Phenomenological Analysis of High-Energy Heavy-Ion Collisions

Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

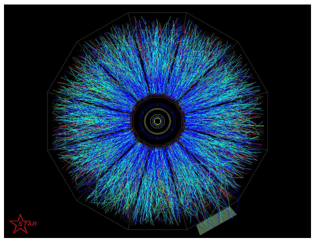
Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe
Department of Physics, Nagoya University
Duke University

Chiho NONAKA

January 6, 2017@KMI2017

Quark-Gluon Plasma

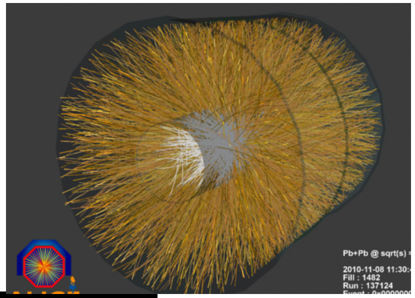
STAR



RHIC:2000

Strongly interacting QGP

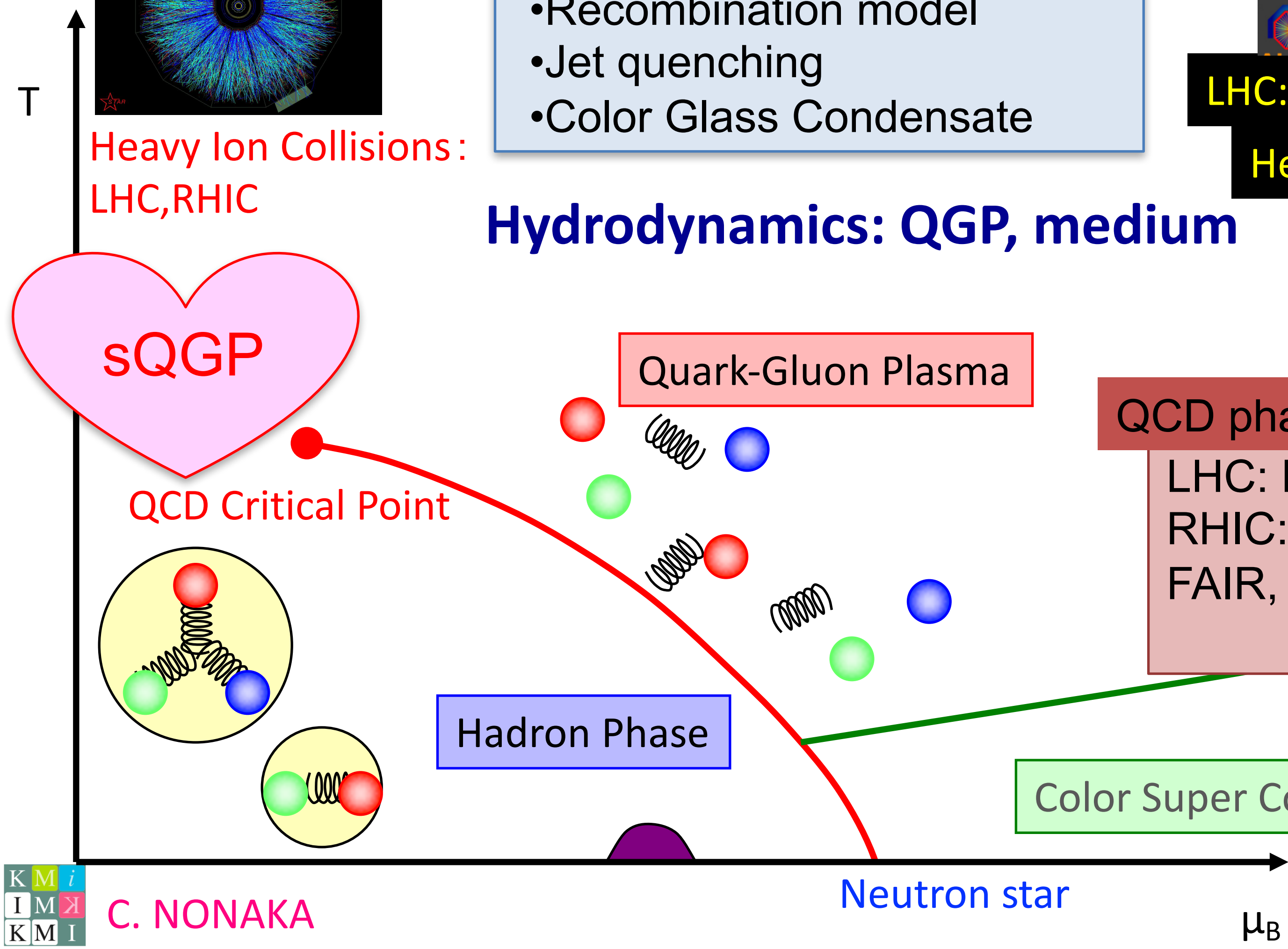
- Relativistic hydrodynamics
- Recombination model
- Jet quenching
- Color Glass Condensate



LHC:2010

Heavy Ion collisions start!

Hydrodynamics: QGP, medium

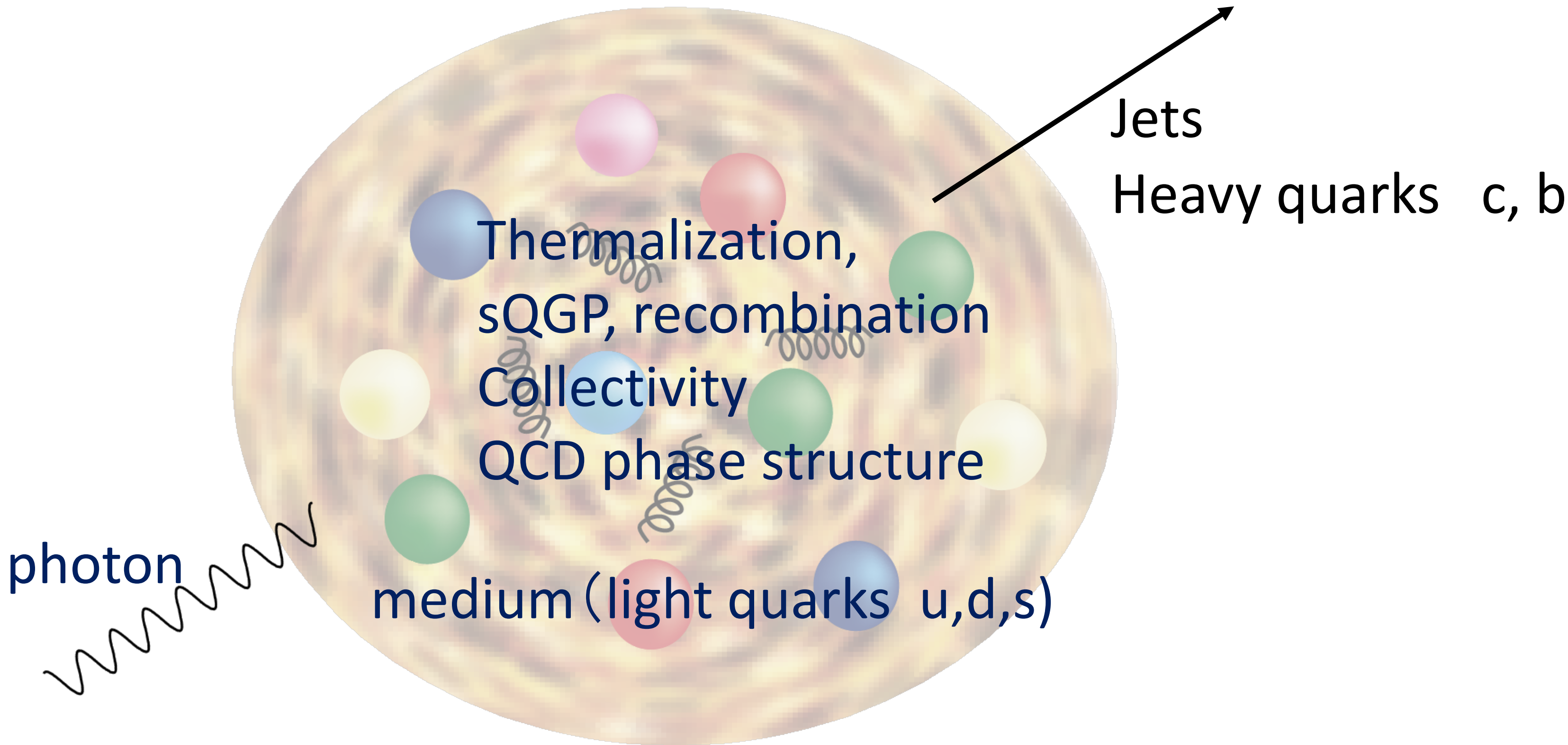


QCD phase diagram

LHC: Energy frontier
RHIC: Beam Energy Scan
FAIR, NICA, J-PARC
: high density

C. NONAKA

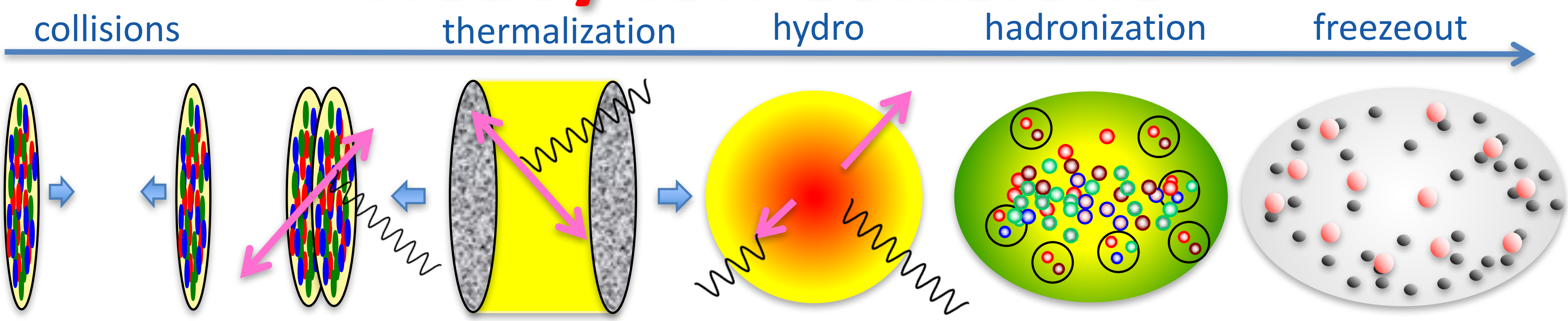
Phenomenology for heavy Ion Collisions



Phenomenological Models

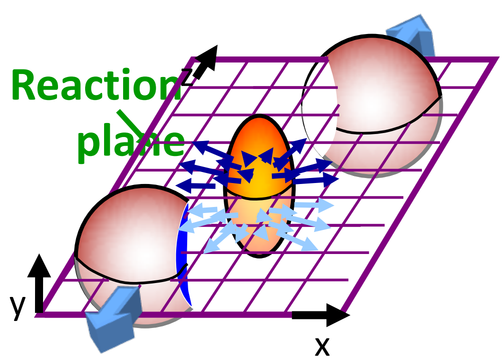
Time evolution of medium ＋ Observables

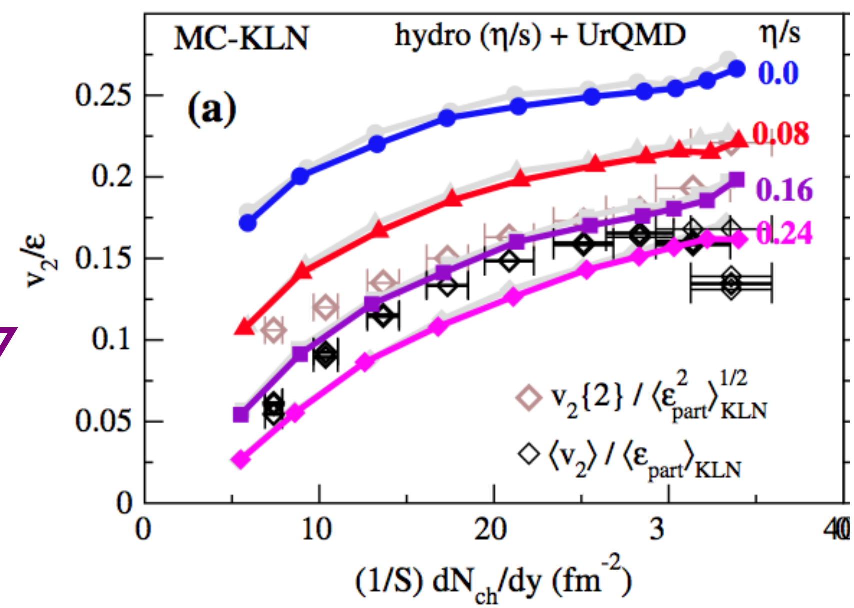
Heavy Ion Collisions



Phenomenological model: **Hydrodynamic model**

- Strong elliptic flow@RHIC ⇨ strongly interacting QGP
- Shear viscosity@RHIC & LHC
- Event-by-event fluctuations
  - Higher harmonics
  - Bulk viscosity





C. NONAKA

Viscous Hydrodynamic Model

- Relativistic viscous hydrodynamic equation for HIC

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \Delta T^{\mu\nu}$$

  - First order in gradient: acausality
  - Second order in gradient: which one is suitable for HIC?
    - Israel-Stewart, Ottinger and Grmela, AdS/CFT, Grad's 14-momentum expansion, Renormalization group…
- Numerical scheme: small numerical viscosity ↔ physical viscosities
  - First order accuracy: large dissipation
  - Second order accuracy : numerical oscillation

    -> artificial viscosity, flux limiter
  - Heavy Ion Collisions: SHASTA, KT, HLLE

    -> Godunov scheme: Riemann solver

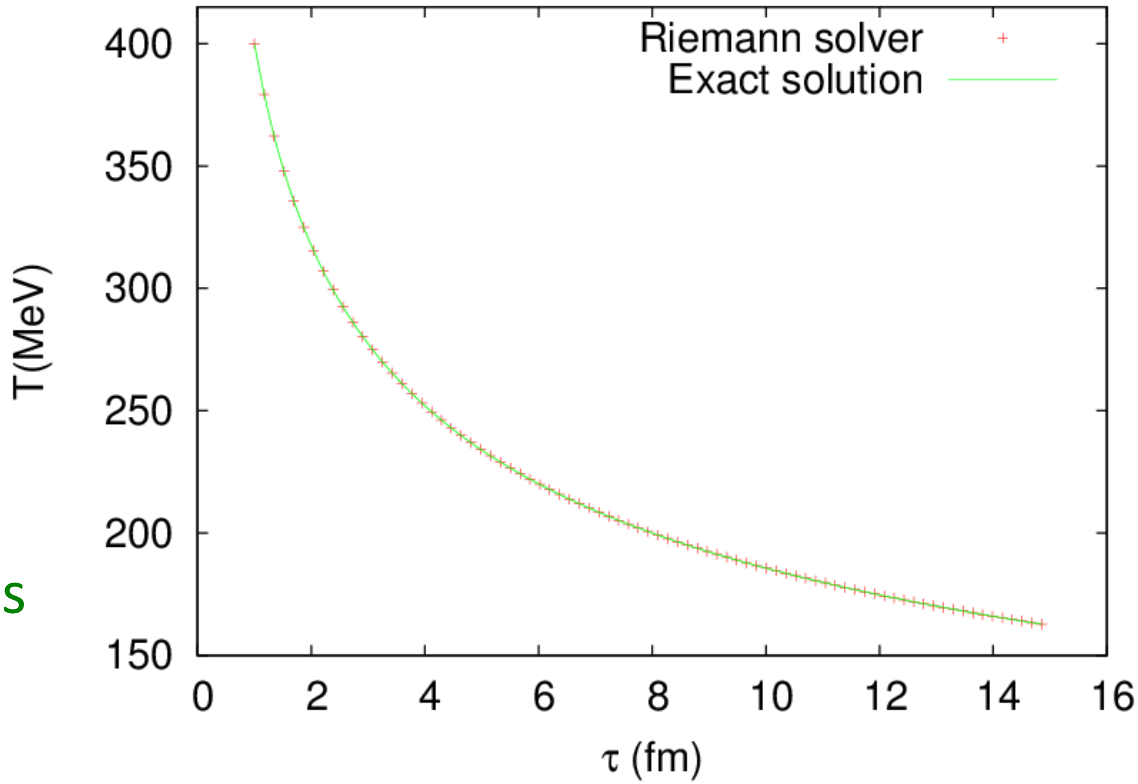
Akamatsu, Inutsuka, CN, Takamoto, arXiv:1302.1665、J. Comp. Phys. (2014)34

K. Okamoto, Y. Akamatsu and CN, Eur. Phys. J. C76 (2016)579

Numerical Tests in 1D

- ✓ Bjorken's scaling solutions
- ✓ Landau-Khalatnikov Solution (1D)
- ✓ Longitudinal fluctuations
- ✓ Conservation property

K. Okamoto, Y. Akamatsu and CN, Eur. Phys. J. C76 (2016)579



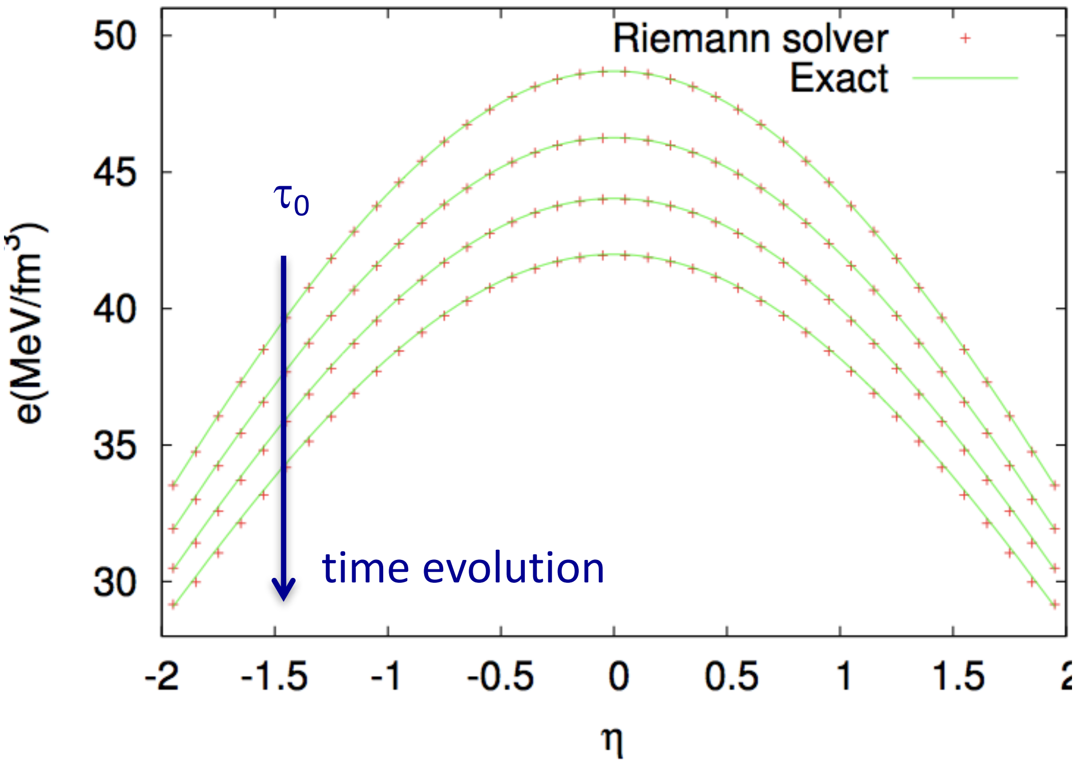
after 8000 steps
error: 0.01 %

C. NONAKA

Numerical Tests in 1D

- ✓ Bjorken's scaling solutions
- ✓ Landau-Khalatnikov Solution (1D)
- ✓ Longitudinal fluctuations
- ✓ Conservation property

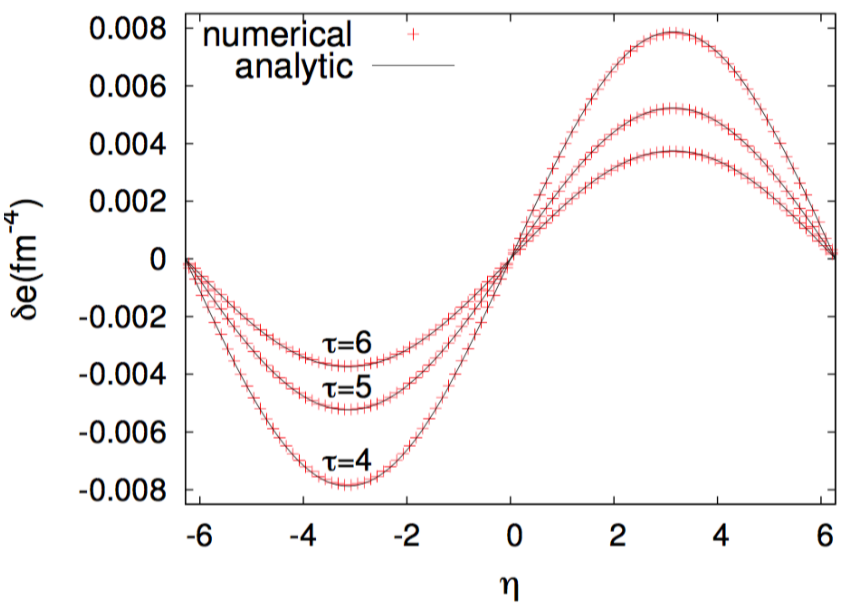
K. Okamoto, Y. Akamatsu and CN, Eur. Phys. J. C76 (2016)579

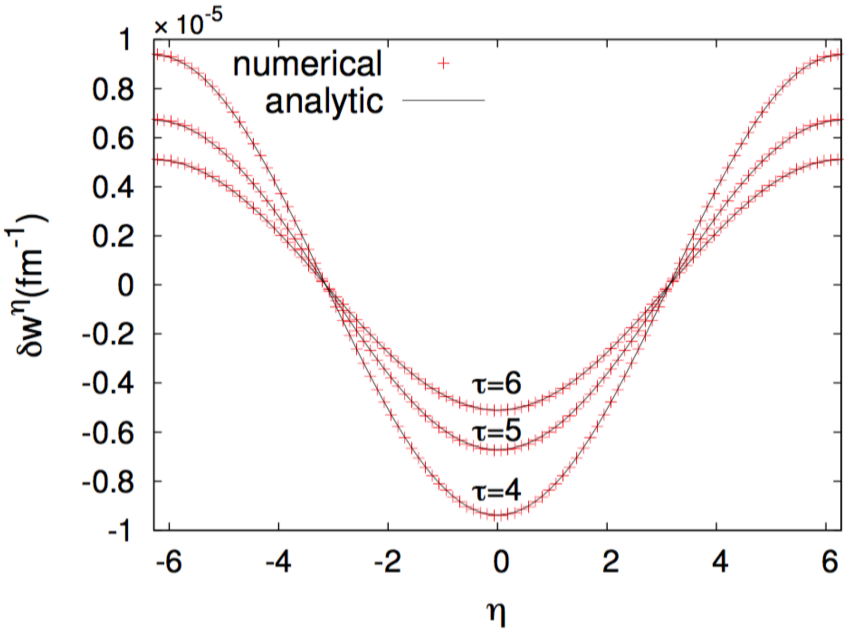


Numerical Tests in 1D

- ✓ Bjorken's scaling solutions
- ✓ Landau-Khalatnikov Solution (1D)
- ✓ **Longitudinal fluctuations**
- ✓ Conservation property

K. Okamoto, Y. Akamatsu and CN, Eur. Phys. J. C76 (2016)579

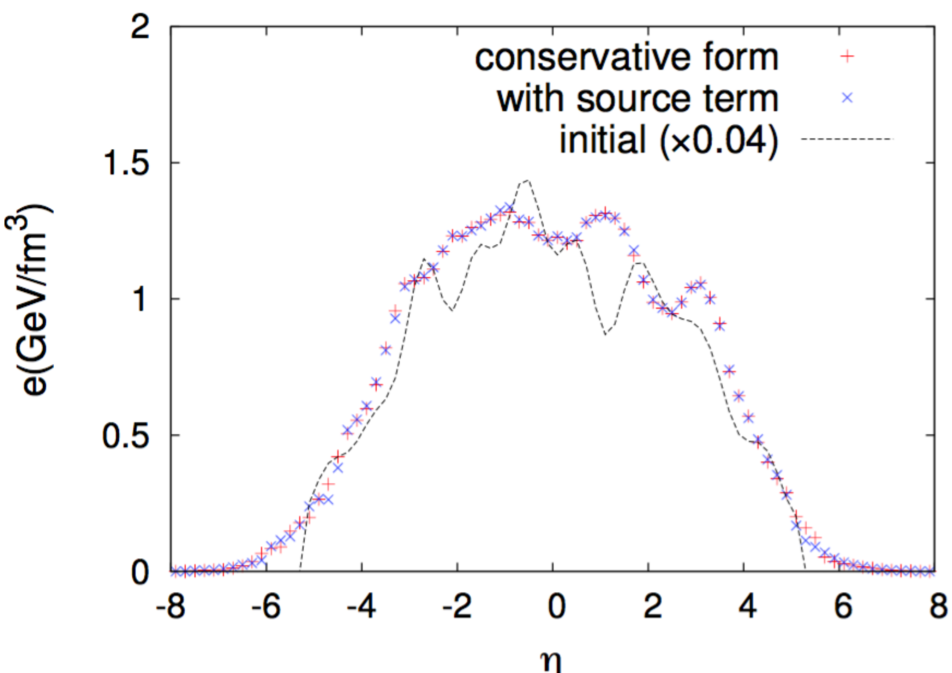




Numerical Tests in 1D

- ✓ Bjorken's scaling solutions
- ✓ Landau-Khalatnikov Solution (1D)
- ✓ Longitudinal fluctuations
- ✓ Conservation property

K. Okamoto, Y. Akamatsu and CN, Eur. Phys. J. C76 (2016)579



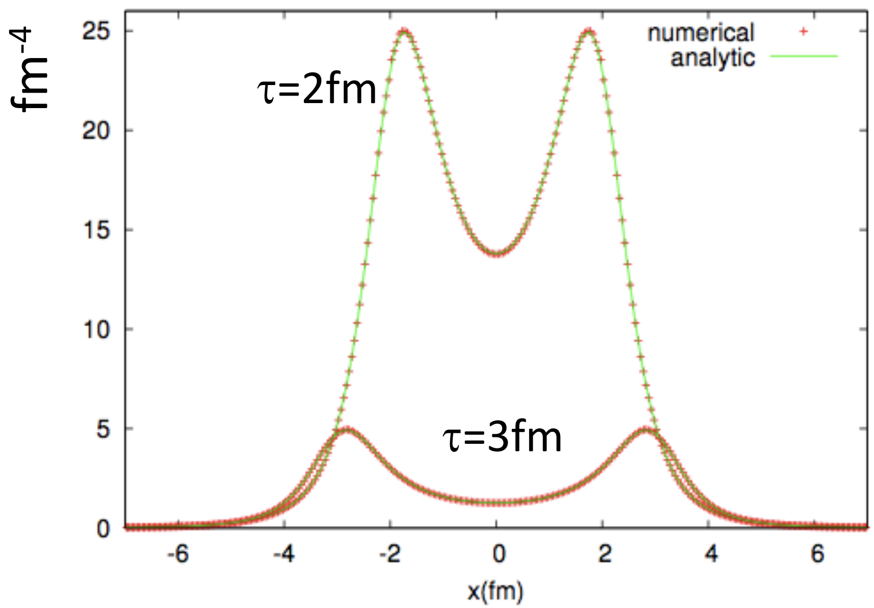
Sum of violation of conservation

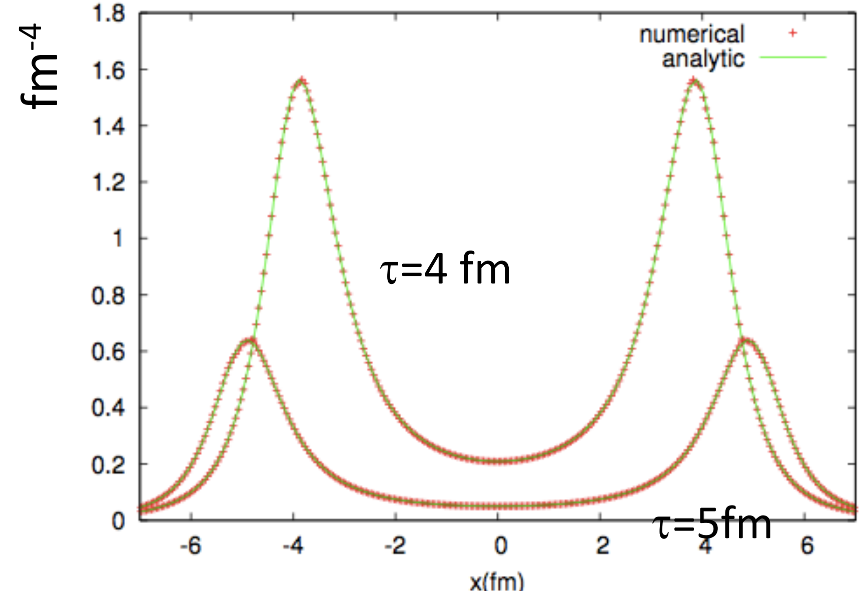
	ε_E	ε_M
conservative	1.38E-09	8.59E-09
with souce	1.27E-02	5.61E-02

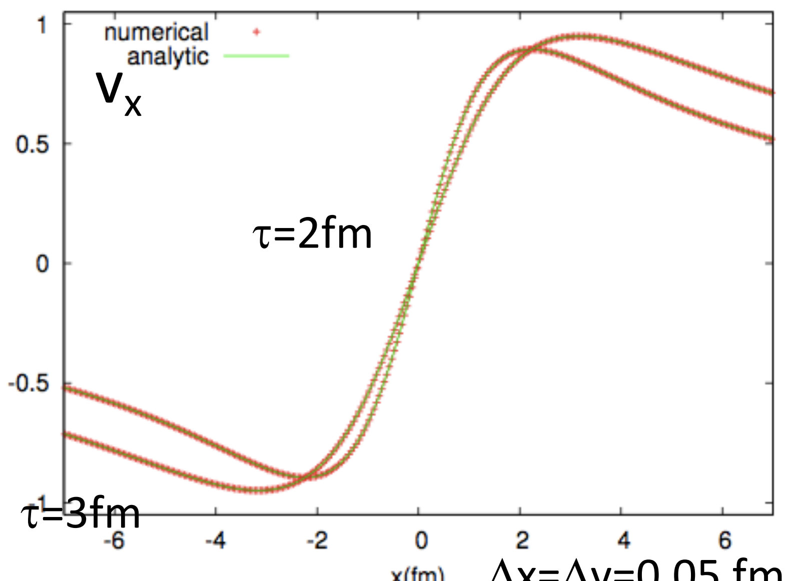
Gubser Flow (3D)

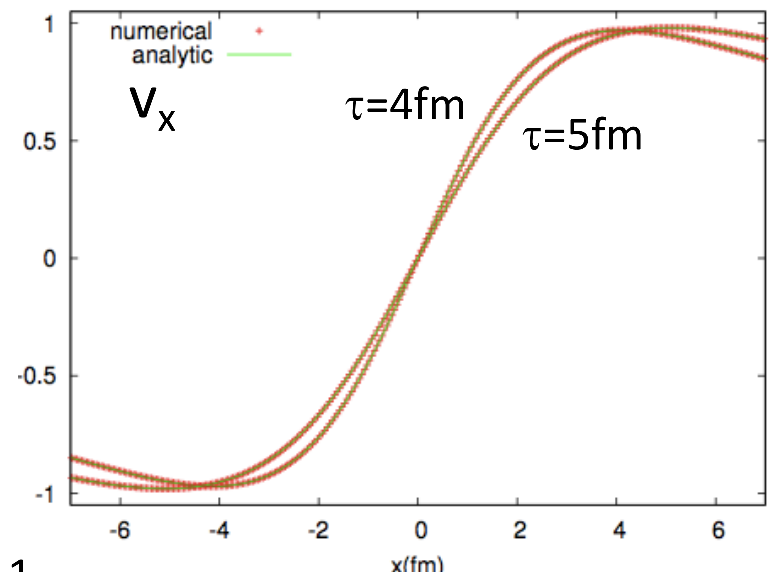
K. Okamoto, Y. Akamatsu and CN,Eur. Phys. J. C76 (2016)579

Energy density *Gubser, PRD82 (2010) 085027*









$\Delta x=\Delta y=0.05$ fm, $\Delta\eta=0.1$

Finite Viscosity

- Israel-Stewart Theory

Akamatsu, Inutsuka, CN, Takamoto, arXiv:1302.1665、J. Comp. Phys. (2014)34

Okamoto and CN, in preparation

1. Dissipative fluid equation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \tau^{\mu\nu}$$
$$= T_{\text{ideal}} + T_{\text{dissip}}$$

Ideal part:

Riemann solver for QGP: Godunov method

Two shock approximation *Mignone, Plewa and Bodo, Astrophys. J. S160, 199 (2005)*

2. Relaxation equation

$$\hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi,$$

$$\hat{D}\pi^{\mu\nu} = \frac{1}{\tau_\pi}(\pi^{\mu\nu}_{NS} - \pi^{\mu\nu}) - I^{\mu\nu}_\pi,$$

$$\hat{D}q^\mu = \frac{1}{\tau_q}(q^\mu_{NS} - q^\mu) - I^\mu_q,$$

$$\hat{D} = u^\mu \partial_u$$

I: second order terms

→

$$\left(\frac{\partial}{\partial t} + v^j \frac{\partial}{\partial x^j}\right)\Pi = -\frac{I_\Pi}{\gamma},$$

advection

+

$$\frac{\partial}{\partial t}\Pi = \frac{1}{\gamma\tau_\Pi}(\Pi_{NS} - \Pi),$$

stiff equation

$$\Delta t < \tau_{\text{relax}} << \tau_{\text{fluid}}$$

$$\tau^{\mu\nu} = \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}$$

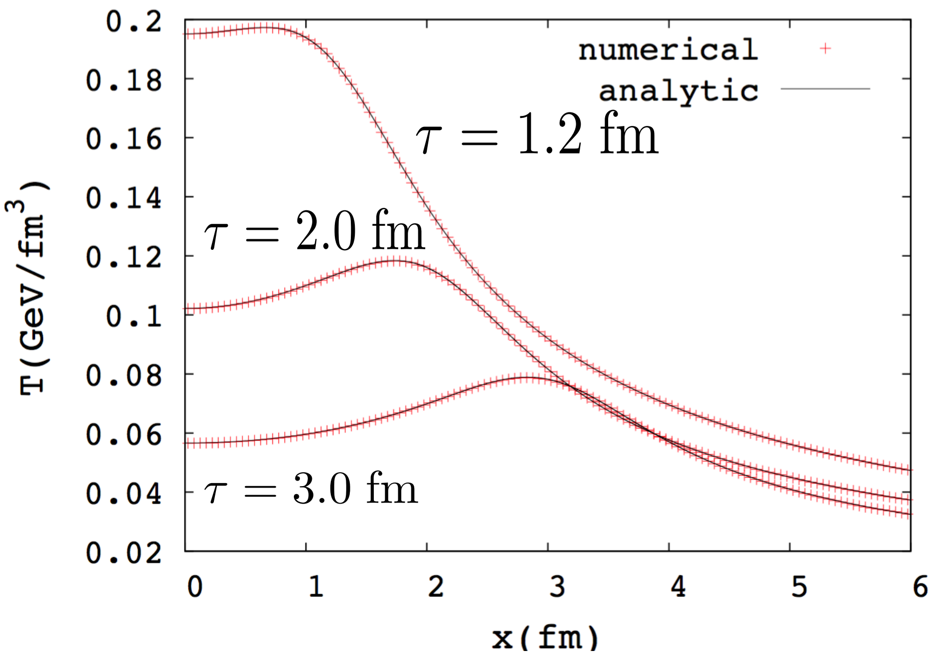
Poster by Okamoto

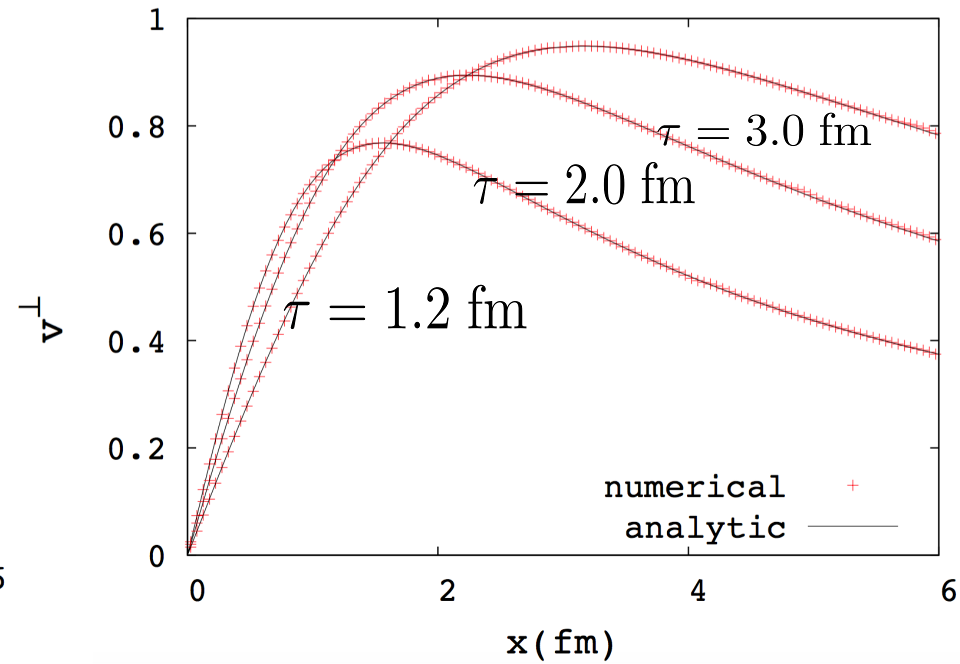
Gubser Flow with Finite η/s

Marrochio et al (MUSIC), PRC91, 014903(2015)

$$\eta/s = 0.2,\ \tau_R = 5\eta/(Ts)$$

at y=0



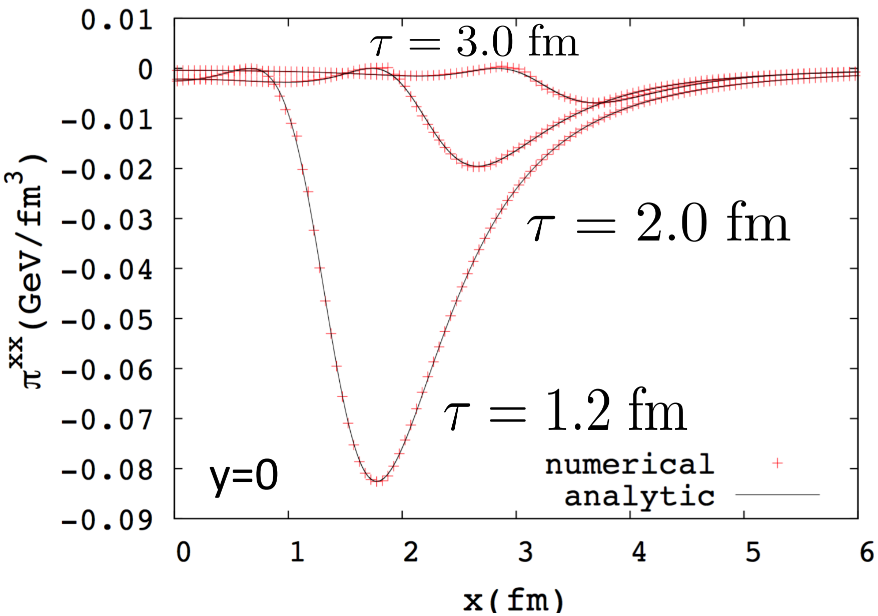


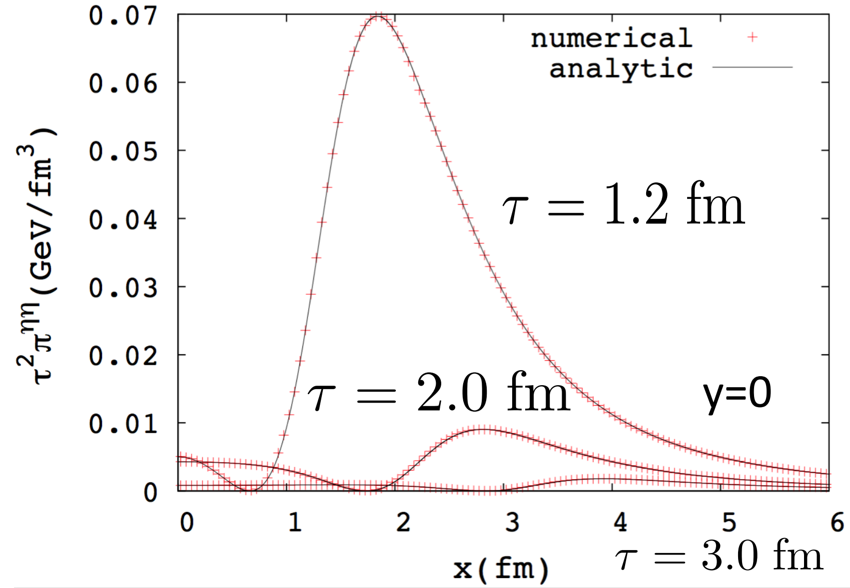
$$dx = dy = 0.05\text{fm}$$

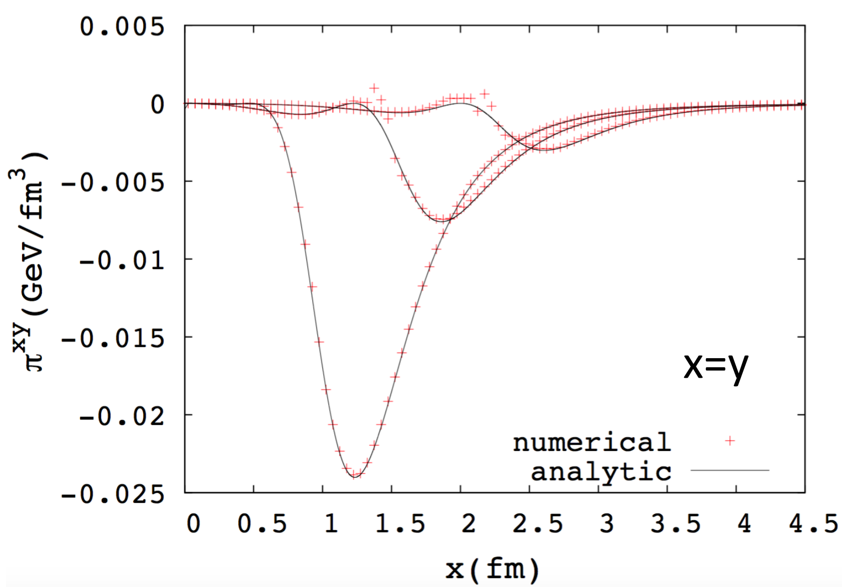
C. NONAKA

Poster by Okamoto

Gubser Flow with Finite η/s



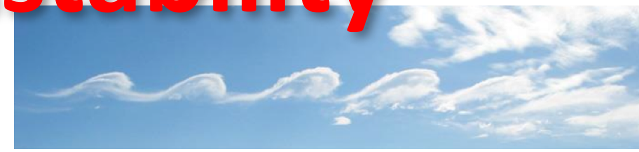


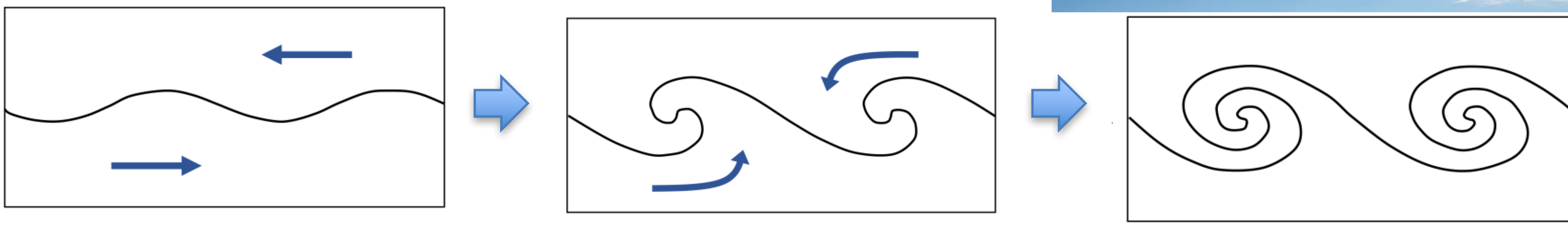


C. NONAKA

Poster by Okamoto

Kelvin-Helmholtz Instability



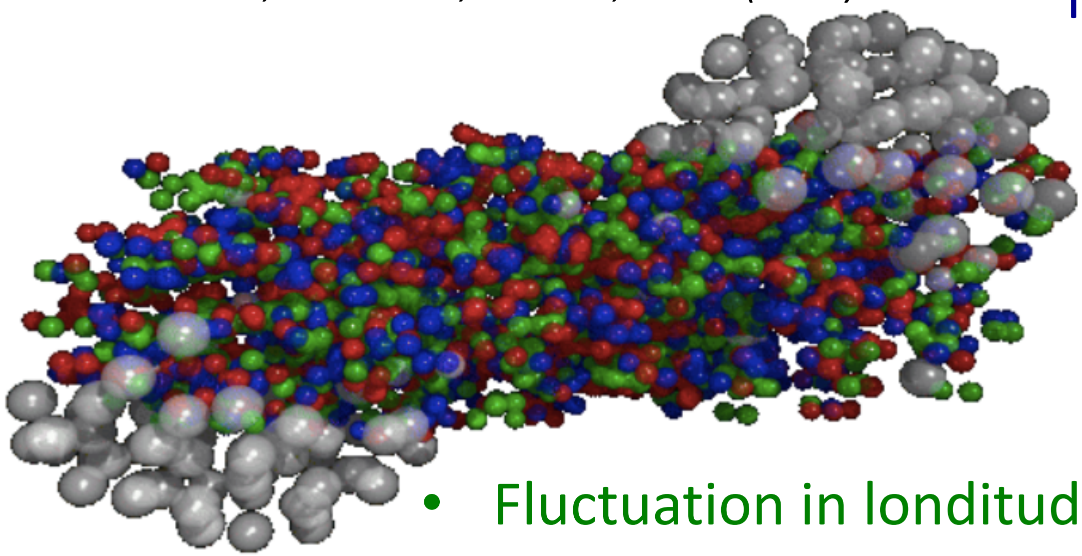


perturbation

vortexes

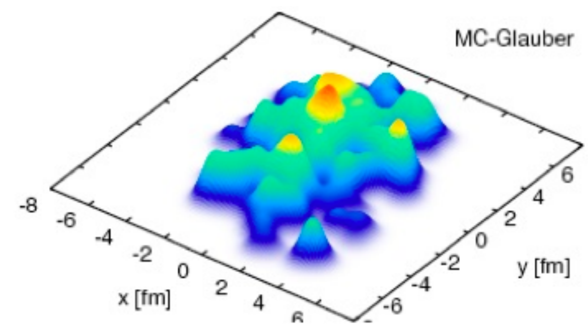
Heavy Ion Collsions

Csernai, Strottman,Anderlik, PRC85(2012)054901



higher harmonics

⇨ event-by-event fluctuation



on transverse plane

- Fluctuation in londitudinal direction?
- KHI occurs in Heavy Ion collisions?
- Effect on collective flow?

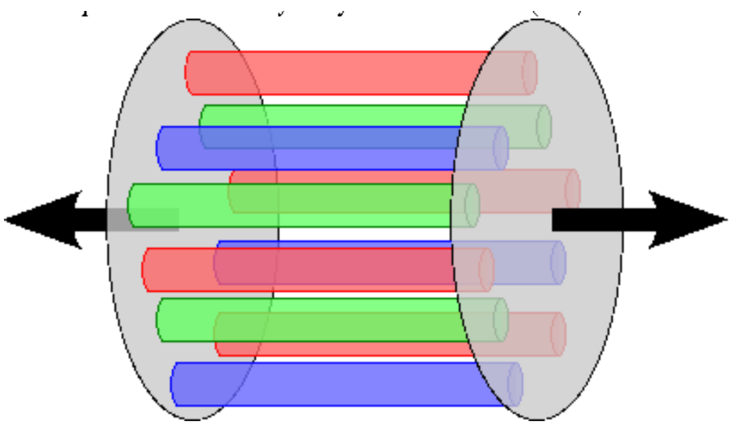
C. NONAKA

Poster by Okamoto

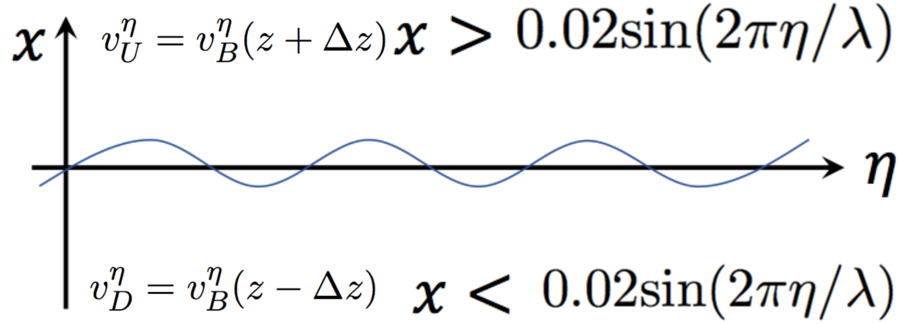
Kelvin-Helmholtz Instability with Bjorken's Flow

Fluctuations in initial conditions
ex color flux tubes

Origin of shear flow and
fluctuations around v_η=0



$\tau_0 = 1$ fm   at mid rapidity



$$\lambda = 0.3.\ \ \Delta z = 0.15$$

Does KHI happen in an expansion system?
What is the effect on physical observables?

C. NONAKA

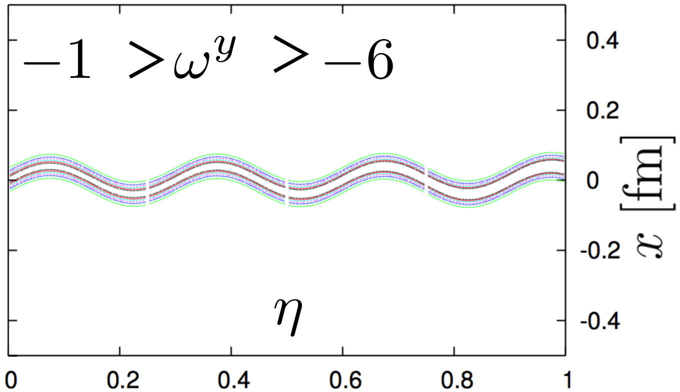
Poster by Okamoto

Kelvin-Helmholtz Instability with Bjorken's Flow

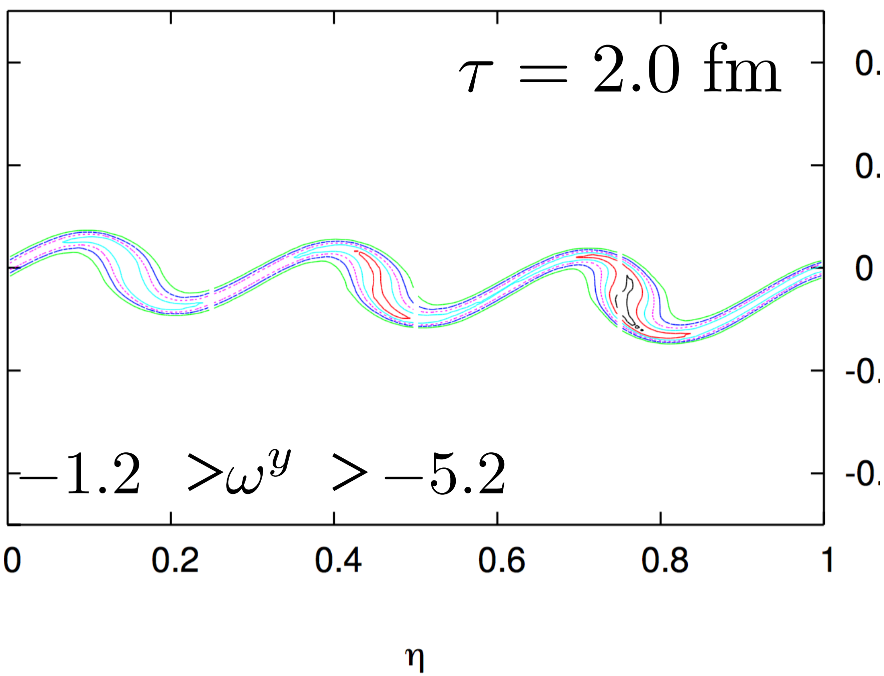
- Ideal

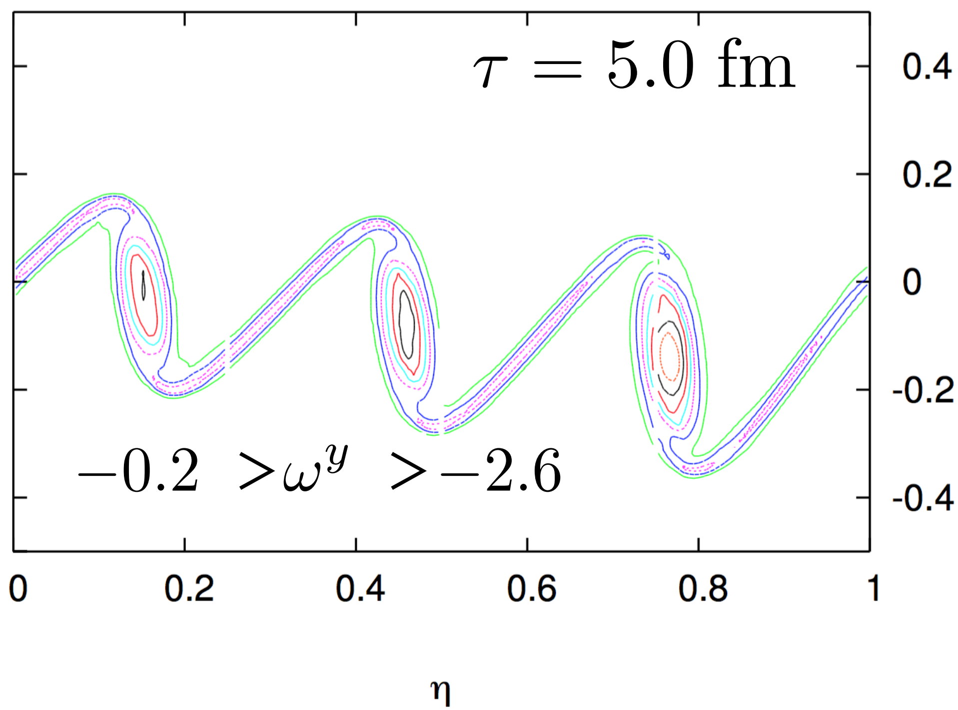
Vorticity vector

$$w^y = \frac{1}{\tau}\left(\frac{\partial u^x}{\partial \eta} - \tau^2 \frac{\partial u^\eta}{\partial x}\right)$$



- KHI grows even with Bjorken's flow.
- Clear vorticities are observed at τ=5 fm.





C. NONAKA

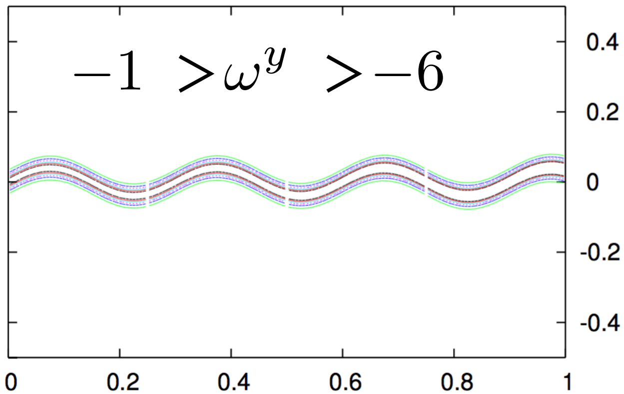
Poster by Okamoto

KHI with Bjorken's Flow

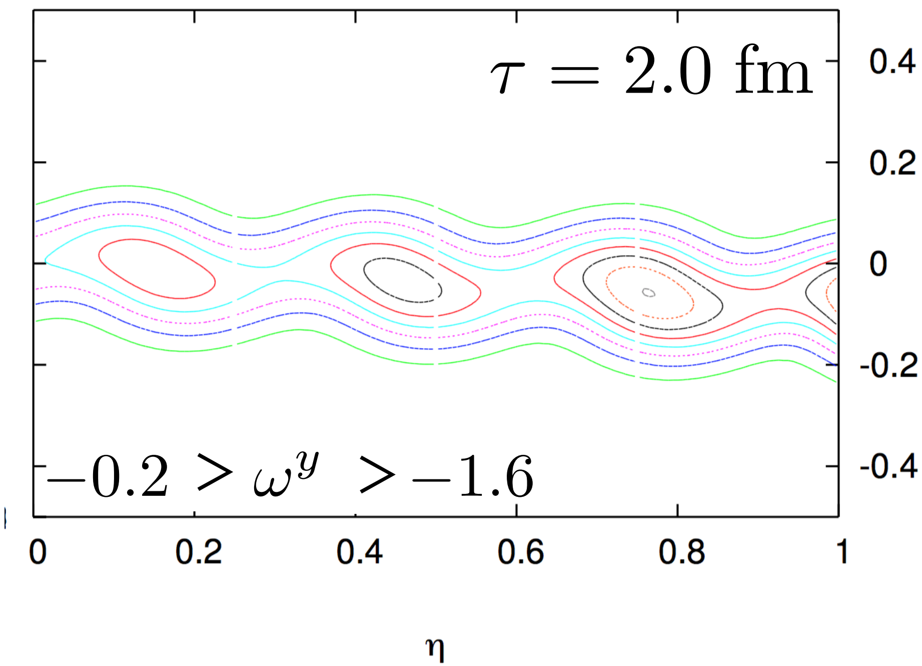
- η/s=0.01

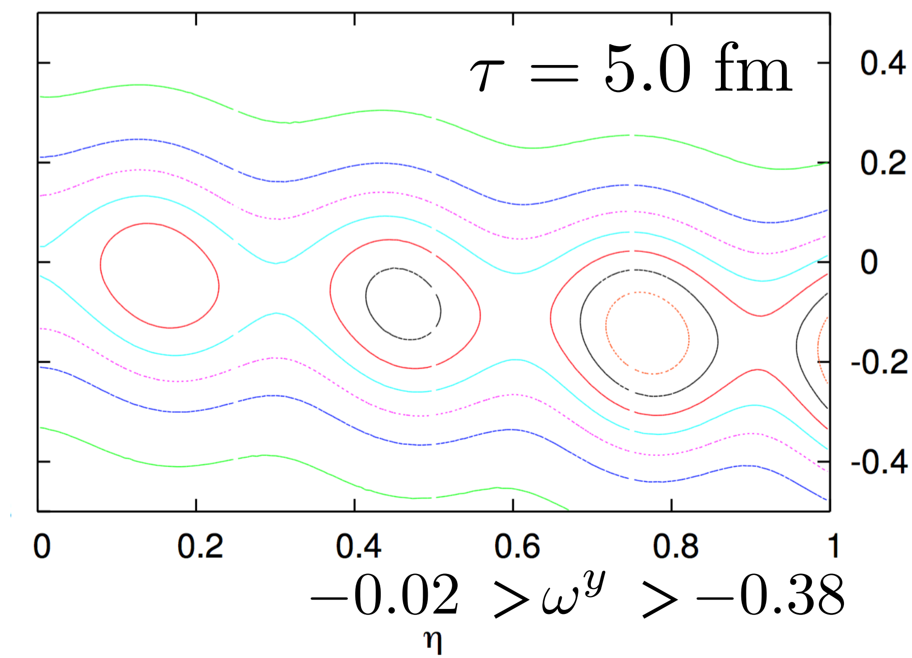
Vorticity vector

$$w^y = \frac{1}{\tau}\left(\frac{\partial u^x}{\partial \eta} - \tau^2 \frac{\partial u^\eta}{\partial x}\right)$$



- Vorticities exist, but they are weaker than those in ideal.
- In the case of η/s=0.08, vorticities are not formed.





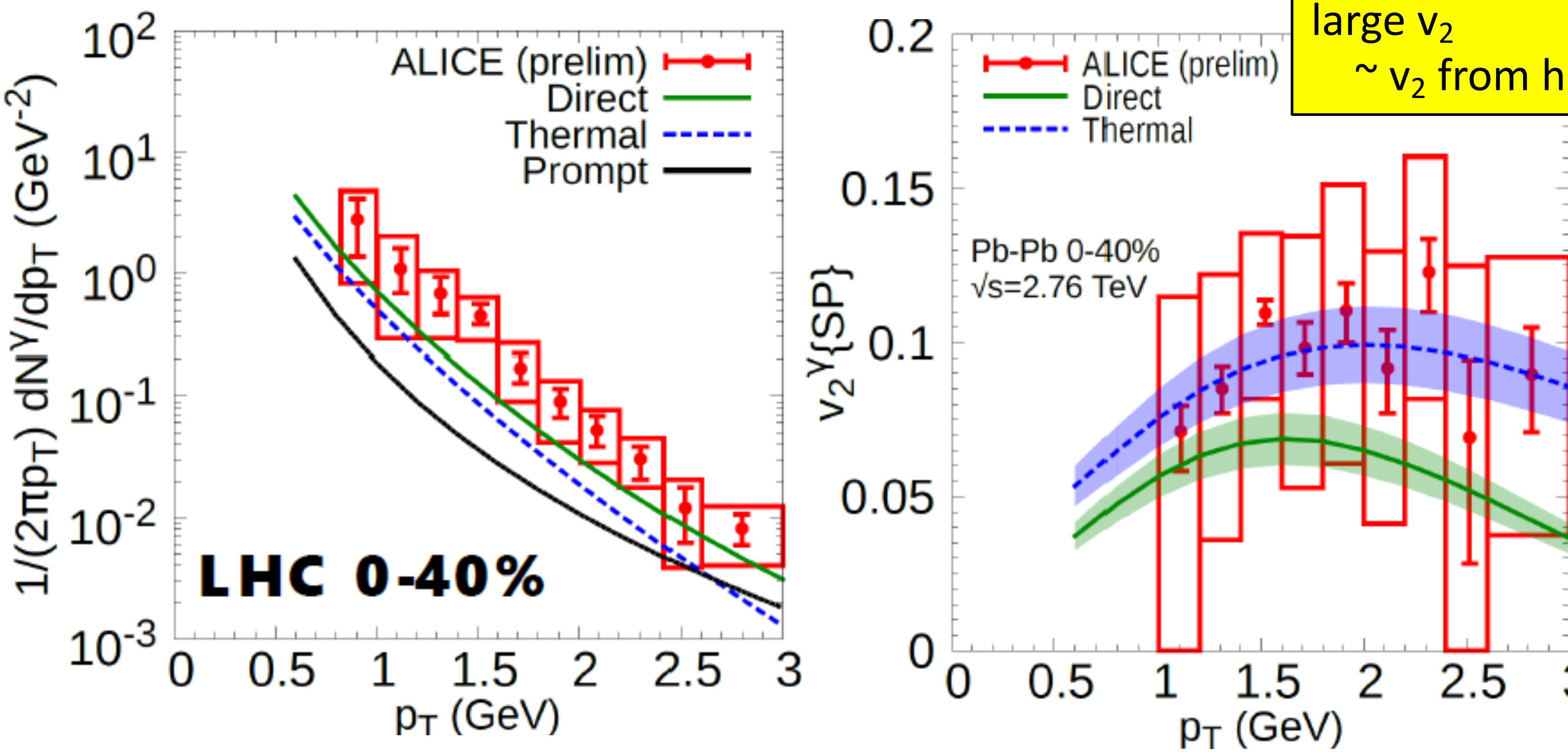
Poster by Okamoto

Photon Puzzle

Itakura, Fuji and CN in preparation

- LHC Pb+Pb $\sqrt{s_{NN}} = 2760$ GeV

large v_2
~ v_2 from hadrons



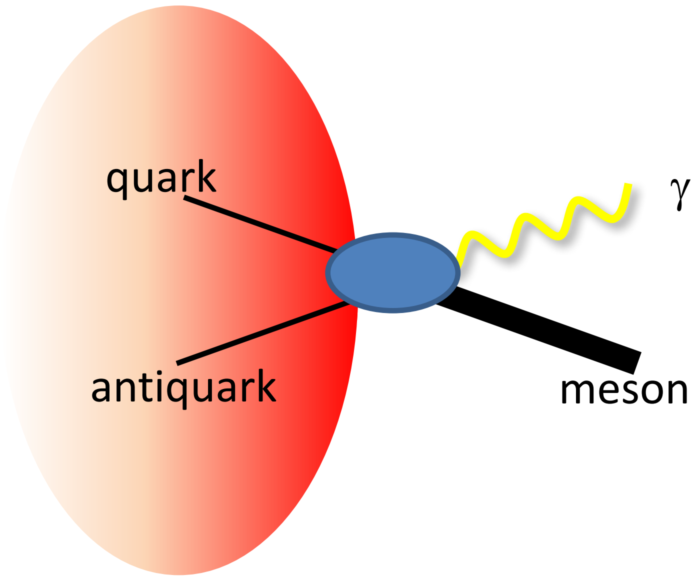
A hydrodynamic model with latest hadronic photon rates can reproduce elliptic flow reasonably, but it fails to explain the yield of photon.

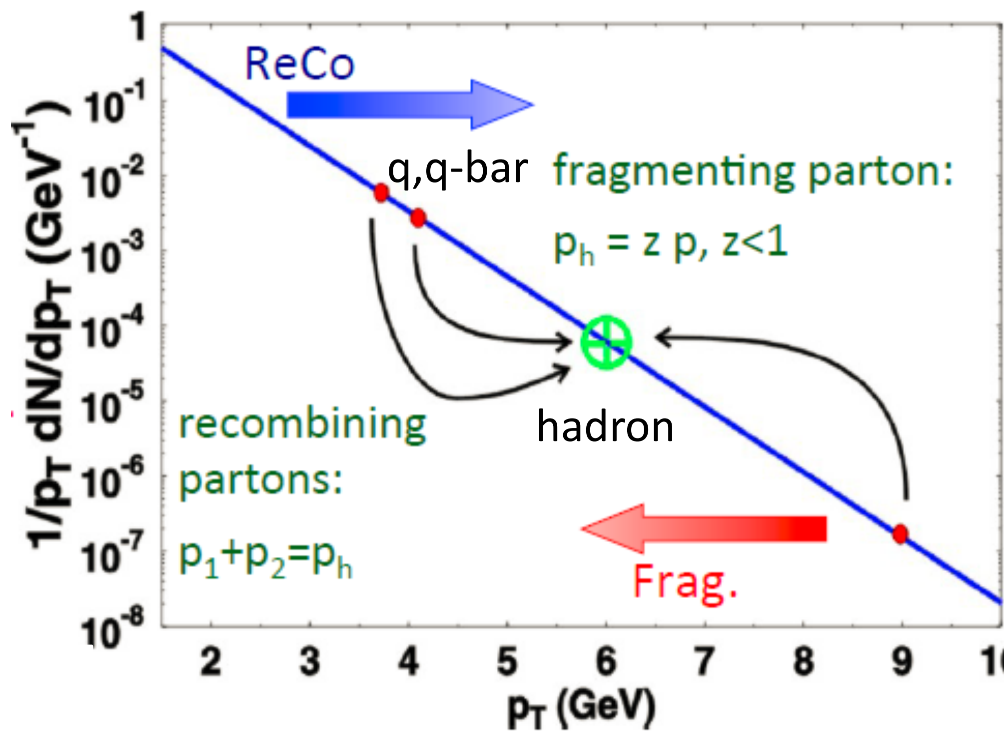
We miss other photon production mechanisms?

Radiative Recombination

Itakura, Fuji and CN in preparation

- Recombination Model





Fries, Mueller, Bass, CN, PRL2003, PRC2003

Solution to

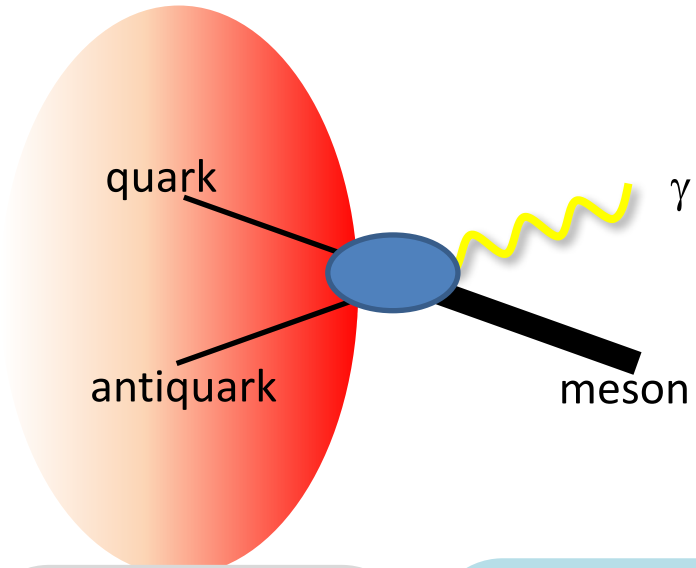
- Baryon anomaly in baryon meson ratio, nuclear modification factor
- Quark number scaling in elliptic flow

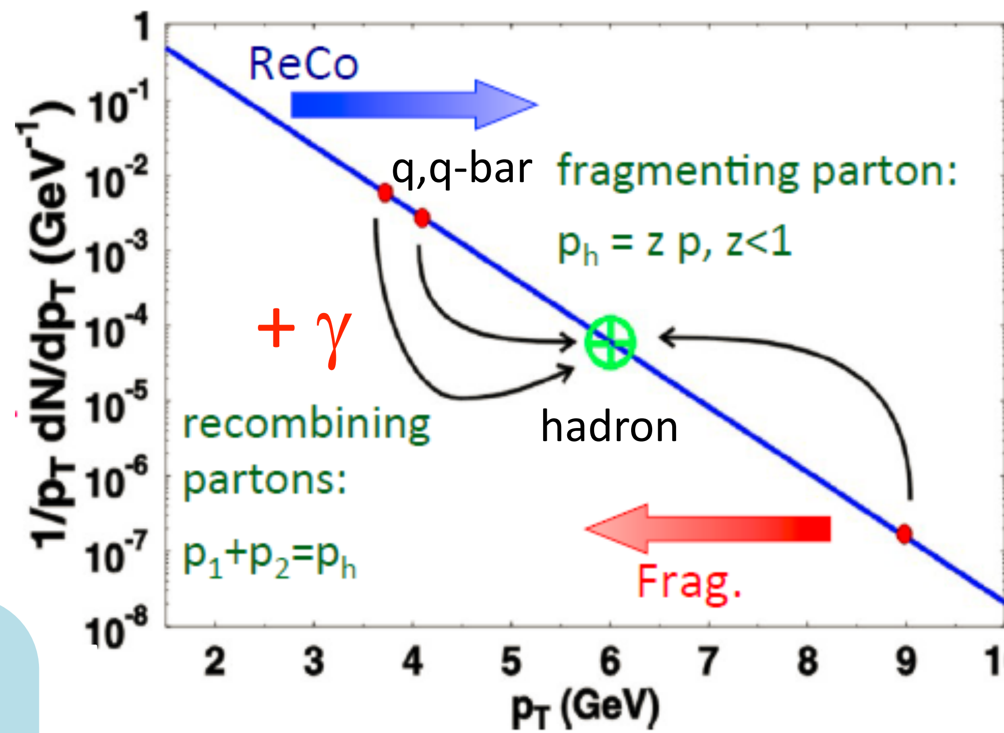
✗ Energy and entropy conservation

Radiative Recombination

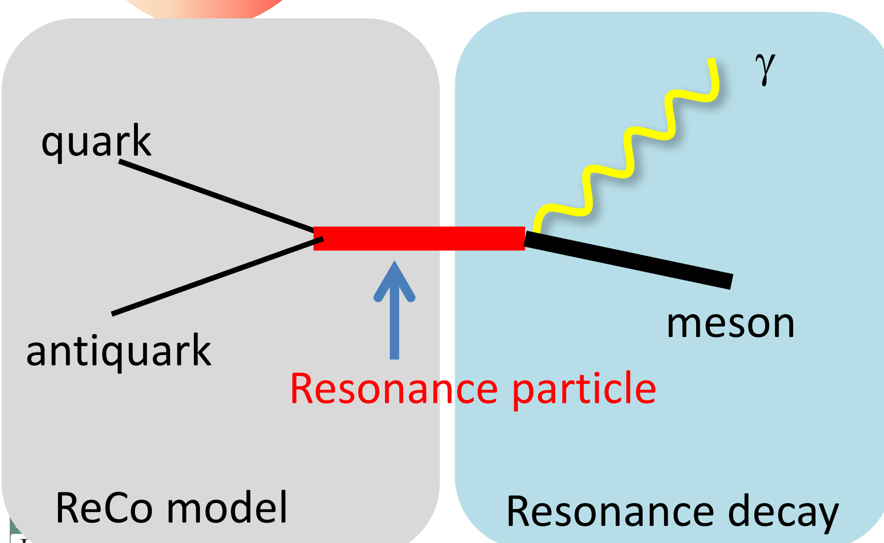
Itakura, Fuji and CN in preparation

- Recombination Model





Fries, Mueller, Bass, CN, PRL2003, PRC2003



Solution to

- Baryon anomaly in baryon meson ratio, nuclear modification factor
- Quark number scaling in elliptic flow

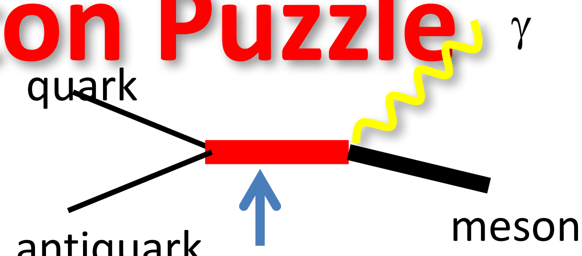
✗ Energy and entropy conservation

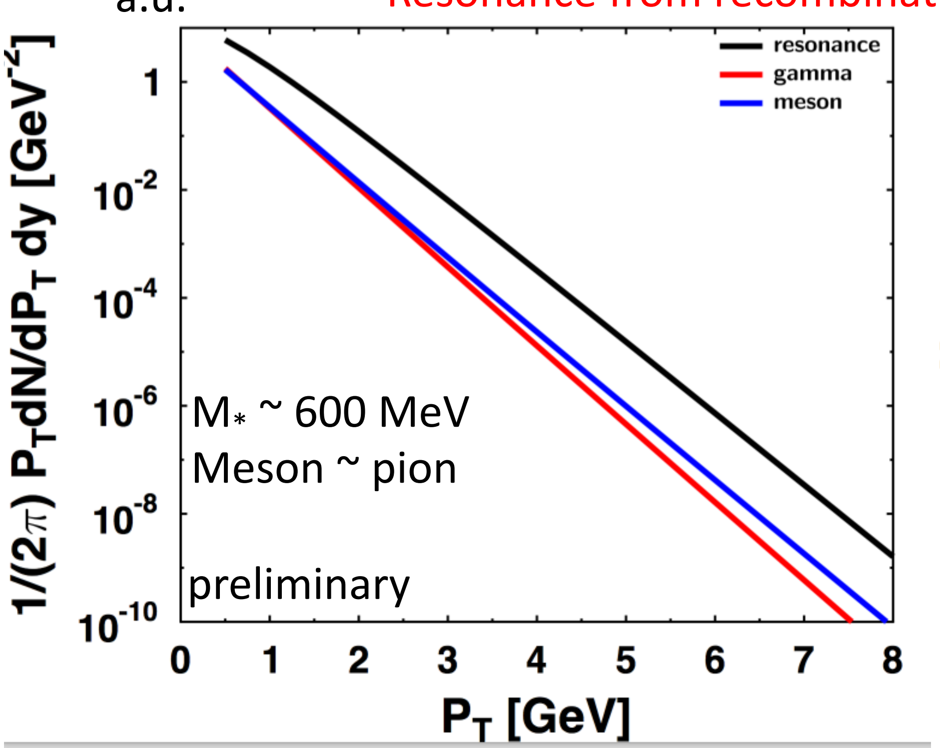
Possible Solution to Photon Puzzle

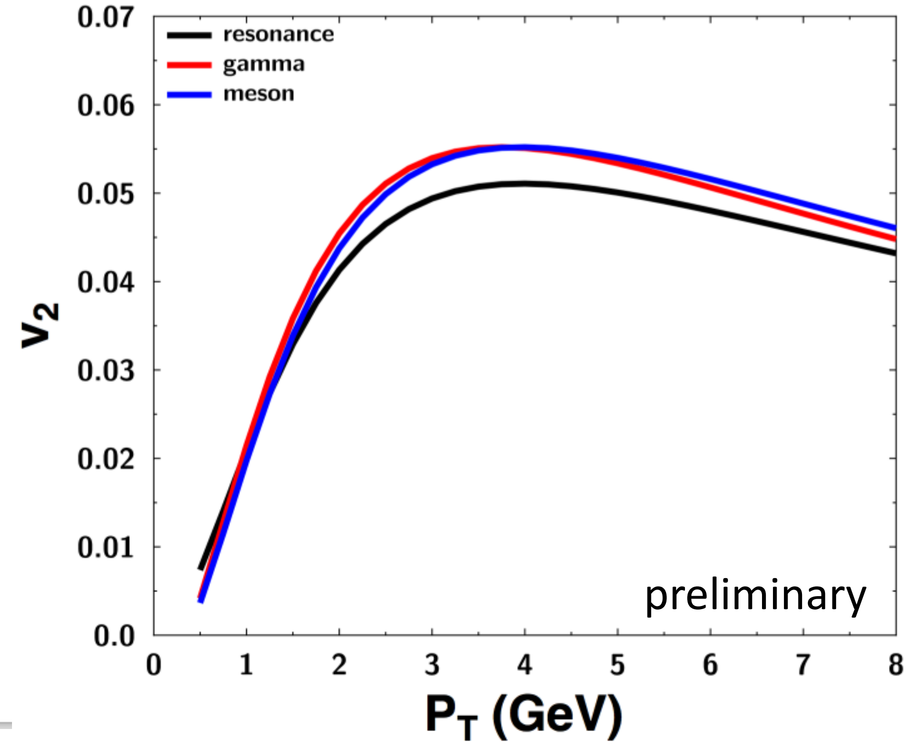
Itakura, Fuji and CN in preparation

$$E_\gamma \frac{dN_\gamma}{d^3\boldsymbol{k}_\gamma} = \kappa \int dM_* \rho(M_*) \int \frac{d^3\boldsymbol{P}_{M_*}}{E_{M_*}} \left(E_{M_*} \frac{dN_{M_*}}{d^3\boldsymbol{P}_{M_*}} \right) \left(\epsilon_\gamma \frac{dn_\gamma(k,\theta)}{d^3\boldsymbol{k}_\gamma} \right)$$

Resonance from recombination







κ=1 (parameter)
Large yield of photon

- v2 of photon is as large as that of meson!
- v2 of photon from resonance decay is larger that that of resonance.

Summary

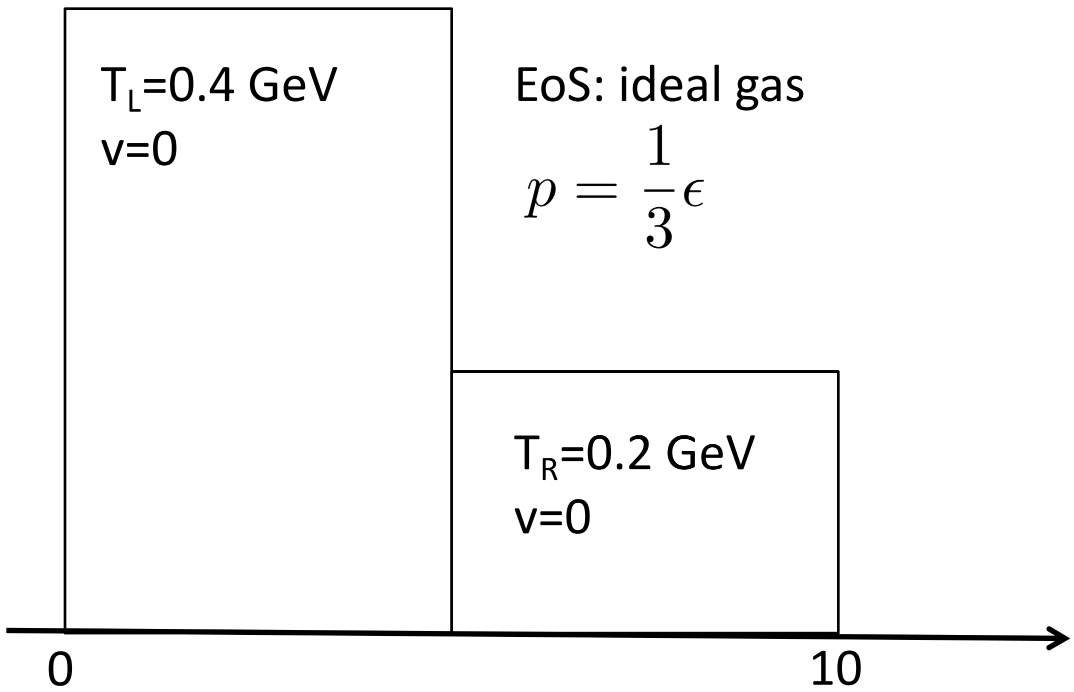
- Hydrodynamic model with state-of-the-art algorithm

Our algorithm

  - Less artificial diffusion: crucial for viscosity analyses
  - Stable for strong shock wave
  - Test calculations: 1D and 3D
  - Kelvin-Helmholtz Instability
  - Now we are ready for experimental analyses @ RHIC and LHC !
    Initial condition (TRENTO by Duke), statistical analyses
  - Future low collision energy experiments at J-PARC and FAIR
- Photon puzzle: large P_T distribution and elliptic flow
  - Radiative Recombination <- Extension of recombination model
    Energy conservation, Deviation of quark number scaling @LHC

Comparison

- Shock Tube Test : *Molnar, Niemi, Rischke*, Eur.Phys.J.C65,615(2010)

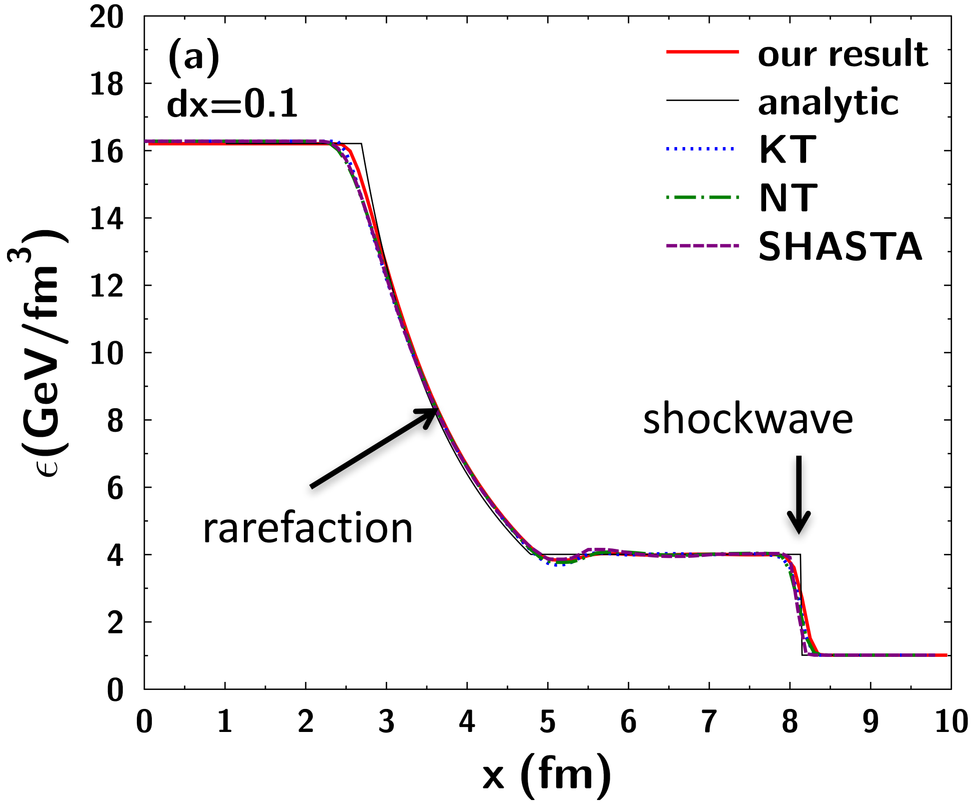


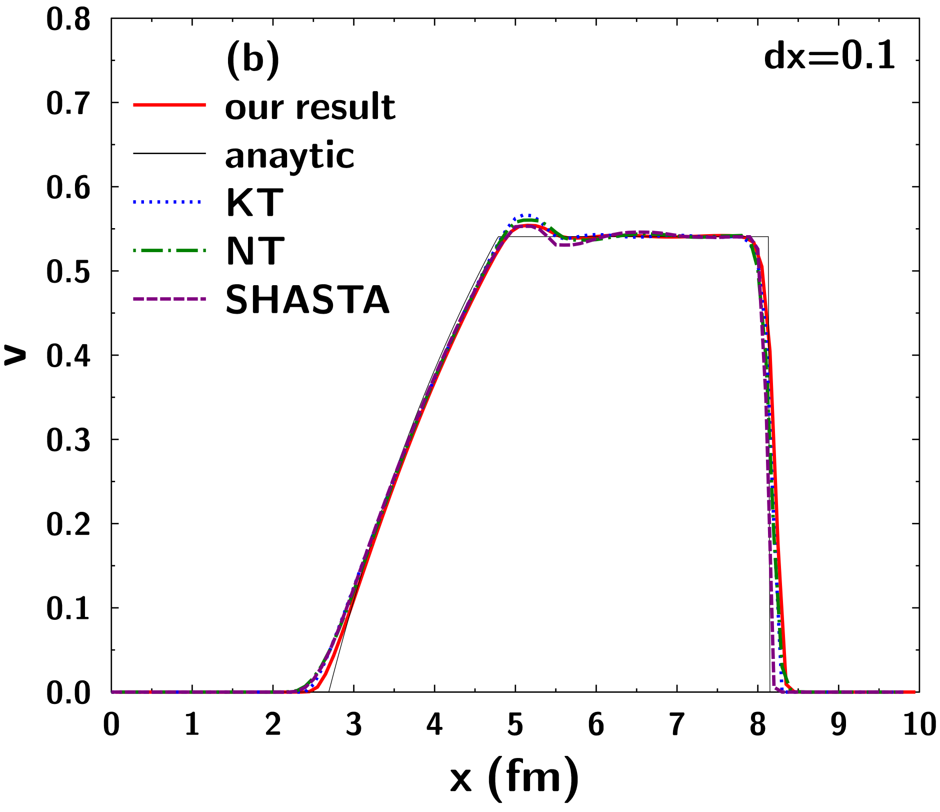
Nx=100, dx=0.1, dt=0.04

- Analytical solution
- Numerical schemes
  SHASTA, KT, NT
  Our scheme

Shocktube Problem

- Ideal case



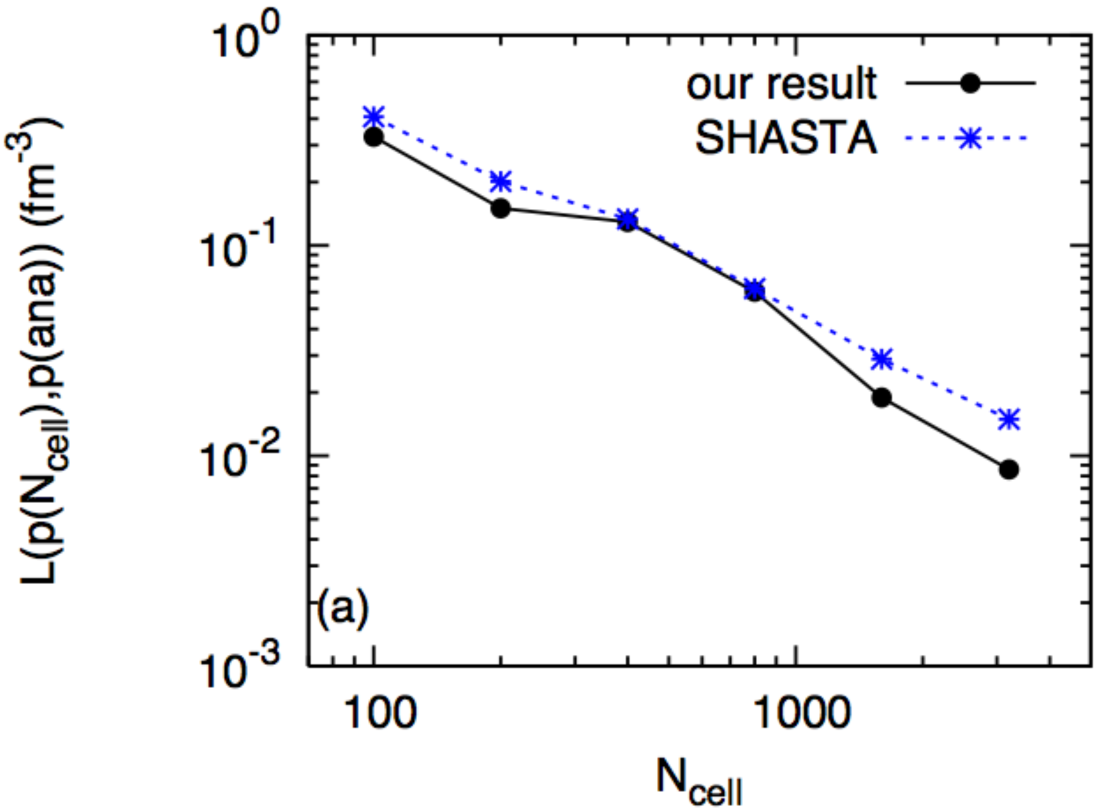


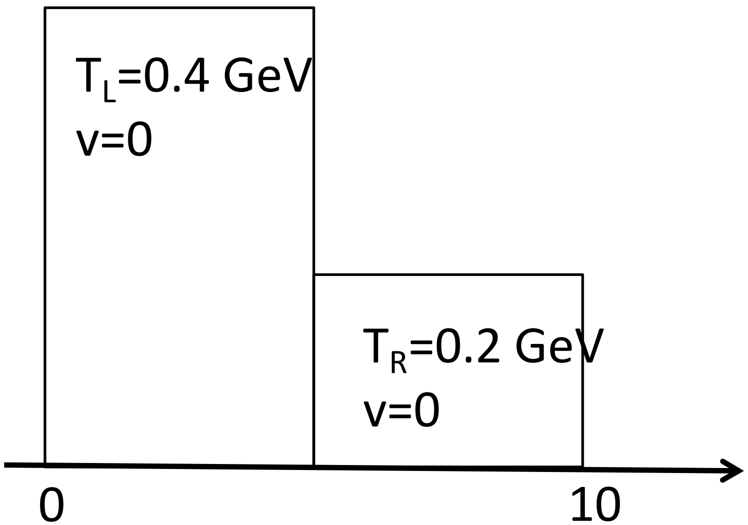
Nx=100, dx=0.1, dt=0.04

C. NONAKA

L1 Norm

- Numerical dissipation: deviation from analytical solution





For analysis of heavy ion collisions

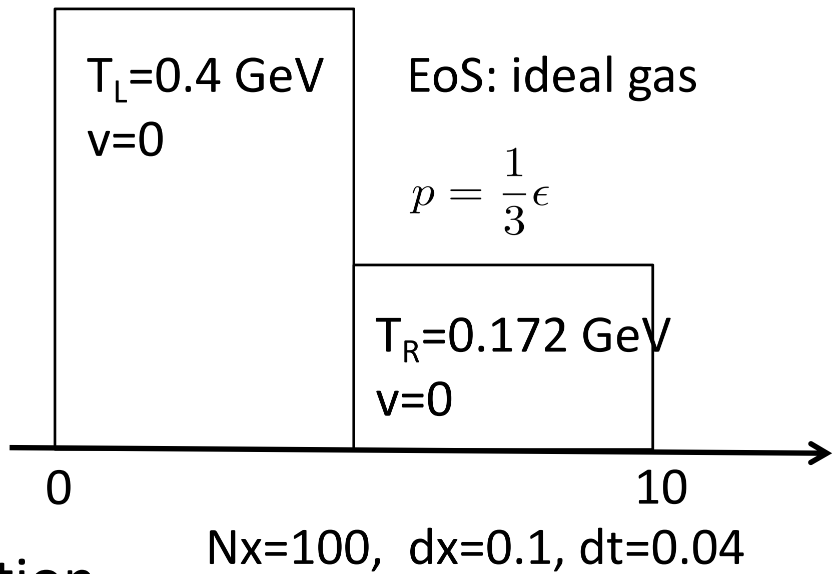
N_{cell}=100: dx=0.1 fm

$$L(p(N_{\rm cell}), p({\rm anaytic})) = \sum_{i=1}^{N_{\rm cell}} |p(N_{\rm cell}) - p({\rm analytic})| \frac{\lambda}{N_{\rm cell}}$$

λ=10 fm

Large ΔT difference

- T_L=0.4 GeV, T_R=0.172 GeV
  - SHASTA becomes unstable.
  - Our algorithm is stable.

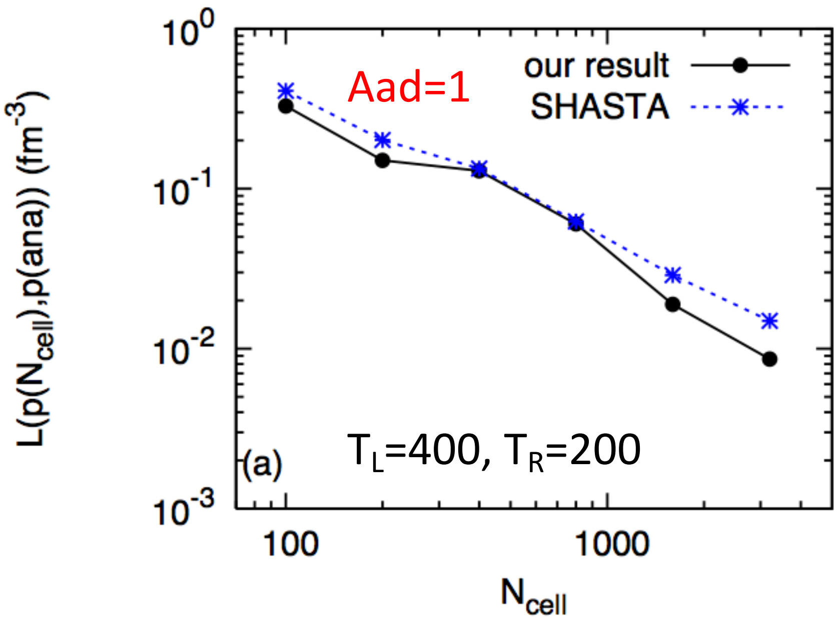


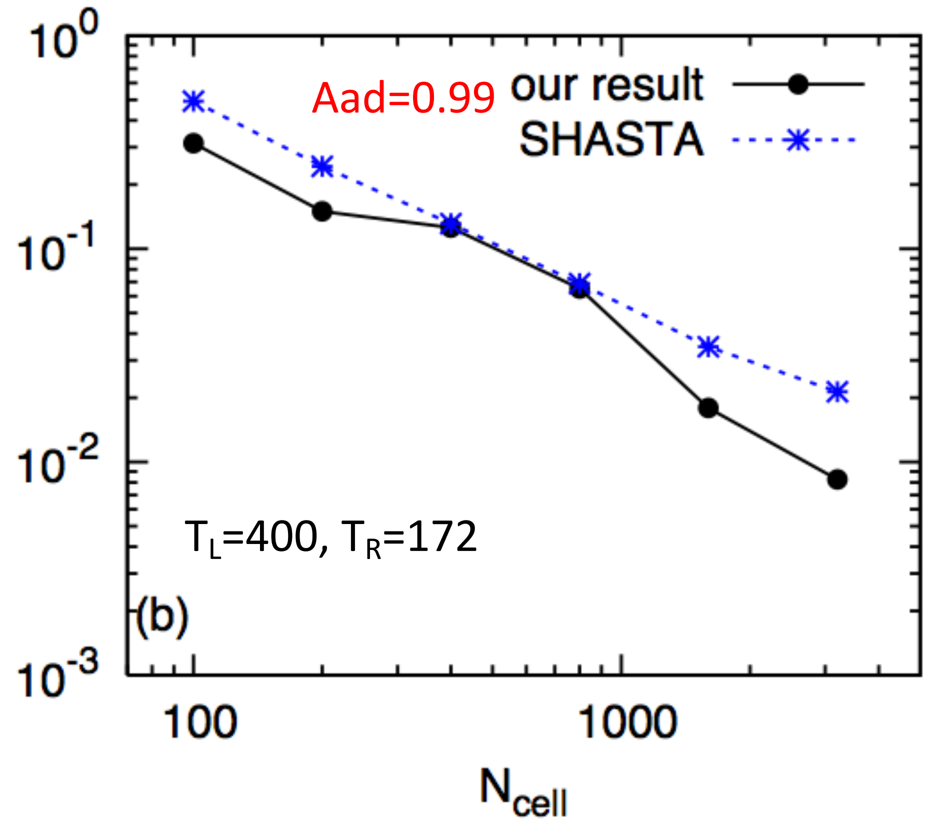
- SHASTA: anti diffusion term, A_{ad}
  - A_{ad} = 1 : default value
  - A_{ad} =0.99: stable,
    more numerical dissipation
- Large fluctuation (ex initial conditions)
  - Our algorithm is stable even with small numerical dissipation.

C. NONAKA

L1 norm

- SHASTA with small A_{ad} has large numerical dissipation





$$L(p(N_{\rm cell}), p({\rm anaytic})) = \sum_{i=1}^{N_{\rm cell}} |p(N_{\rm cell}) - p({\rm analytic})| \frac{\lambda}{N_{\rm cell}}$$

λ=10 fm

C. NONAKA

Longitudinal Fluctuations

- Propagation of longitudinal fluctuations around Bjorken's flow

$$e = e_B + \delta e, \quad w^\eta = \delta w^\eta,$$

$$\partial_\tau \delta e + (1+\lambda) e_B \partial_\eta \delta w^\eta + \frac{1+\lambda}{\tau} \delta e = 0,$$

$$\partial_\tau \delta w^\eta + \frac{\lambda}{1+\lambda} \frac{1}{\tau^2 e_B} \partial_\eta \delta e + \frac{2-\lambda}{\tau} \delta w^\eta = 0.$$

λ=1/3

e_B, $\omega^\eta=0$:
Bjorken's solution

$$D \equiv (1-\lambda)^2 - 4k^2\lambda$$

$D > 0$

$$\delta e(\tau,\eta) = A \left(\frac{\tau}{\tau_0}\right)^{(-3-\lambda-\sqrt{D})/2} \sin(k\eta),$$

$$\delta w^\eta(\tau,\eta) = \frac{\lambda - 1 - \sqrt{D}}{2ke_0(1+\lambda)\tau_0} A \left(\frac{\tau}{\tau_0}\right)^{(-3+\lambda-\sqrt{D})/2} \cos(k\eta),$$

$D < 0$

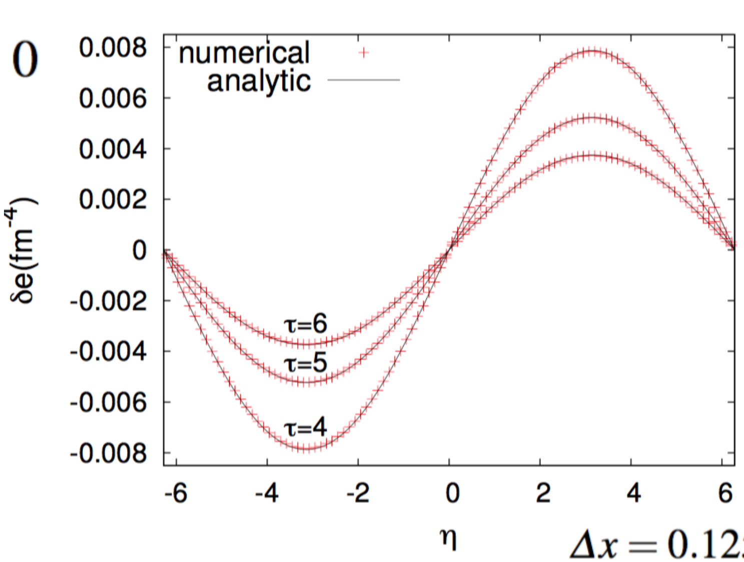
$$\delta e(\tau,\eta) = A \left(\frac{\tau}{\tau_0}\right)^{-(3+\lambda)/2} \sin(k\eta - \theta),$$

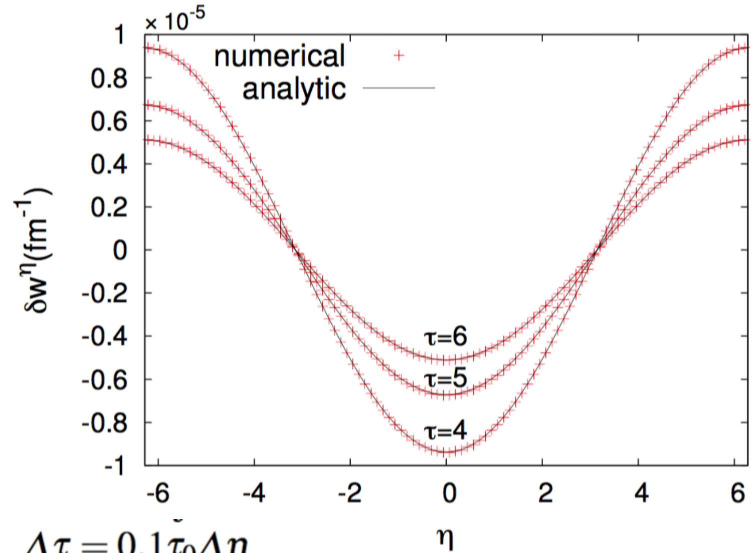
$$\delta w^\eta(\tau,\eta) = \frac{A}{2ke_0(1+\lambda)\tau_0} \left(\frac{\tau}{\tau_0}\right)^{(\lambda-3)/2} \times \left[(\lambda-1)\cos(k\eta-\theta) + \sqrt{-D}\sin(k\eta-\theta)\right],$$

$$\theta \equiv \tfrac{1}{2}\sqrt{-D}\log(\tau/\tau_0).$$

Consistent with Analytical Solutions

$D > 0$

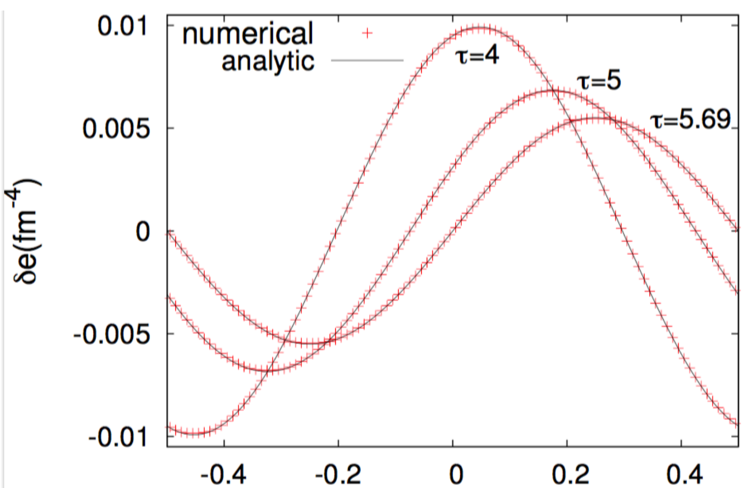


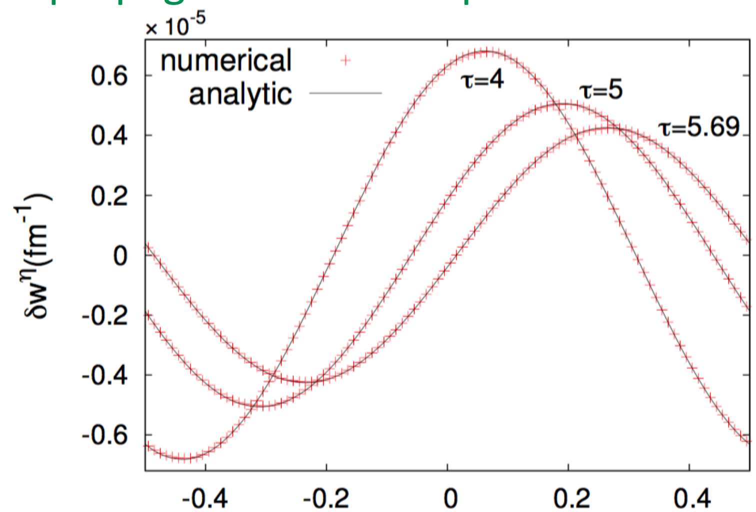


$\Delta x = 0.1256 \text{ fm} \quad \Delta\tau = 0.1\tau_0\Delta\eta$

Fluctuation dose not propagate and its amplitude decreases.

$D < 0$





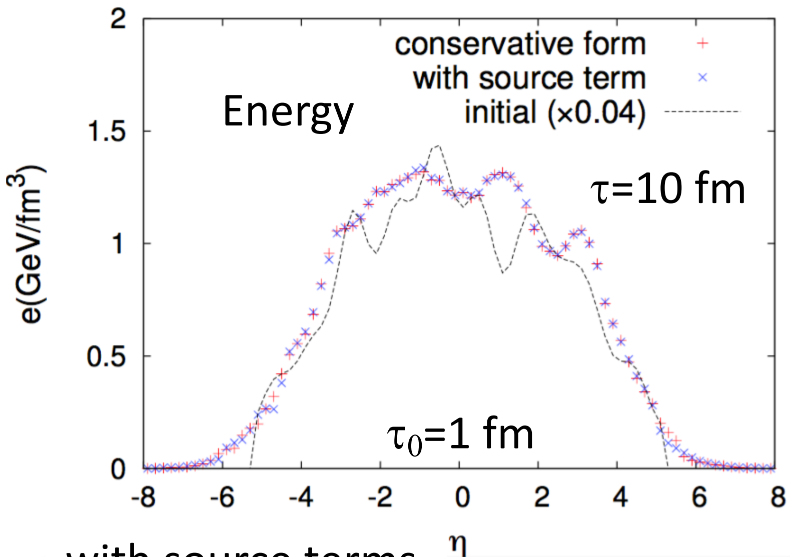
$\Delta x = 0.01 \text{ fm} \quad \Delta\tau = 0.1\tau_0\Delta\eta$

Fluctuation propagates and its amplitude decreases.

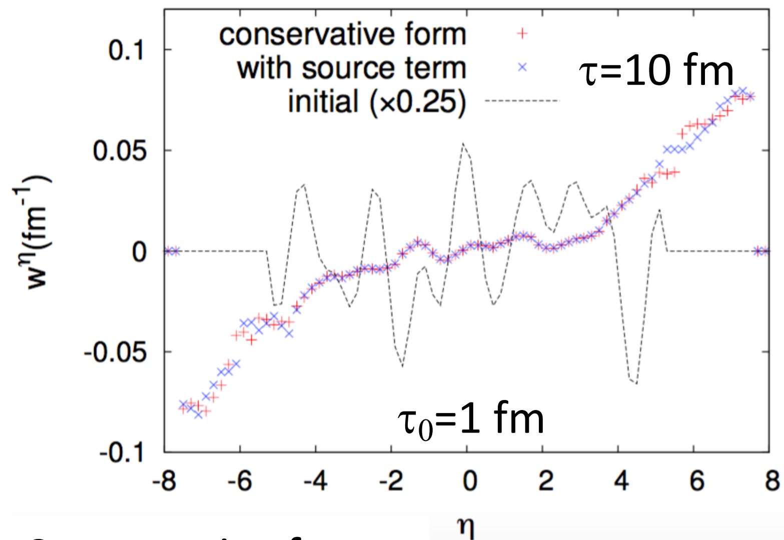
Conservation Property

Okamoto, Akamatsu, CN, arXiv:1607.03630

- Fluctuating initial conditions



Flow



with source terms

$$\partial_\tau T^{\tau\eta} + \partial_i T^{i\eta} + \partial_\eta T^{\eta\eta} = -3T^{\tau\eta}/\tau,$$
$$\partial_\tau T^{\tau\tau} + \partial_i T^{i\tau} + \partial_\eta T^{\eta\tau} = -T^{\tau\tau}/\tau - \tau T^{\eta\eta}.$$

Conservative form

$$\partial_\tau(\tau T^{\tau z}) + \partial_i(\tau T^{iz}) + \partial_\eta(\tau T^{\eta z}) = 0,$$
$$\partial_\tau(\tau T^{\tau t}) + \partial_i(\tau T^{it}) + \partial_\eta(\tau T^{\eta t}) = 0.$$

Sum of violation of conservation $\Delta\eta = 0.2$ $\Delta\tau = 0.1\tau_0\Delta\eta.$

	ε_E	ε_M
conservative	1.38E-09	8.59E-09
with souce	1.27E-02	5.61E-02

The code based on the conservative form keeps conservation property with high accuracy

KIMKMI C. NONAKA

Our Approach

- Israel-Stewart Theory

Takamoto and Inutsuka, arXiv:1106.1732
Akamatsu, Inutsuka, CN, Takamoto, arXiv:1302.1665

(ideal hydro)

1. dissipative fluid dynamics = advection + dissipation

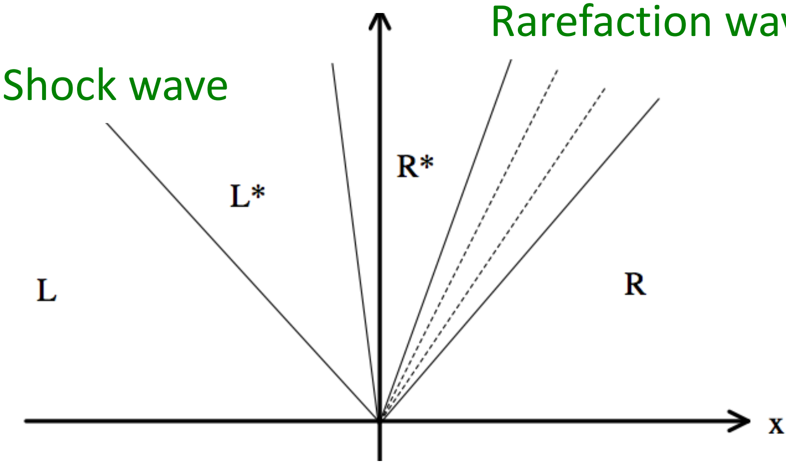
Riemann solver: Godunov method

exact solution

Contact discontinuity

Rarefaction wave

Shock wave



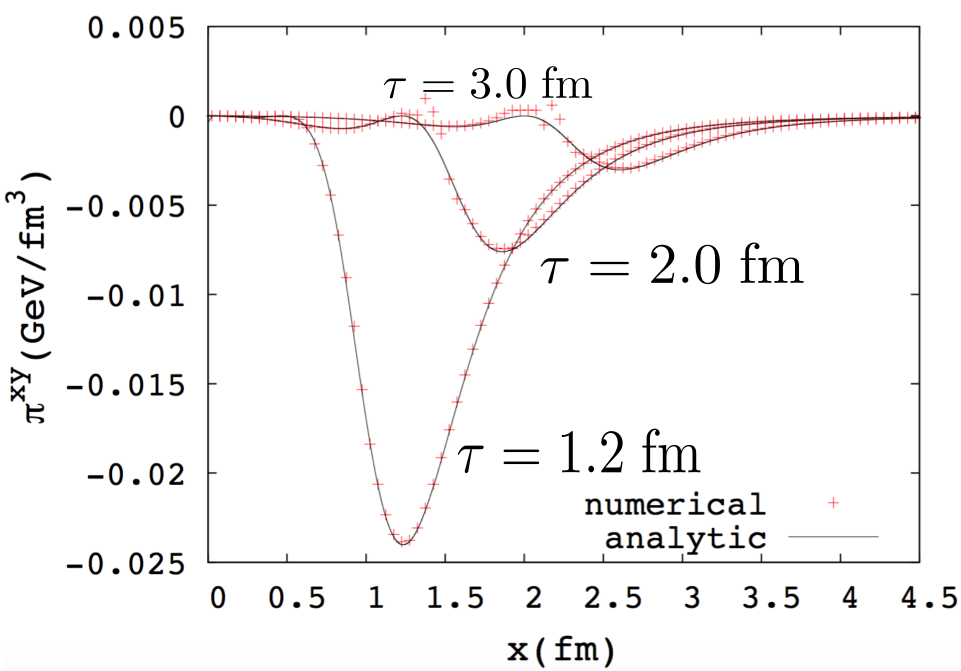
Two shock approximation

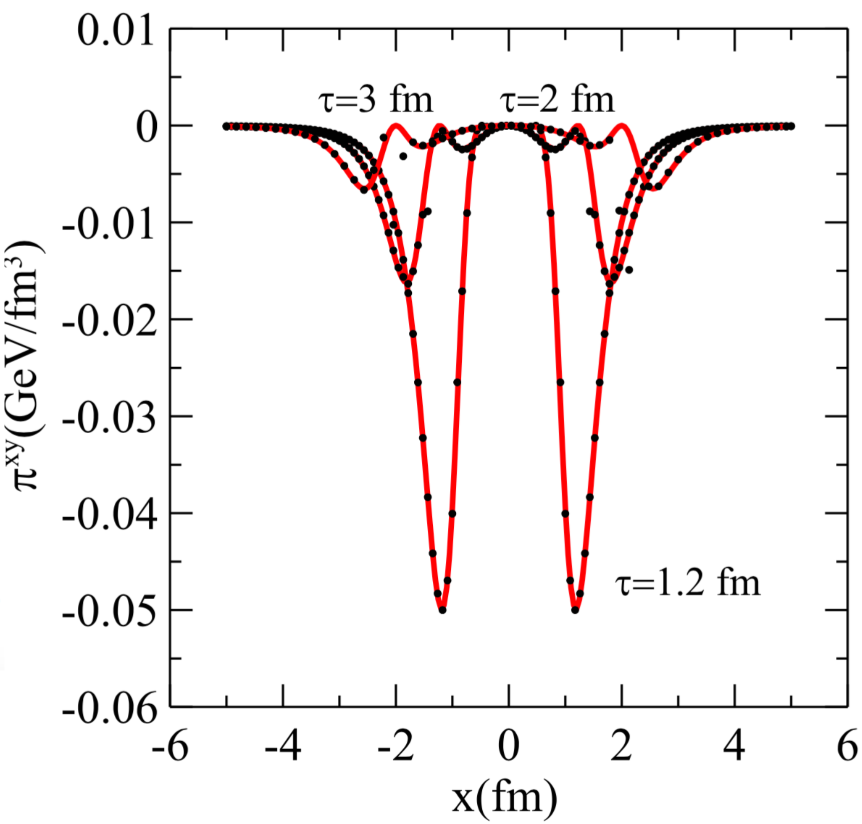
Mignone, Plewa and Bodo, Astrophys. J. S160, 199 (2005)

Rarefaction wave ⟶ shock wave

2. relaxation equation = advection + stiff equation

Gubser Flow with Finite η/s



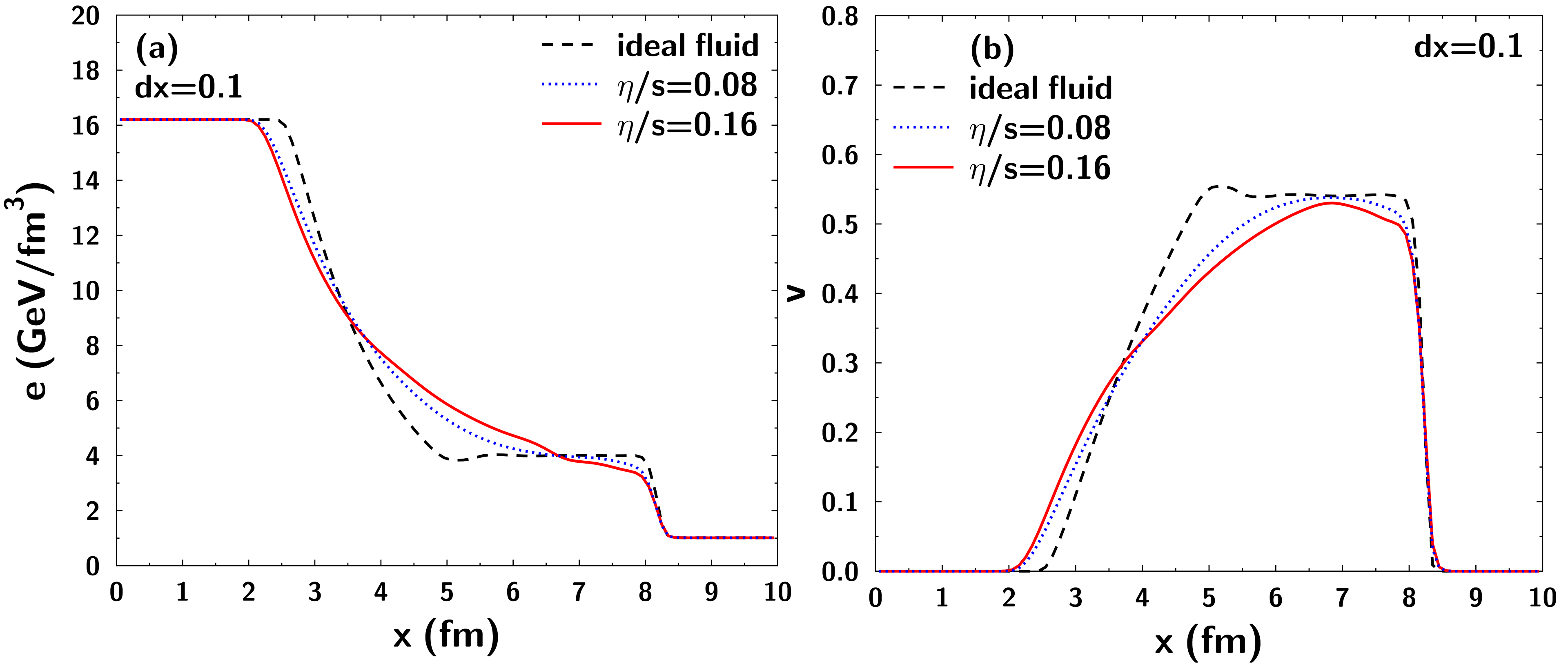


Same conditions in the calculation except for EoS.

MUSIC, Marrochio et al, PRC91, 014903(2015)

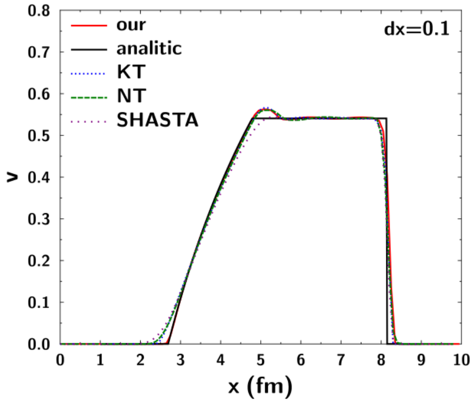
Shocktube problem

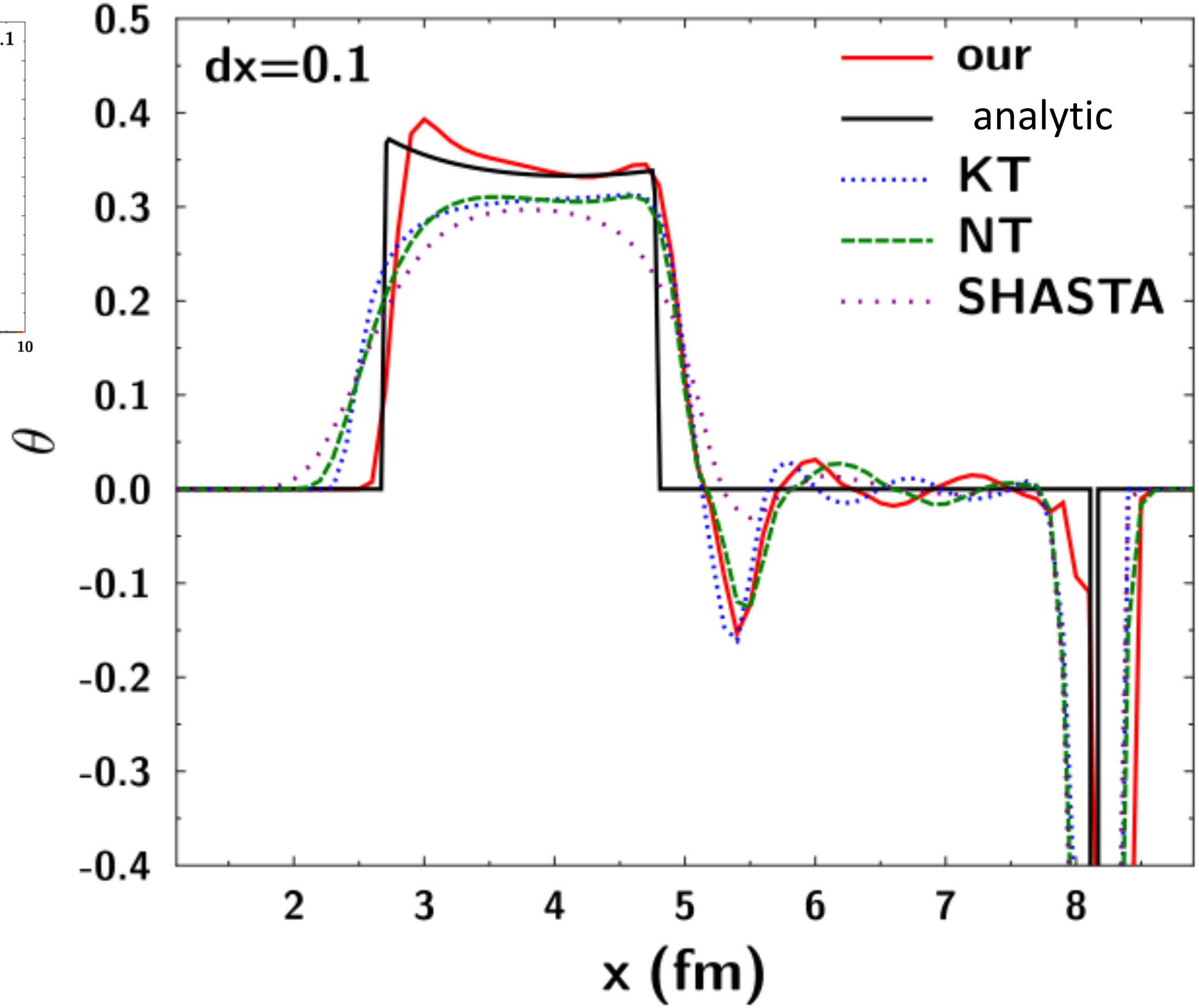
- Viscous case



θ

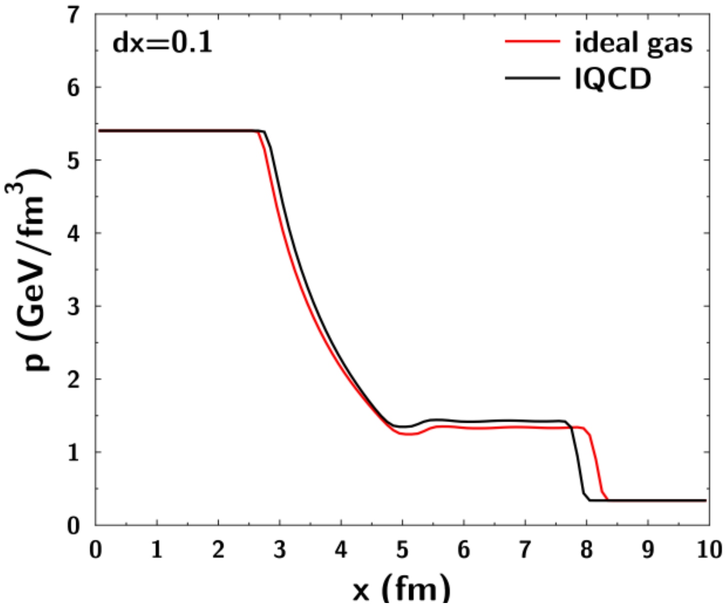
t=4.0 fm dt=0.04, 100 steps

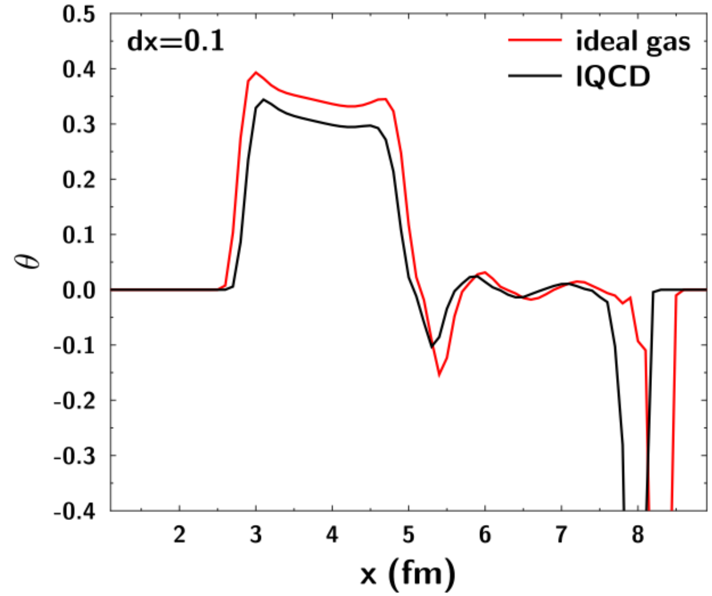


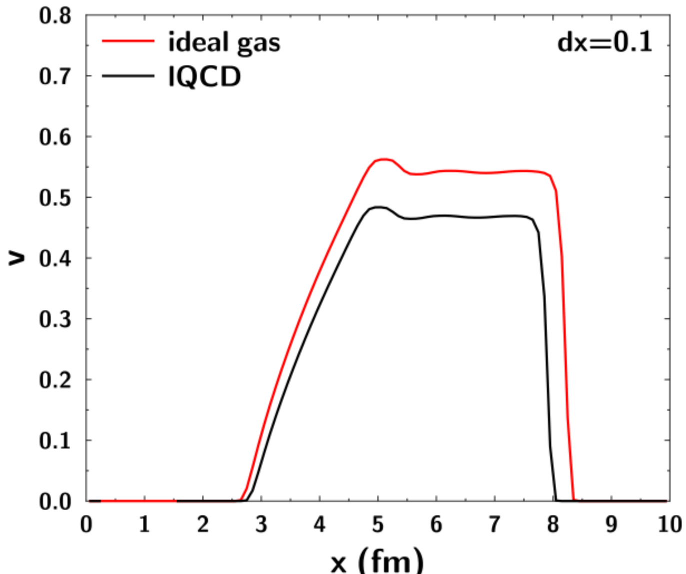


C. NONAKA

EoS Dependence





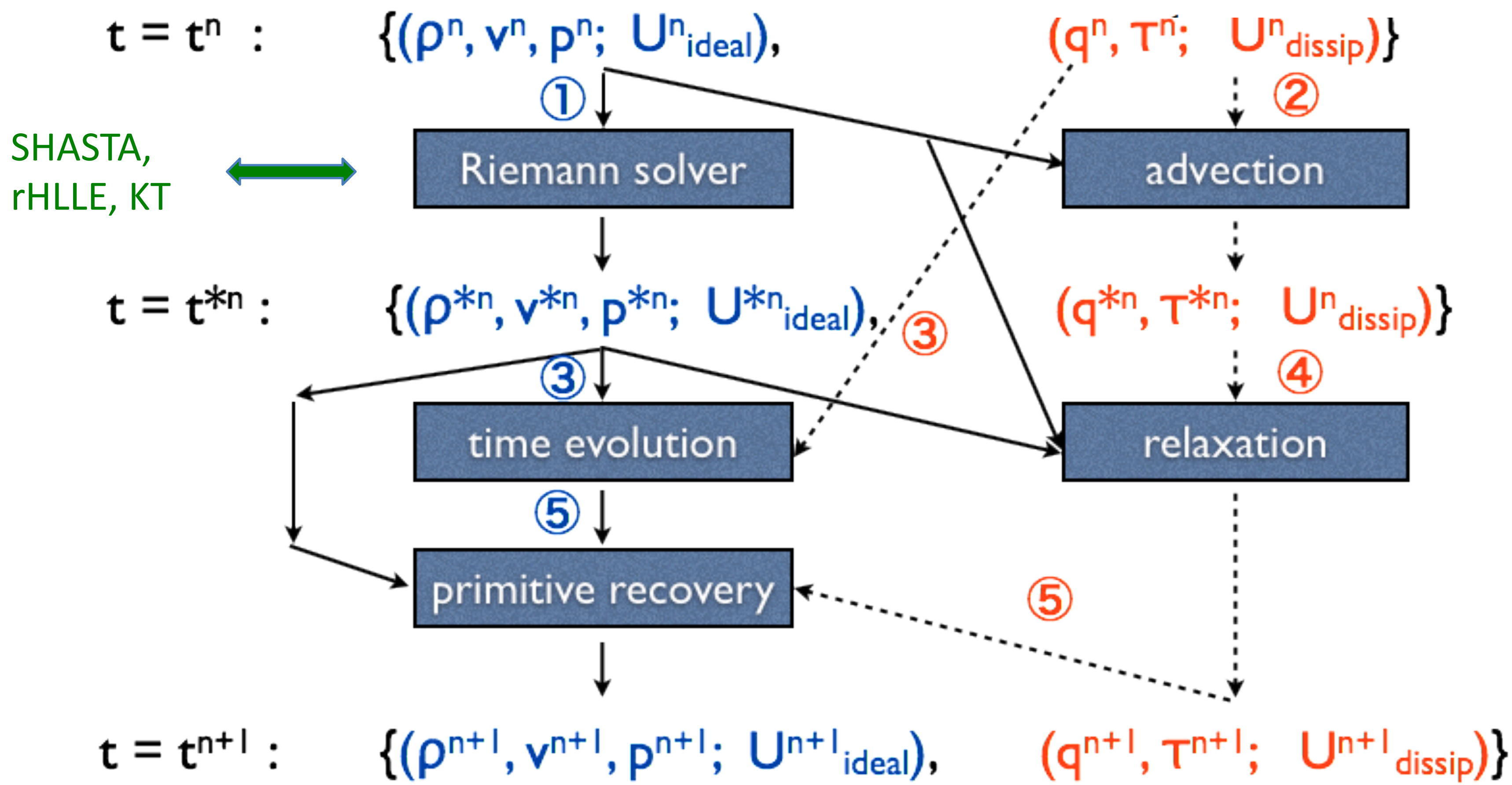


Numerical Method

Takamoto and Inutsuka, arXiv:1106.1732

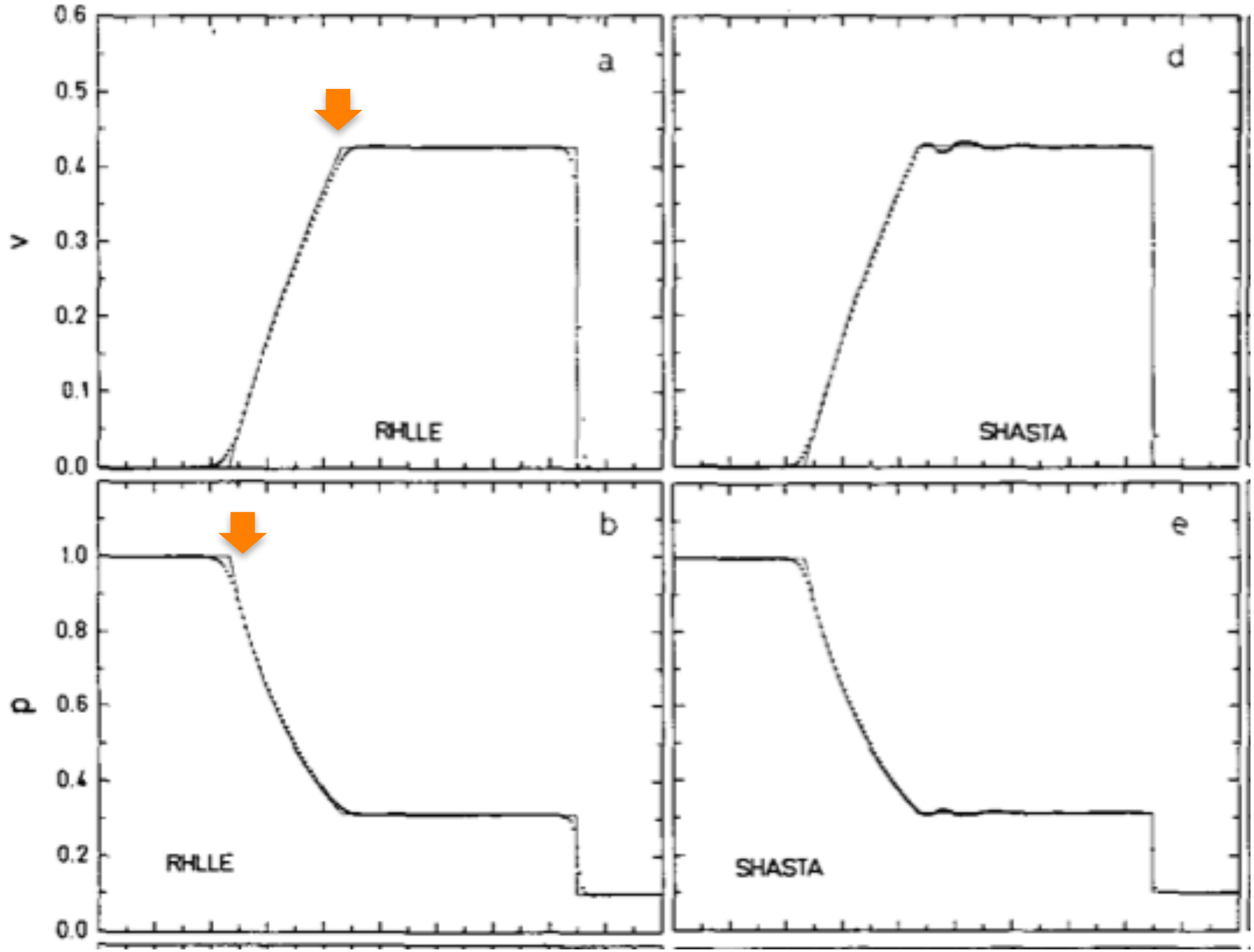
$$\partial_\mu T^{\mu\nu} = 0$$

$$\hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi,$$



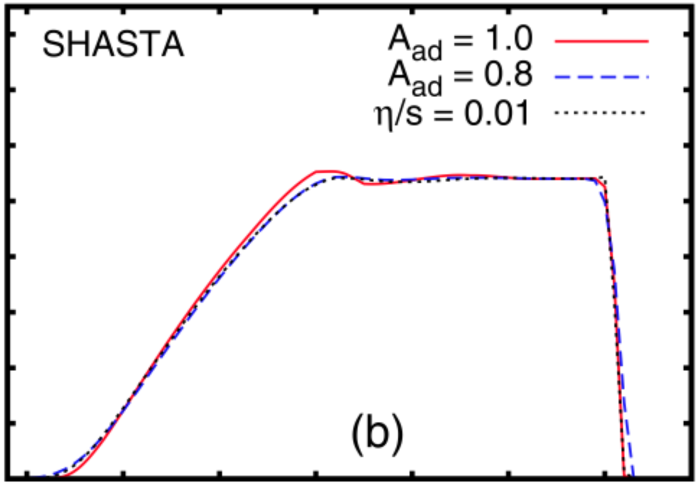
rHLLE vs SHASTA

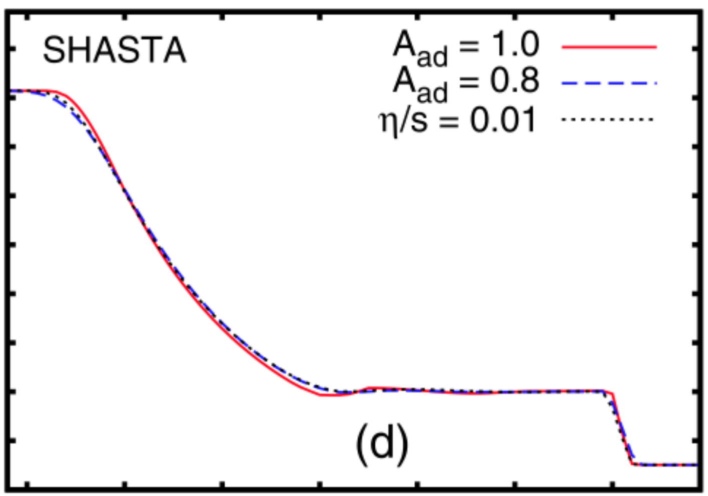
Schneider et al. J. Comp.105(1993)92

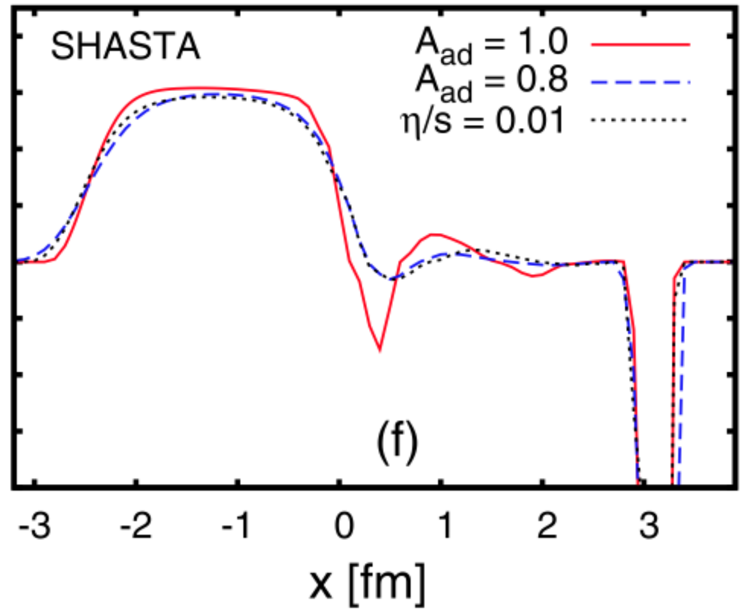


Artificial and Physical Viscosities

Molnar, Niemi, Rischke, Eur.Phys.J.C65,615(2010)





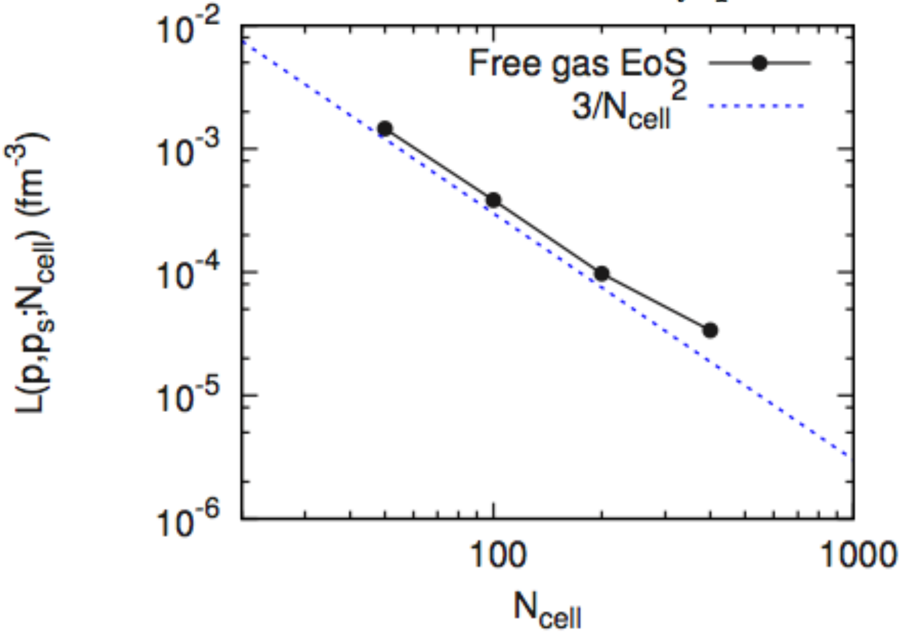


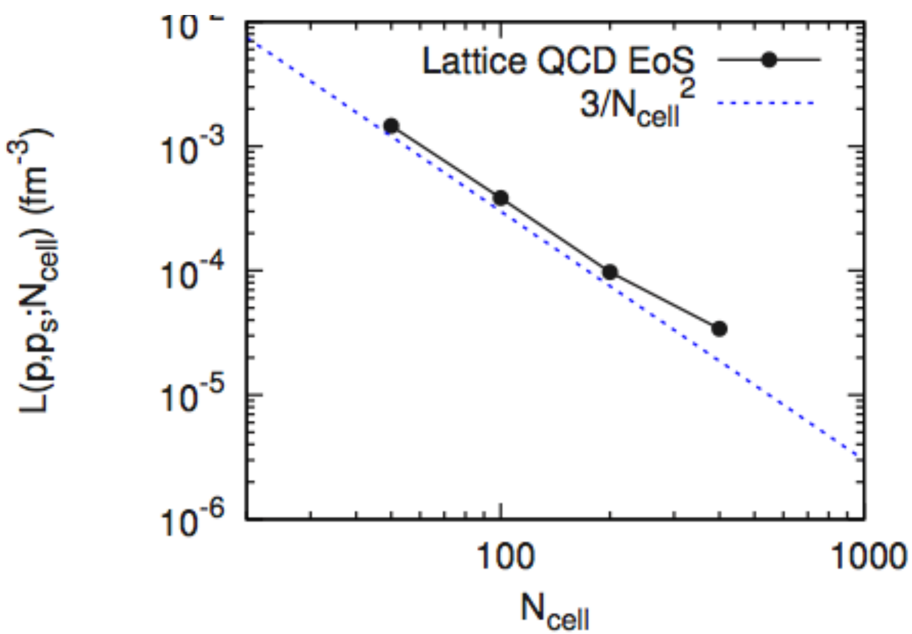
Antidiffusion terms : artificial viscosity stability

$$U_i^{n+1} = \tilde{U_i} - \tilde{A}_i + \tilde{A_{i-1}}$$
$$A_i = A\mathrm{ad}\tilde{\Delta}_i/8$$

Convergence Speed

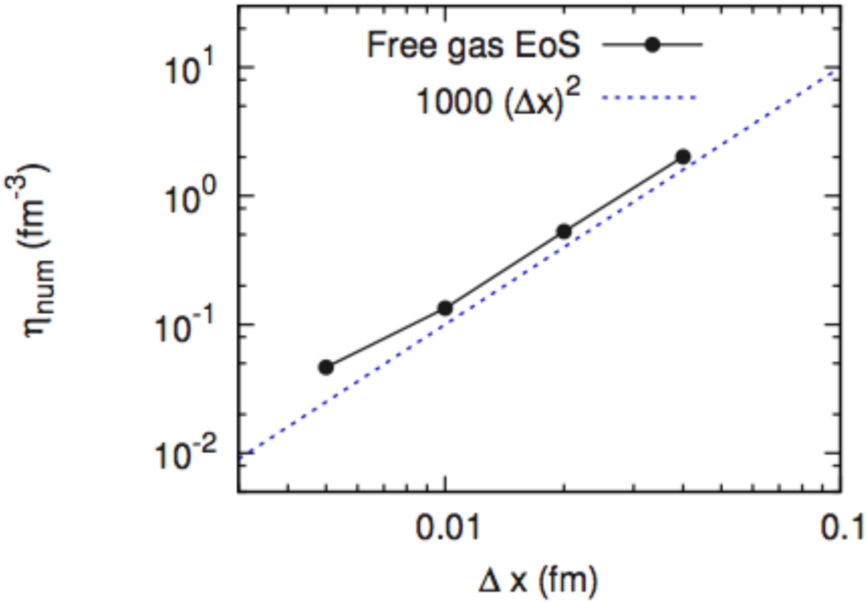
$$L(p, p_s; N_{\rm cell}) = \sum_{i=1}^{N_{\rm cell}} \mid p(x_i, \lambda/c_{s0}) - p_s(x_i, \lambda/c_{s0}) \mid \frac{\lambda}{N_{\rm cell}}$$

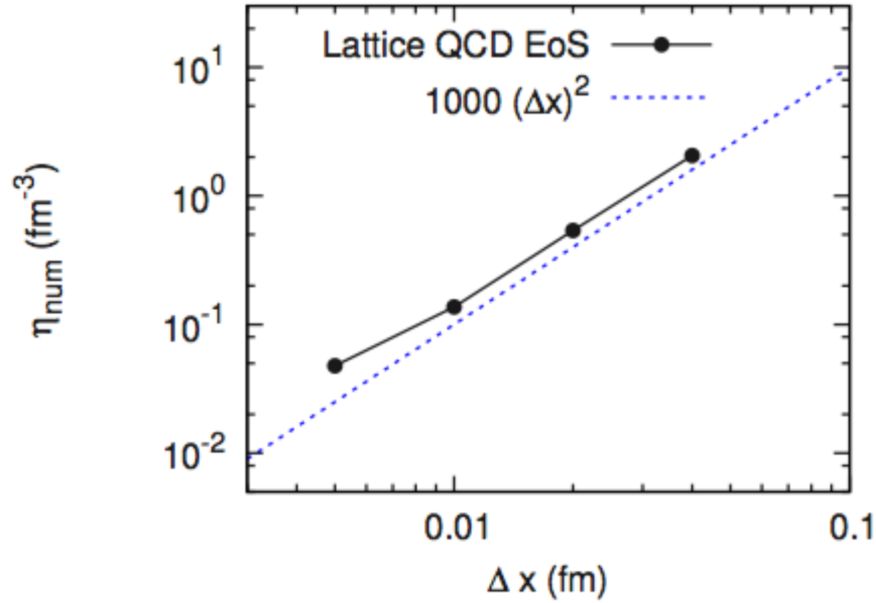




$L(p, p_s; N_{\rm cell}) \propto 1/N_{\rm cell}^2$ ➡ Space and time discretization
Second order accuracy

Numerical Dissipation





- numerical dissipation: $$\eta_{\rm num} = -\frac{3\lambda}{8\pi^2} c_{s0}(e_0 + p_0) \ln\left[1 - \frac{\pi}{2\lambda\delta p} L(p, p_s; N_{\rm cell})\right]$$

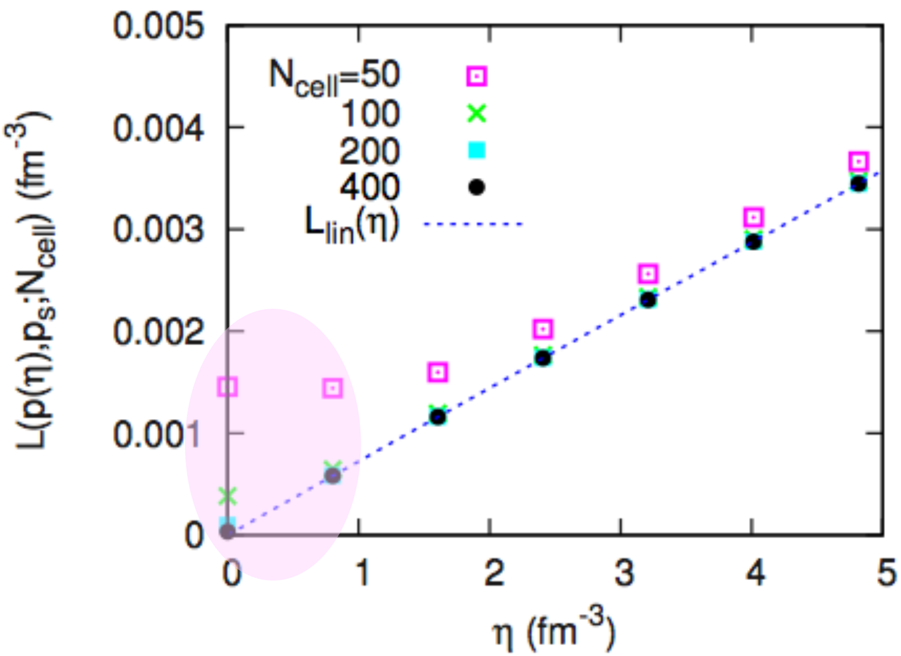
- from fit of calculated data

$$\eta_{\rm num} \approx 1000\,(\Delta x)^2 \quad \longrightarrow \quad \eta_{\rm num} \approx 1 \cdot \frac{c_{s0}(e_0 + p_0)}{\lambda} (\Delta x)^2$$

$$L(p, p_s; N_{\rm cell}) \propto \lambda\delta p / N_{\rm cell}^2 = (\delta p/\lambda) \cdot (\Delta x)^2$$

C. NONAKA

η_{num} vs Grid Size

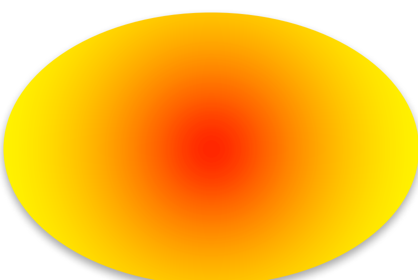


Numerical dissipation:

Deviation from linear analyses (L_{lin})

$$\eta_{\rm num} \approx 1 \cdot \frac{c_{s0}(e_0+p_0)}{\lambda}(\Delta x)^2$$

Ex. Heavy Ion Collisions



λ ~ 10 fm

$0.1<\eta/s<1$

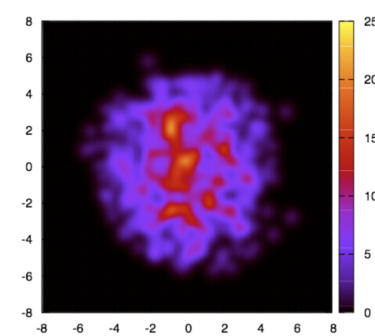
T=500 MeV

Δx << 0.8 – 2.6 fm

Fluctuating initial condition

λ ~ 1 fm

Δx << 0.25 – 0.82 fm

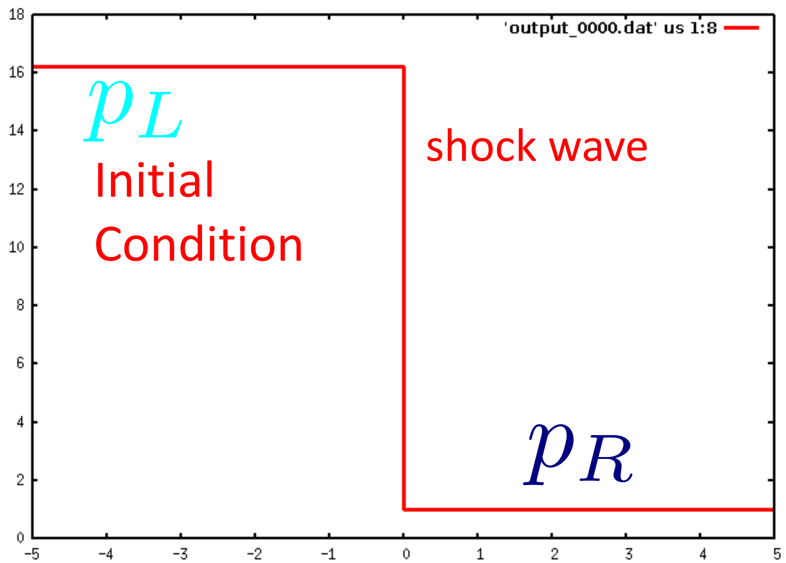


C. NONAKA

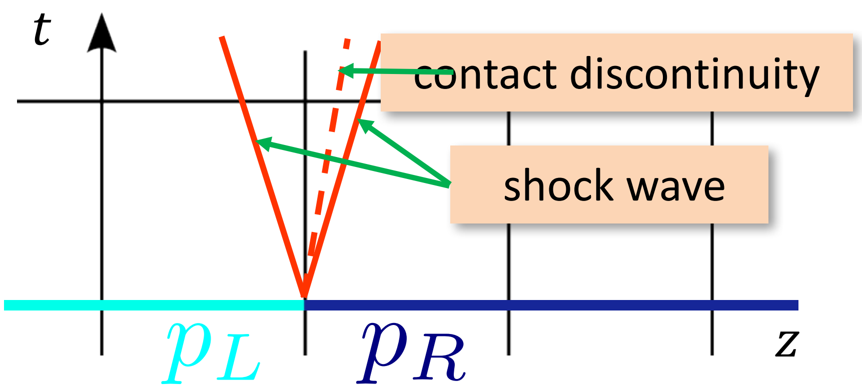
Riemann Solver in (τ,η)-coordinates

Okamoto, Akamatsu, CN, arXiv:1607.03630

- Riemann solution

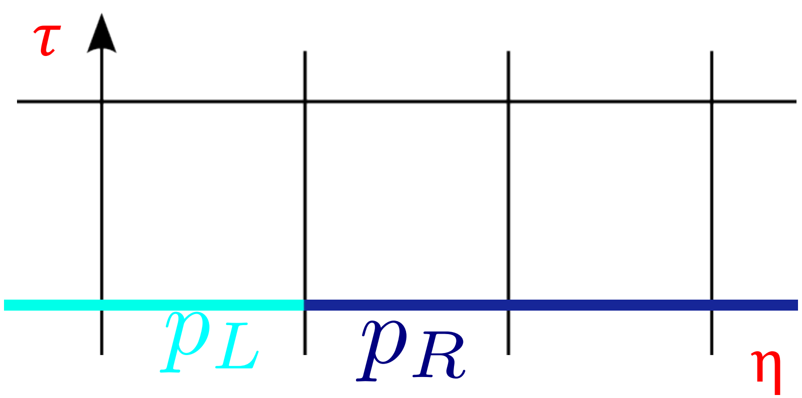


Cartesian



Pons, Marti, Muller, (2000)

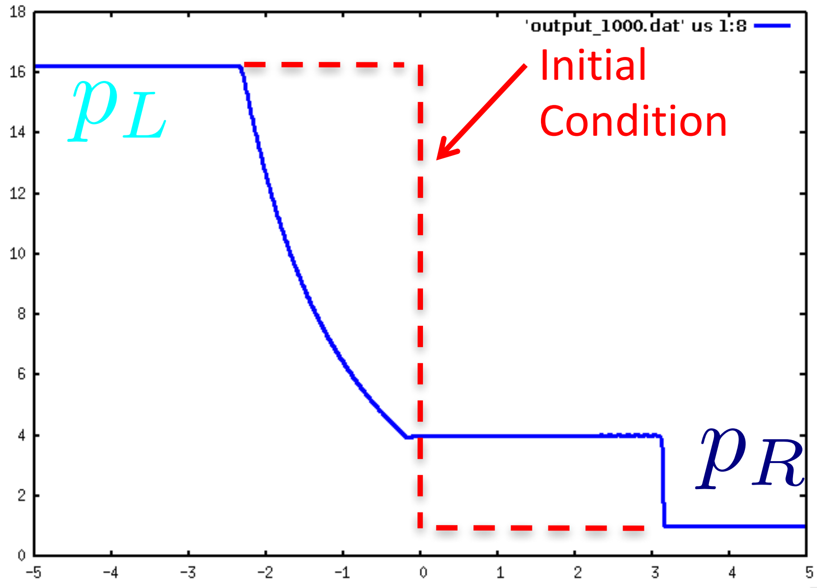
(τ,η) coordinate



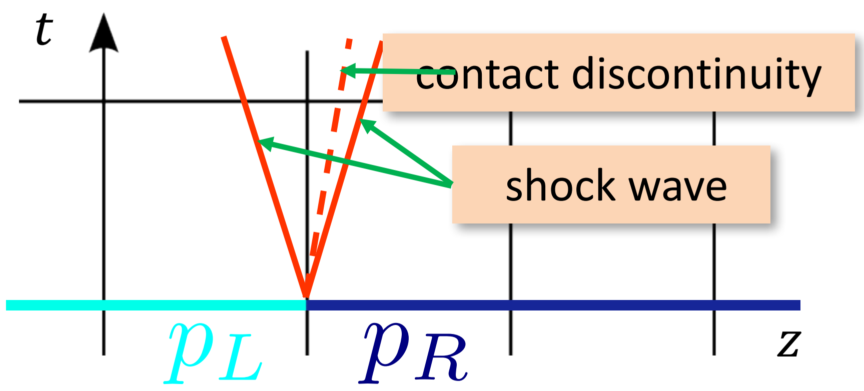
Riemann Solver in (τ,η)-coordinates

- Riemann solution

Okamoto, Akamatsu, CN, arXiv:1607.03630

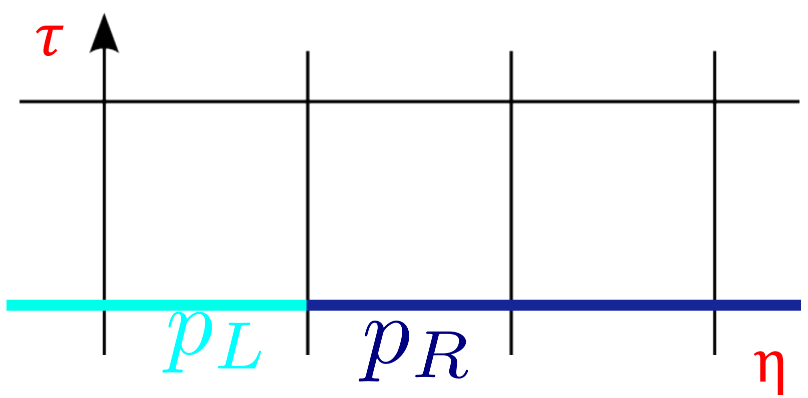


Cartesian



Pons, Marti, Muller, (2000)

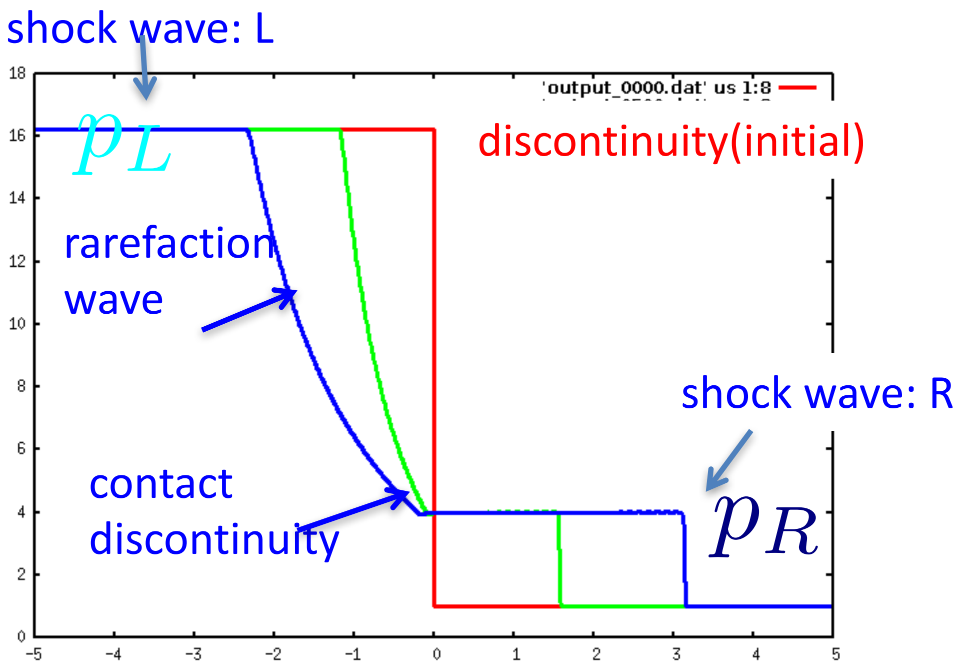
(τ,η) coordinate



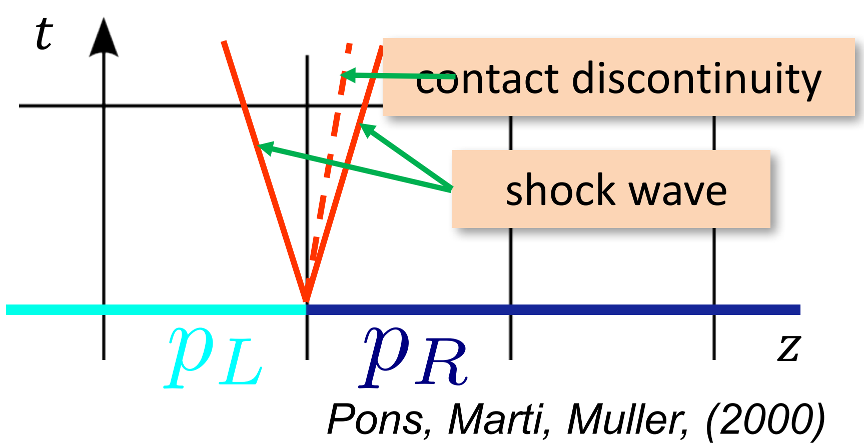
Riemann Solver in (τ,η)-coordinates

Okamoto, Akamatsu, CN, arXiv:1607.03630

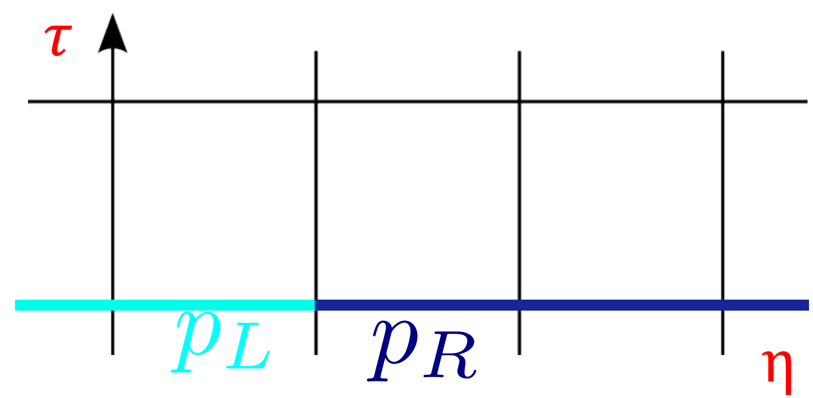
- Riemann solution



Cartesian



(τ,η) coordinates



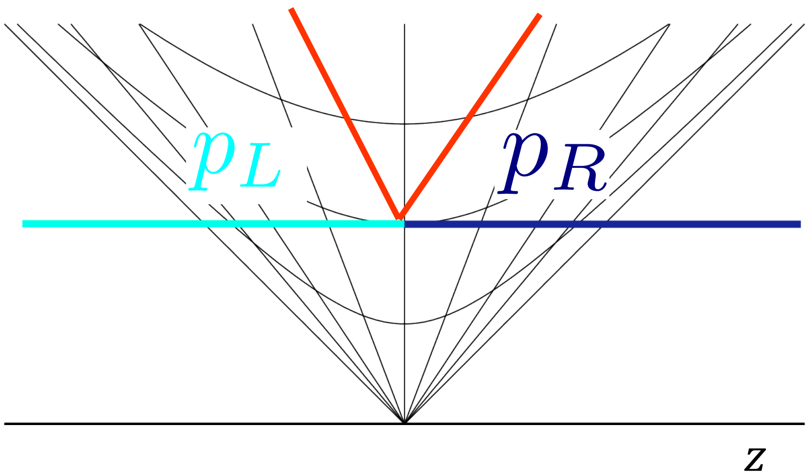
Riemann solution:
Outside of the ligh-cone: Hydrodynamic states (P_L and P_R) are constant.

Riemann Solver in (τ,η)-coordinates

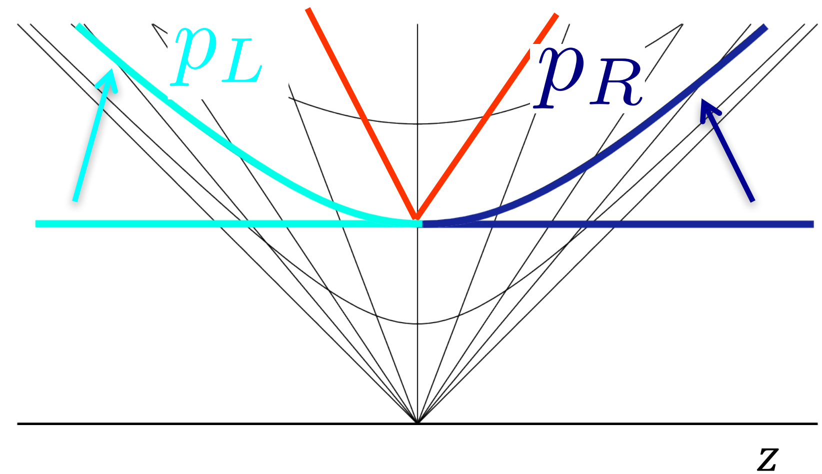
Okamoto, Akamatsu, CN, arXiv:1607.03630

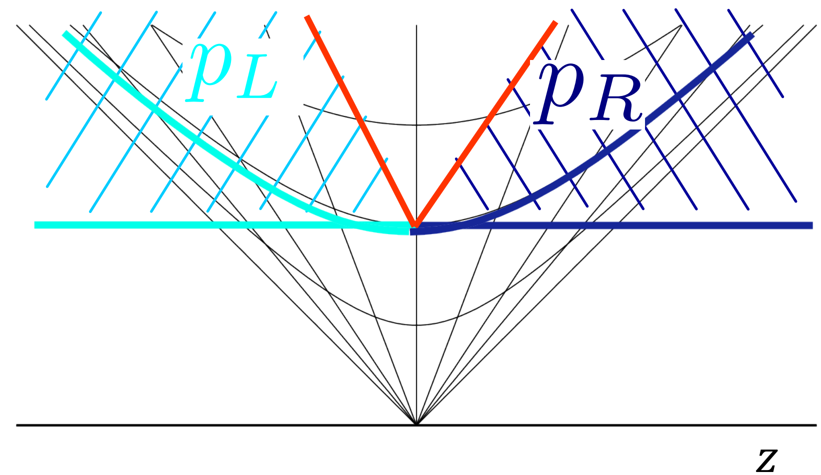
- Riemann solution

Cartesian



(τ,η) coordinates





Riemann solution:
Outside of the ligh-cone:
Hydrodynamic states are constant.

We can construct the Riemann solver in (τ,η) coordiates using that in Cartesian coordinates.