Un-binned Angular Analysis of $B \to D^* \ell \nu$ and the Right-handed Current

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Motivation

- Semileptonic $B \to D^{(*)}\ell\nu$ decay
- $R(D^{(*)})$ anomalies

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}, \quad \textbf{with } \ell = \mu, e$$

- V_{cb} *puzzle*

inclusive decay $B \to X_c\ell\nu$ $(X_c = D, D^*, D_0^* \ldots)$

HQE, Optical theorem, OPE

exclusive decay $B \to D^{(*)}\ell\nu$

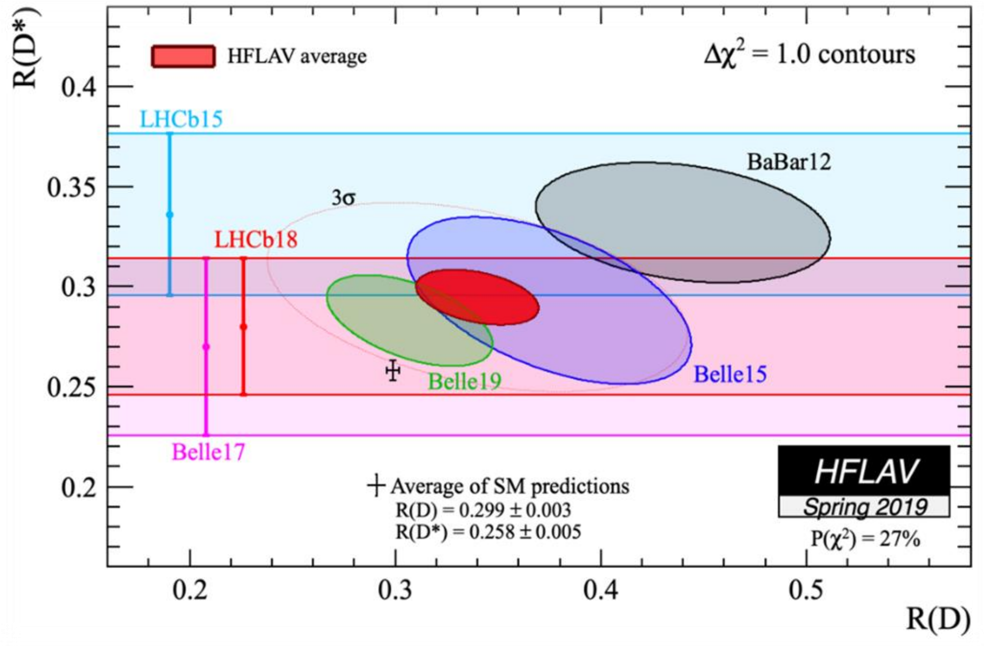
form factor calcualtion: *lattice, LCSR*

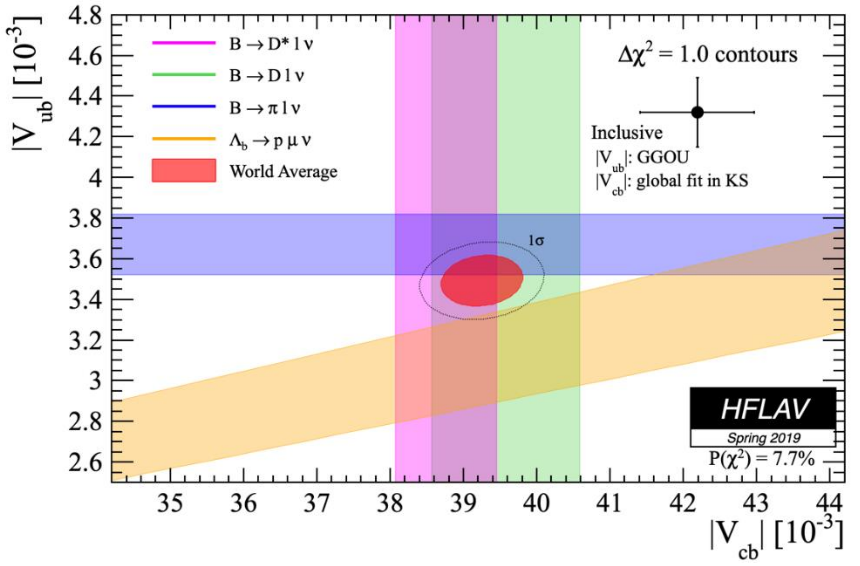
parametrization: CLN(-like)/BGL

in. 42.16(50) vs ex. 39.70(60)

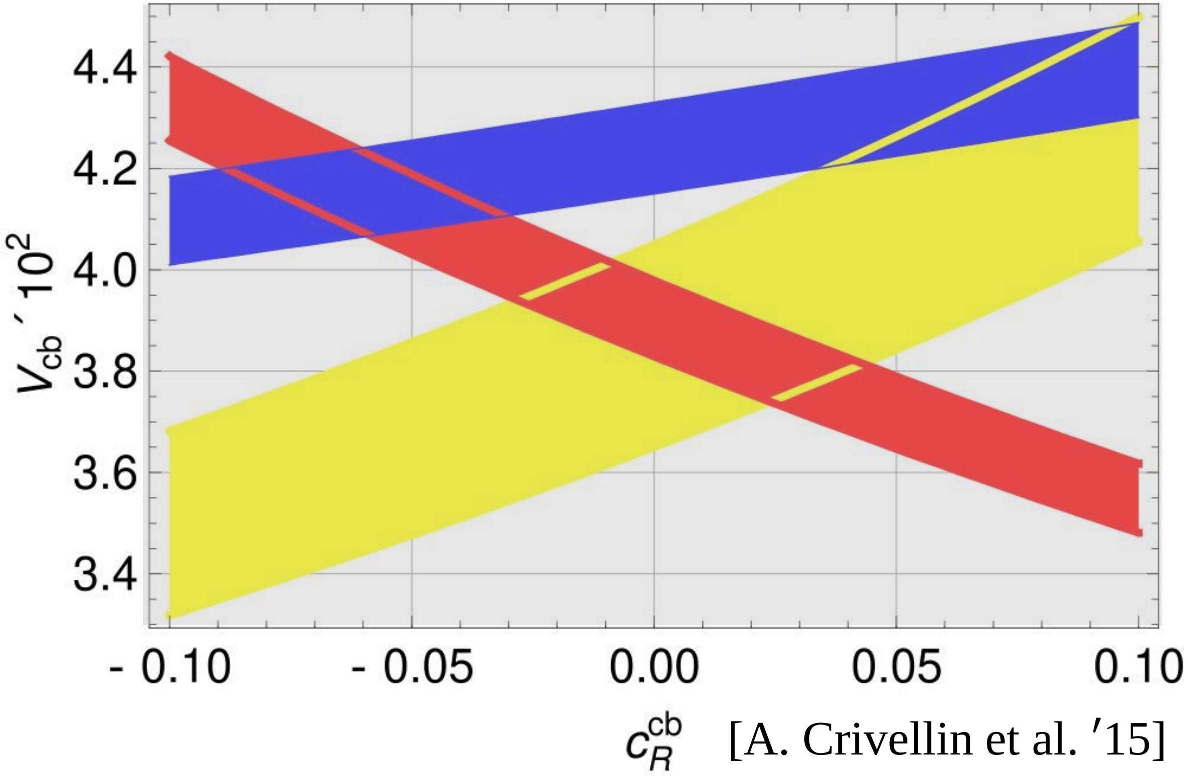
$\sim 3\sigma$ deviation

[M. Bordone et al. ′21] [S. Iguro et al. ′20]





Relation between the R.H. vector current and the V_{cb} puzzle



[A. Crivellin et al. ′15]

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[C_{V_L} \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R} \right] + \text{h.c.}$$

$$\mathcal{O}_{V_L} = (\bar{c}_L \gamma^\mu b_L)(\bar{\ell}_L \gamma_\mu \nu_L) , \quad \mathcal{O}_{V_R} = (\bar{c}_R \gamma^\mu b_R)(\bar{\ell}_L \gamma_\mu \nu_L) .$$

$C_{V_L} = 1$ and $C_{V_R} = 0$ in the SM
$C_{V_R} \neq 0$ in the Left-Right symmetric model from $W_L - W_R$ mixing [E. Kou et al. ′13]

$B \to D^* \ell \nu$ vs $B \to X_c \ell \nu$: $C_{V_R} \sim -5\%$

$B \to D \ell \nu$ vs $B \to X_c \ell \nu$: $C_{V_R} \sim 5\%$

Considerable ex. uncertainties.
Theo. uncertainty from lattice QCD input.
More measurements needed!

Theoretical Framework

Differential decay rate ($m_{\mu,e} \to 0$):

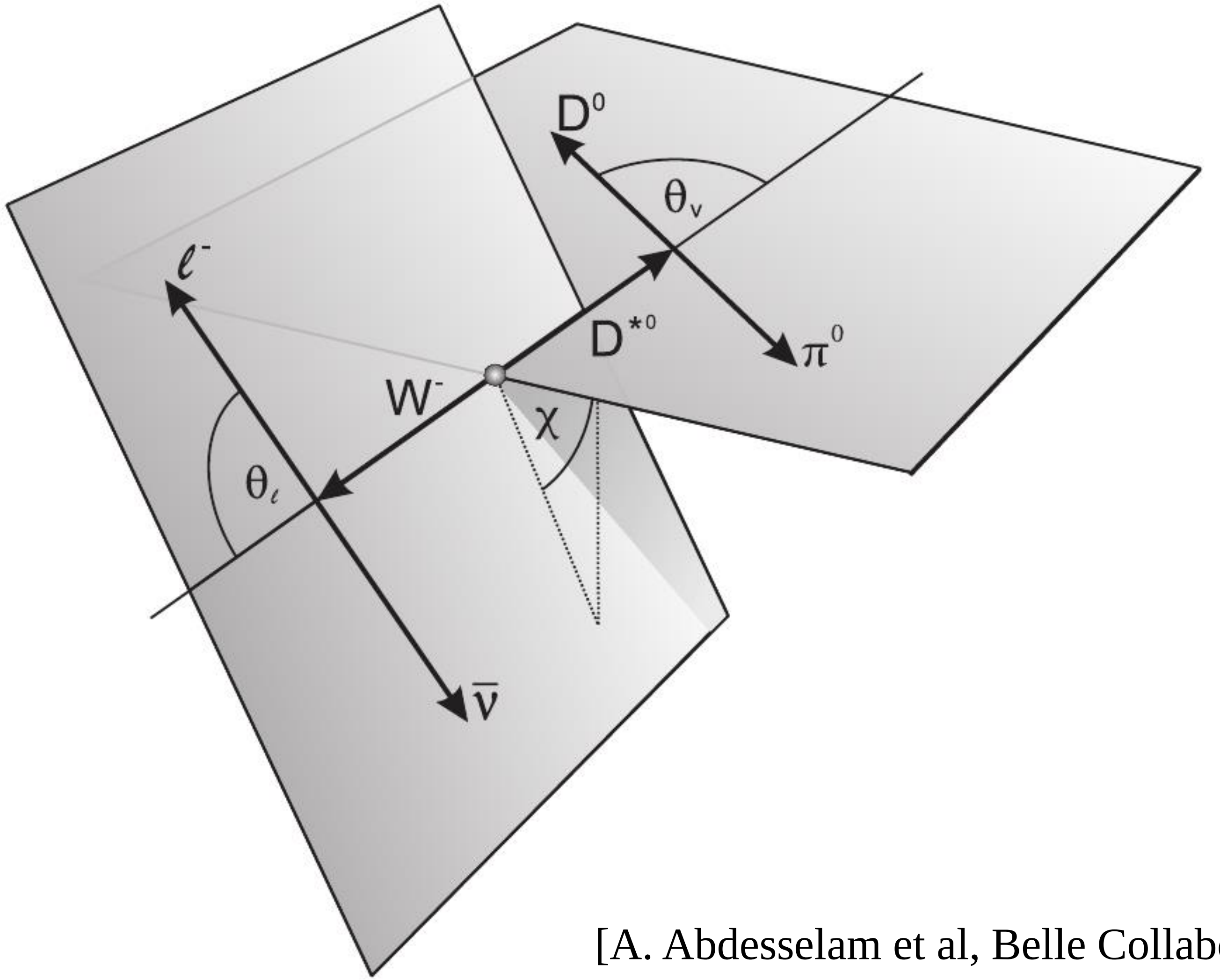
$$
\begin{aligned}
&\frac{\mathrm{d}\Gamma(\bar{B} \to D^*(\to D\pi)\,\ell^-\,\bar{\nu}_\ell)}{\mathrm{d}w\,\mathrm{d}\cos\theta_V\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\chi} \\
&= \frac{6 m_B m_{D^*}^2}{8(4\pi)^4}\sqrt{w^2-1}(1-2\,w\,r+r^2)\,G_F^2\,|V_{cb}|^2\,\mathcal{B}(D^*\to D\pi) \\
&\quad\times\Big\{J_{1s}\sin^2\theta_V + J_{1c}\cos^2\theta_V + (J_{2s}\sin^2\theta_V \\
&\qquad + J_{2c}\cos^2\theta_V)\cos 2\theta_\ell \\
&\qquad + J_3\sin^2\theta_V\sin^2\theta_\ell\cos 2\chi \\
&\qquad + J_4\sin 2\theta_V\sin 2\theta_\ell\cos\chi + J_5\sin 2\theta_V\sin\theta_\ell\cos\chi \\
&\qquad + (J_{6s}\sin^2\theta_V + J_{6c}\cos^2\theta_V)\cos\theta_\ell \\
&\qquad + J_7\sin 2\theta_V\sin\theta_\ell\sin\chi + J_8\sin 2\theta_V\sin 2\theta_\ell\sin\chi \\
&\qquad + J_9\sin^2\theta_V\sin^2\theta_\ell\sin 2\chi\Big\},
\end{aligned}
$$

J_i experimentally measurable, includes H_+, H_-, H_0, C_{V_L} and C_{V_R} (SM and BSM).

J_i functions:

$$
\begin{aligned}
J_{1s} &= \frac{3}{2}(H_+^2+H_-^2)(|C_{V_L}|^2+|C_{V_R}|^2) - 6H_+H_-\mathrm{Re}[C_{V_L}C_{V_R}^*] \\
J_{1c} &= 2H_0^2(|C_{V_L}|^2+|C_{V_R}|^2 - 2\mathrm{Re}[C_{V_L}C_{V_R}^*]) \\
J_{1c} &= 2H_0^2(|C_{V_L}|^2+|C_{V_R}|^2 - 2\mathrm{Re}[C_{V_L}C_{V_R}^*]) \\
J_{2s} &= \frac{1}{2}(H_+^2+H_-^2)(|C_{V_L}|^2+|C_{V_R}|^2) - 2H_+H_-\mathrm{Re}[C_{V_L}C_{V_R}^*] \\
J_{2c} &= -2H_0^2(|C_{V_L}|^2+|C_{V_R}|^2 - 2\mathrm{Re}[C_{V_L}C_{V_R}^*]) \\
J_3 &= -2H_+H_-(|C_{V_L}|^2+|C_{V_R}|^2) + 2(H_+^2+H_-^2)\mathrm{Re}[C_{V_L}C_{V_R}^*] \\
J_4 &= (H_+H_0+H_-H_0)(|C_{V_L}|^2+|C_{V_R}|^2 - 2\mathrm{Re}[C_{V_L}C_{V_R}^*]) \\
J_5 &= -2(H_+H_0-H_-H_0)(|C_{V_L}|^2-|C_{V_R}|^2) \\
J_{6s} &= -2(H_+^2-H_-^2)(|C_{V_L}|^2-|C_{V_R}|^2) \\
J_{6c} &= 0 \\
J_7 &= 0 \\
J_8 &= 2(H_+H_0-H_-H_0)\mathrm{Im}[C_{V_L}C_{V_R}^*] \\
J_9 &= -2(H_+^2-H_-^2)\mathrm{Im}[C_{V_L}C_{V_R}^*]
\end{aligned}
$$

Kinematic variables in $B \to D^*(\to D\pi)\ell\nu$



θ_ℓ the angle between the lepton and the direction opposite the B-meson in the virtual W-boson rest frame;

θ_ν the angle between the D meson and the direction opposite the B meson in the D^* rest frame;

χ the tilting angle between the two decay planes spanned by the W and D systems in the B meson rest frame;

w the dimensionless four-momentum transfer.

[A. Abdesselam et al, Belle Collaboration ′17]

Helicity amplitudes in CLN and BGL parametrizations

$$H_\pm(w) = m_B\sqrt{r}(w+1)h_{A_1}(w) \times \left[1 \mp \sqrt{\frac{w-1}{w+1}}R_1(w)\right]$$

$$H_0(w) = m_B^2\sqrt{r}(w+1)\frac{1-r}{\sqrt{q^2}}h_{A_1}(w)\times \left[1+\frac{w-1}{1-r}(1-R_2(w))\right]$$

CLN parametrization (HQE based)

$$h_{A_1}(w) = h_{A_1}(1)(1-8\rho_{D*}^2 z + (53\rho_{D*}^2 - 15)z^2 - (231\rho_{D*}^2 - 91)z^3)$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

$$H_\pm(w) = f(w) \mp m_B|\mathbf{p}_{D*}|g(w)$$

$$H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{q^2}}$$

BGL parametrization (analyticity based)

$$g(z) = \frac{1}{P_g(z)\phi_g(z)}\sum_{n=0}^{N} a_n^g z^n$$

$$f(z) = \frac{1}{P_f(z)\phi_f(z)}\sum_{n=0}^{N} a_n^f z^n$$

$$\mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)}\sum_{n=0}^{N} a_n^{\mathcal{F}_1} z^n$$

Blaschke factors:

P_g, P_f, P_{F_1}

outer functions:

$\phi_g, \phi_f, \phi_{F_1}$

Un-binned Angular Analysis

Exsiting binned analysis (projected χ^2 fit): Belle '17 '18; BaBar '19

Normalised PDF:

$$\hat{f}_{\langle\vec{g}\rangle}(\cos\theta_V,\cos\theta_\ell,\chi) = \frac{9}{8\pi}$$
$$\times\left\{\frac{1}{6}(1-3\langle g_{1c}\rangle+2\langle g_{2s}\rangle+\langle g_{2c}\rangle)\sin^2\theta_V+\langle g_{1c}\rangle\cos^2\theta_V\right.$$
$$+(\langle g_{2s}\rangle\sin^2\theta_V+\langle g_{2c}\rangle\cos^2\theta_V)\cos 2\theta_\ell$$
$$+\langle g_3\rangle\sin^2\theta_V\sin^2\theta_\ell\cos 2\chi$$
$$+\langle g_4\rangle\sin 2\theta_V\sin 2\theta_\ell\cos\chi+\langle g_5\rangle\sin 2\theta_V\sin\theta_\ell\cos\chi$$
$$+(\langle g_{6s}\rangle\sin^2\theta_V+\langle g_{6c}\rangle\cos^2\theta_V)\cos\theta_\ell$$
$$+\langle g_7\rangle\sin 2\theta_V\sin\theta_\ell\sin\chi+\langle g_8\rangle\sin 2\theta_V\sin 2\theta_\ell\sin\chi$$
$$\left.+\langle g_9\rangle\sin^2\theta_V\sin^2\theta_\ell\sin 2\chi\right\},$$

$$\langle g_i\rangle \equiv \frac{\langle J'_i\rangle}{6\langle J'_{1s}\rangle+3\langle J'_{1c}\rangle-2\langle J'_{2s}\rangle-\langle J'_{2c}\rangle}$$

$$J'_i \equiv J_i\sqrt{w^2-1}(1-2wr+r^2)$$

The experimental determination of $< g_i >$ can be pursued by the *maximum likelihood method*:

$$\mathcal{L}(\langle\vec{g_i}\rangle) = \sum_{i=1}^{N}\ln\hat{f}_{\langle\vec{g_i}\rangle}(e_i)$$

Angular observables allow to determine C_{V_R} without the intervention of the V_{cb} puzzle!

Pseudo data generation

Pseudo data generated using CLN parameters fitted by Belle [E. Waheed et al, ′18]

$$N_{event} = (5306, 8934, 10525, 11241, 11392, 11132, 10555, 9726, 8693, 7497)$$

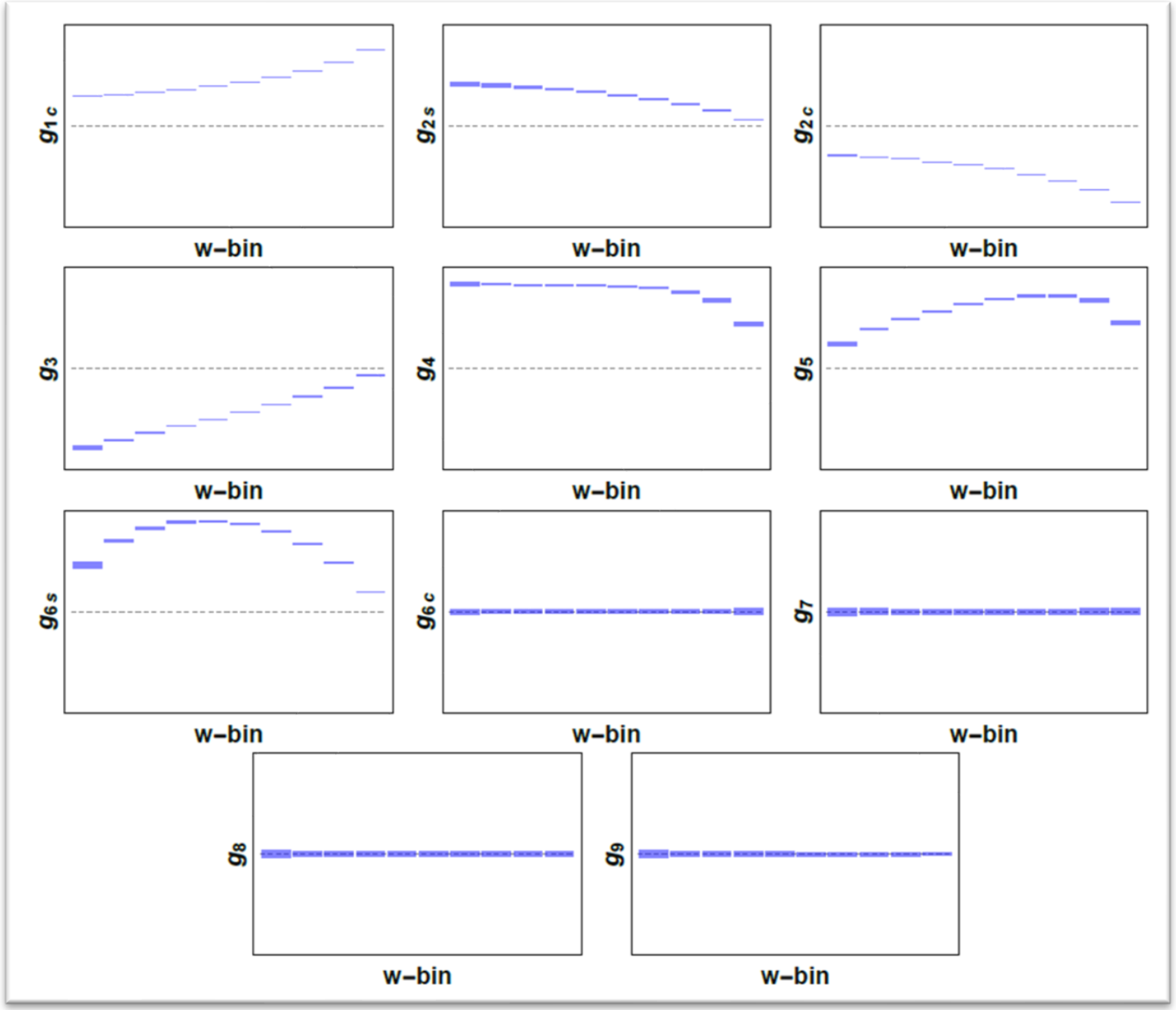
Pseudo data generated using BGL parameters fitted by Belle [E. Waheed et al, ′18]

$$N_{event} = (5306, 8934, 10525, 11241, 11392, 11132, 10555, 9726, 8693, 7497)$$

$< g_i >$ generated in 10 bins with covariance matrices by toy Monte-Carlo method

Total event number: 95k as in Belle analysis

Using pseudo data we fit theoretical formula including C_{V_R} (on top of form factors). Note V_{cb} is not possible to fit any more because it cancels in g_i!



$< g_i >$ generated in 10 w-bins

χ^2 utilized in the CLN/BGL fit

$$\chi^2(\vec{v}) = \chi^2_{\text{angle}}(\vec{v}) + \chi^2_{\text{lattice}}(\vec{v})$$

$$\chi^2_{\text{angle}}(\vec{v}) = \sum_{w-\text{bin}=1}^{10} \Big[\sum_{ij} N_{\text{event}} \hat{V}_{ij}^{-1} (\langle g_i \rangle^{\text{exp}} - \langle g_i^{\text{th}}(\vec{v}) \rangle)(\langle g_j \rangle^{\text{exp}} - \langle g_j^{\text{th}}(\vec{v}) \rangle) \Big]_{w-\text{bin}}$$

We include the lattice input by introducing

$$\chi^2_{\text{lattice}}(v_i) = \left(\frac{v_i^{\text{lattice}} - v_i}{\sigma_{v_i}^{\text{lattice}}} \right)^2$$

with $h_{A_1}(1) = 0.906 \pm 0.013$
by Fermilab/MILC

Notes:
1.) C_{V_R} and V_{cb} are dependent using decay rates in the fit as the changes in both parameters directly impact $Br(B \to D^* \ell \nu)$

2.) the angular fit does not converge as C_{V_R} is not independent of the vector form factor

Lattice input of the vector form factor is crucial for determining C_{V_R}!

$R_1(1)$ ~4% error $h_V(1)$ ~7% error

[T. Kaneko et al, '19]

Fit of C_{V_R}

CLN fit:

$$R_1(1) = \frac{h_V(1)}{h_{A_1}(1)}$$

$$\vec{v} = (\rho^2_{D^*}, R_1(1), R_2(1), C_{V_R})$$
$$= (1.106, 1.229, 0.852, 0)$$
$$\sigma_{\vec{v}} = (3.177, 0.049, 0.018, 0.021)$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & -0.016 & -0.763 & 0.095 \\ -0.016 & 1. & 0.006 & \underline{-0.973} \\ -0.763 & 0.006 & 1. & -0.117 \\ 0.095 & -0.973 & -0.117 & 1. \end{pmatrix}$$

C_{V_R} can be determined to a precision of ~2 (4)% in CLN (BGL) parametrization.

BGL fit:

$$h_V(1) = \frac{m_B\sqrt{r}}{P_g(0)\phi_g(0)} a_0^g$$

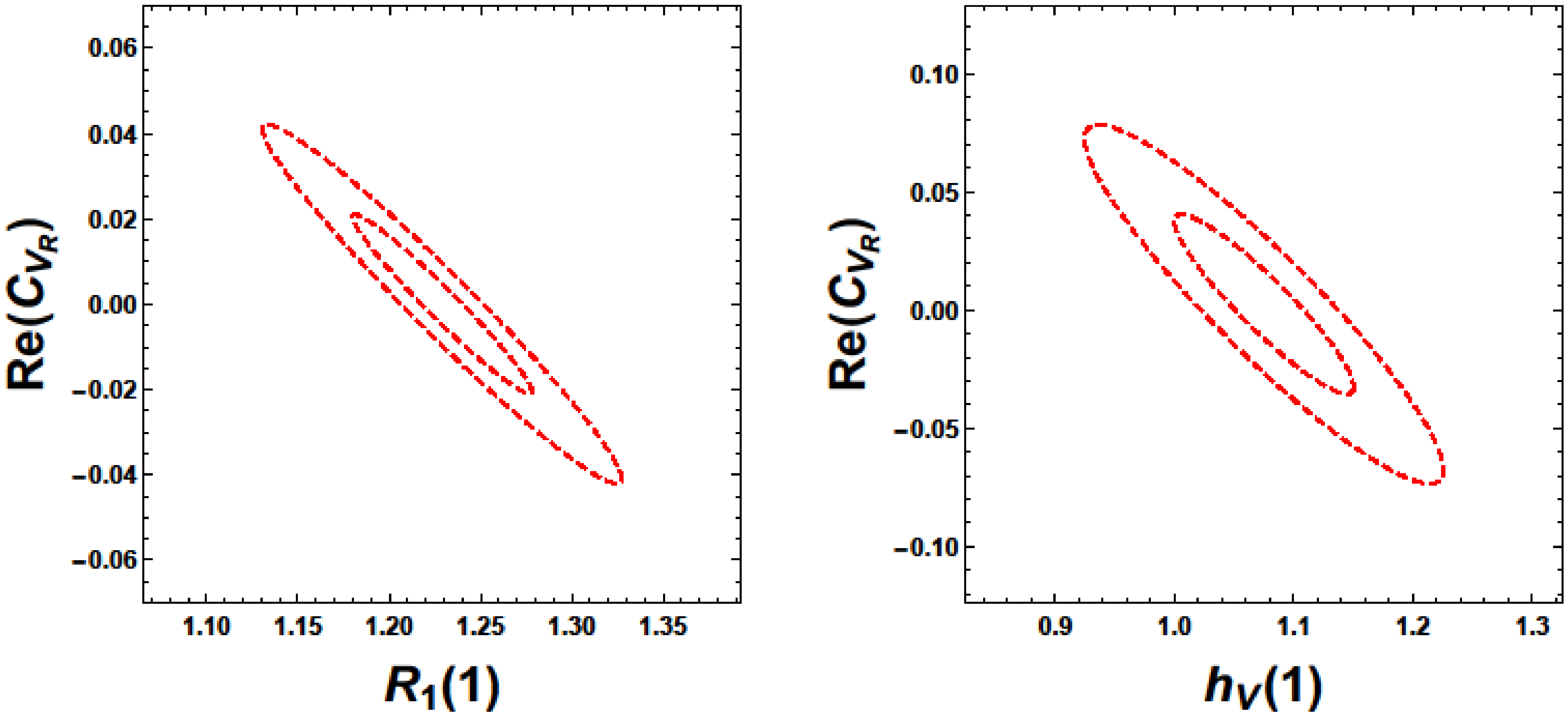
$$\vec{v} = (a_0^f, a_1^f, a_1^{\mathcal{F}_1}, a_2^{\mathcal{F}_1}, a_0^g, C_{V_R})$$
$$= (0.0132, 0.0169, 0.0070, -0.0852, 0.0241, 0.0024)$$
$$\sigma_{\vec{v}} = (0.0002, 0.0109, 0.0026, 0.0352, 0.0017, 0.0379)$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & 0.022 & 0.039 & -0.035 & 0.000 & 0.189 \\ 0.022 & 1. & 0.860 & -0.351 & 0.000 & 0.316 \\ 0.039 & 0.860 & 1. & -0.762 & 0.000 & 0.283 \\ -0.035 & -0.351 & -0.762 & 1. & 0.000 & -0.119 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1. & \underline{-0.923} \\ 0.189 & 0.316 & 0.283 & -0.119 & -0.923 & 1. \end{pmatrix}$$

Central value of C_{V_R} is ~0 because of Belle measurement.

C_{V_R} and the vector form factor are highly correlated!

Contour Plots



If new lattice results turn out to be different from the experimental fitted value (assuming SM), non-zero C_{V_R} can be hinted.

Fit of C_{V_R} using forward-backward asymmetry (FBA) only

$FBA \sim < g_{6s} >$
Advantage: one angle measurement

$$\langle \mathcal{A}_{FB} \rangle \equiv \frac{\int_0^1 \frac{d\Gamma}{d\cos\theta_\ell} \mathrm{d}\cos\theta_\ell - \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_\ell} \mathrm{d}\cos\theta_\ell}{\int_0^1 \frac{d\Gamma}{d\cos\theta_\ell} \mathrm{d}\cos\theta_\ell + \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_\ell} \mathrm{d}\cos\theta_\ell} = 3\langle g_{6s} \rangle$$

$$\vec{v} = (\rho^2_{D^*}, R_1(1), R_2(1), C_{V_R}) = (1.106, 1.229, 0.852, 0.000)$$

$$\sigma_{\vec{v}} = (2.200, 0.049, 0.031, 0.022)$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & 0.008 & -0.873 & 0.262 \\ 0.008 & 1. & -0.040 & \underline{-0.931} \\ -0.873 & -0.040 & 1. & -0.296 \\ 0.262 & -0.931 & -0.296 & 1. \end{pmatrix}$$

C_{V_R} can be determined at a precision of 2.2% using FBA alone! Almost as good as the full set of $< g_i >$!

$Im(C_{V_R})$ can also be determined at precision of 0.7% for both CLN and BGL!

Summary & Conclusions

- The normalized angular observables $< g_i >$ for $B \to D^*(D\pi)\ell\nu$ determined in the un-binned analysis are useful for the precision measurement of C_{V_R} by circumventing the V_{cb} puzzle.
- C_{V_R} is highly dependent on the vector form factor, thus it can only be determined with the vector form factor calculated by lattice.
- The real (imaginary) part of C_{V_R} can be determined at precision of 2-4 (1) % using the full set of $< g_i >$.
- FBA ($< g_{6s} >$) can determine C_{V_R} at almost equally good precision, thus it is highly proposed to be measured in the near future.

Thank you!

Backup

SM fit including V_{cb}

$$\chi^2(\vec{v}) = \chi^2_{\text{angle}}(\vec{v}) + \chi^2_{\text{lattice}}(\vec{v}) + \chi^2_{w-\text{bin}}(\vec{v})$$

w dependence in χ^2:

$$\chi^2_{w-\text{bin}}(\vec{v}) = \sum_{w-\text{bin}=1}^{10} \frac{([N]_{w-\text{bin}} - \alpha\langle\Gamma\rangle_{w-\text{bin}})^2}{[N]_{w-\text{bin}}}$$

The factor α is a constant, which relates the number of events and the decay rate:

$$\alpha \equiv \frac{4N_{B\overline{B}}}{1+f_{+0}}\tau_{B^0} \times \epsilon\mathcal{B}(D^0 \to K^-\pi^+)$$

$N_{B\overline{B}}$: number of $B\overline{B}$ pairs produced from $\Upsilon(4S)$

τ_{B^0}: B^0 lifetime

f_{+0}: B^+/B^0 production ratio at Belle

SM fit results in CLN parametrization

$$\vec{v} = (h_{A_1}(1), \rho^2_{D^*}, R_1(1), R_2(1), V_{cb})$$
$$= (0.906, 1.106, 1.229, 0.852, 0.0387)$$

$$\sigma_{\vec{v}} = (0.013, 0.019, 0.011, 0.011, 0.0006)$$

SM fit results in BGL parametrization

$$\vec{v} = (a_0^f, a_1^f, a_1^{\mathcal{F}_1}, a_2^{\mathcal{F}_1}, a_0^g, V_{cb})$$
$$= (0.0132, 0.0169, 0.0070, -0.0853, 0.0242, 0.0384)$$

$$\sigma_{\vec{v}} = (0.0002, 0.0028, 0.0011, 0.0199, 0.0004, 0.0006)$$

$\alpha = 6.616(6.613) \times 10^{18}$ in CLN (BGL) parametrization

Experimental efficiency: $\epsilon = \sim 4.8 \times 10^{-2}$