Theoretical constraint on modified gravity
-revisiting great era of GR-

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Modified gravity

Motivation from UV

General relativity (GR)

BH Singularities
Initial singularity

quantization

Not renormalizable
Not unitary

cf. Gauge theories

Classical singularity
at the source

quantization

Renormalizable
Unitary

GR should be modified in UV
to quantize gravity theory.

String theory, loop quantum gravity......

Modified gravity

Motivation from IR

In present cosmology

Cosmic microwave background
Baryon acoustic oscillation
Galaxy rotational curve
⋮

↓

Dark energy
Dark matter

(Invisible gravitational source)

There is a possibility:

Einstein GR

↓

New theory of gravity??

Scalar tensor theory, f(R),
Massive gravity, Bigravity,
Torsion gravity......

cf. In Solar system at end of 19c.

Perihelion precession of Mercury

↓

Dark Planet Vulcan???
(Invisible gravitational source)

But, reality is

Newtonian gravity

↓

Einstein GR

Mathematical structures

(end of 60s - beginning of 80s)

GR + standard model in particle physics

① Causality (Cauchy problems)

Gravitational waves (GWs) propagate to null direction
(fastest speed of propagation is the same as that of photons).
This gives well-defined Cauchy developments.

② Positivity of total energy

In GR (with dominant energy condition)
ADM energy in asymptotically flat spacetime is positive.
Therefore, Hamiltonian is bounded from below and
the (Minkowski) vacuum is semiclassically stable.

③ Properties of Black Holes

Uniqueness of BHs
Penrose inequality

Mathematical structures

GR + standard model in particle physics

+ modification term of gravity or/and exotic matters

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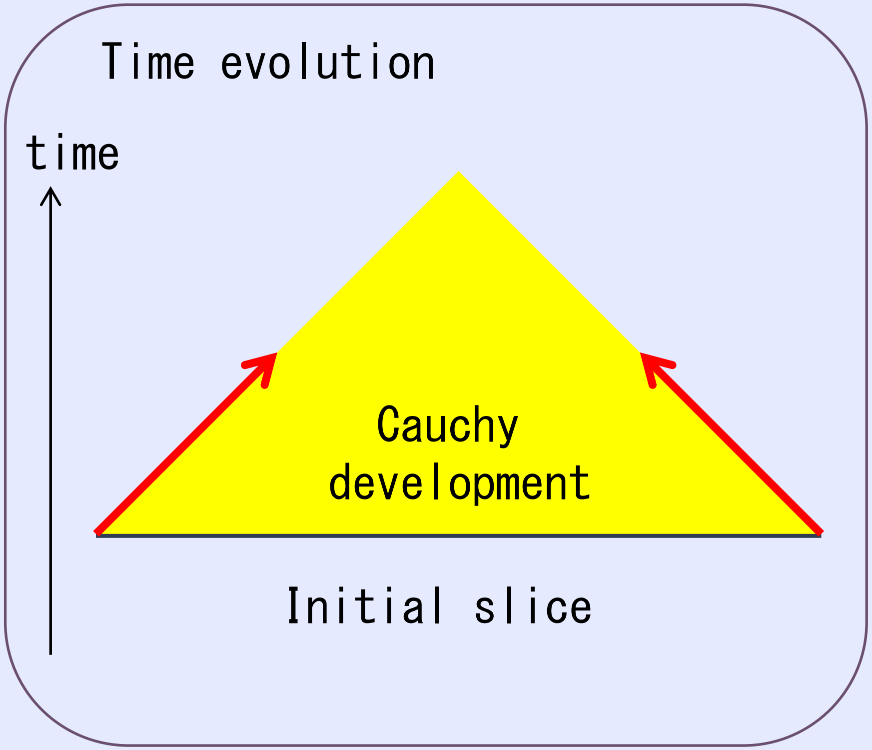
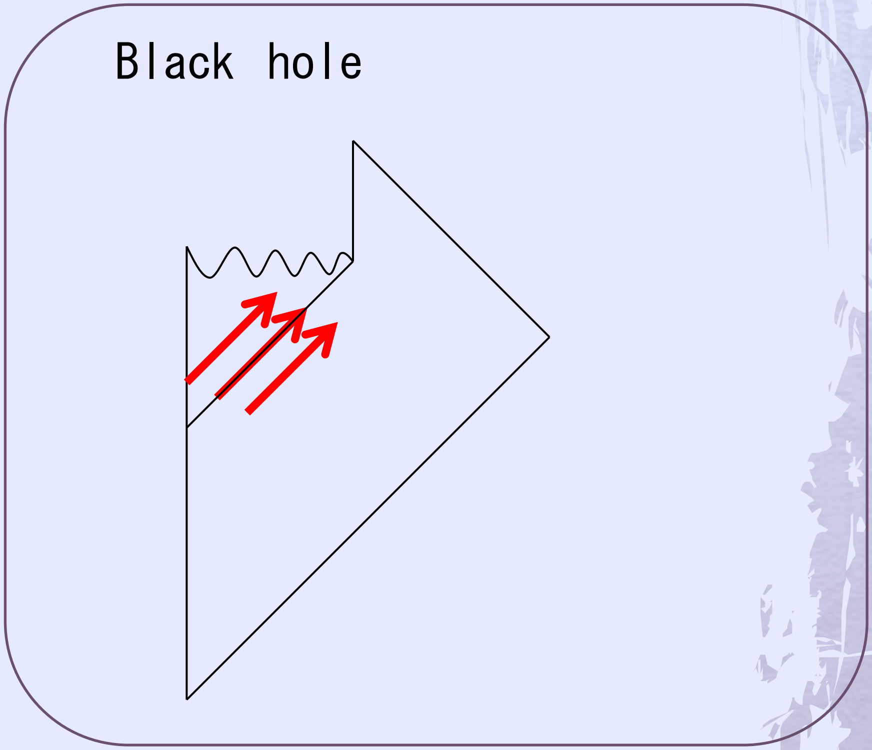
1,Causality

1、 Fastest propagations are null
(GR + Standard Model Particles)





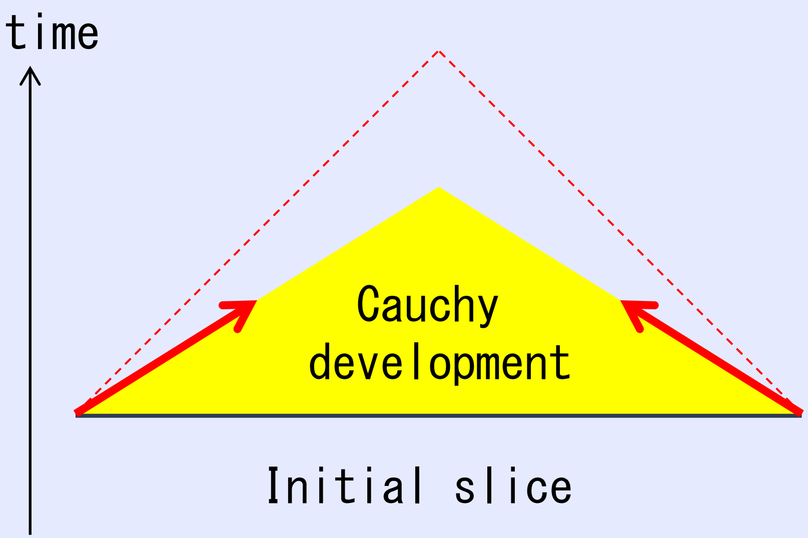
1,Causality

Superluminality

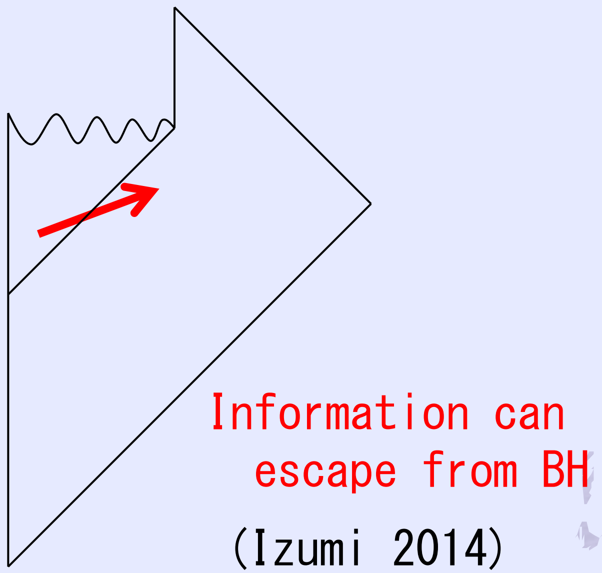
2、 Some propagations are faster than null.
(GR + Standard Model Particles + Gauss Bonnet term (string correction))

C. Aragone (1988)
Choquet-Bruhat (1988)

Time evolution



Black hole



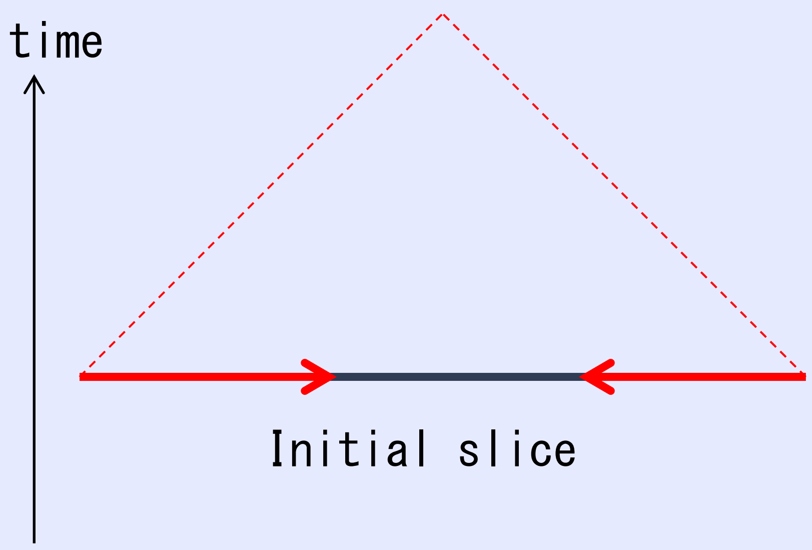
1,Causality

3、 Some propagations have infinite speed.
(Massive gravity or Torsion gravity)

Acausal

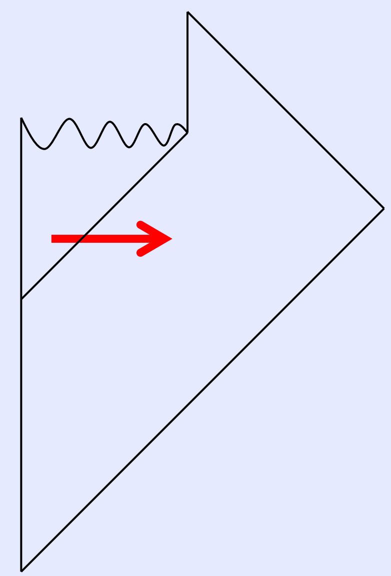
(Izumi, Ong, Nester, Chen, Gu, 2013, 2014, 2015)

Time evolution



No Cauchy development
Acausal structures

Black hole



2,Positive energy theorem

Existence of lowest energy state is important
for the system to be stable.

The positivity of energy in GR was not so trivial.
Positive energy theorem

Deformation of minimal surface (Schoen, Yau 1979)

Spinor (Witten 1981)

- Asymptotically AdS
- Bogomolny bound $M \geq \sqrt{Q_e^2 + Q_m^2}$ (Gibbons, Hull 1982)
- Constraint on the scalar field potential (Townsend 1984)

In cosmology
scalar field with non-trivial kinetic term is discussed.
: k-essence $\mathcal{L} = f(\nabla_\mu\phi\nabla^\mu\phi, \phi)$

But only canonical kinetic term $\nabla_\mu\phi\nabla^\mu\phi$
matches with Witten's Positive energy proof.

(Nozawa, Shiromizu 2014)

3.1,Uniqueness theorem of BH

BH is a good object to check the validity of GR
in strong gravitational field.

Stationary BH is characterized only by total mass
total angular momentum
total charge.

Suppose BHs are relaxed to be stationary,
we can know the spacetime structure of BH in reality.

We can test general relativity
from the observation of BH.

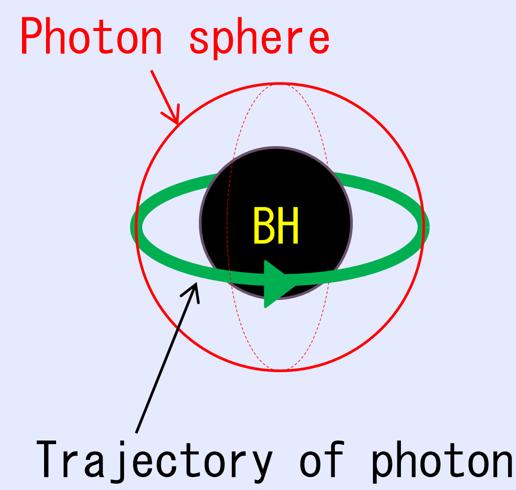
But the geometry on horizon never can be seen by definition.

3.1,Uniqueness theorem of PS

Is there other object to investigate the strong gravity region?

Photon Sphere (PS)

If the gravity field is enough strong,
even photon can have circular orbits.



Photon surface can be seen in principle.

- Uniqueness of Photon sphere (Yazadjiev, Stoytcho 2015)
- Perturbative uniqueness of Photon sphere (Yoshino 2016)
- Uniqueness of photon sphere with hair of conformally coupled scalar field.

(Tomikawa, Shiromizu, Izumi arXiv:1612.01228)

3.2,Penrose inequality

The size of BH is bounded from above.

<u>Penrose inequality</u> (Penrose 1973)

$$A_{AH} \leq 4\pi(2mG)^2 \qquad \text{(on maximal hypersurface)}$$

Equality happens iff the spacetime is Schwarzschild

This is the condition for horizon, that we can never see.

<u>Penrose inequality for "photon sphere"</u>

(Shiromizu, Tomikawa, Izumi, Yoshino, arXiv:1701.00564)

$$A_{LT} \leq 4\pi(3mG)^2 \qquad \text{(on maximal hypersurface)}$$

Area of Loosely trapped hypersurface (LTH)

Def: $r^a \nabla_a k \ (\simeq \frac{d^2}{dr^2} \det g_{ij}) \geq 0$

Equality happens iff the spacetime is Schwarzschild
and then LTH is the photon sphere.

Conclusion

Many modified theories of gravity are proposed.

We should revisit the properties and theorems in GR.
It would give a hint for the real theory of gravity.

Thank you!