HVP contribution to $(g-2)_\mu$: status of the Mainz calculation

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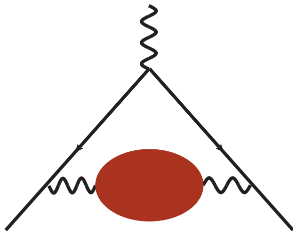
- Time-momentum representation [Bernecker, Meyer '11 '13]

$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty {\rm d}t\ \widetilde{K}(t)\ G(t)$$

$$G(t) = -\frac{1}{3}\sum_{k=1}^{3}\sum_{\vec{x}} \langle V_k(x) V_k(0)\rangle$$



Electromagnetic current : $V_\mu(x) = \frac{2}{3}\overline{u}(x)\gamma_\mu u(x) - \frac{1}{3}\overline{d}(x)\gamma_\mu d(x) - \frac{1}{3}\overline{s}(x)\gamma_\mu s(x) + \cdots$

- $N_f = 2+1$ with Wilson fermions : O(a)-improved (action + currents)

- Two different discretizations of the vector current

$$J_\mu^{(l),a}(x) = \overline{\psi}(x)\gamma_\mu \frac{\lambda^a}{2}\psi_j(x)\,,$$

$$J_\mu^{(c),a}(x) = \frac{1}{2}\left(\overline{\psi}(x+a\hat{\mu})(1+\gamma_\mu)U_\mu^\dagger(x)\frac{\lambda^a}{2}\psi(x) - \overline{\psi}(x)(1-\gamma_\mu)U_\mu(x)\frac{\lambda^a}{2}\psi(x+a\hat{\mu})\right)$$

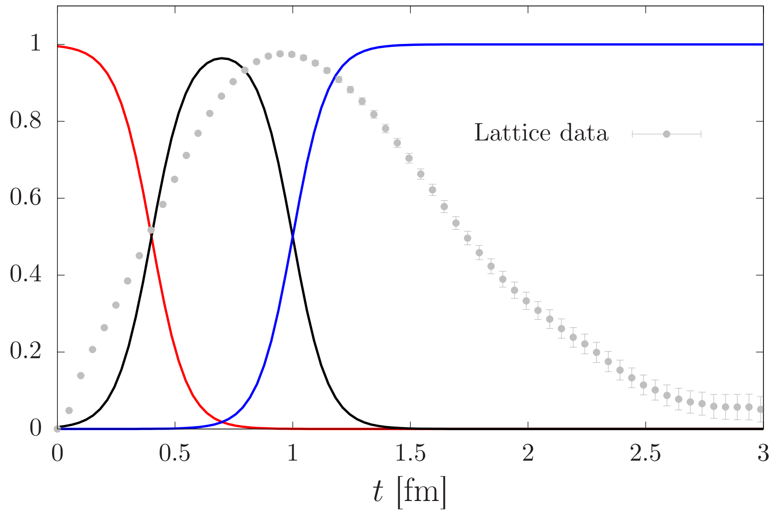
$\rightarrow$ combined continuum extrapolation

▶ Time-momentum representation
[Bernecker, Meyer '11 '13]

$$a_\mu^{\rm win} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t G(t)\, \widetilde{K}(t)\; W(t; t_0, t_1)\,,$$

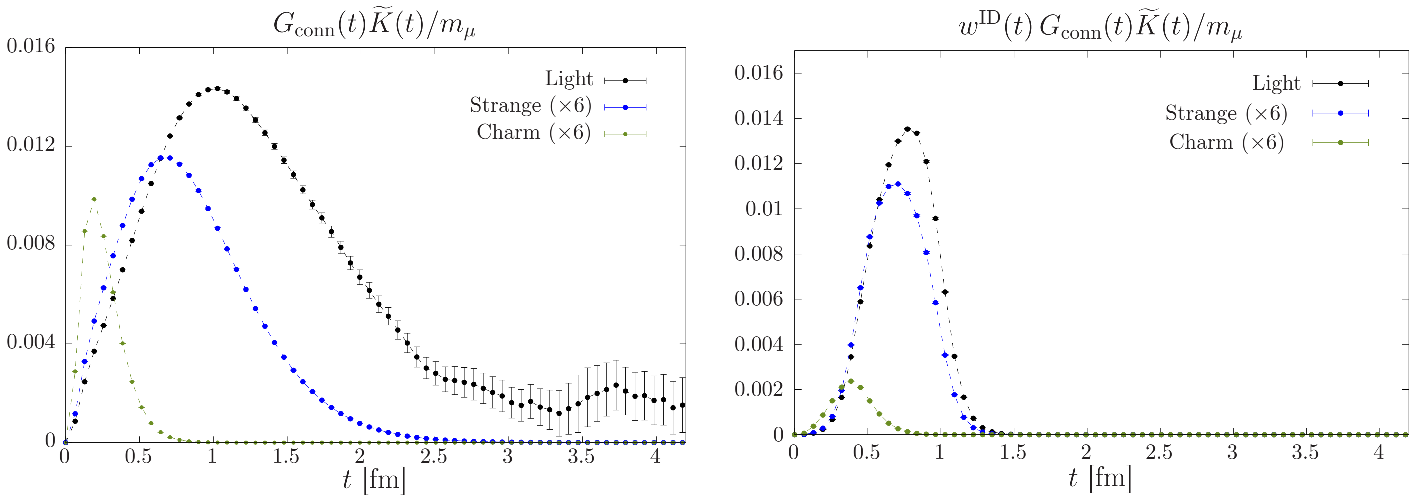
- Short distances : $W^{\rm SD}(t; t_0) = [1 - \Theta(t, t_0, \Delta)]$
- Intermediate distances : $W^{\rm ID}(t; t_0, t_1) = [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]$
- Long distances : $W^{\rm LD}(t; , t_1) = \Theta(t, t_1, \Delta)$

Smooth step function : $\Theta(t, t', \Delta) = [1 + \tanh[(t - t')/\Delta]]\,/2$



▶ This talk focuses on the intermediate window

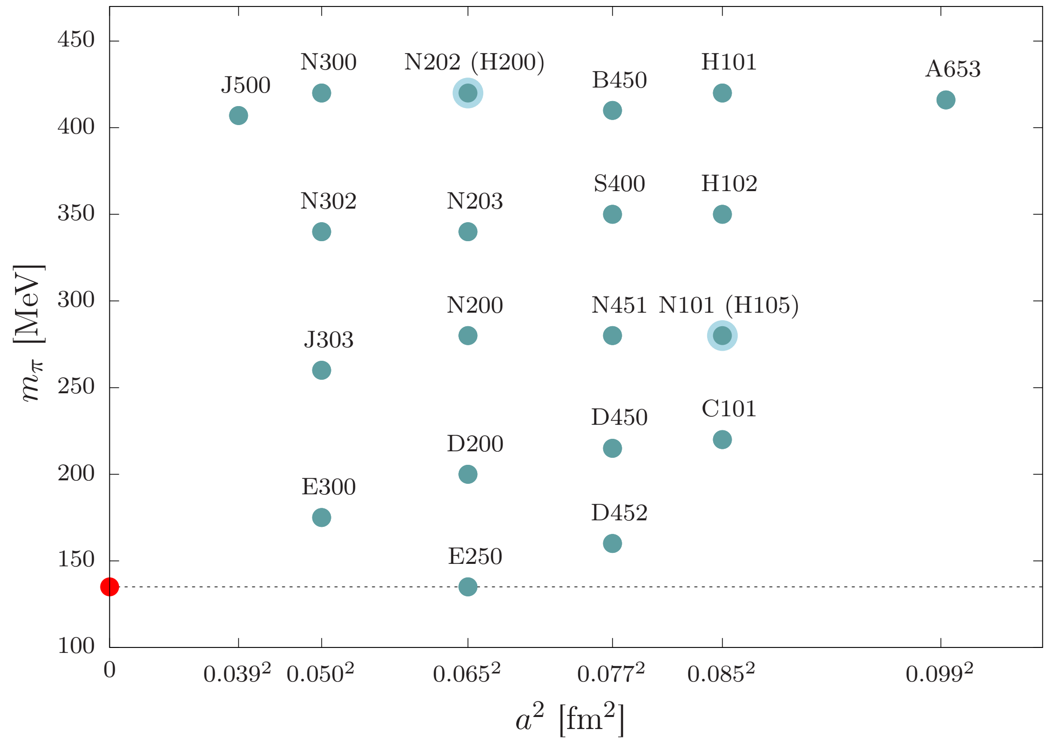
- no signal-to-noise ratio issue
  - $\rightarrow$ 1-2 permille statistical precision can be reached on the integrand
  - $\rightarrow$ the treatment of the noisy long-distance part of the correlator is irrelevant here
  - $\rightarrow$ it means that the statistical error is not the main source of uncertainty



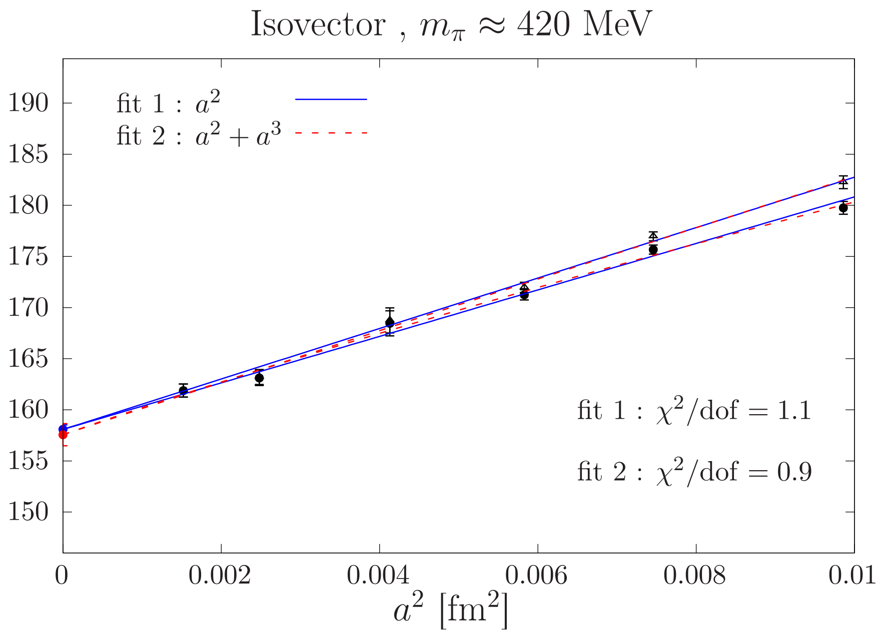
- need to have a good control over other systematics :

  continuum extrapolation, finite-size effects

- $> 35\,000$ gauge field configurations for 22 ensembles
- include the physical pion mass
- 6 lattice spacings



- The continuum extrapolation is one of the main sources of error
  - → Dedicated study performed at $m_\pi = m_K \approx 420$ MeV
  - → 6 lattice spacings < 0.1 fm
  - → 2 different discretizations of the vector current to check $O(a)-$improvement



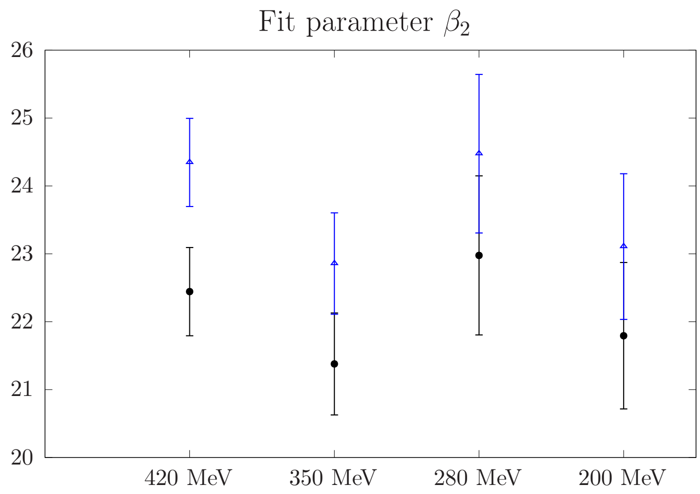
- We observe a very good scaling within a large range of lattice spacings

- What about lighter pion masses ?

  $\rightarrow$ same fit ansatz :

$$a_\mu^{\mathrm{hvp,I1}}(a,d) = a_\mu^{\mathrm{hvp,I1}}(0) + \beta_2^{(d)}\,\widetilde{a}^2 + \beta_3^{(d)}\,\widetilde{a}^3$$

  $\rightarrow$ dimensionless parameter $\widetilde{a} = a/a_{\beta=3.34}$ and $d = (ll), (lc)$



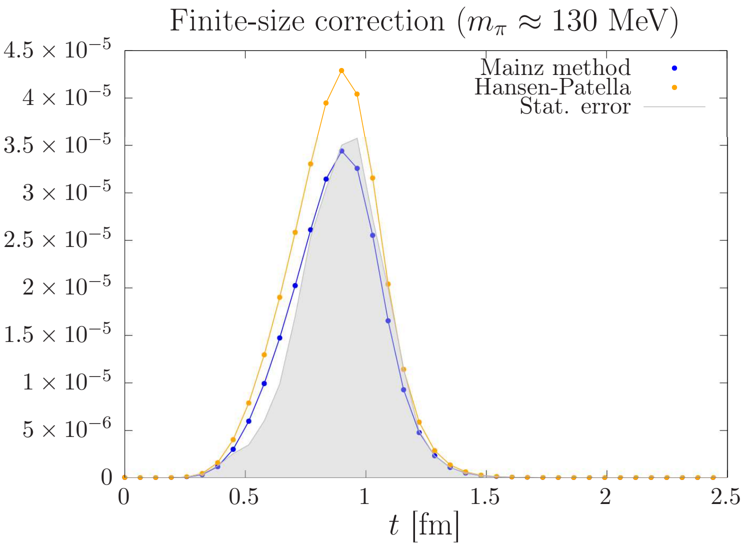
Global fit (22 ensembles)

$$\beta_2^{(ll)} = 27(5)\,, \quad \beta_3^{(ll)} = -5(5)$$
$$\beta_2^{(lc)} = 25(5)\,, \quad \beta_3^{(lc)} = -0(5)$$

$\rightarrow$ compatible values

$\rightarrow$ β_3 compatible with zero

- No significant pion mass dependence of slope parameter β_2
- We are currently accumulating statistics on a second ensemble at our finest lattice spacing with smaller pion mass



- Mainz method [D.Bernecker, H.B.Meyer '11]
  - $\rightarrow$ used in [Phys.Rev.D 100 (2019)]
  - $\rightarrow$ NLO ChPT for $t < 1.5$ fm
- Hansen-Patella method [M.T Hansen, A. Patella '19 '20]

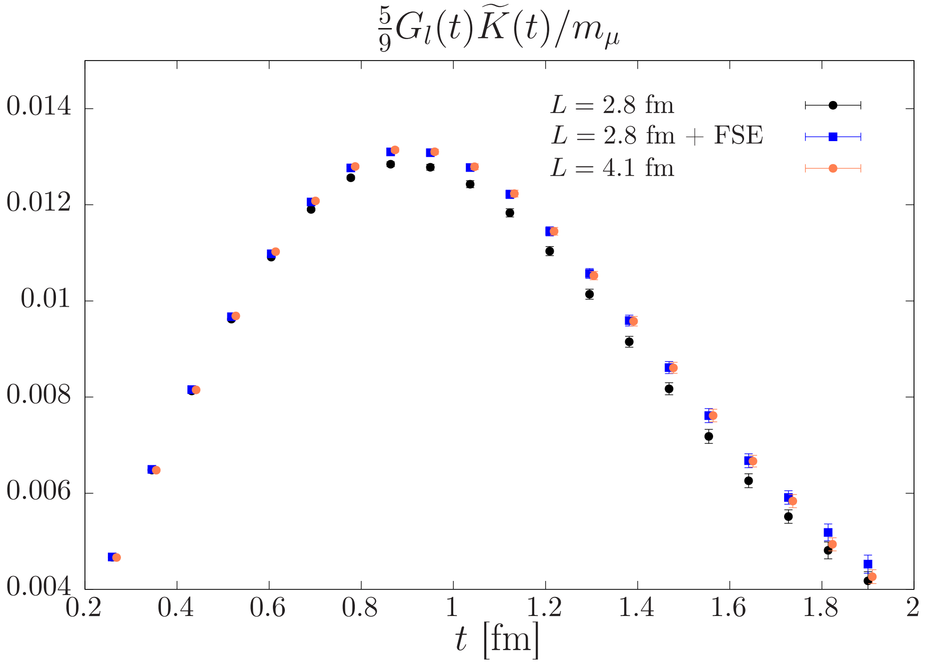
▶ Finite-volume correction for full a_μ^{hvp} :

$$m_\pi L = 4 \quad (L = 6.2 \text{ fm}) \quad \Rightarrow \quad \Delta a_\mu = 22.6 \times 10^{-10} \quad (3\%)$$

▶ Intermediate window : $\Delta a_\mu = 0.6 \times 10^{-10}$ (0.25%)

$\rightarrow$ FSE are much smaller for the intermediate window
$\rightarrow$ but not negligible compared to the statistical precision ($\sim 0.2\%$)

▶ Wilson quarks : this is the only correction applied to the raw lattice data !



- Our procedure works remarkably well at $m_\pi = 280$ MeV [PRD 100 (2019) 014510]

$$m_\pi L = 4 \quad (L = 6.2 \text{ fm}) \quad \Rightarrow \quad \Delta a_\mu = 22.6 \times 10^{-10} \quad (3\%)$$

- In agreement with other direct lattice calculations [E. Shintani, Y. Kuramashi '19] , [BMW '20]

- Scale setting

  $\rightarrow$ one of the main challenges for $a_\mu^{\rm hvp}$: $\frac{\delta a_\mu^{\rm hvp}}{a_\mu^{\rm hvp}} = \left| \frac{m_\mu}{a_\mu^{\rm hvp}} \frac{{\rm d}a_\mu^{\rm hvp}}{{\rm d}m_\mu} \right| \frac{\delta a}{a} \approx 1.8 \frac{\delta a}{a}$

  $\rightarrow$ window quantities : the absolute scale also enters the definition of the window

$$\frac{\delta a_\mu^{\rm hvp,win}}{a_\mu^{\rm hvp,win}} \approx \frac{m_\mu}{a_\mu^{\rm hvp,win}} \frac{{\rm d}a_\mu^{\rm hvp,win}}{{\rm d}m_\mu} \frac{\delta a}{a} + \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty {\rm d}t \, \frac{\delta w(t)}{\delta a} G(t) \widetilde{K}(t) \times \frac{\delta a}{a_\mu^{\rm hvp,win}}$$

  - significantly reduces the sensitivity to the scale for the isovector contribution
  - but makes it worse for the (small !) charm quark contribution

- Preliminary error budget

Table – Isovector

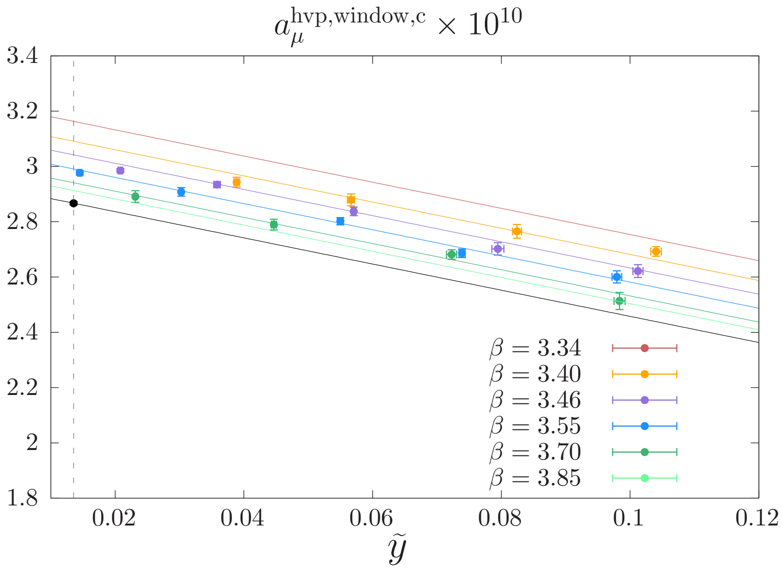
Type	%
Renormalization	< 0.01
Correlator	0.2
Finite-volume	≤ 0.2
Extrap.	0.61
Scale setting	0.38
Total	0.77

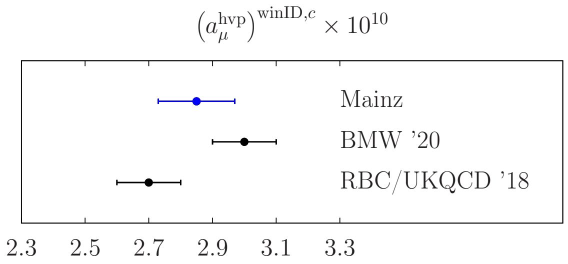
Table – Isoscalar

Type	%
Renormalization	< 0.01
Correlator	0.3
Extrap.	1.3
Scale setting	0.86
Total	1.59

- Final stage of the analysis

- Extrapolation to the physical point $\widetilde{y} \sim m_\pi^2$



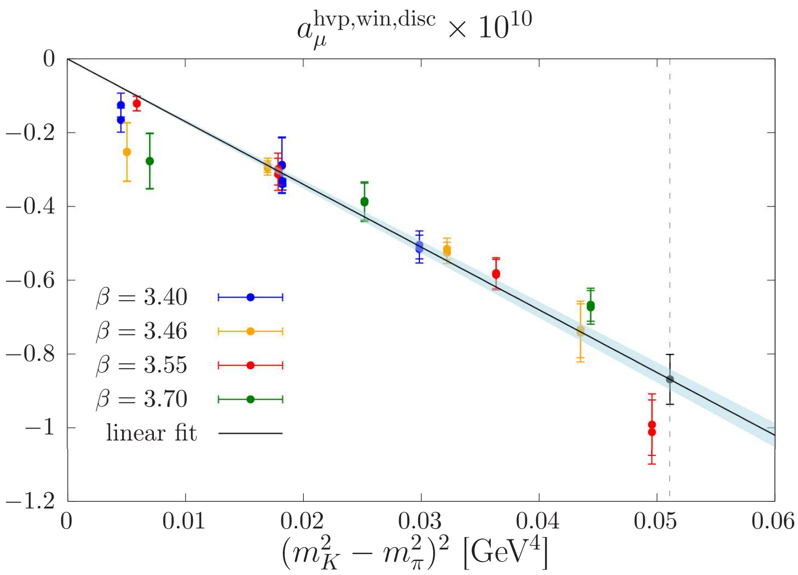


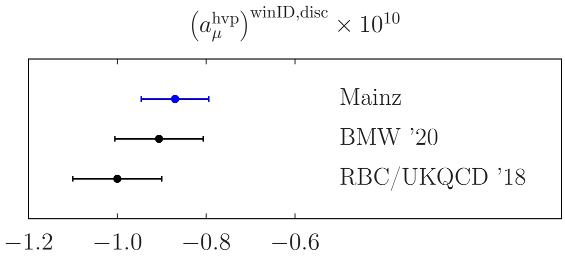
- Error budget for the charm quark contribution

Type	%
Renormalization	< 0.05
Tuning of κ_c	< 0.2
Correlator	< 0.05
Extrap.	1.7
Scale setting	3.9
Total	4.3

$\longrightarrow$ much more sensitive to the scale than $a_\mu^{\mathrm{hvp,c}}$

Quark disconnected contribution





- as for the light quark contribution : there is no signal/noise problem here
- small contribution (about 7% of the total disconnected contribution to full a_μ^{hvp})
- small discretization effects

Conclusion

- Large statistics : 22 ensembles including the physical pion mass

  $\rightarrow$ statistical error is sub-dominant for the intermediate window

  $\rightarrow$ the treatment of the tail of the correlator is also irrelevant here

- Continuum limit

  $\rightarrow$ 6 lattice spacings. All of them < 0.1 fm.

  $\rightarrow$ our 2 finest lattice spacings are smaller than any other lattice collaboration (for $g-2$)

  $\rightarrow$ fully non-perturbative O(a)-improvement

  $\rightarrow$ two discretizations of the vector current to test the consistency of the continuum limit and O(a)-improvement

- Ensembles with different volumes to check FSE

  $\rightarrow$ this is the only correction applied to the raw lattice data

- Ongoing works

  $\rightarrow$ estimate of the quenching of the charm

  $\rightarrow$ isospin-breaking corrections

  $\rightarrow$ running of alpha / weak mixing angle

Thank you.