Lepton Universality Violation at LHCb

LHCbでのレプトン普遍性違反

"Les quarks beauté bousculent l'universalité leptonique"

CERN Courier March 2018

Steve Playfer

on behalf of the LHCb collaboration

Workshop on Hints of New Physics in Heavy Flavours

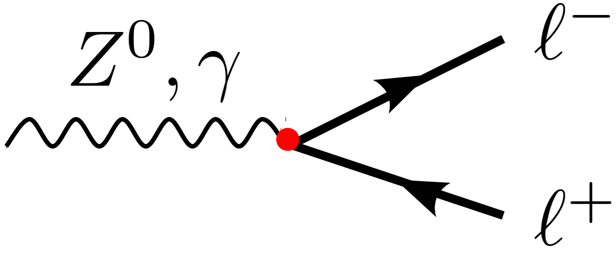
Nagoya University, November 15th 2018

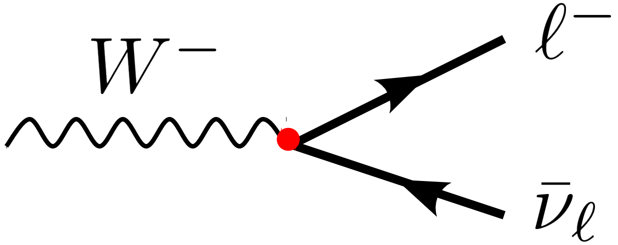
Outline

- Introduction to Lepton Flavour Universality
- The LHCb detector
- Electroweak penguin decays
- The ratios R_K and R_{K^*}
- Anomalies in Branching Fractions and angular distributions
- The semileptonic ratios R_{D^*} and $R_{J/\psi}$
- Outlook for the future

Lepton Flavour Universality

EW bosons in SM:



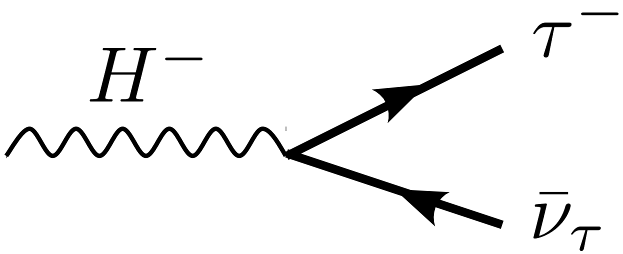


Couplings of Z,W and γ to $\ell \in \{e, \mu, \tau\}$ do not depend on lepton flavour

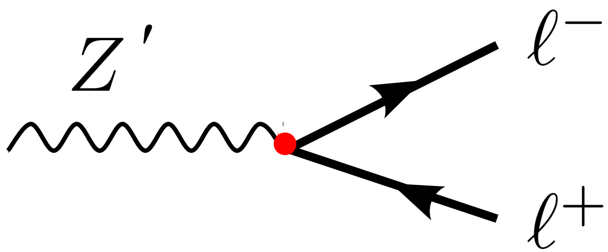
Differences in decay rates are driven by the different masses
m_e = 0.511MeV, m_μ =105MeV, m_τ = 1777MeV

Semileptonic b decays to e and μ almost identical
Leptonic $B \to \ell\nu$, $B \to \ell\ell$ helicity-suppressed by m_ℓ^2

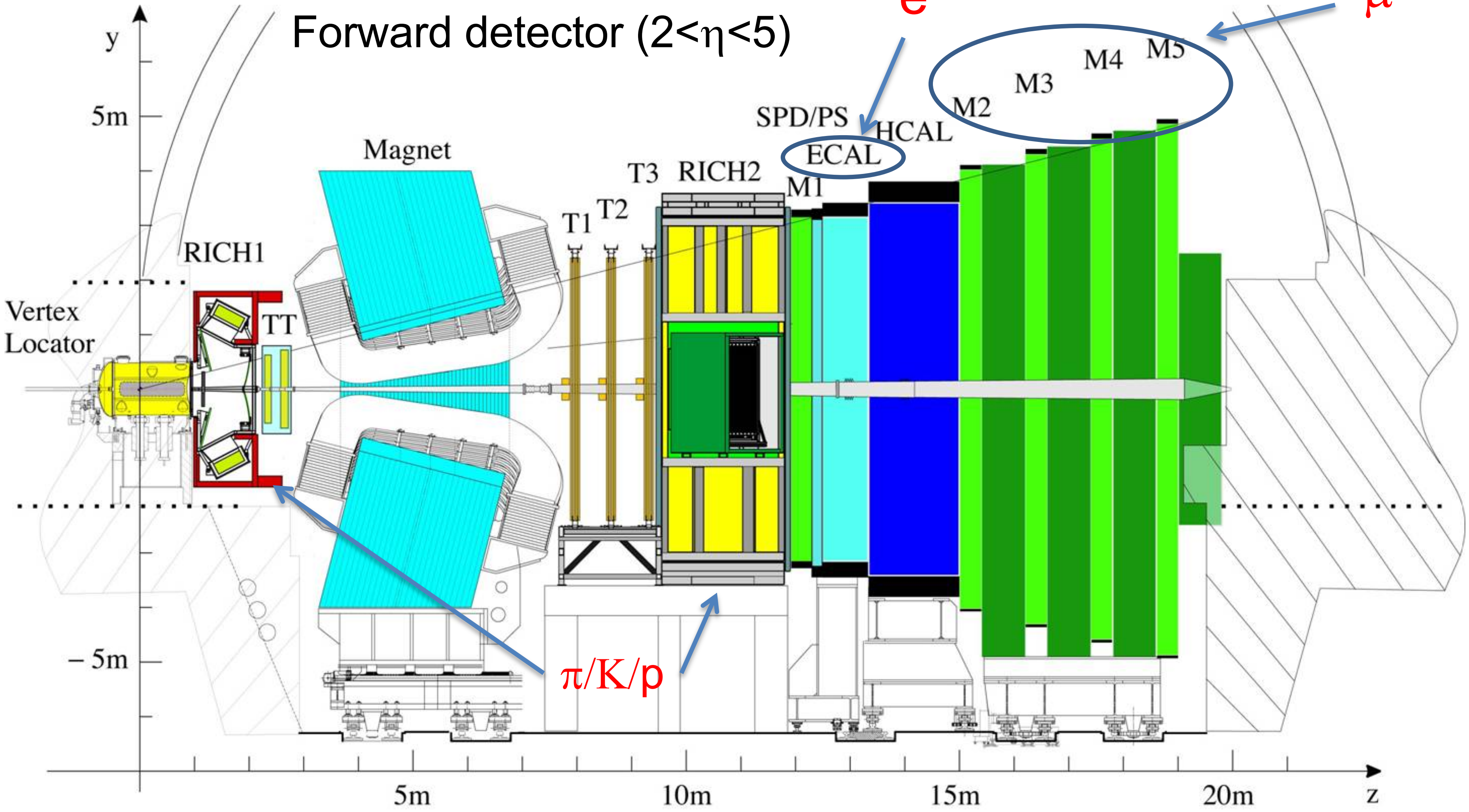
Charged Higgs in NP:



Heavy Z′ boson in NP:



The LHCb Detector



The LHCb detector is not lepton flavour universal!

Triggering at LHCb

Hardware (Level 0)

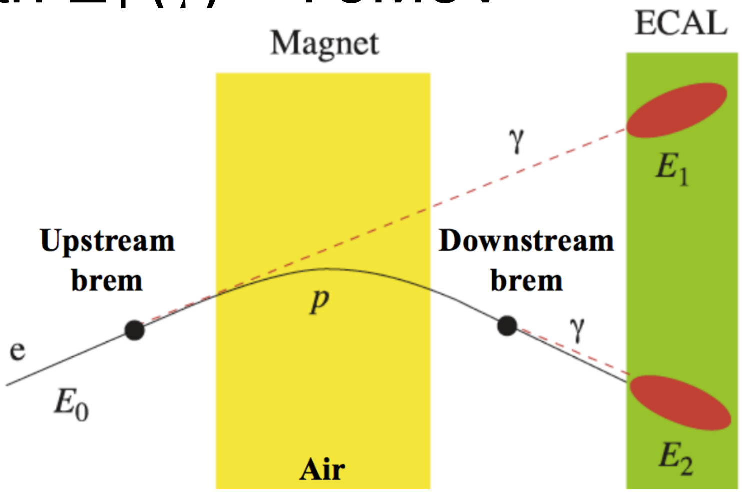
- L0M: Muons identified with p_T > 1.5 to 1.8 GeV/c
- L0E: Electrons identified with E_T > 2.5 to 3.0 GeV
- L0H: Any π/K from the signal decay with E_T > 3.5 GeV
- L0I: Other high p_T tracks independent of the signal decay

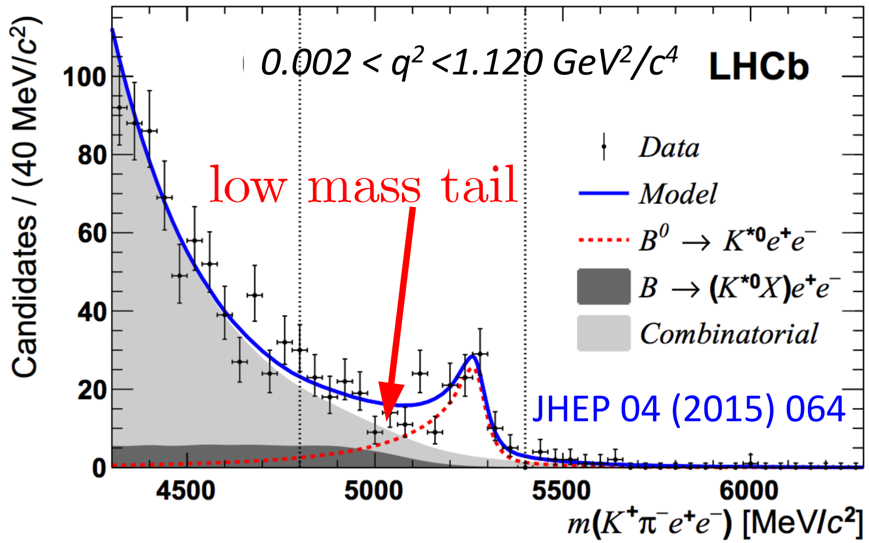
Software

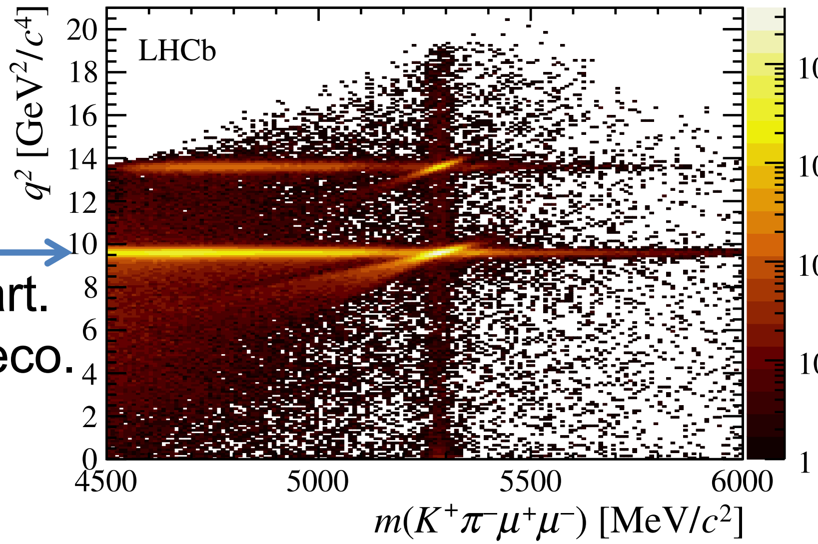
- 2,3 or 4-track vertices displaced from the primary vertex and consistent with the signal decay mode

Electron Reconstruction at LHCb

Bremsstrahlung recovery < 100%
with $E_T(\gamma)$ > 75MeV







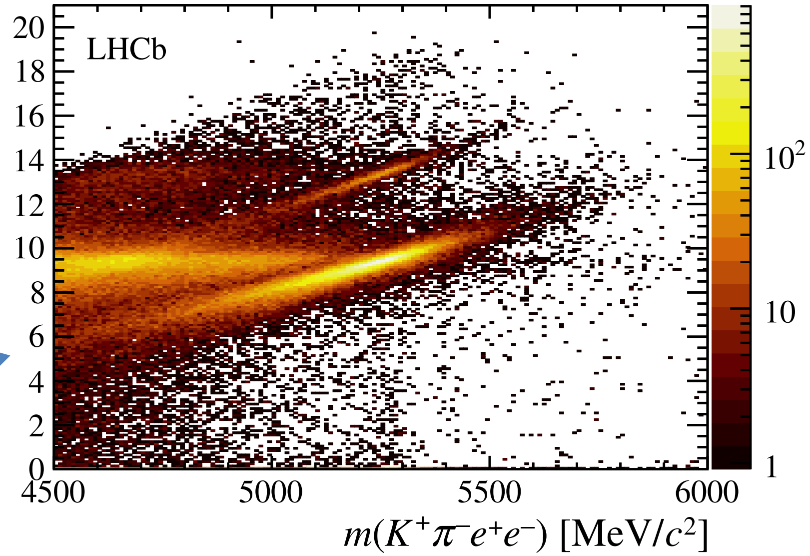
Part. Reco.

JHEP 08 (2017) 055

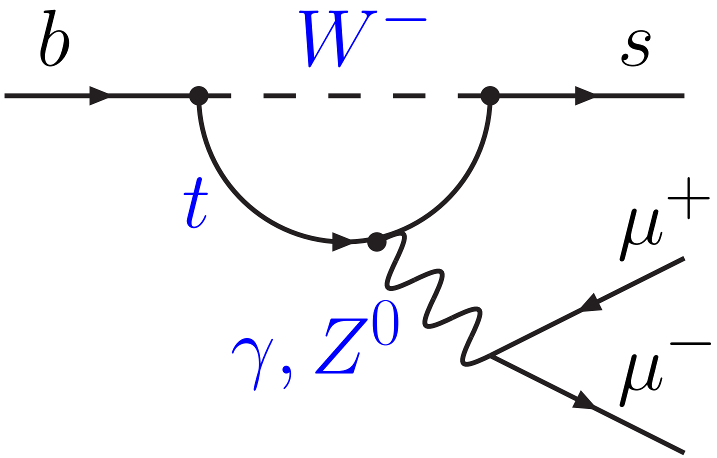
ψ′

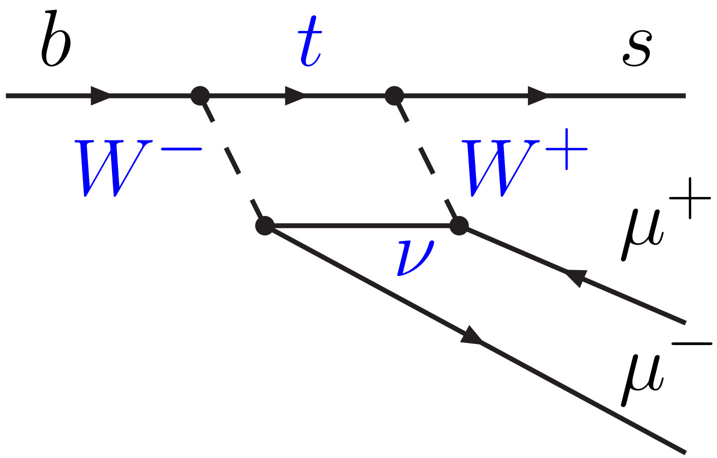
J/ψ

Brems strahlung



Electroweak Penguin Decays

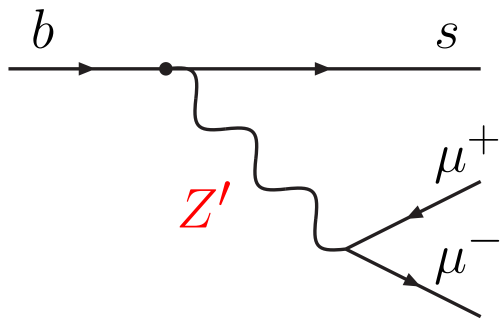




Flavour changing neutral current transitions require loops/boxes in the SM

Can replace W, Z, t with charged Higgs, Z′, SUSY partners, leptoquarks or other NP

Could in principle have tree level FCNC couplings, but these are strongly constrained by other measurements



Effective Theory

$$\mathcal{H}_{\text{eff}} = -\frac{4\,G_F}{\sqrt{2}} V_{tb} V_{ts}^{*} \frac{\alpha_e}{4\pi} \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

Wilson coefficients — Four-fermion operators

Integrate out scales above $\mu \sim m_b$

SM calculations of inclusive rates give (10% accuracy):

$C_7 \sim -0.3$ from the photon

$C_9 \sim +4$ from EW vector

$C_{10} \sim -4$ from EW axial-vector

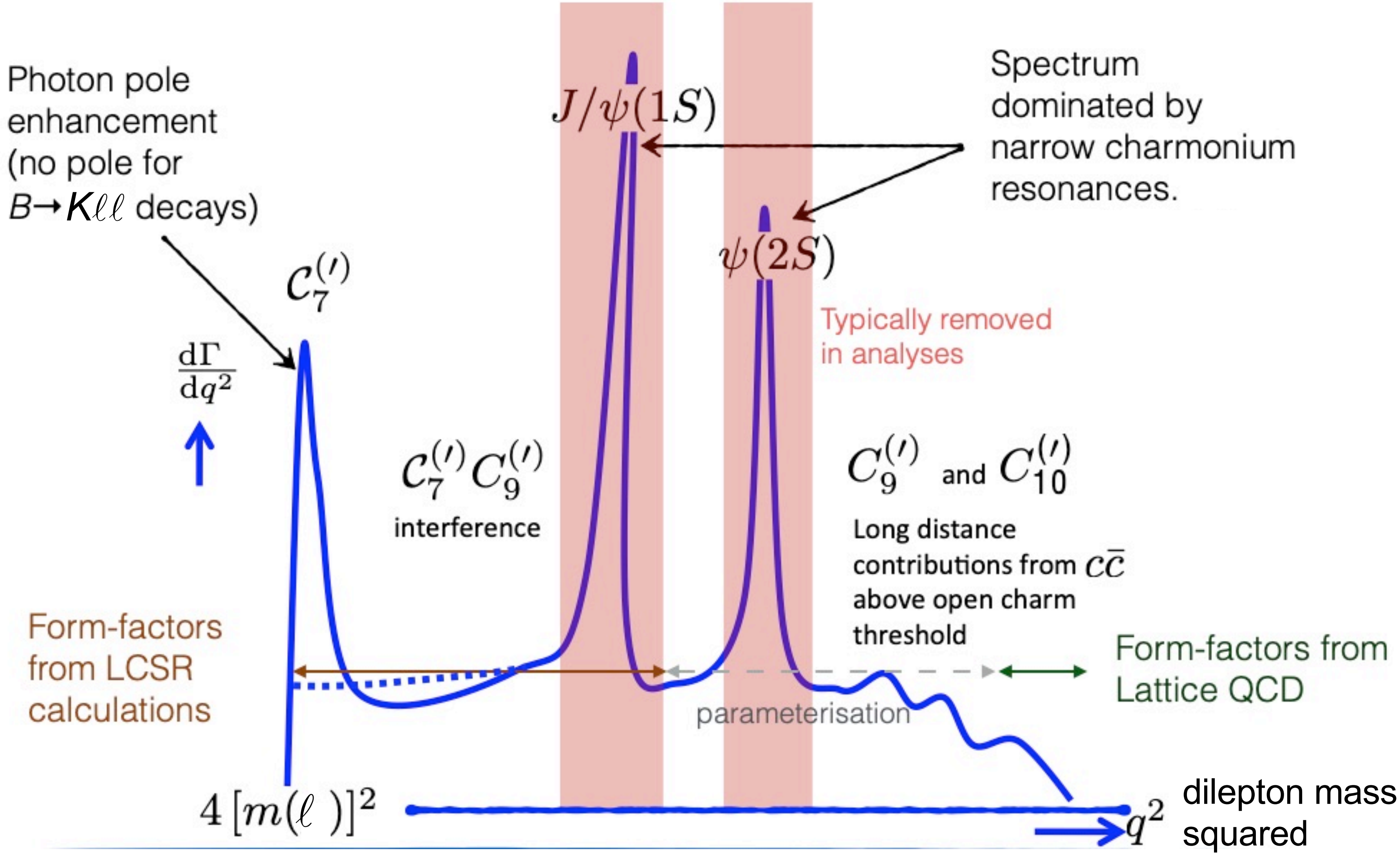
$$\mathcal{O}_{7\gamma}^{(\prime)} = \frac{m_b}{e}(\bar{s}\sigma^{\mu\nu} P_{R(L)} b) F_{\mu\nu}$$

$$\mathcal{O}_{9V}^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_{10A}^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

(′) indicate RH contributions (suppressed by m_s/m_b in SM)

Map of K(*)$\ell\ell$ Contributions

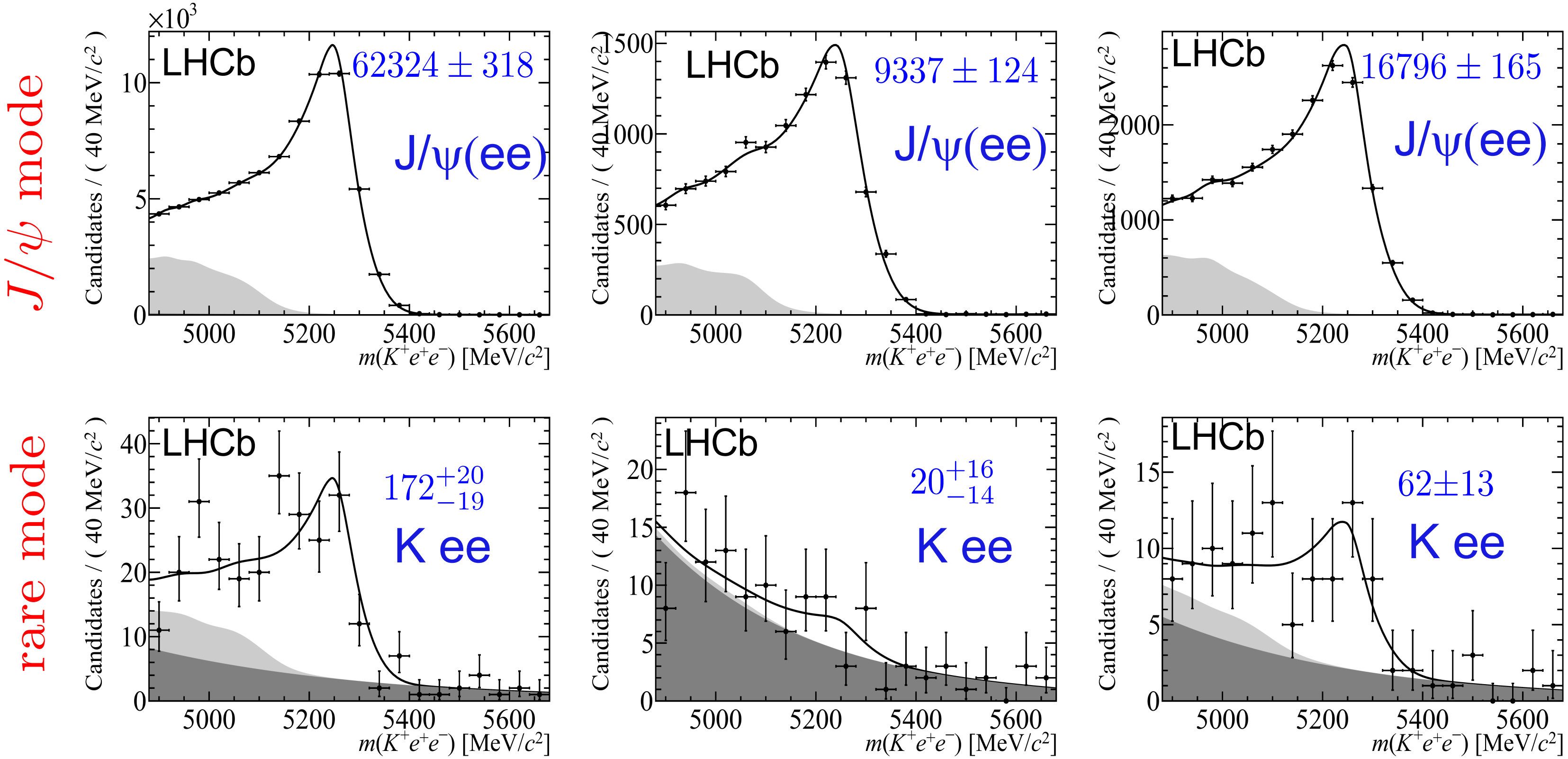


Measure Double Ratios

Reduces dependence on simulation for selection and reconstruction efficiency.

$$R_K = \left(\frac{\mathcal{N}_{K^+\mu^+\mu^-}}{\mathcal{N}_{K^+e^+e^-}}\right)\left(\frac{\mathcal{N}_{J/\psi\,(e^+e^-)K^+}}{\mathcal{N}_{J/\psi\,(\mu^+\mu^-)K^+}}\right)\left(\frac{\epsilon_{K^+e^+e^-}}{\epsilon_{K^+\mu^+\mu^-}}\right)\left(\frac{\epsilon_{J/\psi\,(\mu^+\mu^-)K^+}}{\epsilon_{J/\psi\,(e^+e^-)K^+}}\right)$$

L0 electron | L0 hadron | L0 signal independent | L0 muon



667(1)k J/ψ(μμ) events

3/fb at 7-8TeV

1226(41) K μμ events $1<q^2<6\,\mathrm{GeV}^2$

PRL 113, 151601 (2014)

Result for R_K

LHCb PRL 113, 151601 (2014): $R_K = 0.745^{+0.09}_{-0.07}$ (stat) $\pm$ 0.036 (syst)

3/fb at 7-8TeV Window $1 < q^2 < 6$ GeV^2 2.6σ away from SM R_K=1(10^{-2})

Bordone, Isidori & Pattori
EPJC 76, 440 (2016)

Errors are almost entirely from Kee samples.
Dominant systematics from fit shapes and bremsstrahlung correction.

For comparison:

BaBar PRD86, 032012 (2012): $R_K = 0.74^{+0.31}_{-0.25}$ (stat) $\pm$ 0.07 (syst)

Window $0.1 < q^2 < 8$ GeV^2

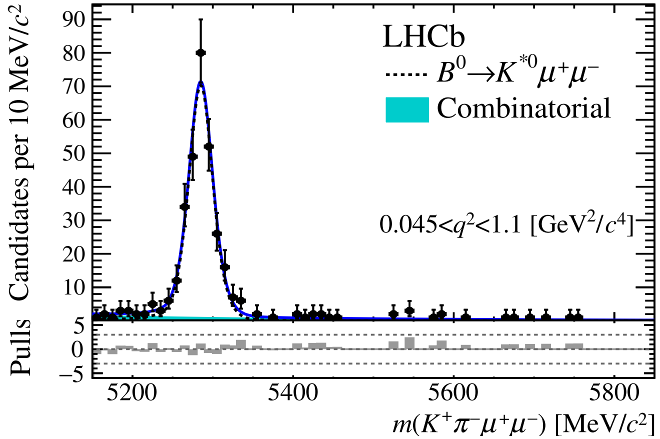
Belle PRL103, 171801 (2009): $R_K = 1.03 \pm 0.19$ (stat) $\pm$ 0.06 (syst)

All q^2 apart from J/ψ and ψ' regions

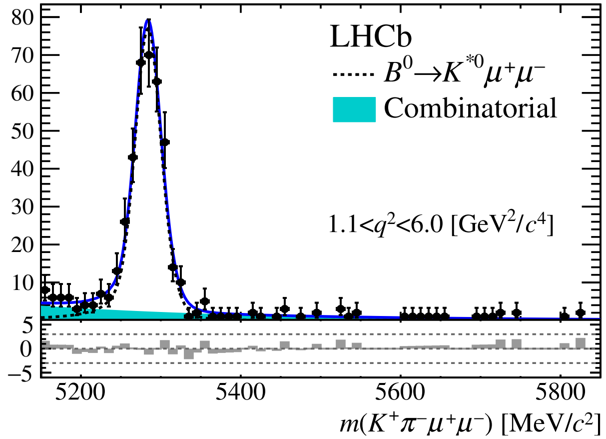
K* ℓℓ signals

3/fb at 7-8TeV

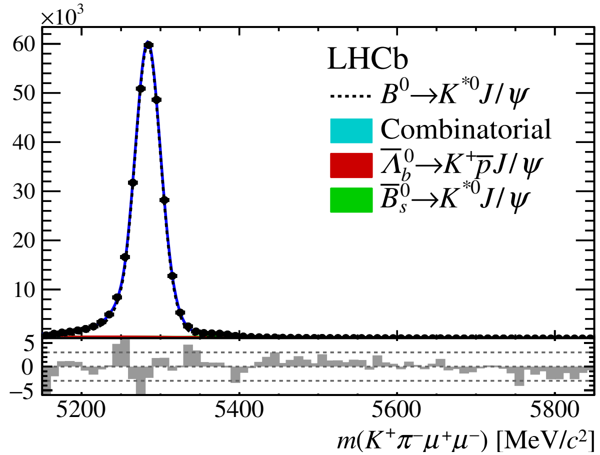
285 K* μμ events



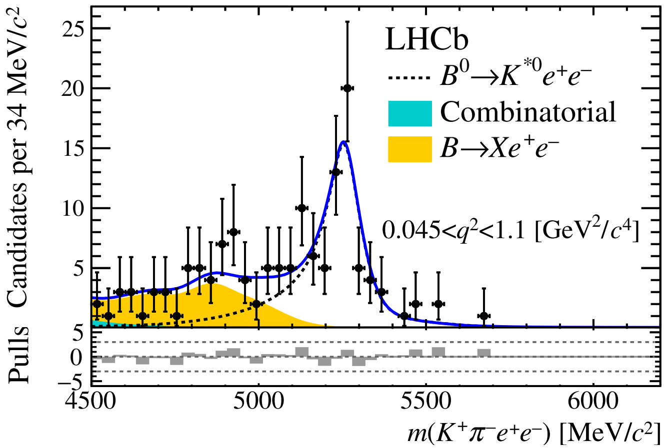
353 K* μμ events



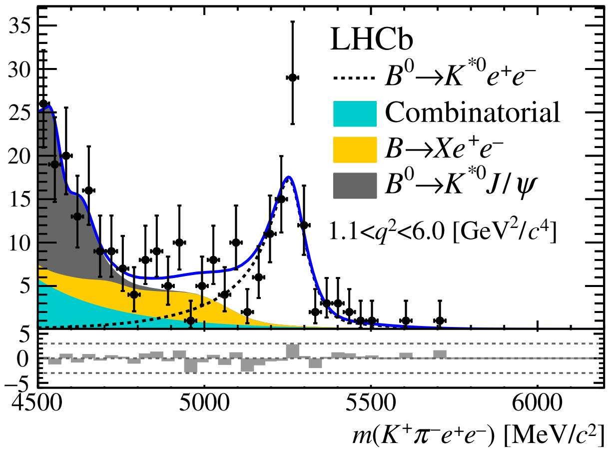
274k J/ψ(μμ) events



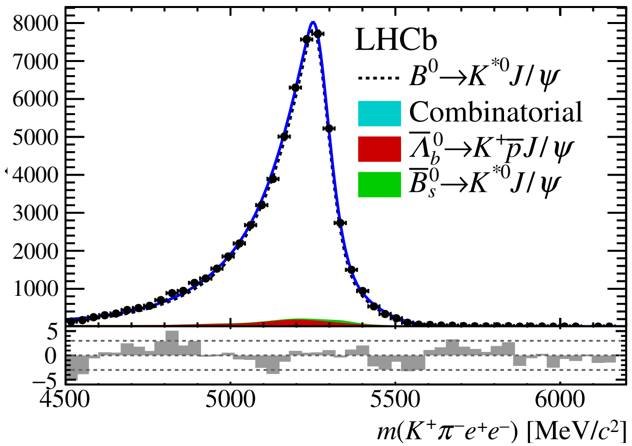
89 K*ee events



111 K*ee events



58k J/ψ(ee) events



JHEP 08,055 (2017)

Results for R_{K^*}

LHCb JHEP 08, 055 (2017) 3/fb at 7-8TeV

$R_{K^*} = 0.66^{+0.11}_{-0.07}$ (stat) $\pm$ 0.03 (syst) for $0.045 < q^2 < 1.1$ GeV^2

2.2σ away from SM prediction of R_{K^*}=0.926(4) Altmannshofer et al EPJC 77, 377 (2017)

$R_{K^*} = 0.69^{+0.11}_{-0.07}$ (stat) $\pm$ 0.05 (syst) for $1.1 < q^2 < 6$ GeV^2

2.5σ away from SM prediction of R_{K^*}=1(10^{-2}) Bordone, Isidori & Pattori EPJC 76, 440 (2016)

For comparison:

BaBar PRD86, 032012 (2012): $R_{K^*} = 1.06^{+0.48}_{-0.33}$ (stat) $\pm$ 0.08 (syst)

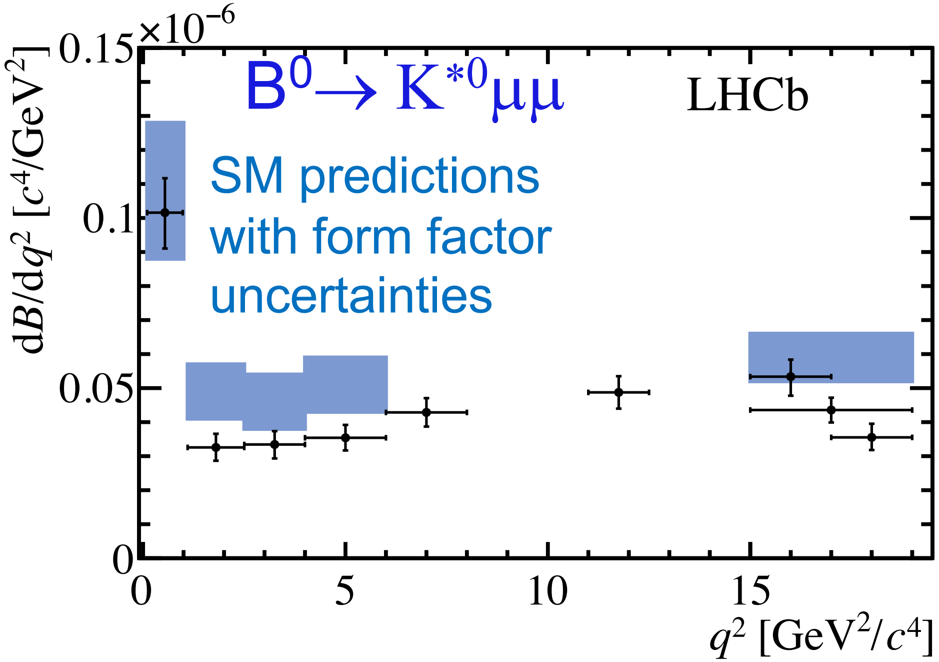
Window $0.1 < q^2 < 8$ GeV^2

Belle PRL103, 171801 (2009): $R_{K^*} = 0.83 \pm 0.17$ (stat) $\pm$ 0.05 (syst)

All q^2 apart from J/ψ and ψ' regions

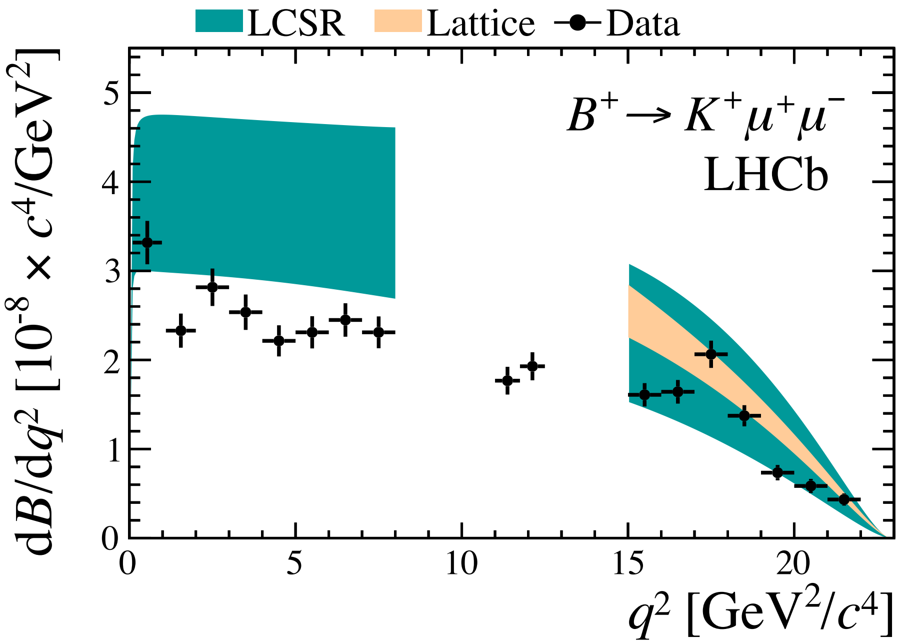
K(*) μμ Branching Fractions

3/fb at 7-8TeV



JHEP 11, 017 (2016)

Corrected for 10% S-wave Kπ using angular/mass fit

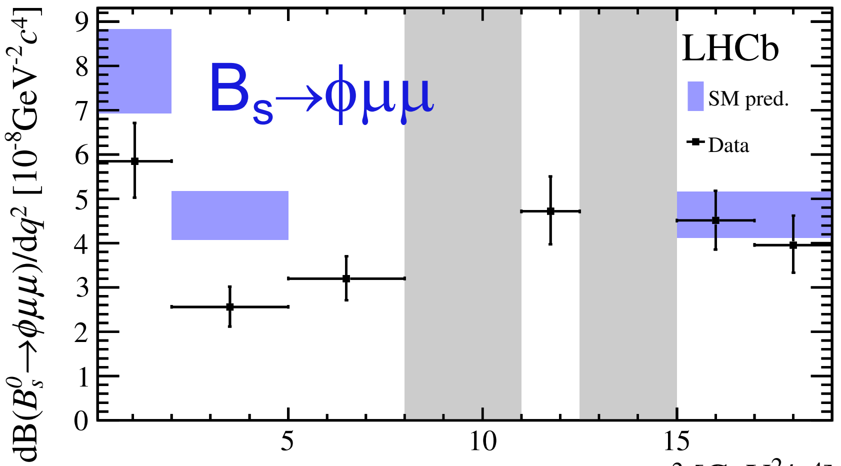


JHEP 06, 133 (2014)

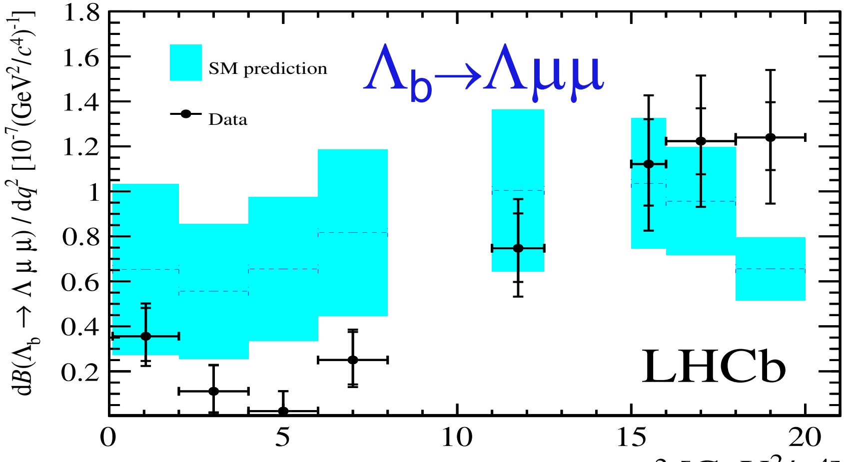
Are K(*)μμ BFs below SM?

K(*)ee BFs agree with SM!

More Branching Fractions

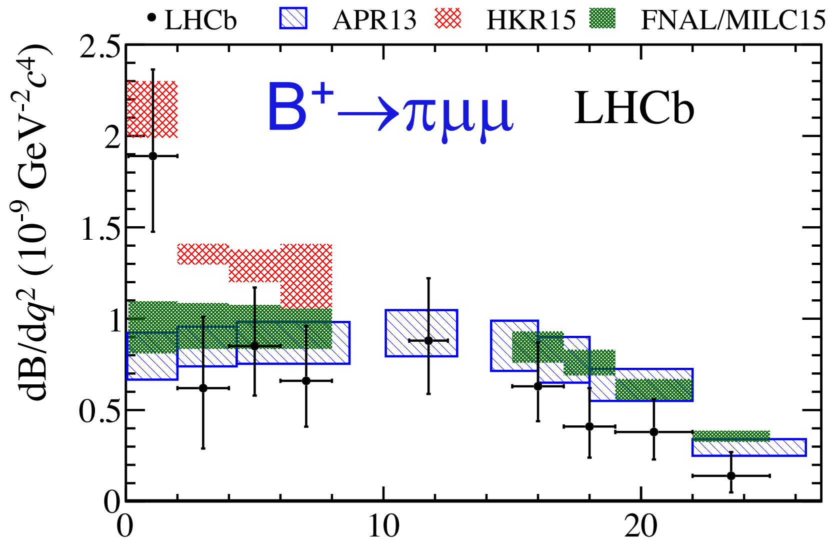


JHEP 11, 017 (2016)



JHEP 06, 115 (2015)

3/fb at 7-8TeV

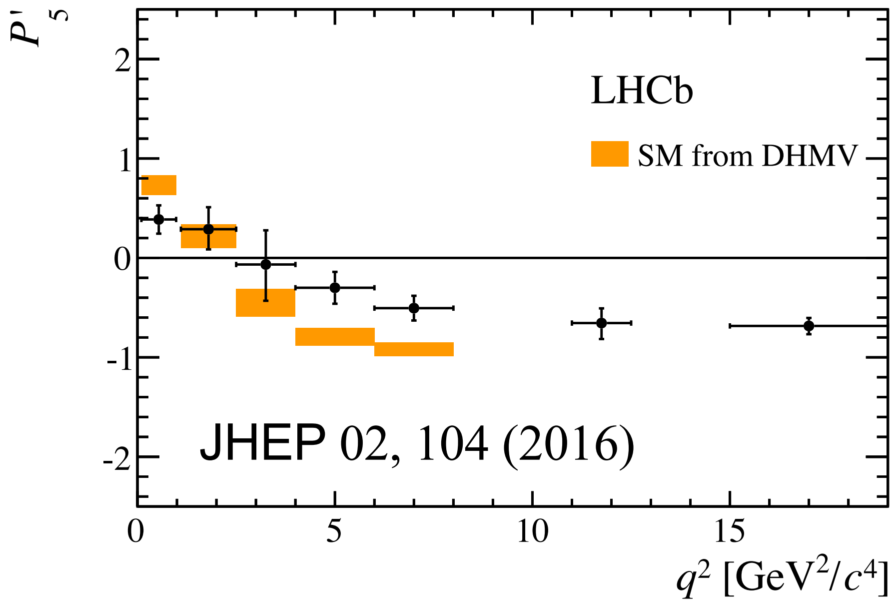


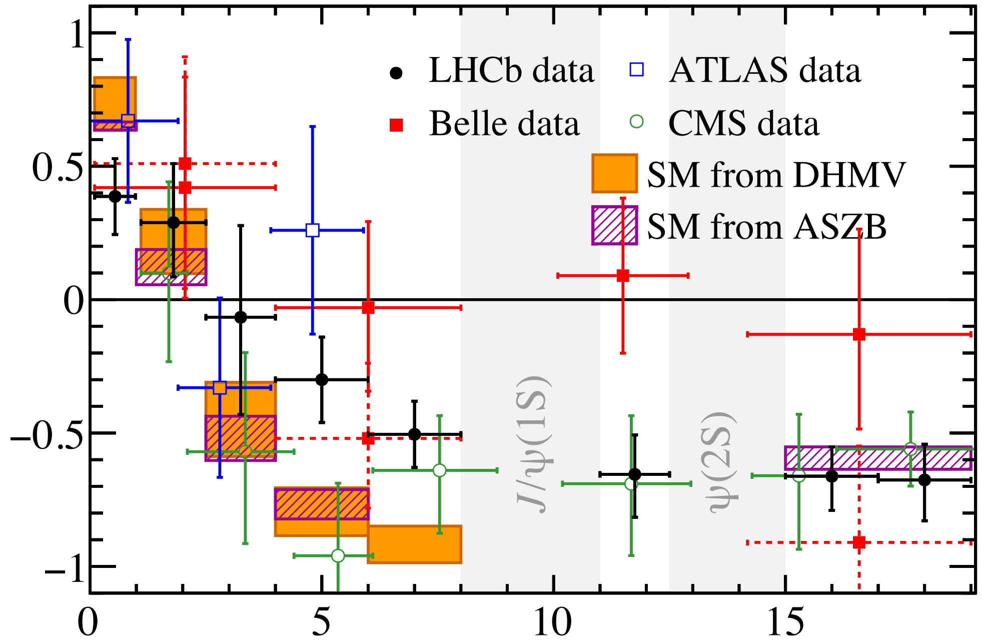
JHEP 06, 133 (2014)

These BFs also below SM?

Angular analysis of K*μμ

3/fb at 7-8TeV





$P_5' = S_5/\sqrt{[F_L(1-F_L)]}$

Is an angular coefficient that is designed to be insensitive to form factors

DHMV = Descotes-Genon et al JHEP 12, 125 (2014)

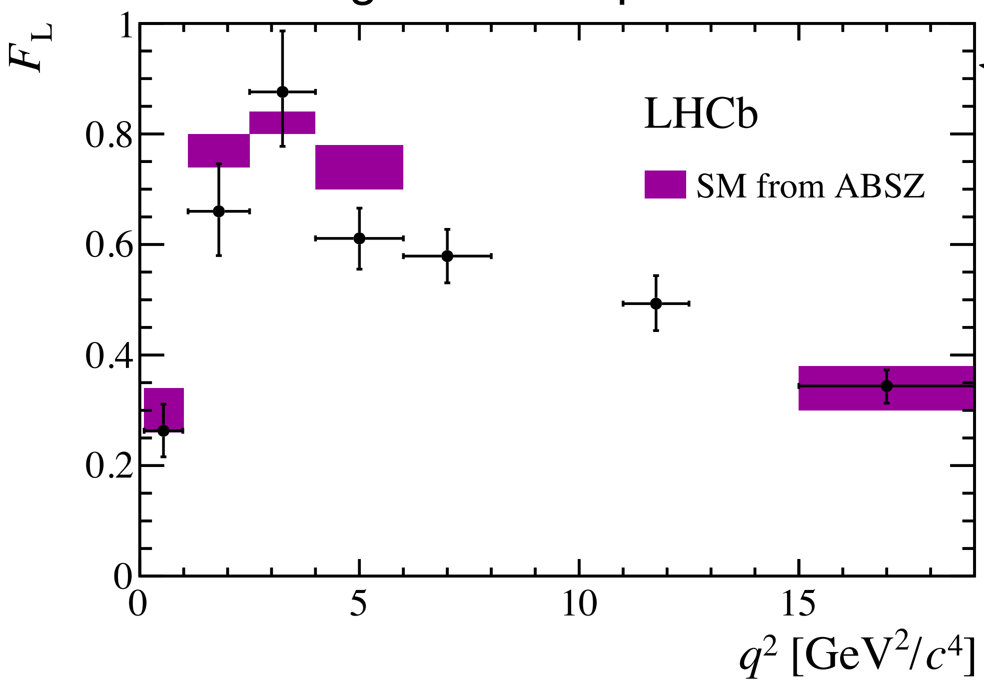
LHCb says disagreement with SM is at the level of 3.4σ

Supported by Belle and maybe ATLAS.
Not confirmed by CMS.

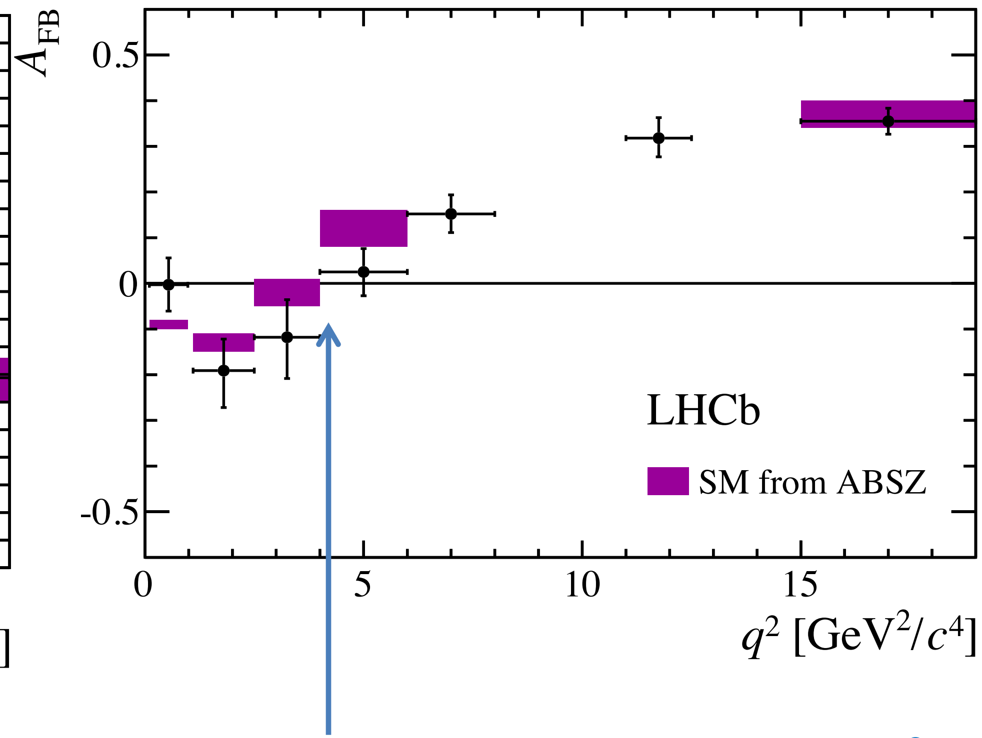
K*μμ F_L and A_{FB}

3/fb at 7-8TeV   JHEP 02,014 (2016)

Longitudinal K* polarisation



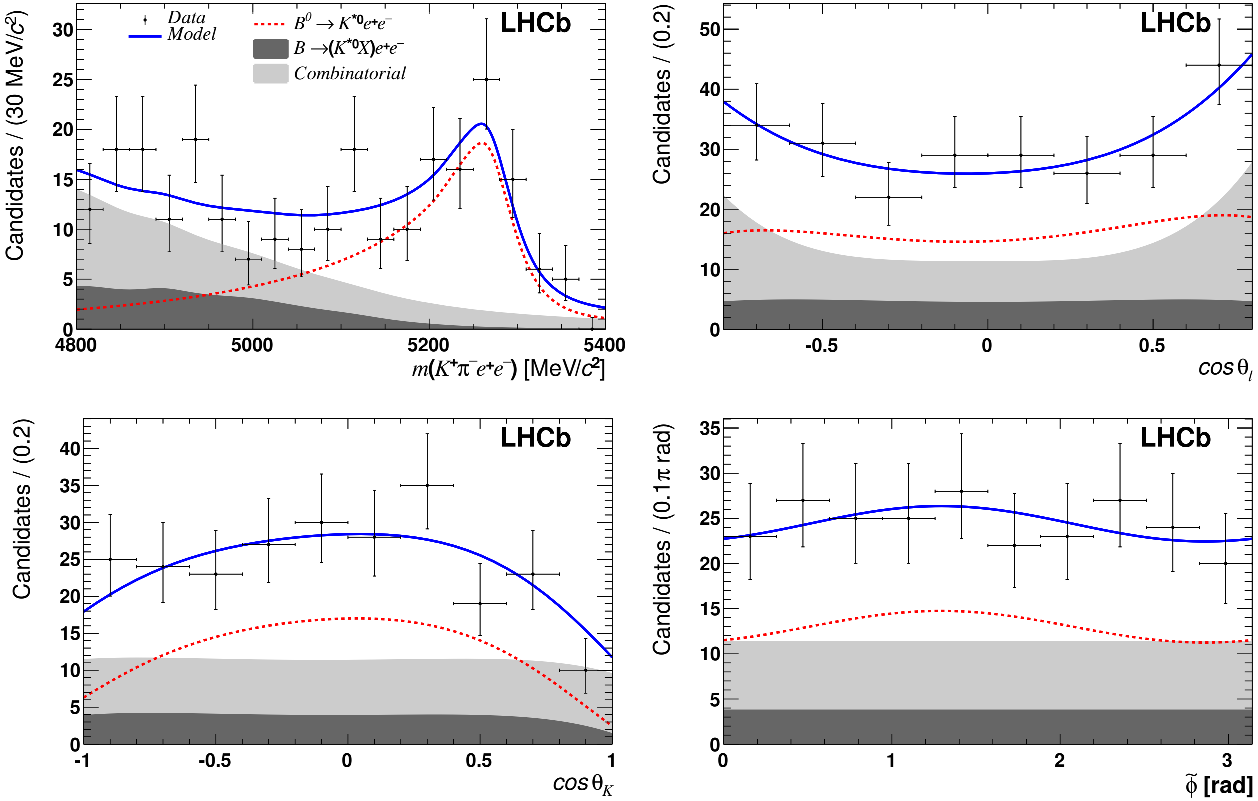
Lepton forward-backward asymmetry



ABSZ = Altmannshofer & Straub
EPJC 75, 882 (2015)

Zero-crossing point shifted up in q^2

Angular analysis of K*ee



$$F_{\rm L} = 0.16 \pm 0.06 \pm 0.03$$
$$A_{\rm T}^{(2)} = -0.23 \pm 0.23 \pm 0.05$$
$$A_{\rm T}^{\rm Im} = +0.14 \pm 0.22 \pm 0.05$$
$$A_{\rm T}^{\rm Re} = +0.10 \pm 0.18 \pm 0.05$$

“For SM values of C7 the ratio C7′/C7 is compatible with zero”

3/fb at 7-8TeV

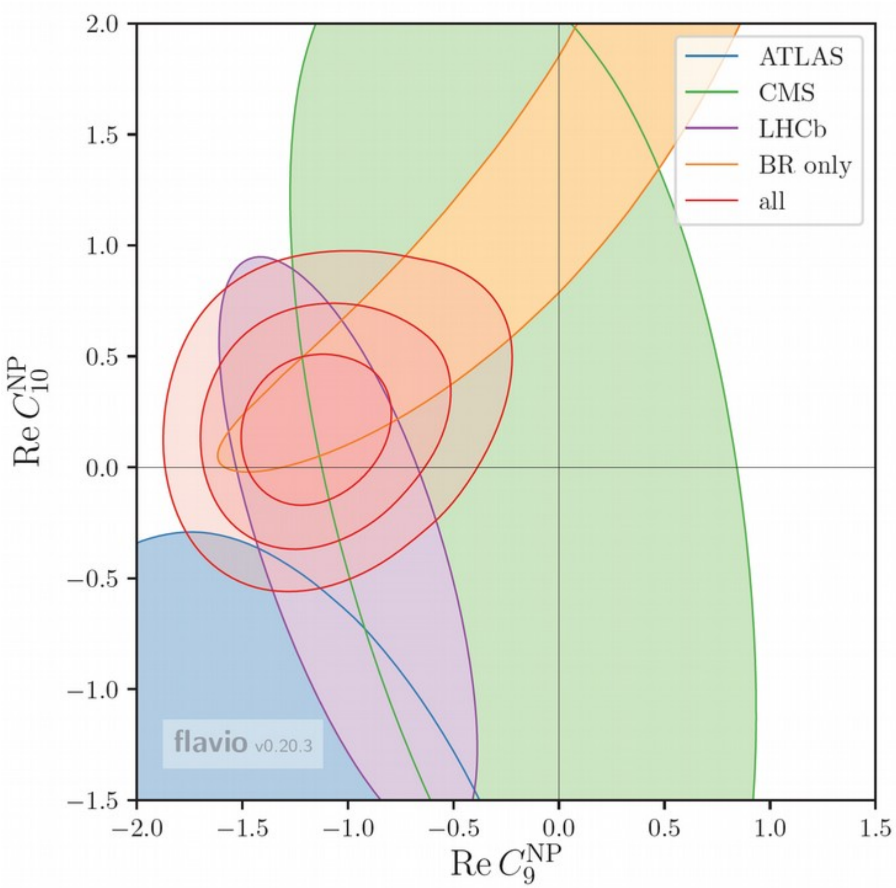
JHEP 04, 064 (2015)

$0.002 < q^2 < 1.12 GeV^2$

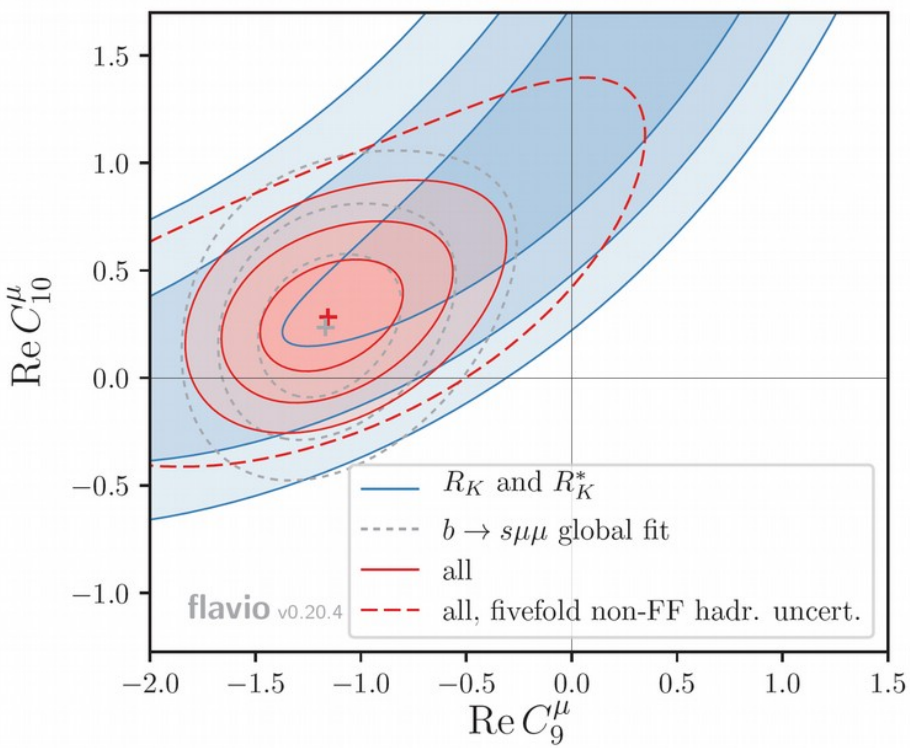
Low q^2 region is described by photon pole.

Global Fits for Wilson coefficients

BF and angular fits



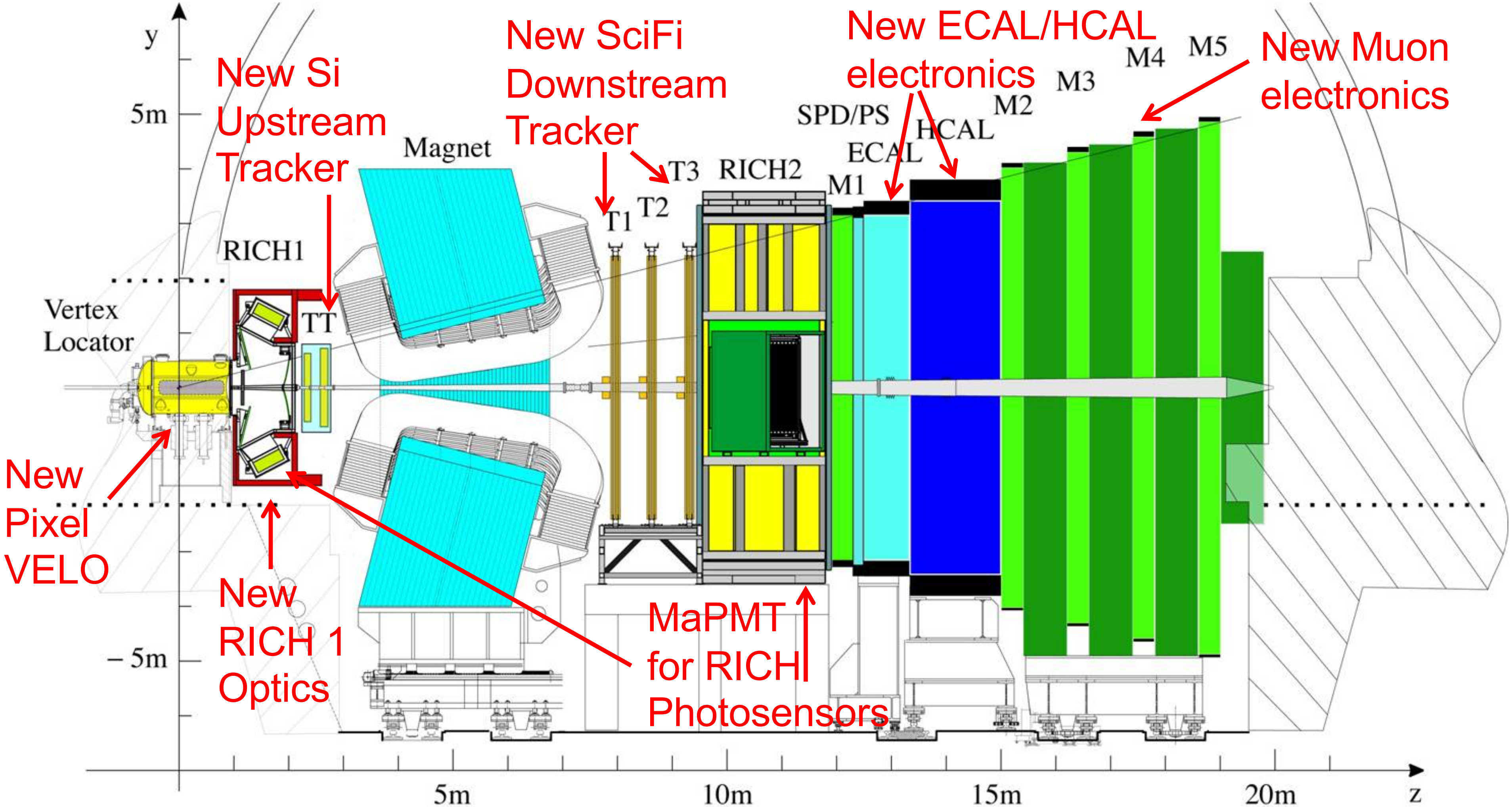
Including R_K and R_K*



Altmannshofer et al 1703.09189, 1704.05435

Consistent with $\Delta C_9 \sim -1$ due to NP. Could also be a small shift in ΔC_{10} .

LHCb Upgrade 2019-2020



Designed for 40MHz readout with a full trigger in software.
Instantaneous luminosity $2x10^{33}/cm^2/s$ (increase x5)

LHCb Upgrade TDR
CERN-LHCC-2012-007

Outlook for R_K and R_{K^*}

Physics case for LHCb Upgrade II arXiv:1808.08865 (2018)

	$\sigma(R_K)$	$\sigma(R_{K^*})$
$1 < q^2 < 6$ GeV^2		
➢ All results based on **3/fb** at 7-8TeV (Run 1 2010-2012)	0.09	0.11 (stat)
	0.036	0.050 (syst)
➢ We have another **6/fb** at 13TeV (Run 2 2015-2018) x4 in B statistics due to increased production X-section	0.043	0.052
➢ LHCb upgrade during shutdown (2019-2020) 40MHz readout and trigger entirely in software Better calorimeter granularity and timing for electrons	After upgrade can reduce syst errors	
➢ Integrated luminosity **50/fb** in Runs 3 & 4 Higher instantaneous luminosity $2 \times 10^{33}/cm^2/s$	0.017	0.020
➢ Possible major upgrade in ~2030 Much higher luminosity 2×10^{34}, with target of **300/fb**	0.007	0.008
	similar to σ(SM)	

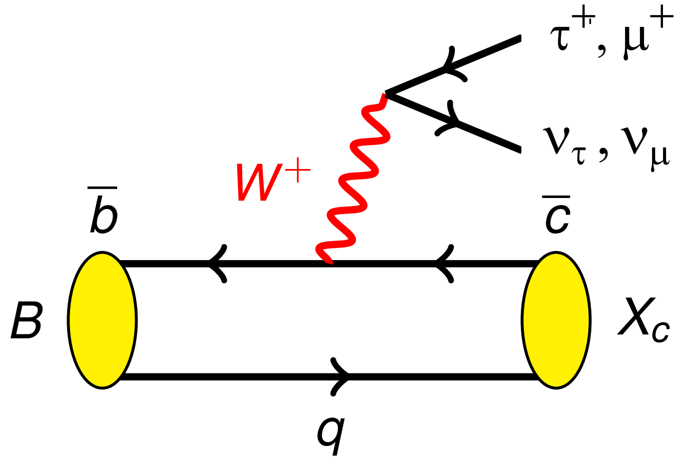
More on upgrades in talk by Eugeni Grauges Pous

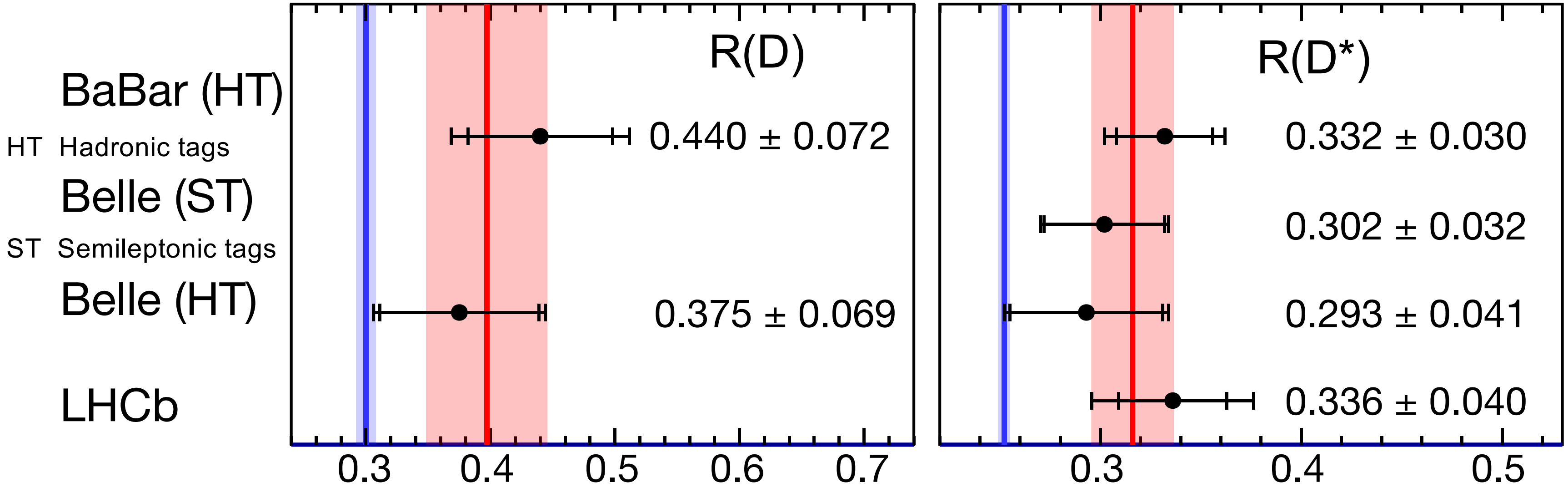
Semileptonic D(*)$\ell\nu$ decays

Review by Ciazarek et al
Nature 546, 227 (2017)

SM predictions of R(D$^{(*)}$) = D(*)$\tau\nu$/D(*)$\mu\nu$

R(D) = 0.299(6), R(D*) = 0.252(3)





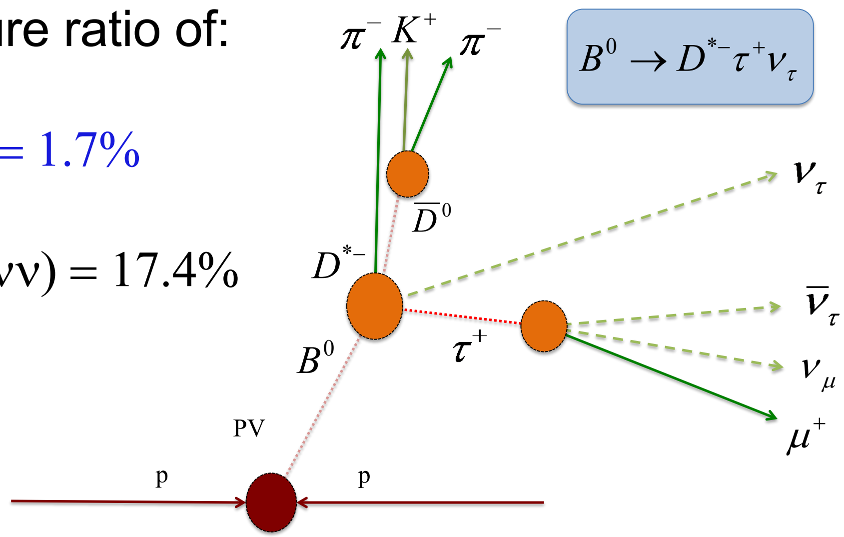
World averages (2017): R(D) = 0.397(49), R(D*) = 0.316(19) are 2σ/3σ above SM

R(D*) with $\tau \to \mu\nu\nu$

Measure ratio of:

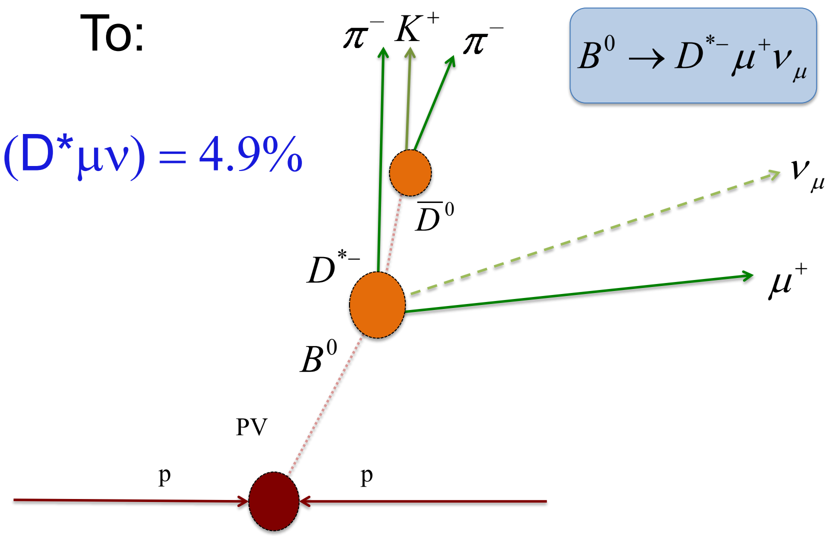
$B(D^*\tau\nu) = 1.7\%$

$B(\tau \to \mu\nu\nu) = 17.4\%$



To:

$B(D^*\mu\nu) = 4.9\%$

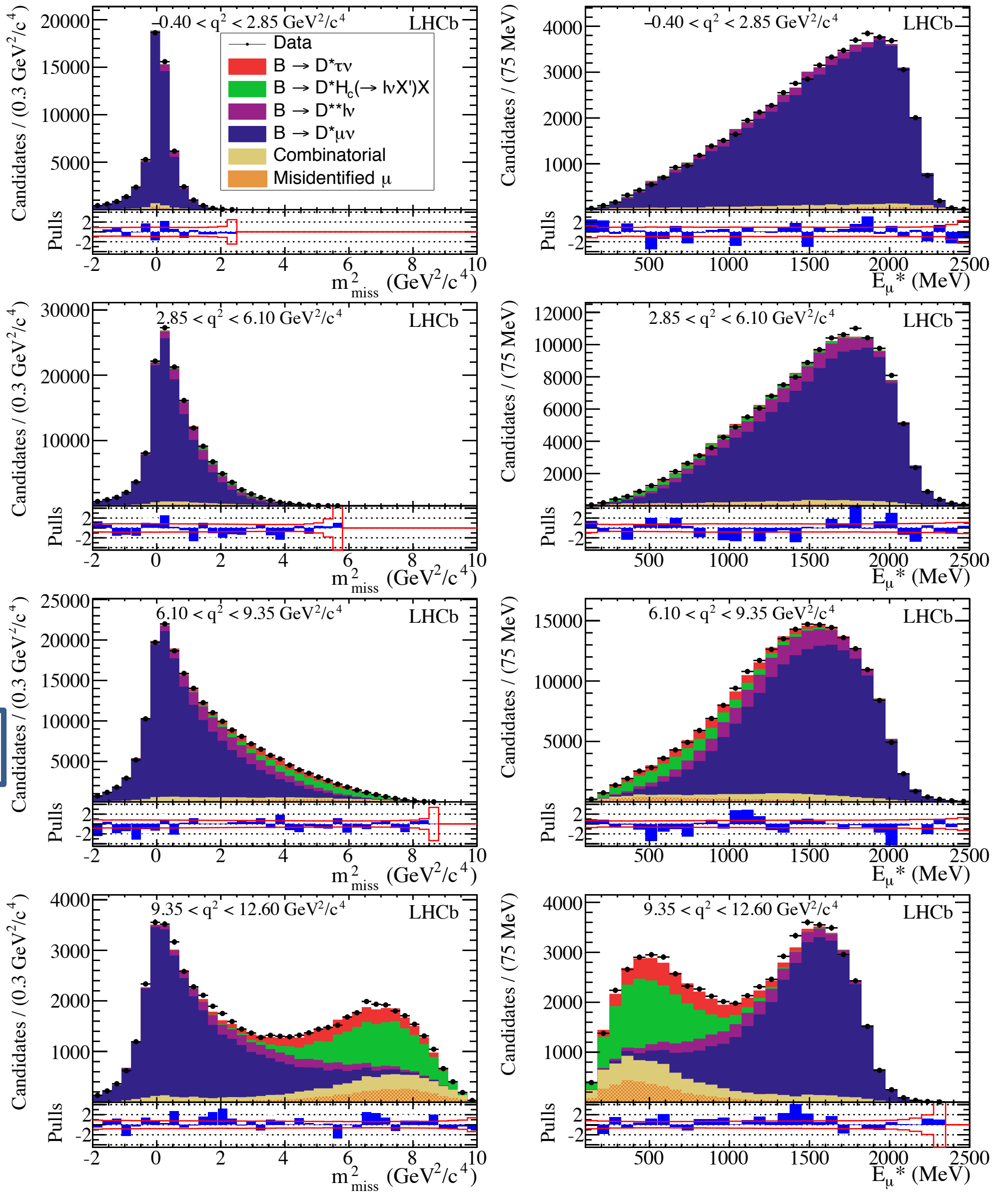


- Same visible final state particles
- 3ν in signal mode, 1ν in normalisation mode
- Separated by fit to missing mass m_{miss}^2, muon energy E_μ^*, leptonic q^2

- Backgrounds from D**, $B \to D_{(s)}D^*$, combinatorics, muon mis-ID
- Mostly dealt with by control samples, e.g. wrong-sign combinations, additional charged track at B decay vertex …

Result for R(D*) with $\tau \to \mu\nu\nu$

- $B \to D^* \mu\nu$ dominates at low q^2
- $B \to D^* \tau\nu$ is visible at high q^2, high m_{miss}^2, low E_μ^*
- Backgrounds from $D^{**}\mu\nu$, $B \to D_{(s)}D^*$ combinatorics, muon mis-ID



$R(D^*) = 0.336 \pm 0.027 \text{ (stat)} \pm 0.030 \text{ (syst)}$

Systematic limited by size of MC sample!

1.9σ above SM

LHCb PRL 115, 112991 (2015)

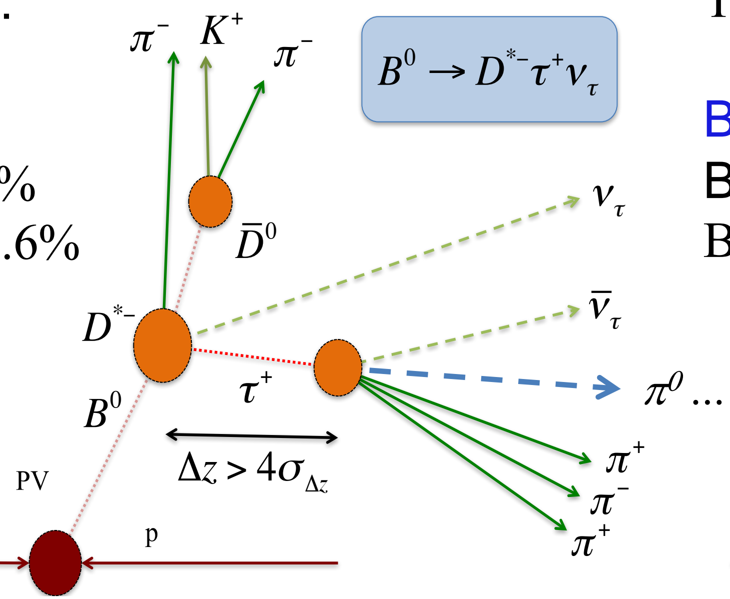
R(D*) with $\tau \to 3\pi(\pi^0)\nu$

Measure ratio of:

$B \to D^* \tau\nu$

$B(\tau \to 3\pi\nu) = 9.3\%$

$B(\tau \to 3\pi\pi^0\nu) = 4.6\%$

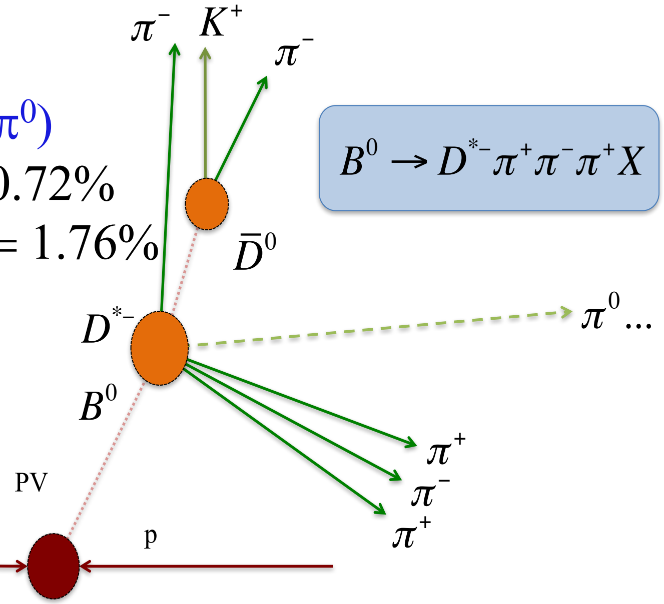


To:

$B \to D^*3\pi(\pi^0)$

$B(D^*3\pi) = 0.72\%$

$B(D^*3\pi\pi^0) = 1.76\%$



- Same visible final state particles (we don't require the π^0)
- 2ν in signal mode, 0ν in normalisation mode
- Signal extracted by fit to q^2, τ lifetime, and a BDT (to suppress D_sD^*)

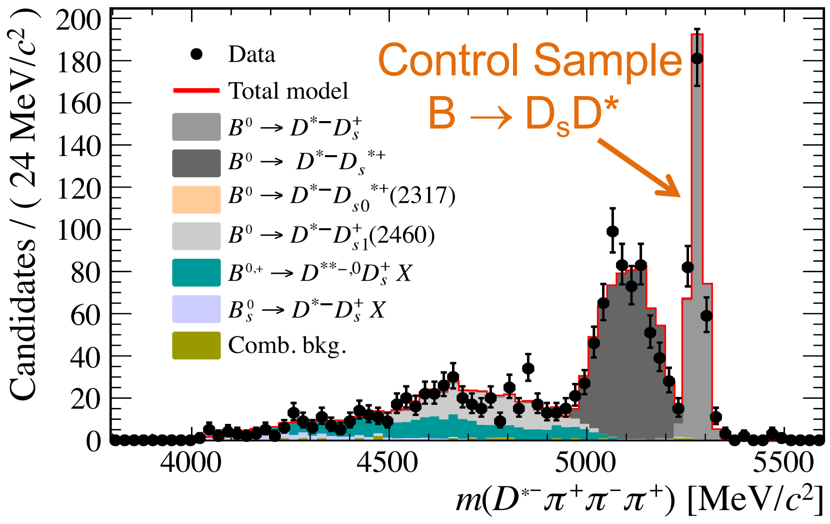
- Backgrounds from D**, $B \to D_{(s)}D^*$, $B \to D^*3\pi X$, combinatorics
- Mostly dealt with by control samples

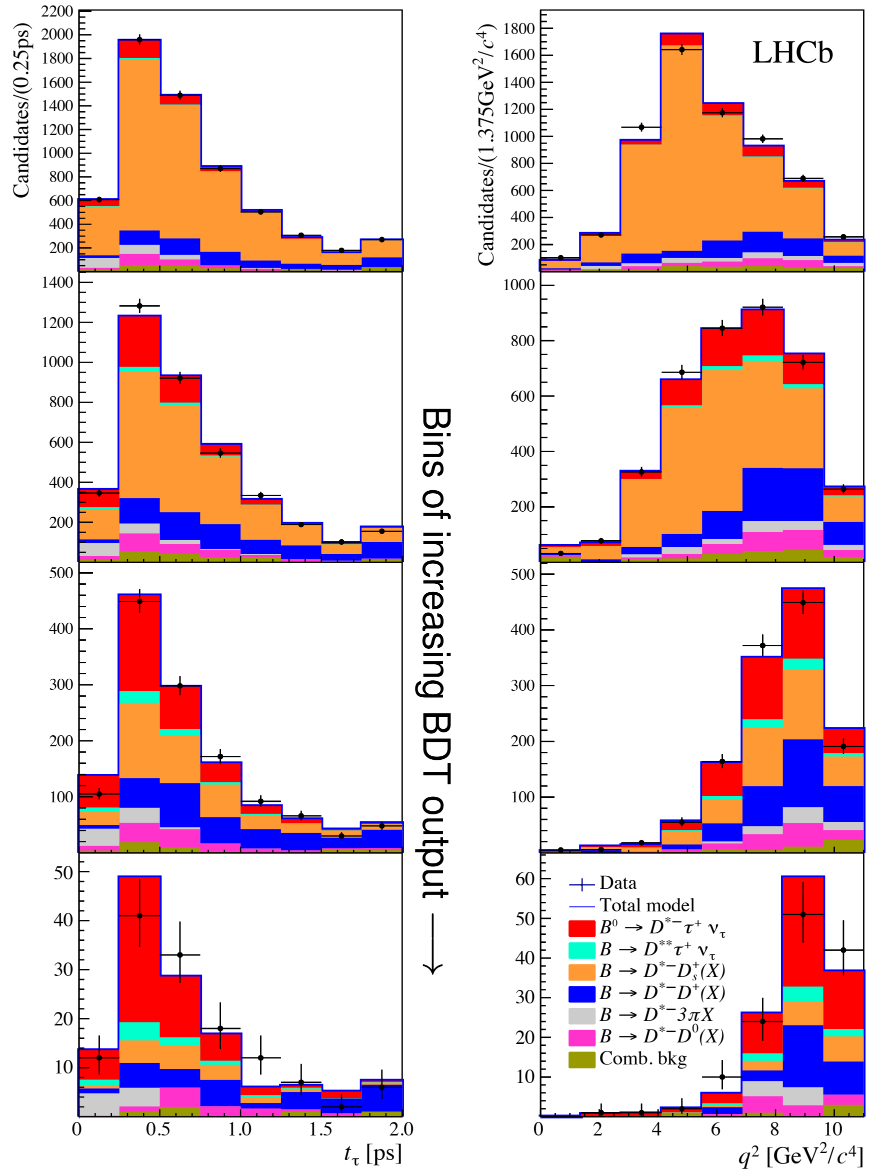
Result for R(D*) with $\tau \rightarrow 3\pi\nu$

- $B \rightarrow D^* \tau\nu$ is visible at high q^2, high BDT, and with non-zero t_τ
- Backgrounds from D^{**}, $B \rightarrow DD^*$, $B \rightarrow D_sD^*$, combinatorics

R(D*) = 0.291 ± 0.019 (stat) ± 0.026 (syst) ± 0.054 (norm)

Consistent with SM and R(D*) from $\tau \rightarrow \mu\nu\nu$





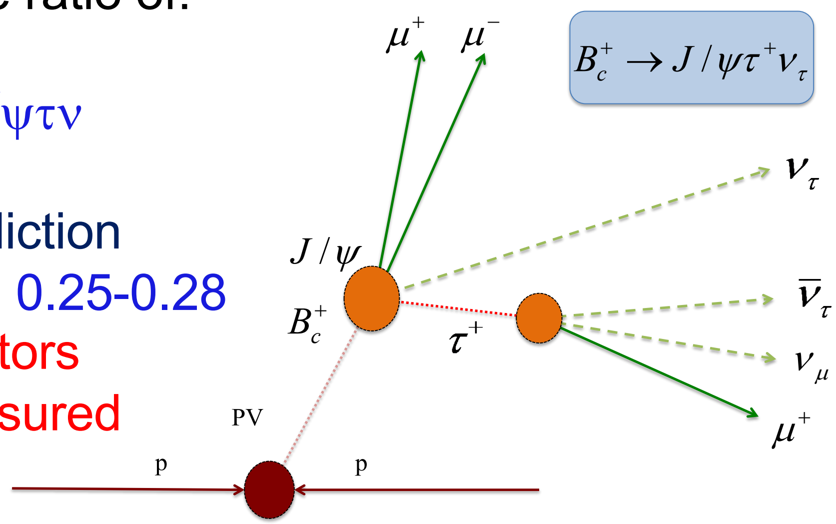
LHCb PRL 120, 171802 & PRD 97,07213 (2018)

R(J/ψ) with $\tau \to \mu\nu\nu$

Measure ratio of:

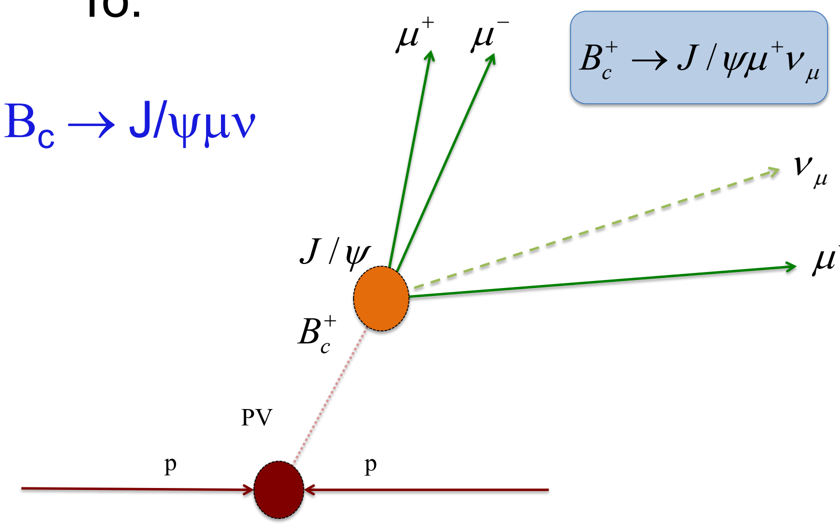
$B_c \to J/\psi\tau\nu$

SM prediction
R(J/ψ) = 0.25-0.28
form factors
not measured



To:

$B_c \to J/\psi\mu\nu$

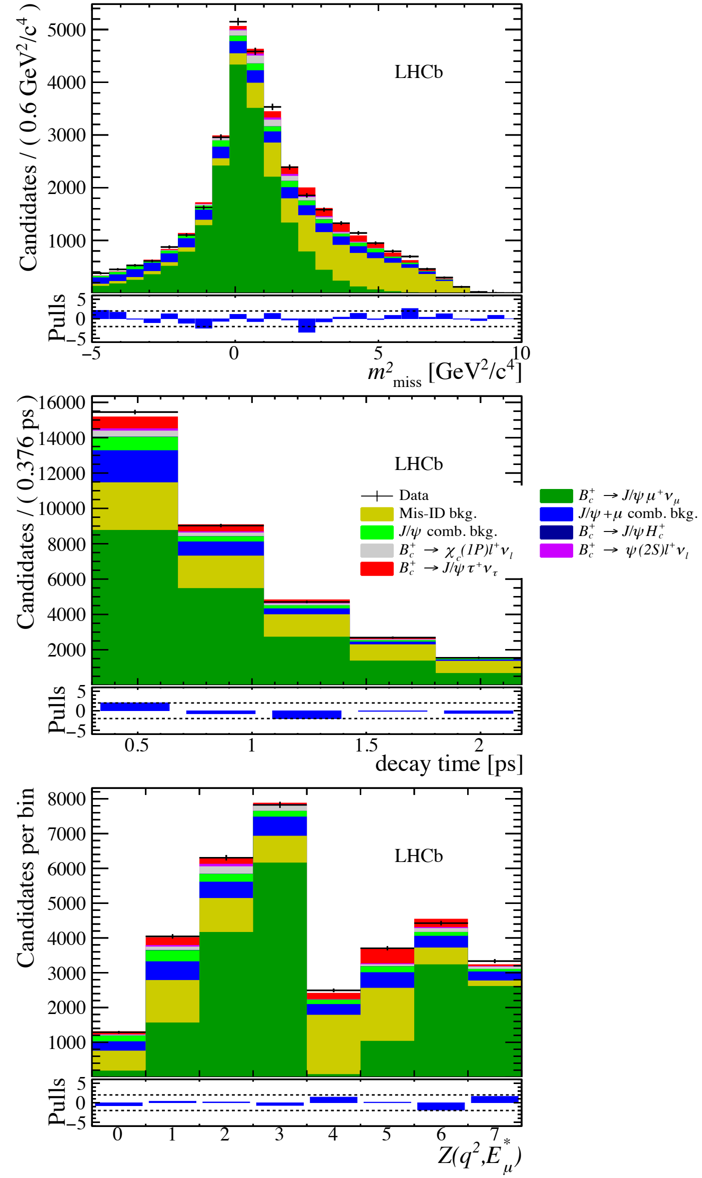


- Same visible final state particles
- 3ν in signal mode, 1ν in normalisation mode
- Separated by fit to m_{miss}^2, E_μ^*, q^2 and using $\tau(B_c)$=0.5ps

- Backgrounds from other charmonium, combinatorics, muon mis-ID
- Mostly dealt with by control samples

Result for R(J/ψ) with $\tau \to \mu\nu\nu$

- $B_c \to J/\psi\ \mu\nu$ is visible at zero m_{miss}^2, and with small $\tau(B_c)$
- $B_c \to J/\psi\ \tau\nu$ is visible at high q^2, high m_{miss}^2, and with small $\tau(B_c)$
- Main backgrounds from mis-ID, combinatorics



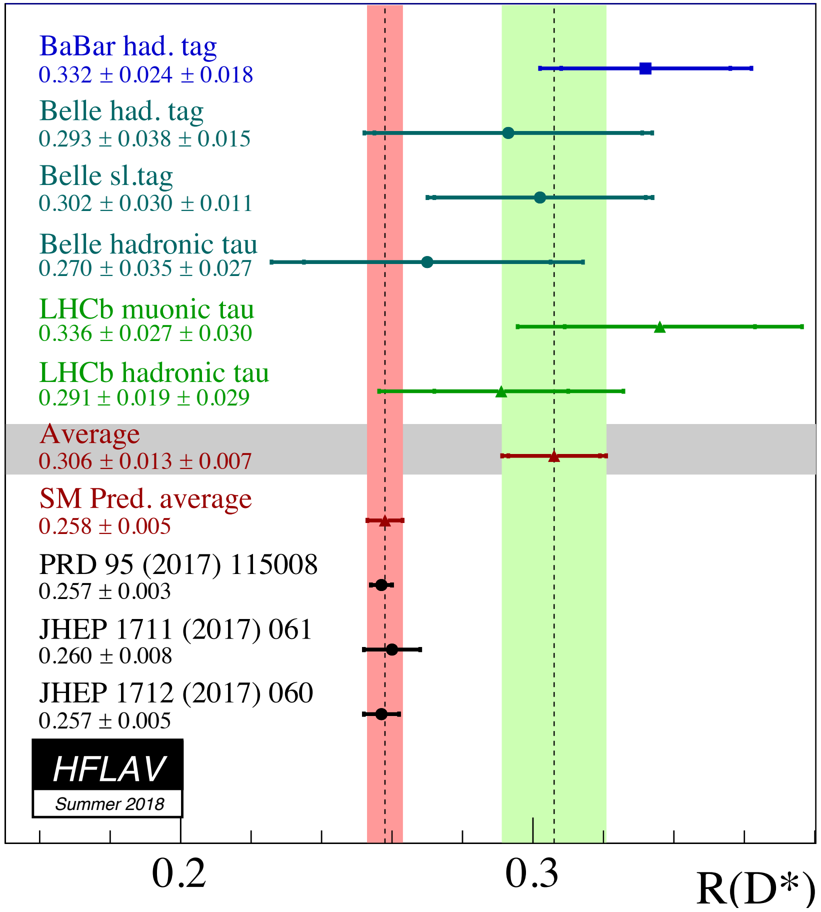
$R(J/\psi) = 0.71 \pm 0.17\ (stat) \pm 0.18\ (syst)$

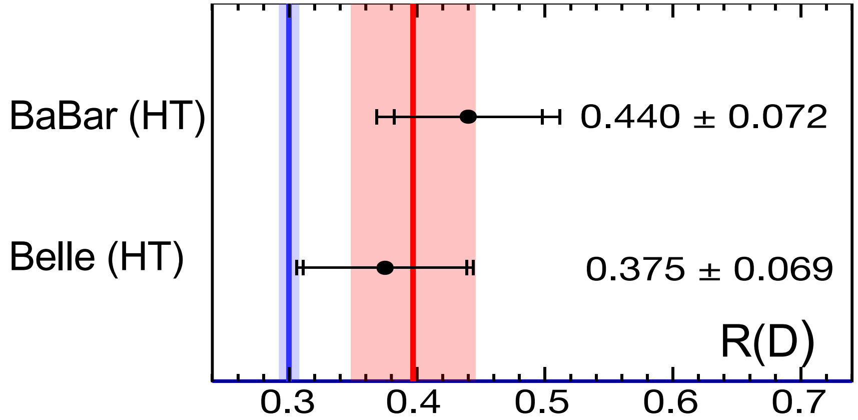
Systematic dominated by form factors

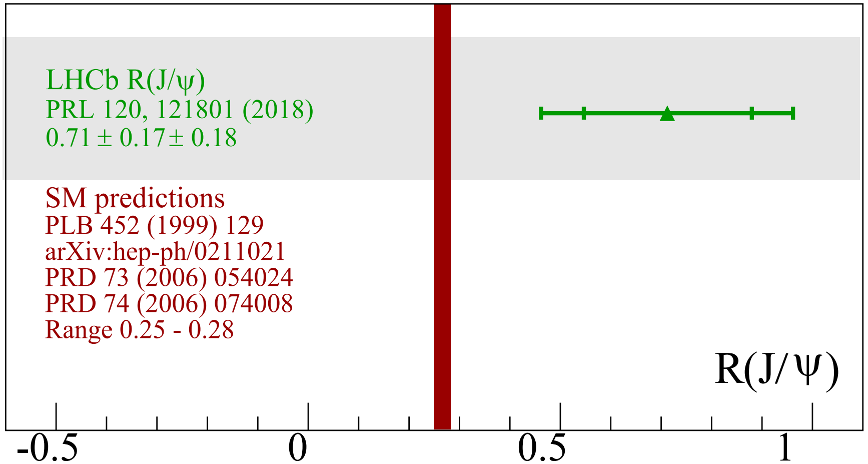
2σ above SM

LHCb PRL 120, 121801 (2018)

Summary of R(D),R(D*) and R(J/ψ)







Three experiments
Nine measurements all above SM
Different techniques and final states

R(D), R(D*), R(J/ψ) are 2/3/2σ above SM
Combined significance 4σ
SM uncertainties are small

Outlook for R(D*) and R(J/ψ)

Physics case for LHCb Upgrade II arXiv:1808.08865 (2018)	$\sigma(R_{D^*})$	$\sigma(R_{J/\psi})$
➢ All results based on **3/fb** at 7-8TeV (Run 1 2010-2012)	0.027 0.030	0.17 (stat) 0.18 (syst)
➢ We have another **6/fb** at 13TeV (Run 2 2015-2018) x4 in B statistics due to increased production X-section	0.014	0.10
➢ LHCb upgrade during shutdown (2019-2020) 40MHz readout and trigger entirely in software Better vertexing for reducing backgrounds to τ, D and B	After upgrade can reduce syst errors	
➢ Integrated luminosity **50/fb** in Runs 3 & 4 Higher instantaneous luminosity $2\times10^{33}/cm^2/s$	0.007	0.07
➢ Possible major upgrade in ~2030 Much higher luminosity 2×10^{34}, with target of **300/fb**	0.002	0.02
	similar to σ(SM)	

More on upgrades in talk by Eugeni Grauges Pous

More analyses to come …

- $b \to s\ell\ell$: $R(B_s \to \phi)$, $R(\Lambda_b \to \Lambda)$, $R(K^{**})$,
- Full angular analysis of K^*ee

- $b \to d\ell\ell$: $R(\pi)$, $R(\rho)$, $R(B_s \to K^*)$

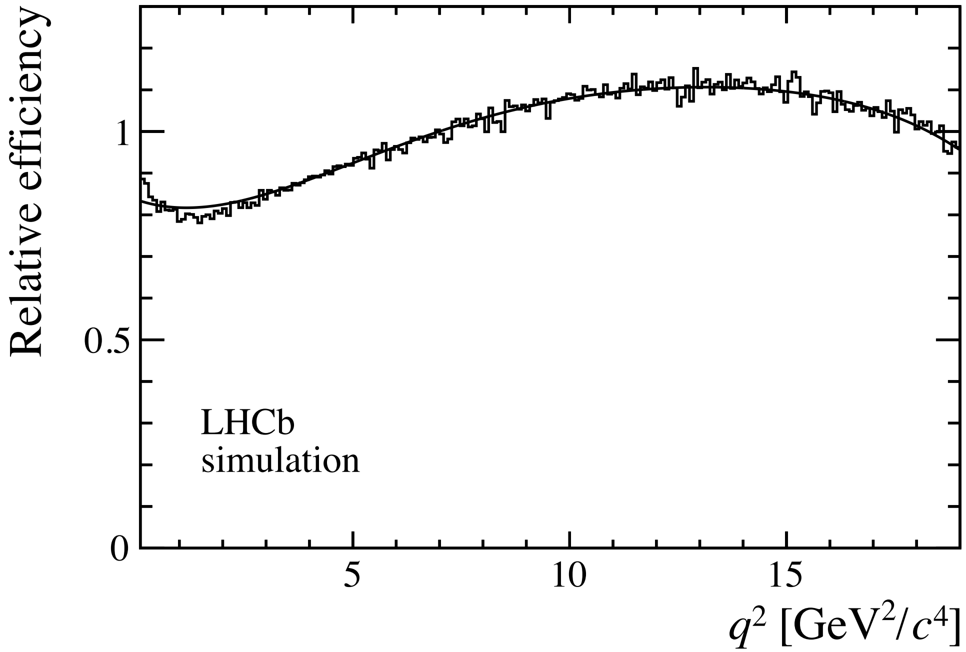
- $b \to c\tau\nu$: $R(D)$, $R(B_s \to D_s^{(*)})$, $R(\Lambda_b \to \Lambda_c)$
- Angular analysis of $\tau \to 3\pi\nu$ to determine spin structure of NP in $R(D^*)$
- $b \to u\tau\nu$: $\Lambda_b \to p\tau\nu$, $B \to p\bar{p}\tau\nu$

Summary and Conclusions

- There are a number of 2-3σ anomalies that have appeared in $b \rightarrow s\ell\ell$ and $b \rightarrow c\ell\nu$ since 2012
- R(K) and R(K*) both suggest a 30% deficit in muons compared to electrons in $b \rightarrow s\ell\ell$ ($1<q^2<6\,GeV^2$)
- R(D), R(D*) and R(J/ψ) all suggest an enhancement in τ compared to μ in $b \rightarrow c\ell\nu$
- LHCb can push these lepton universality tests to the % level or better in the next 10-20 years

BACKUP

K*$\ell\ell$ Efficiency Ratios



K*μμ as a function of q^2 after integrating over decay angles

K*ee as a function of q^2 bin and L0 trigger after integrating over decay angles

Normalised to J/ψ = 1

	$\varepsilon_{\ell^+\ell^-}/\varepsilon_{J/\psi(\ell^+\ell^-)}$	
	low-q^2	central-q^2
$\mu^+\mu^-$	0.679 ± 0.009	0.584 ± 0.006
e^+e^- (L0E)	0.539 ± 0.013	0.522 ± 0.010
e^+e^- (L0H)	2.252 ± 0.098	1.627 ± 0.066
e^+e^- (L0I)	0.789 ± 0.029	0.595 ± 0.020

K*$\ell\ell$ Systematics

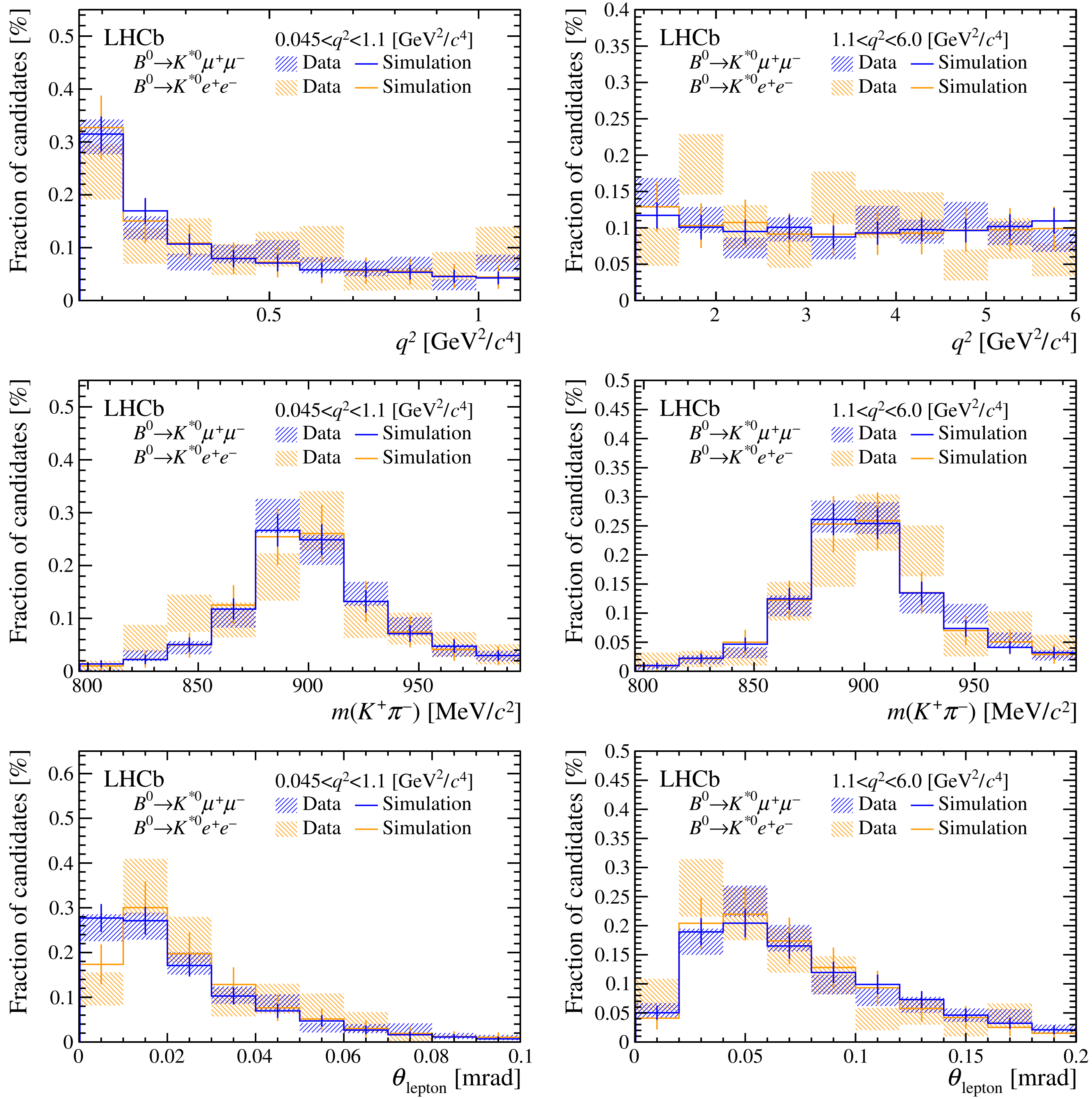
	$\Delta R_{K^{*0}}/R_{K^{*0}}$ [%]					
	low-q^2			central-q^2		
Trigger category	L0E	L0H	L0I	L0E	L0H	L0I
Corrections to simulation	2.5	4.8	3.9	2.2	4.2	3.4
Trigger	0.1	1.2	0.1	0.2	0.8	0.2
PID	0.2	0.4	0.3	0.2	1.0	0.5
Kinematic selection	2.1	2.1	2.1	2.1	2.1	2.1
Residual background	—	—	—	5.0	5.0	5.0
Mass fits	1.4	2.1	2.5	2.0	0.9	1.0
Bin migration	1.0	1.0	1.0	1.6	1.6	1.6
$r_{J/\psi}$ ratio	1.6	1.4	1.7	0.7	2.1	0.7
Total	4.0	6.1	5.5	6.4	7.5	6.7

Checks of K(*)$\ell\ell$ Results

- R_{K^*} (J/ψ) = 1.043 $\pm$ 0.006 (stat) $\pm$ 0.045 (syst)
- BF(J/ψ K) = 1.01x10^{-3} and BF(J/ψ K*) = 1.27x10^{-3}
- BF(K*$\mu\mu$) = 0.342$\pm$ 0.006 (stat) $\pm$ 0.045 (syst) x10^{-7} $1.1<q^2<6 GeV^2$
- BF(K*γ) = 4.2x10^{-5} from photon contribution to low q^2 region
- Take double ratios with respect to ψ'
- Compare kinematic distributions and other selection variables (sPlot method)

All checks are ok to better than 10%

K*ℓℓ sPlots

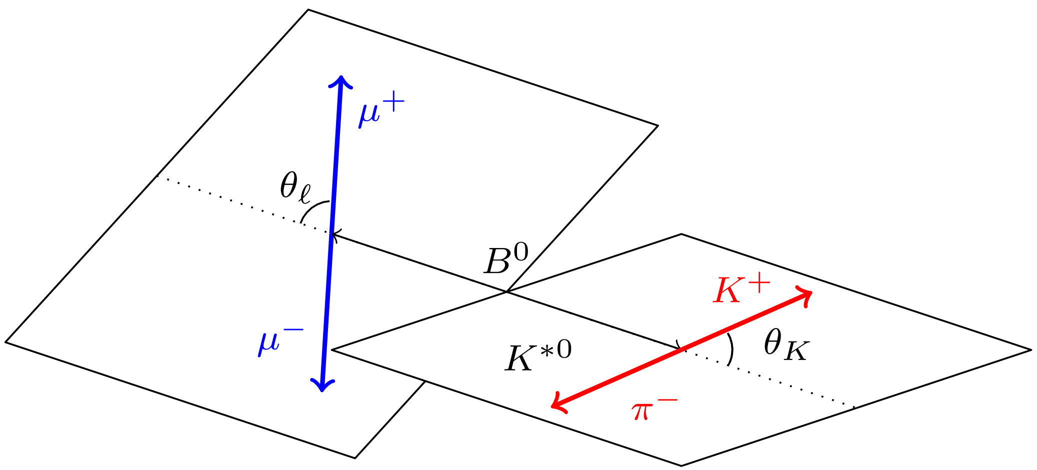


q^2

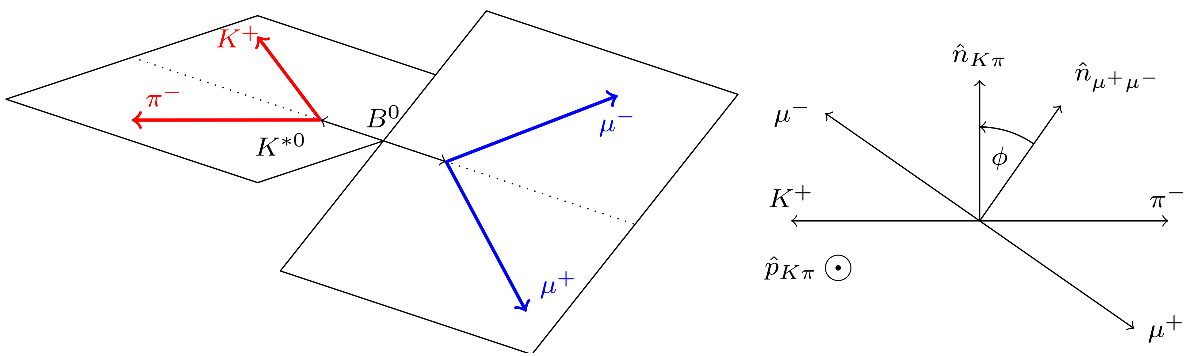
$M(K\pi)$

θ_ℓ

$K^*\ell\ell$ Angular Analysis



Lepton decay angle θ_ℓ
and K* decay angle θ_K
defined in B rest frame



Acoplanarity angle ϕ
defined in B rest frame

Reverses sign for $\overline{B}$

K*$\ell\ell$ Angular Coefficients

i	I_i	f_i
$1s$	$\frac{3}{4}\left[\lvert\mathcal{A}_\parallel^\mathrm{L}\rvert^2+\lvert\mathcal{A}_\perp^\mathrm{L}\rvert^2+\lvert\mathcal{A}_\parallel^\mathrm{R}\rvert^2+\lvert\mathcal{A}_\perp^\mathrm{R}\rvert^2\right]$	$\sin^2\theta_K$
$1c$	$\lvert\mathcal{A}_0^\mathrm{L}\rvert^2+\lvert\mathcal{A}_0^\mathrm{R}\rvert^2$	$\cos^2\theta_K$
$2s$	$\frac{1}{4}\left[\lvert\mathcal{A}_\parallel^\mathrm{L}\rvert^2+\lvert\mathcal{A}_\perp^\mathrm{L}\rvert^2+\lvert\mathcal{A}_\parallel^\mathrm{R}\rvert^2+\lvert\mathcal{A}_\perp^\mathrm{R}\rvert^2\right]$	$\sin^2\theta_K\cos 2\theta_l$
$2c$	$-\lvert\mathcal{A}_0^\mathrm{L}\rvert^2-\lvert\mathcal{A}_0^\mathrm{R}\rvert^2$	$\cos^2\theta_K\cos 2\theta_l$
3	$\frac{1}{2}\left[\lvert\mathcal{A}_\perp^\mathrm{L}\rvert^2-\lvert\mathcal{A}_\parallel^\mathrm{L}\rvert^2+\lvert\mathcal{A}_\perp^\mathrm{R}\rvert^2-\lvert\mathcal{A}_\parallel^\mathrm{R}\rvert^2\right]$	$\sin^2\theta_K\sin^2\theta_l\cos 2\phi$
4	$\sqrt{\frac{1}{2}}\mathrm{Re}(\mathcal{A}_0^\mathrm{L}\mathcal{A}_\parallel^\mathrm{L*}+\mathcal{A}_0^\mathrm{R}\mathcal{A}_\parallel^\mathrm{R*})$	$\sin 2\theta_K\sin 2\theta_l\cos\phi$
5	$\sqrt{2}\mathrm{Re}(\mathcal{A}_0^\mathrm{L}\mathcal{A}_\perp^\mathrm{L*}-\mathcal{A}_0^\mathrm{R}\mathcal{A}_\perp^\mathrm{R*})$	$\sin 2\theta_K\sin\theta_l\cos\phi$
$6s$	$2\mathrm{Re}(\mathcal{A}_\parallel^\mathrm{L}\mathcal{A}_\perp^\mathrm{L*}-\mathcal{A}_\parallel^\mathrm{R}\mathcal{A}_\perp^\mathrm{R*})$	$\sin^2\theta_K\cos\theta_l$
7	$\sqrt{2}\mathrm{Im}(\mathcal{A}_0^\mathrm{L}\mathcal{A}_\parallel^\mathrm{L*}-\mathcal{A}_0^\mathrm{R}\mathcal{A}_\parallel^\mathrm{R*})$	$\sin 2\theta_K\sin\theta_l\sin\phi$
8	$\sqrt{\frac{1}{2}}\mathrm{Im}(\mathcal{A}_0^\mathrm{L}\mathcal{A}_\perp^\mathrm{L*}+\mathcal{A}_0^\mathrm{R}\mathcal{A}_\perp^\mathrm{R*})$	$\sin 2\theta_K\sin 2\theta_l\sin\phi$
9	$\mathrm{Im}(\mathcal{A}_\parallel^\mathrm{L*}\mathcal{A}_\perp^\mathrm{L}+\mathcal{A}_\parallel^\mathrm{R*}\mathcal{A}_\perp^\mathrm{R})$	$\sin^2\theta_K\sin^2\theta_l\sin 2\phi$
10	$\frac{1}{3}\left[\lvert\mathcal{A}_\mathrm{S}^\mathrm{L}\rvert^2+\lvert\mathcal{A}_\mathrm{S}^\mathrm{R}\rvert^2\right]$	1
11	$\sqrt{\frac{4}{3}}\mathrm{Re}(\mathcal{A}_\mathrm{S}^\mathrm{L}\mathcal{A}_0^\mathrm{L*}+\mathcal{A}_\mathrm{S}^\mathrm{R}\mathcal{A}_0^\mathrm{R*})$	$\cos\theta_K$
12	$-\frac{1}{3}\left[\lvert\mathcal{A}_\mathrm{S}^\mathrm{L}\rvert^2+\lvert\mathcal{A}_\mathrm{S}^\mathrm{R}\rvert^2\right]$	$\cos 2\theta_l$
13	$-\sqrt{\frac{4}{3}}\mathrm{Re}(\mathcal{A}_\mathrm{S}^\mathrm{L}\mathcal{A}_0^\mathrm{L*}+\mathcal{A}_\mathrm{S}^\mathrm{R}\mathcal{A}_0^\mathrm{R*})$	$\cos\theta_K\cos 2\theta_l$
14	$\sqrt{\frac{2}{3}}\mathrm{Re}(\mathcal{A}_\mathrm{S}^\mathrm{L}\mathcal{A}_\parallel^\mathrm{L*}+\mathcal{A}_\mathrm{S}^\mathrm{R}\mathcal{A}_\parallel^\mathrm{R*})$	$\sin\theta_K\sin 2\theta_l\cos\phi$
15	$\sqrt{\frac{8}{3}}\mathrm{Re}(\mathcal{A}_\mathrm{S}^\mathrm{L}\mathcal{A}_\perp^\mathrm{L*}-\mathcal{A}_\mathrm{S}^\mathrm{R}\mathcal{A}_\perp^\mathrm{R*})$	$\sin\theta_K\sin\theta_l\cos\phi$
16	$\sqrt{\frac{8}{3}}\mathrm{Im}(\mathcal{A}_\mathrm{S}^\mathrm{L}\mathcal{A}_\parallel^\mathrm{L*}-\mathcal{A}_\mathrm{S}^\mathrm{R}\mathcal{A}_\perp^\mathrm{R*})$	$\sin\theta_K\sin\theta_l\sin\phi$
17	$\sqrt{\frac{2}{3}}\mathrm{Im}(\mathcal{A}_\mathrm{S}^\mathrm{L}\mathcal{A}_\perp^\mathrm{L*}+\mathcal{A}_\mathrm{S}^\mathrm{R}\mathcal{A}_\perp^\mathrm{R*})$	$\sin\theta_K\sin 2\theta_l\sin\phi$

$$S_i = (I_i + \bar{I}_i) \Big/ \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} + \frac{\mathrm{d}\bar{\Gamma}}{\mathrm{d}q^2}\right)$$

$$A_i = (I_i - \bar{I}_i) \Big/ \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} + \frac{\mathrm{d}\bar{\Gamma}}{\mathrm{d}q^2}\right)$$

F_L = S1c $\quad$ A_{FB} = ¾ S6s

$$P_1 = \frac{2\,S_3}{(1-F_\mathrm{L})} = A_\mathrm{T}^{(2)}\,,$$

$$P_2 = \frac{2}{3}\frac{A_\mathrm{FB}}{(1-F_\mathrm{L})}\,,$$

$$P_3 = \frac{-S_9}{(1-F_\mathrm{L})}\,,$$

$$P'_{4,5,8} = \frac{S_{4,5,8}}{\sqrt{F_\mathrm{L}(1-F_\mathrm{L})}}\,,$$

$$P'_6 = \frac{S_7}{\sqrt{F_\mathrm{L}(1-F_\mathrm{L})}}\,.$$

$D^*\tau\nu$ Control Samples

- $D^{*-}h^+$ for muon mis-identification
  using $D^0(K\pi)$, $\Lambda(p\pi)$ to calibrate particle identification

- $D^{*-}\mu^-$ for combinatorial background

- Additional charged track at B vertex for D^{**}
  and partially reconstructed backgrounds in hadronic final states

- Additional neutral energy in ECAL about D^* or τ direction

- $D^{*-}D_s^+(KK\pi)$, $D^{*-}D^+(K\pi\pi)$, $D^{*-}D^0(K\pi)$ for double charm

- $D^{*-}D_s^+(3\pi)$, $D^{*-}D^+(3\pi)$ for dominant backgrounds in $\tau \rightarrow 3\pi\nu$