★ will need to include
 - ◆ structure-dependent QED effects
 - program being developed for kaon quantities, muon $g-2$
- ★ Include effects of $D^* \rightarrow D\pi$ directly in the LQCD calculation. Theoretical framework for semileptonic B decays to vector meson final states under development (Briceño et al, arXiv:1406.5965, 2015 PRD; Agadjanov et al, arXiv:1605.03386).
 - LQCD pilot studies are underway for $B_s \rightarrow K^* l\nu$, $B \rightarrow K^* ll, \dots$



ありがとうございます

Thank you!

Backup slides



Lattice QCD Introduction

$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \mathcal{O}(\psi, \bar{\psi}, A) e^{-S} \quad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F_{\mu\nu}^a)^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, \mathcal{O} , are then written in terms of $(\not{D} + m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D} + m) e^{-S}$
2. calculate quark propagators, $(\not{D} + m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, from correlation functions

5. systematic error analysis

Heavy Quark Treatment

- For light quarks ($m_\ell < \Lambda_{\text{QCD}}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{\text{QCD}})^n$
- For heavy quarks, leading discretization errors $\sim \alpha_s^k (am_h)^n$
with currently available lattice spacings
 - for b quarks $am_b > 1$
 - for charm $am_c \sim 0.15-0.6$

⇒ need effective field theory methods for b quarks
for charm can use light quark methods, if action is sufficiently improved

- avoid errors of $(am_b)^n$ in the action by using EFT:
 - ◆ relativistic HQ actions (Fermilab, Columbia [aka RHQ], Tsukuba)
 - ◆ HQET
 - ◆ NRQCD

or

- use improved light quark actions for charm (HISQ, tmWilson, NP imp. Wilson,...)
and for b :
 - ◆ use same LQ action as for charm but keep $am_h < 1$,
 - ◆ use HQET and/or static limit to extrapolate/interpolate to b quark mass

chiral-continuum extrapolation

Some ensembles still have

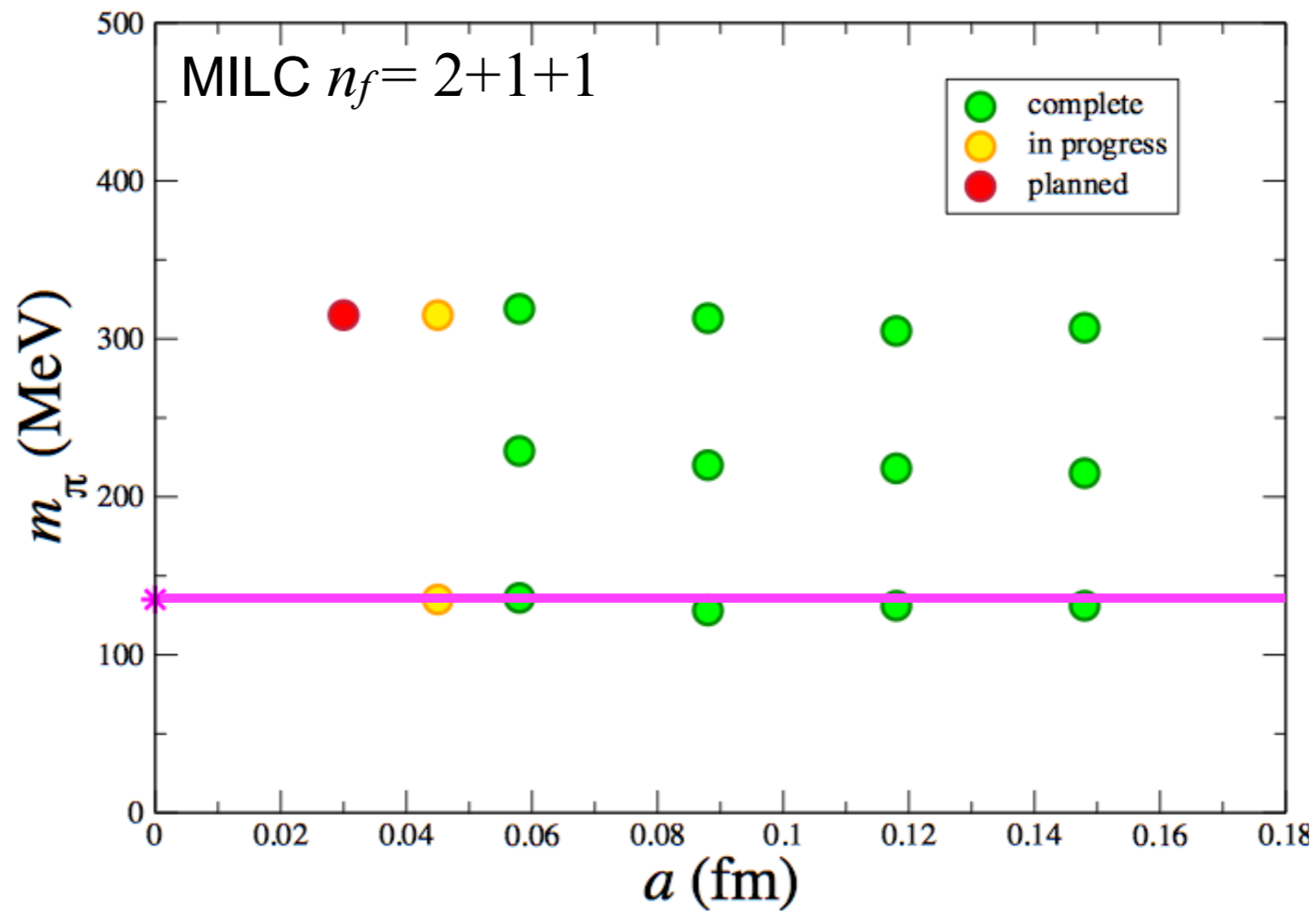
$$m_{\text{light}} > 1/2 (m_u + m_d)_{\text{phys}}$$

χ^{PT} guides the extrapolation/interpolation to the physical point.

- include (light quark) discretization effects (for example, staggered χ^{PT})
- can also add HQ discretization terms to chiral-continuum fits
- combined chiral-continuum extrapolation/interpolation
- for B, D meson processes use Heavy Meson χ^{PT} : $\chi^{\text{PT}} + 1/M$ expansion

chiral-continuum extrapolation

Example: Set of ensembles by MILC collaboration



Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses: [PACS-CS](#), [BMW](#), [MILC](#), [RBC/UKQCD](#), [ETM](#)

finite volume effects

One stable hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_\pi L}$

• keep $m_\pi L \gtrsim 4$

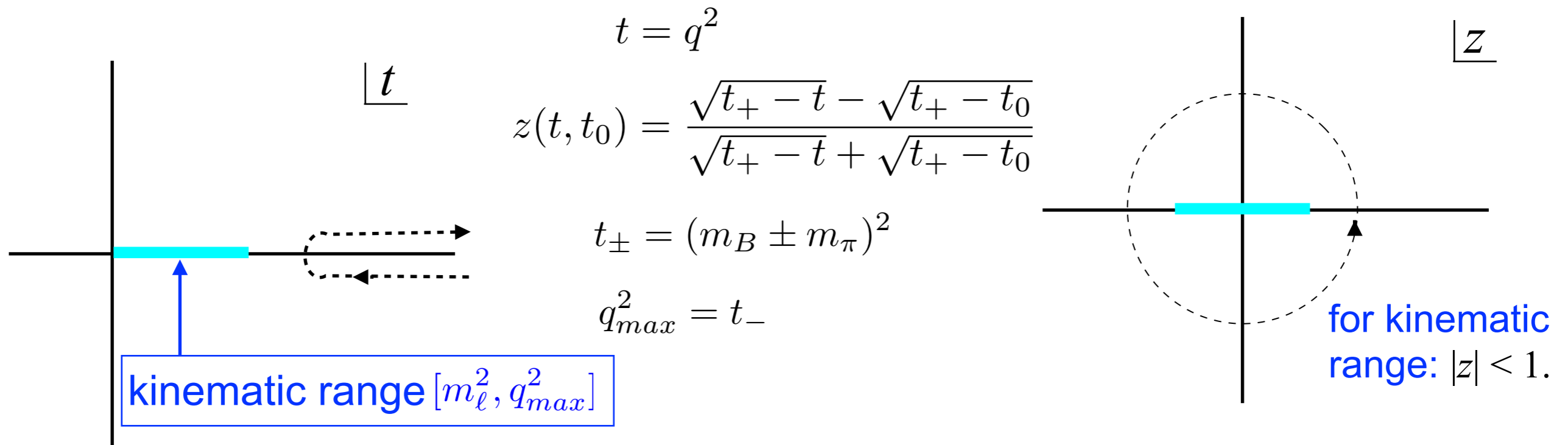
To quantify residual error:

• include FV effects in χ PT

• compare results at several L s (with other parameters fixed)

The story changes completely with two or more hadrons in initial/final state!
(or if there are two or more intermediate state hadrons)

The z -expansion



Bourelly et al (Nucl.Phys. B189 (1981) 157)
 Boyd, Grinstein, Lebed (hep-ph/9412324,
 PRL 95; hep-ph/9504235, PLB 95; hep-ph/
 9508211, NPB 96; hep-ph/9705252, PRD 97)
 Lellouch (arXiv:hep-ph/9509358, NPB 96)
 Boyd & Savage (hep-ph/9702300, PRD 97)
 Bourelly et al (arXiv:0807.2722, PRD 09)

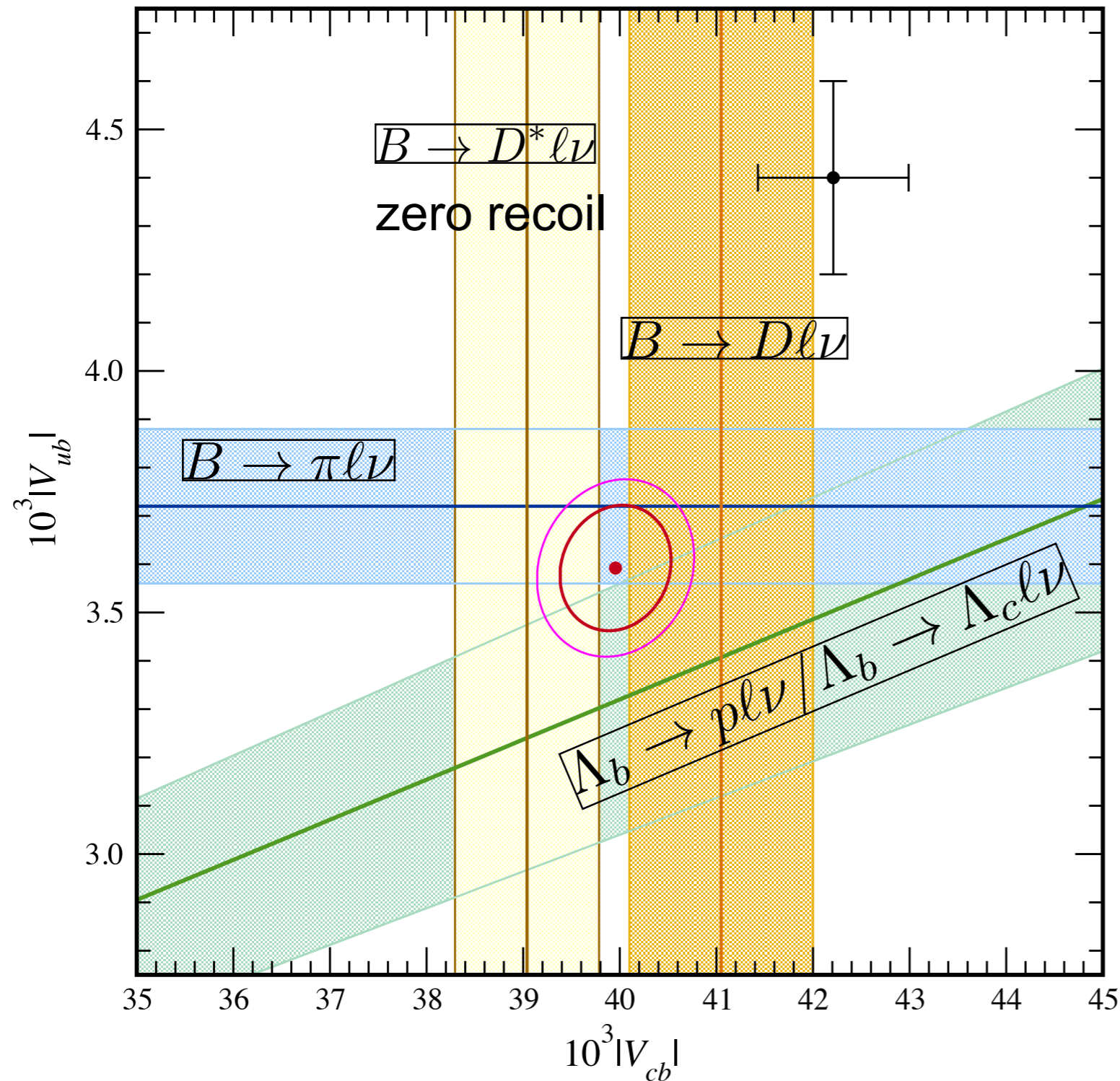
The form factor can be expanded as:

$$f(t) = \frac{1}{P(t)\phi(t, t_0)} \sum_{k=0} a_k(t_0) z(t, t_0)^k$$

- $P(t)$ removes poles in $[t_-, t_+]$
- The choice of outer function ϕ affects the unitarity bound on the a_k .
- In practice, only first few terms in expansion are needed.

Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$

A. Kronfeld (priv. communication)



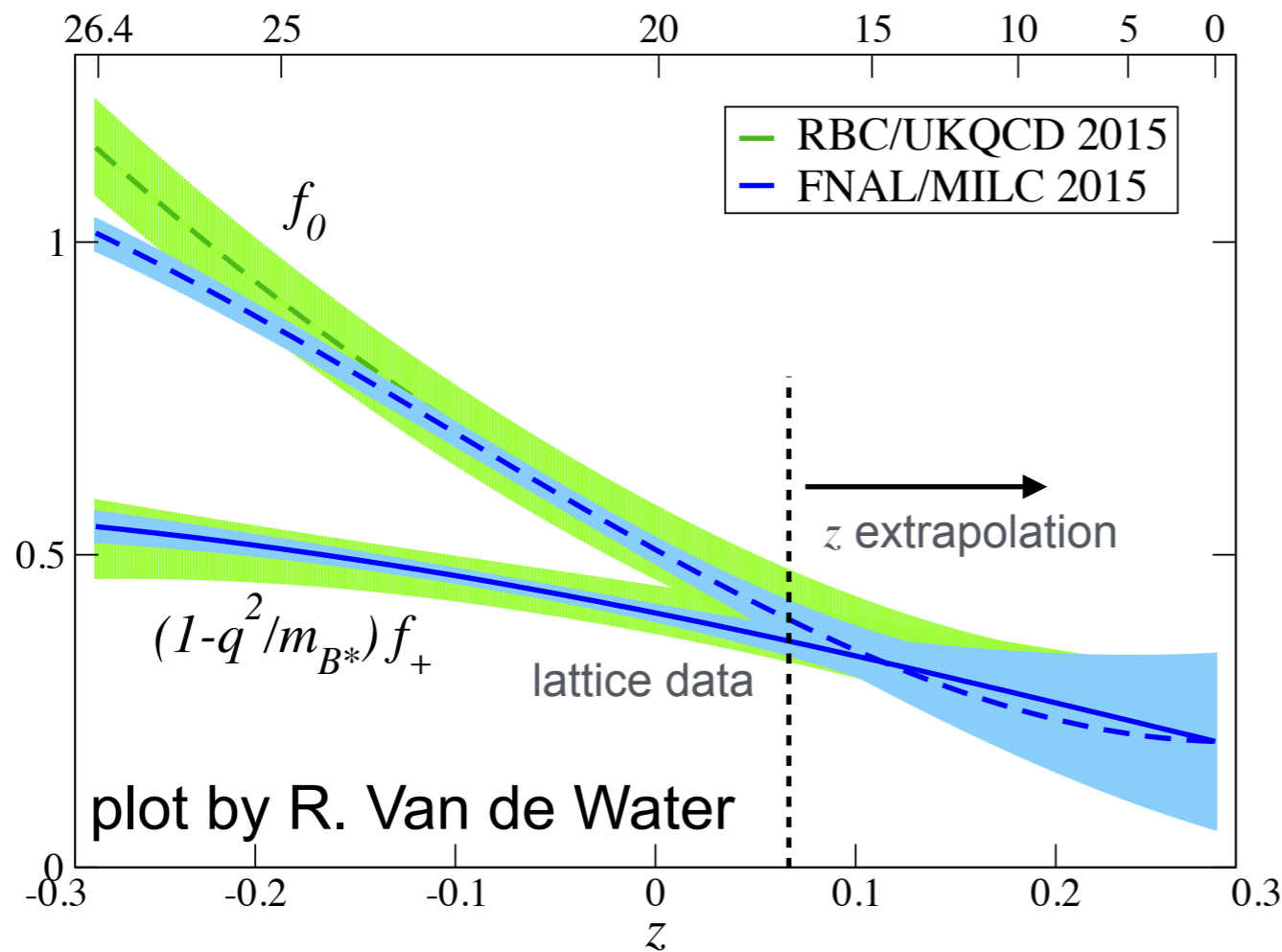
- $|V_{ub}|/|V_{cb}|$ (latQCD + LHCb)
- $|V_{ub}|$ (latQCD + BaBar + Belle)
- $|V_{cb}|$ (latQCD + BaBar + Belle)
- $|V_{cb}|$ (latQCD + HFAG, $w = 1$)
- $p = 0.19$
- $\Delta\chi^2 = 1$
- $\Delta\chi^2 = 2$
- inclusive $|V_{xb}|$

$\sim 3\sigma$ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

New in 2015:

- $|V_{cb}|$ from $B \rightarrow D l \nu$
- $|V_{ub}|$ from $B \rightarrow \pi l \nu$
- $|V_{ub}/V_{cb}|$ from $\Lambda_b \rightarrow p l \nu / \Lambda_b \rightarrow \Lambda_c l \nu$

form factors for $B \rightarrow \pi \ell \nu$ & V_{ub}

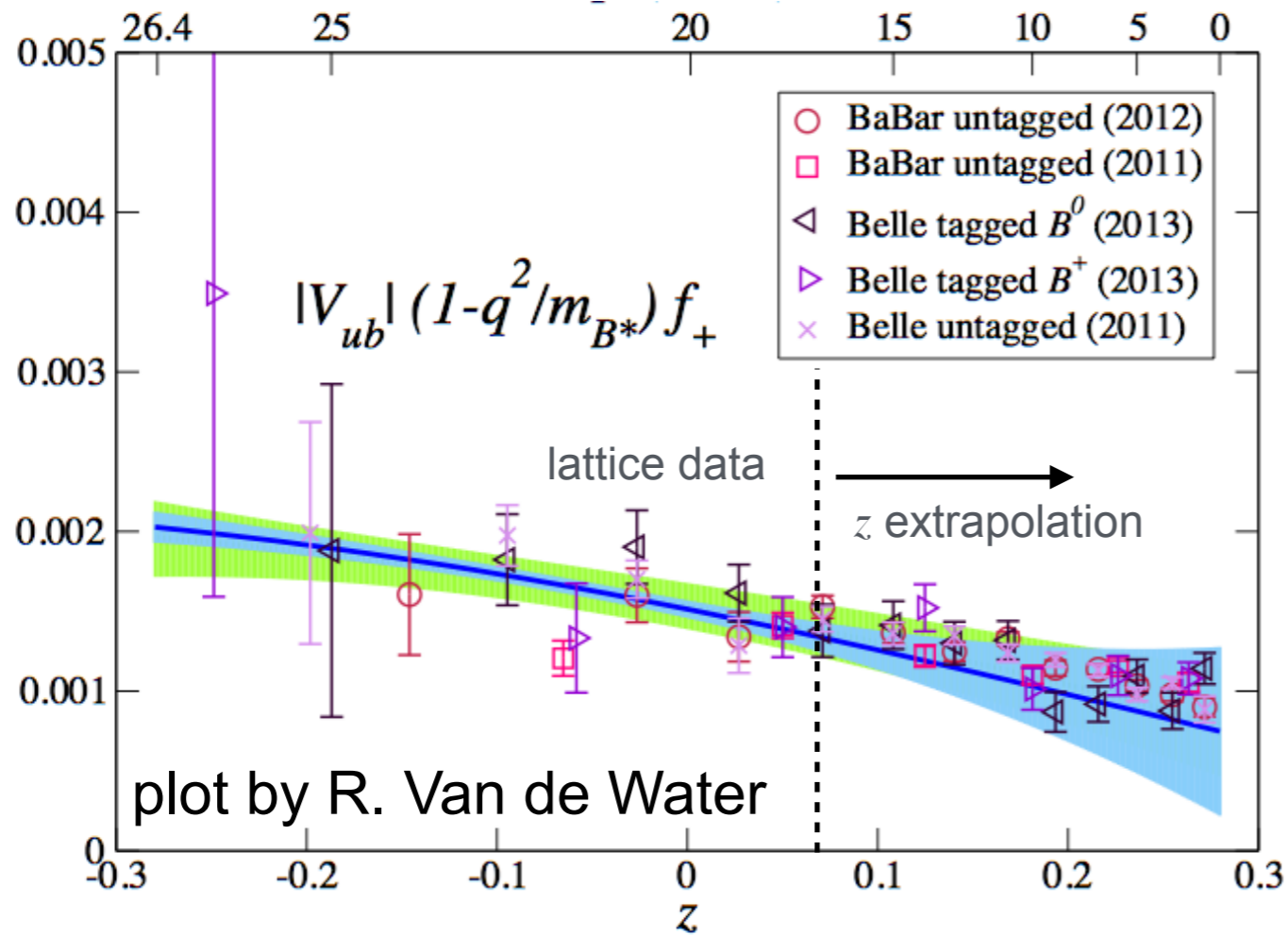


RBC (arXiv:1501.05373, PRD 2015)

FNAL/MILC (arXiv:1503.07839, PRD 2015)

- ★ FNAL/MILC & RBC form factors are in good agreement
- ★ HPQCD (arXiv:1510.07446, PRD 2016): f_0 with physical light quarks at zero recoil satisfies soft-pion theorem
- ★ Note: two independent LQCD **predictions** for $B_s \rightarrow K \ell \nu$ form factors (HPQCD, arXiv:1406.2279, PRD 2014; RBC, arXiv:1501.05373, PRD 2015)
+ ongoing work by ALPHA (Banerjee, Koren @ Lattice 2016), FNAL/MILC, ...

form factors for $B \rightarrow \pi \ell \nu$ & V_{ub}



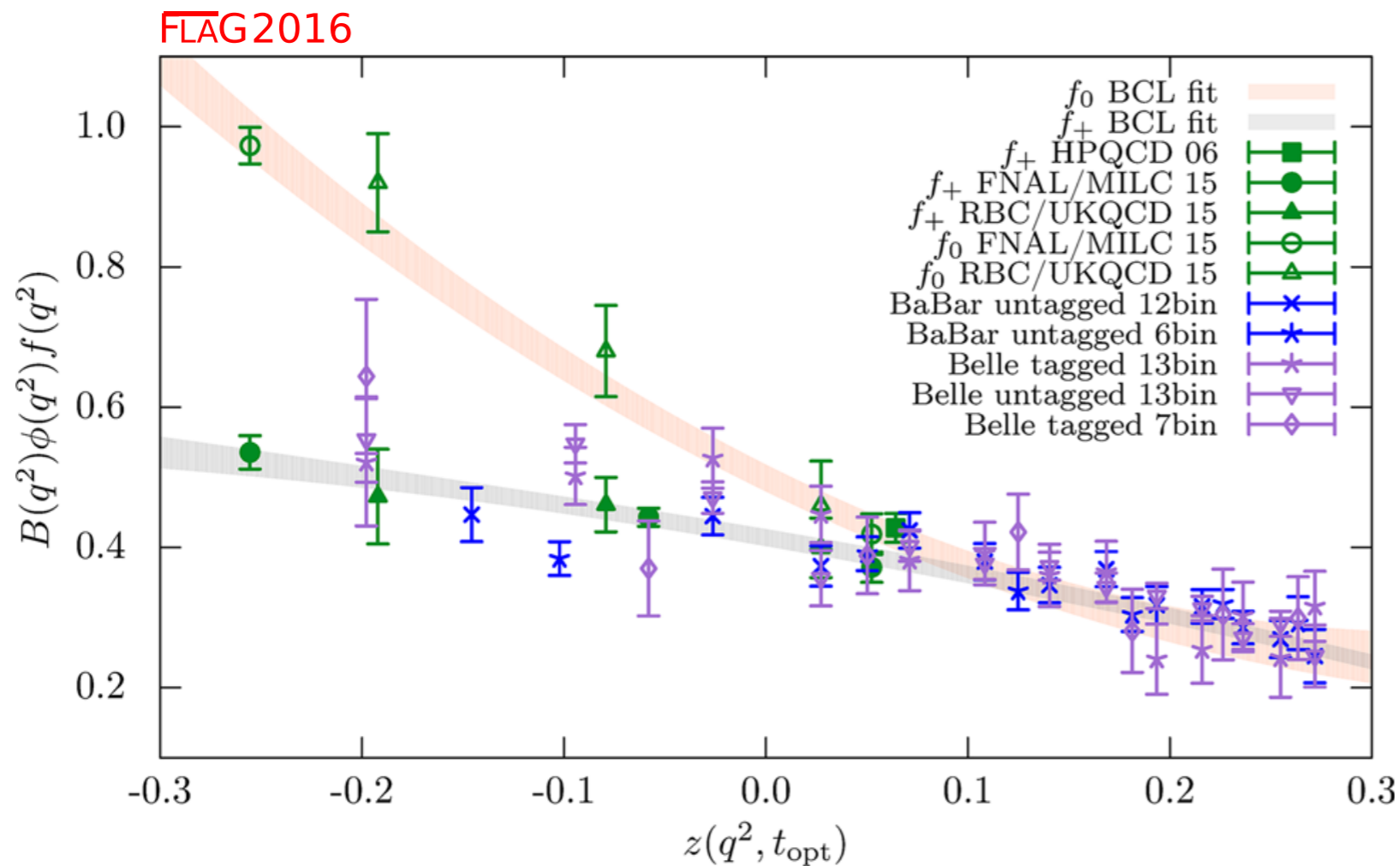
RBC (arXiv:1501.05373, PRD 2015)

FNAL/MILC (arXiv:1503.07839, PRD 2015)

$$|V_{ub}| = 3.72 (16) 10^{-3}$$

- ★ shape of f_+ agrees with experiment and uncertainties are commensurate
- ★ fit lattice form factors together with experimental data to determine $|V_{ub}|$ **and** obtain form factors (f_+, f_0) with improved precision...

form factors for $B \rightarrow \pi \ell \nu$ & V_{ub}

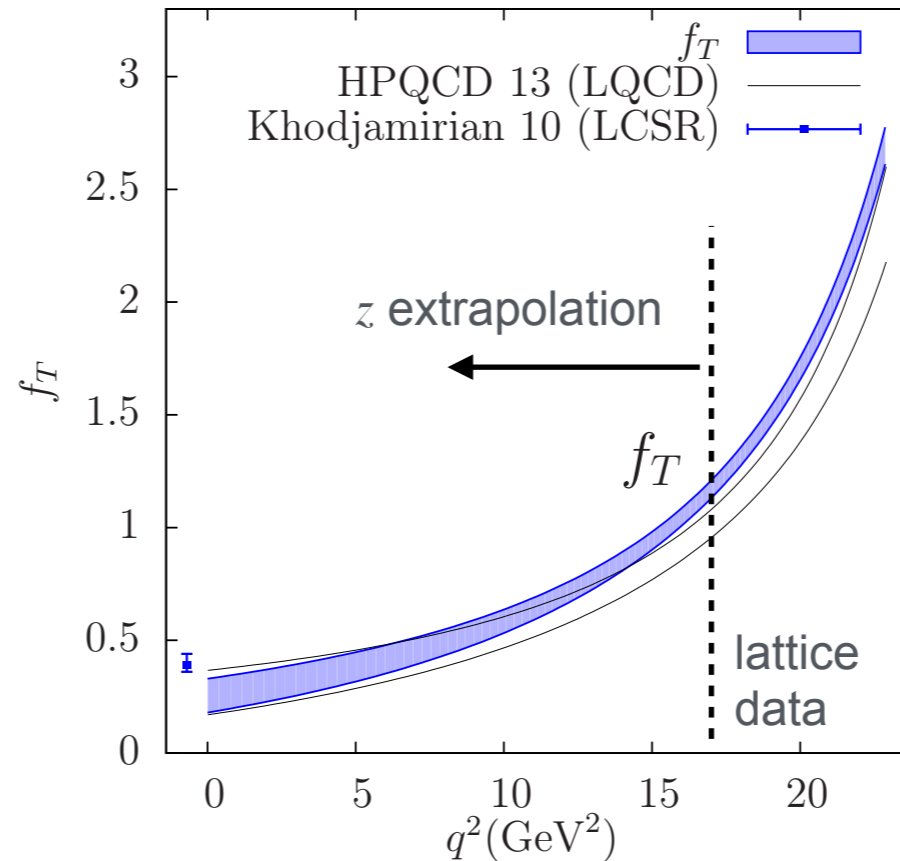
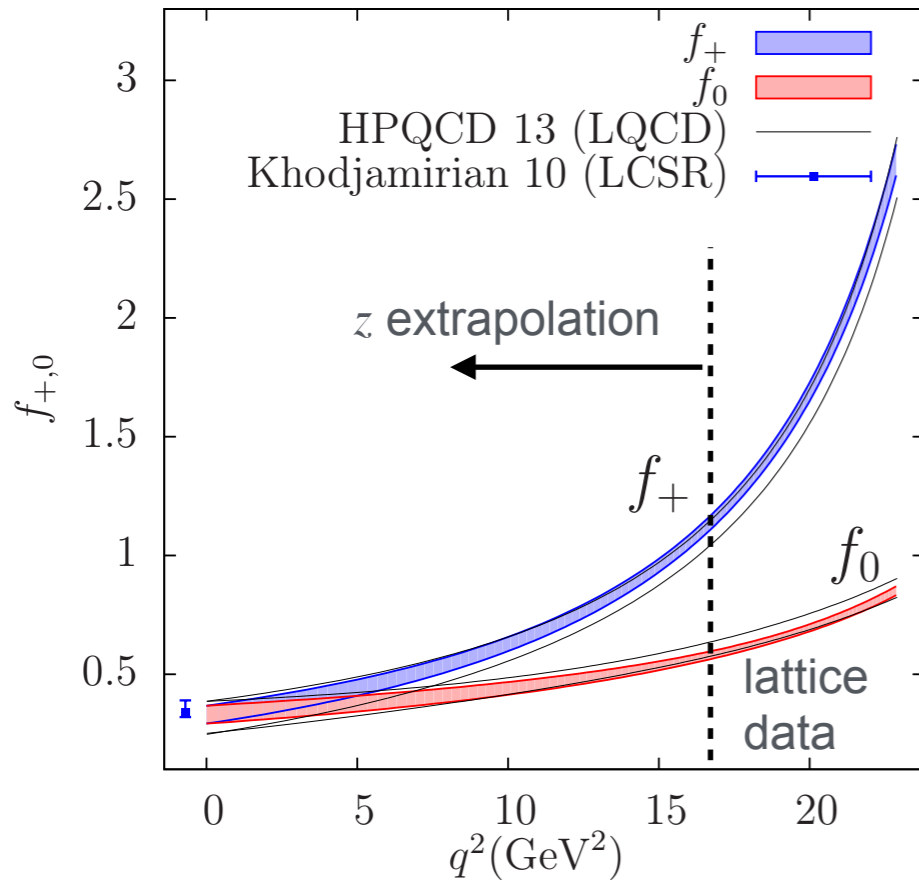


S. Aoki et al (FLAG-3 review,
arXiv:1607.00299, 2017 EJPC)

- ★ shape of f_+ agrees with experiment and uncertainties are commensurate
- ★ fit lattice form factors together with experimental data to determine $|V_{ub}|$ **and** obtain form factors (f_+, f_0) with improved precision...



form factors for $B \rightarrow K \ell \ell$



HPQCD (arXiv:1306.0434,
1306.2384, PRL 2013)

FNAL/MILC
(arXiv:1509.06235, PRD 2016)

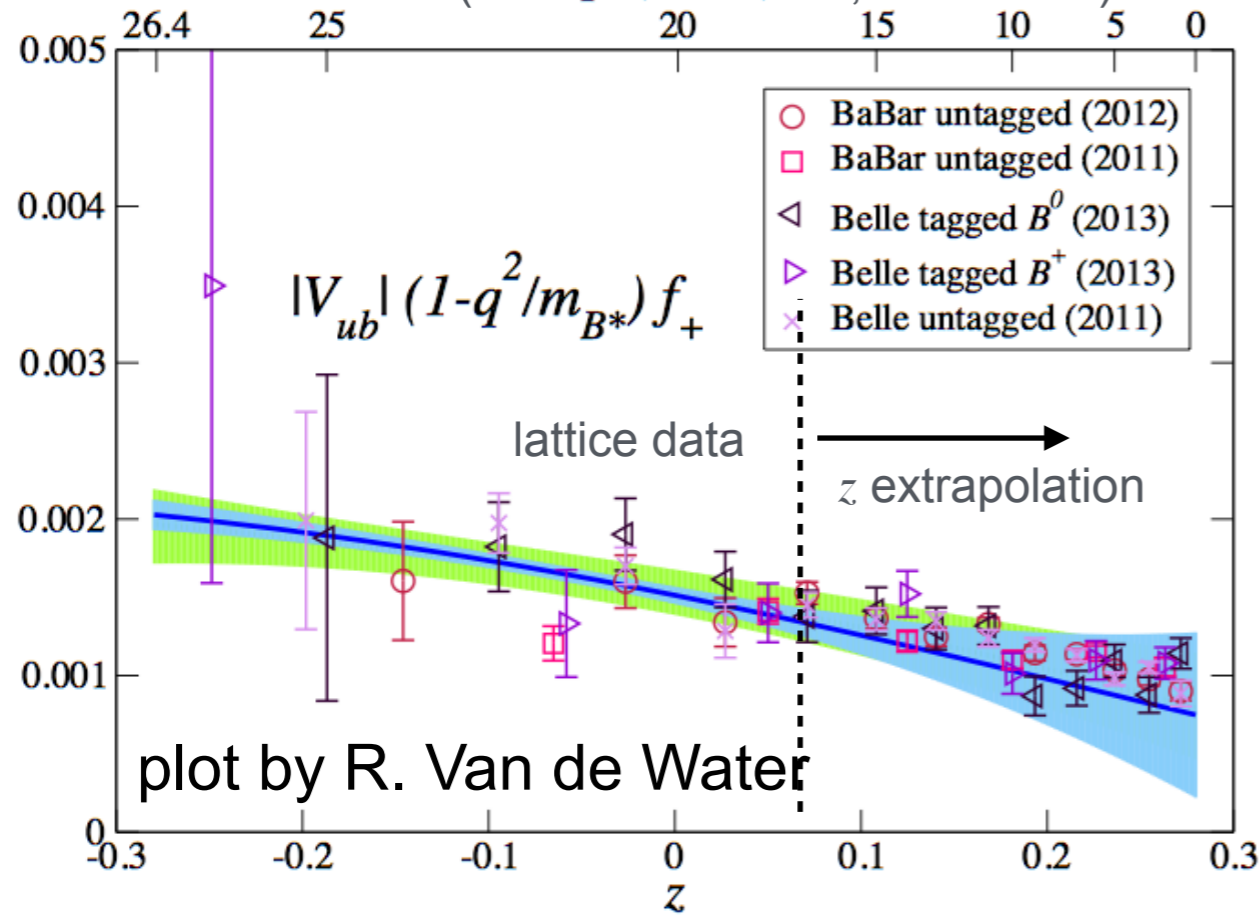
- ★ Two LQCD calculations (on overlapping ensemble sets, different valence actions):
HPQCD (NRQCD b + HISQ), FNAL/MILC (Fermilab b + asqtad)
- ★ consistent results for all three form factors
- ★ also consistent with LCSR (Khodjamirian et al, arXiv:1006.4945, JHEP 2010)
- ★ Note: First LQCD calculation of $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ form factors (10 total)
(see Meinel talk)



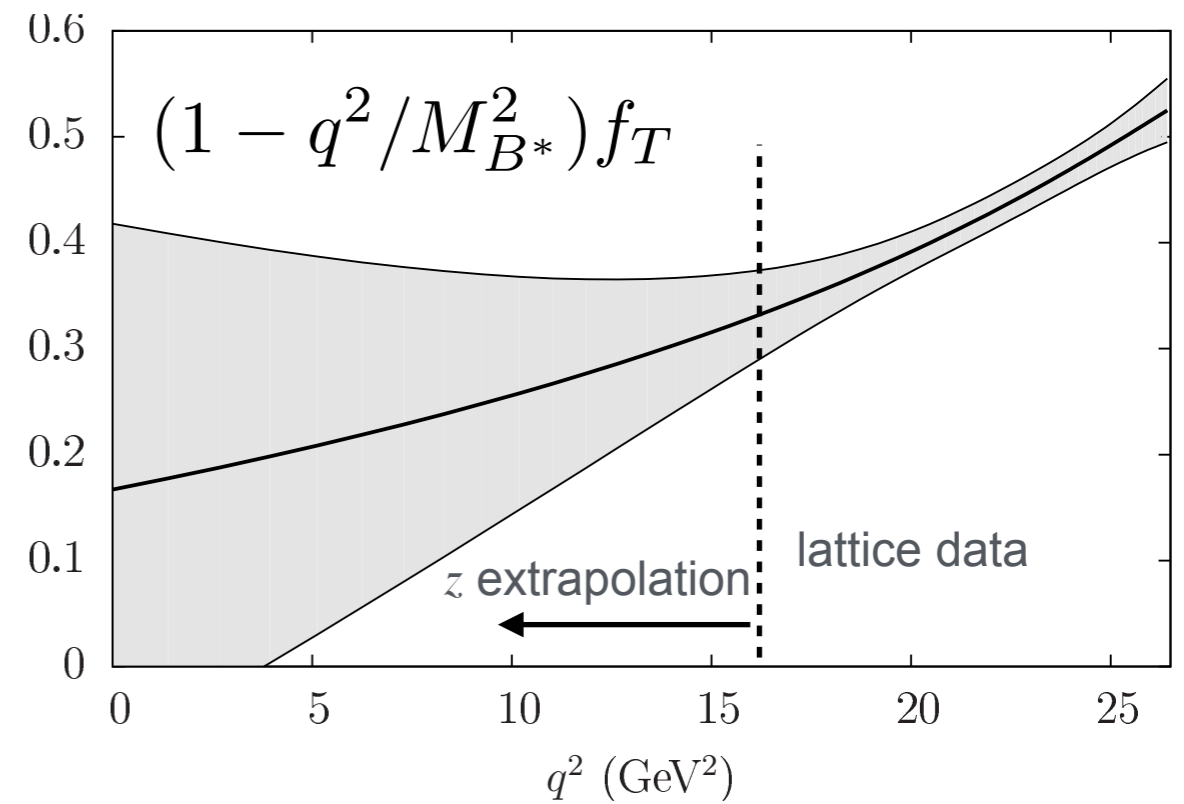
form factors for $B \rightarrow \pi \ell \ell$

RBC (arXiv:1501.05373, PRD 2015)

FNAL/MILC (arXiv:1503.07839, PRD 2015)



FNAL/MILC (arXiv:1507.01618, PRL 2015)



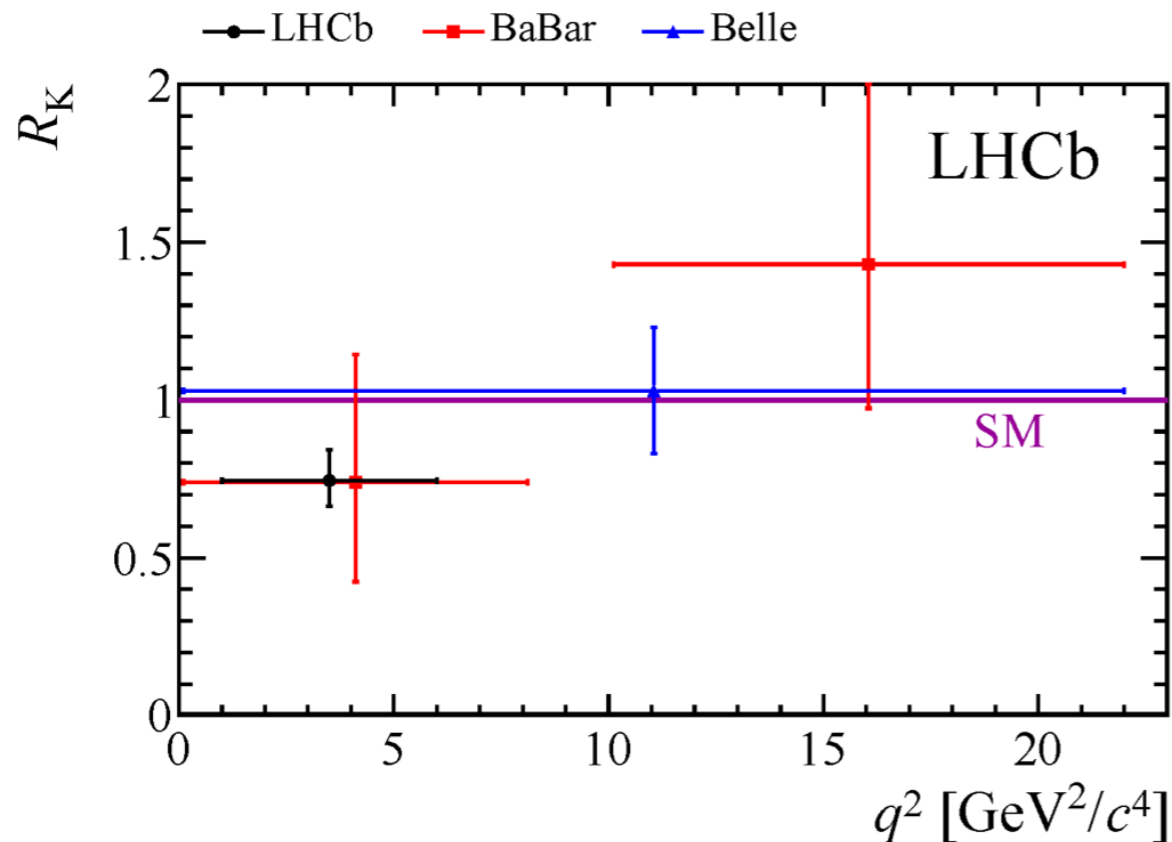
★ First LQCD calculation of f_T by FNAL/MILC

★ Take f_+, f_0 from combined fit of lattice form factors + experimental data for $d\mathcal{B}(B \rightarrow \pi \ell \nu)/dq^2$



BSM phenomenology: LFU μ/e

Lepton universality test: $B \rightarrow K \mu^+ \mu^- / B \rightarrow K e^+ e^-$



LHCb (arXiv:1406.6482, PRL 2014):

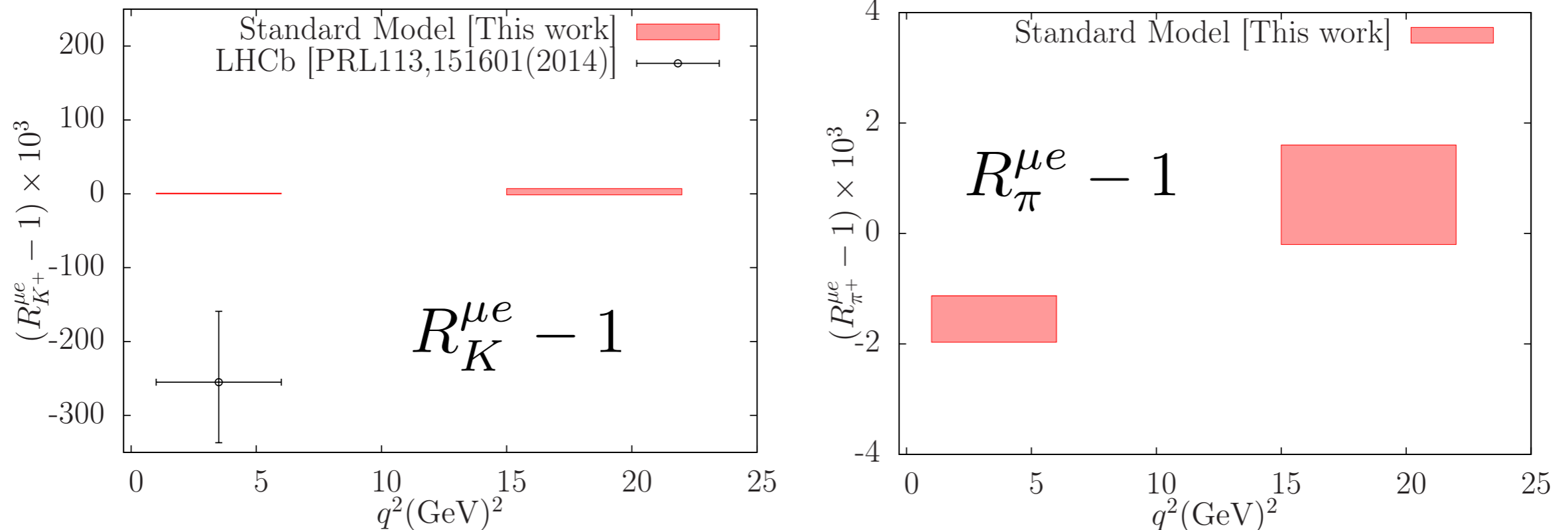
$$R_K = 0.745 \left(\begin{matrix} 90 \\ 74 \end{matrix} \right) (36)$$

$\sim 2.6 \sigma$ tension between LHCb measurement and SM theory



BSM phenomenology: LFU μ/e

D. Du et al (arXiv:1510.02349, PRD 2016)



$\sim 2.6 \sigma$ tension between LHCb measurement and SM theory

In the SM these ratios are insensitive to the form factors
(see also C. Bouchard et al, arXiv:1303.0434, PRL 2013)